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Mode localization and frequency loci veering in a disordered coupled beam system

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Abstract. Vibration mode localization and frequency loci veering in disordered coupled beam system are studied in this paper using finite element analysis. Two beams coupled with transverse and rotational springs are examined. Small disorders in the physical parameters such as Young's modulus, mass density or span length of the substructure are introduced in the investigation of the mode localization and frequency loci veering phenomena. The effect of disorder in the elastic support on the mode localization phenomenon is also discussed. It is found that an asymmetric disorder in the weakly coupled system will lead to the occurrence of mode localization and frequency loci phenomena.

Keywords: coupled beam; disorder; mode localization; finite element; vibration; eigenvalue; mode shape.

1. Introduction

In structural analysis, it is common to make the assumption that the structures are perfectly periodic, or completely symmetric. However, a dynamic model may be far from the assumed prototype due to small disorders in the structure, such as mistuned parameters, geometrical irregularities and manufacturing errors. It is known that when the degrees of freedom of a nominally periodic structure are weakly coupled and there are some small disorders in the structure, then the free vibration modes will typically be spatially localized, resulting in confined regions of the structure where the vibration energy is concentrated. This is the so called vibration mode localization phenomenon (Anderson 1958, Hodges 1982, Pierre and Dowel 1987, Pierre *et al*. 1987, Pierre 1988, Cai *et al*. 1995, Kang and Tan 1999, Kim and Lee 2000, Huang and Kuang 2001, Xie and Chen 2002, Huang and Kuang 2005, Jacques and Potier-Ferry 2005). Besides, two frequency loci approach each other and do not cross but veer away from each other with high local curvature. This is known as frequency loci veering phenomenon (Kuttle and Sigillito 1981, Pierre 1988, Chen and Ginsberg 1992, Liu *et al*. 1995, Yang 1997, Chan and Liu 2000, Lacarbonara *et al*. 2005). These two phenomena have attracted much attention the areas of mechanical and aeronautic engineering in the past thirty years.

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The mode localization phenomenon was first investigated in the field of solid state physics by Anderson (1958), who showed that in a randomly disordered linear chain of particles, the wave function of the chain can exhibit spatially confined modes of motion. One of the earliest studies of the phenomenon of localization in the field of structural dynamics was made by Hodges (1982). Pierre and Dowell (1987) studied localization phenomena for a chain of coupled oscillators. Pierre *et al*. (1988) applied a modified perturbation method to study the mode localization and verified the existence of localized modes by carrying out an experiment on a disordered dual-span Euler-Bernoulli beam. Chen and Ginsberg (1992) investigated the relationship between mode localization and eigenvalue loci veering of nearly periodic structures by applying a perturbation method to a general eigenvalue problem and found that small disorder results in strong mode localization in the eigenvalue veering zone. A comprehensive survey on mode localization phenomenon in practical engineering structures was given by Bendikisen (2000). Xie and Chen (2002) studied the vibration mode localization in rib-stiffened plates with randomly misplaced stiffener in one direction.

There are also many articles in the literature on mode localization phenomenon in non-linear systems (Chao and Shaw 1997, King and Layne 1998, Jiang and Vakakis 2003). Vakakis and Cetinkaya (1993) carried out a study on the free vibrations of n-degree-of-freedom nonlinear systems with cyclic symmetry and week coupling between substructures. It was shown that nonlinear mode localization occurs in the perfectly symmetric, weakly coupled structures. In contrast to linear mode localization, which exists only in the presence of substructure 'mistuning'. Vakakis *et al*. (1993) examined the nonlinear localized modes of an n-DOF nonlinear cyclic system by the averaging method of multiple scales. In addition, the transition from mode localization to mode nonlocalization in a nonlinear periodic system is analytically studied for the first time. An investigation on mode localization in a distributed system of coupled flexible beams with geometric nonlinearities was conducted by King and Vakakis (1995a). King and Vakakis (1995b) investigated the forced periodic and transient responses of a cyclic system with nonlinear mode localization. The effects of the nonlinear localized mode on the forced responses were studied.

Mode localization and frequency loci veering have been studied by some researchers using the perturbation method (Chen and Ginsberg 1992, Chan and Liu 2000). The purpose of this paper is to investigate the vibration mode localization and frequency loci veering phenomena using the finite element analysis. Several disorder cases are included: a disorder in the Young's modulus, a disorder in the mass density, a disorder in the span length of a substructure, and also, a disorder in the elastic support. The sensitivities of these parameters on mode localization are compared and discussed.

2. Equation of motion of the coupled beam system

Fig. 1 shows the dual-span coupled beam system under study. Two Euler-Bernoulli beams are

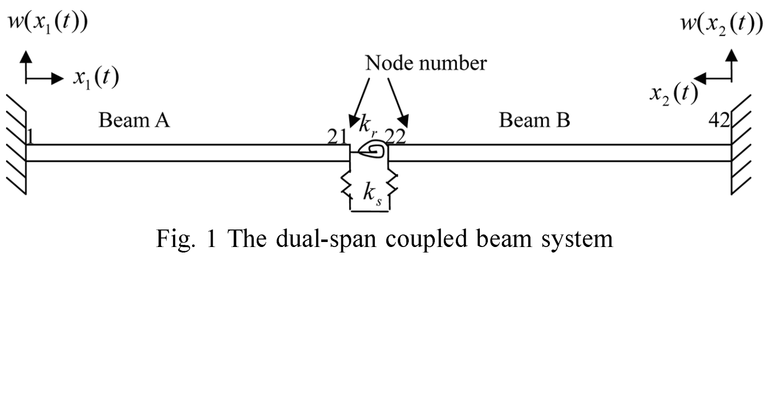

Fig. 1 The dual-span coupled beam system

coupled with a transverse spring and a rotational in the middle span and clamped at the two ends. The equation of motion of free vibration of the original system and the corresponding boundary conditions can be expressed as follows

For Beam A, we have

$$\rho A_{01}(x_1)\frac{\partial^2 w(x_1, t)}{\partial t^2} + c\frac{\partial w(x_1, t)}{\partial t} + \frac{\partial^2}{\partial x_1^2}\left[EI_{01}(x_1)\frac{\partial^2 w(x_1, t)}{\partial x_1^2}\right] = 0 \tag{1}$$

and the corresponding boundary conditions are:

$$w(x_1)\big|_{x_1 = 0} = 0, \quad \left.\frac{\partial w(x_1, t)}{\partial x_1}\right|_{x_1 = 0} = 0 \tag{2a}$$

$$EI_{01}(x_1)\left.\frac{\partial^2 w(x_1, t)}{\partial x_1^2}\right|_{x_1 = l_{01}} + k_{r0}\left[\left.\frac{\partial w(x_1, t)}{\partial x_1}\right|_{x_1 = l_{01}} + \left.\frac{\partial w(x_2, t)}{\partial x_2}\right|_{x_2 = l_{02}}\right] = 0 \tag{2b}$$

$$\left.\frac{\partial}{\partial x_1}\left(EI_{01}(x_1)\frac{\partial^2 w(x_1, t)}{\partial x_1^2}\right)\right|_{x_1 = l_{01}} - k_{s0}\left[w(x_1, t)\big|_{x_1 = l_{01}} - w(x_2, t)\big|_{x_2 = l_{02}}\right] = 0 \tag{2c}$$

For Beam B, we have

$$\rho A_{02}(x_2)\frac{\partial^2 w(x_2, t)}{\partial t^2} + c\frac{\partial w(x_2, t)}{\partial t} + \frac{\partial^2}{\partial x_2^2}\left[EI_{02}(x_2)\frac{\partial^2 w(x_2, t)}{\partial x_2^2}\right] = 0 \tag{3}$$

and the corresponding boundary conditions are:

$$w(x_2, t)\big|_{x_2 = 0} = 0, \quad \left.\frac{\partial w(x_2, t)}{\partial x_2}\right|_{x_2 = 0} = 0 \tag{4a}$$

$$EI_{02}(x_2)\left.\frac{\partial^2 w(x_2, t)}{\partial x_2^2}\right|_{x_2 = l_{02}} + k_{r0}\left[\left.\frac{\partial w(x_1, t)}{\partial x_1}\right|_{x_1 = l_{01}} + \left.\frac{\partial w(x_2, t)}{\partial x_2}\right|_{x_2 = l_{02}}\right] = 0 \tag{4b}$$

$$\left.\frac{\partial}{\partial x_2}\left(EI_{02}(x_2)\frac{\partial^2 w(x_2, t)}{\partial x_2^2}\right)\right|_{x_2 = l_{02}} - k_{s0}\left[w(x_1, t)\big|_{x_1 = l_{01}} - w(x_2, t)\big|_{x_2 = l_{02}}\right] = 0 \tag{4c}$$

where ρA_{01} and ρA_{02} are the mass per unit length of the original left and right beam respectively, EI_{01} and EI_{02} are the flexural rigidities of the original left and right beam respectively, l_{01} and l_{02} are the lengths of the left and right beam respectively, k_{s0} and k_{r0} are the coefficients of the coupling transverse and rotational spring of the original system respectively, c is the viscous damping of the beam. It is assumed after small perturbation, these system parameters are changed as

$$EI(x_1) = EI_{01}(1 + \varepsilon) \tag{5a}$$

$$EI(x_2) = EI_{02}(1 + \varepsilon) \tag{5b}$$

$$\rho A(x_1) = \rho A_{01}(1 + \varepsilon) \tag{6a}$$

$$\rho A(x_2) = \rho A_{02}(1 + \varepsilon) \tag{6b}$$

$$k_s = k_{s0}(1 + \varepsilon) \tag{7a}$$

$$k_r = k_{r0}(1 + \varepsilon) \tag{7b}$$

where $\varepsilon(|\varepsilon|<<1)$ is the perturbation parameter in different system parameters. The natural frequencies and modes shapes of the perturbed system can be obtained from the classical matrix perturbation method. However, in the present study, the natural frequencies and mode shapes of the original and perturbed systems are calculated using the finite element method.

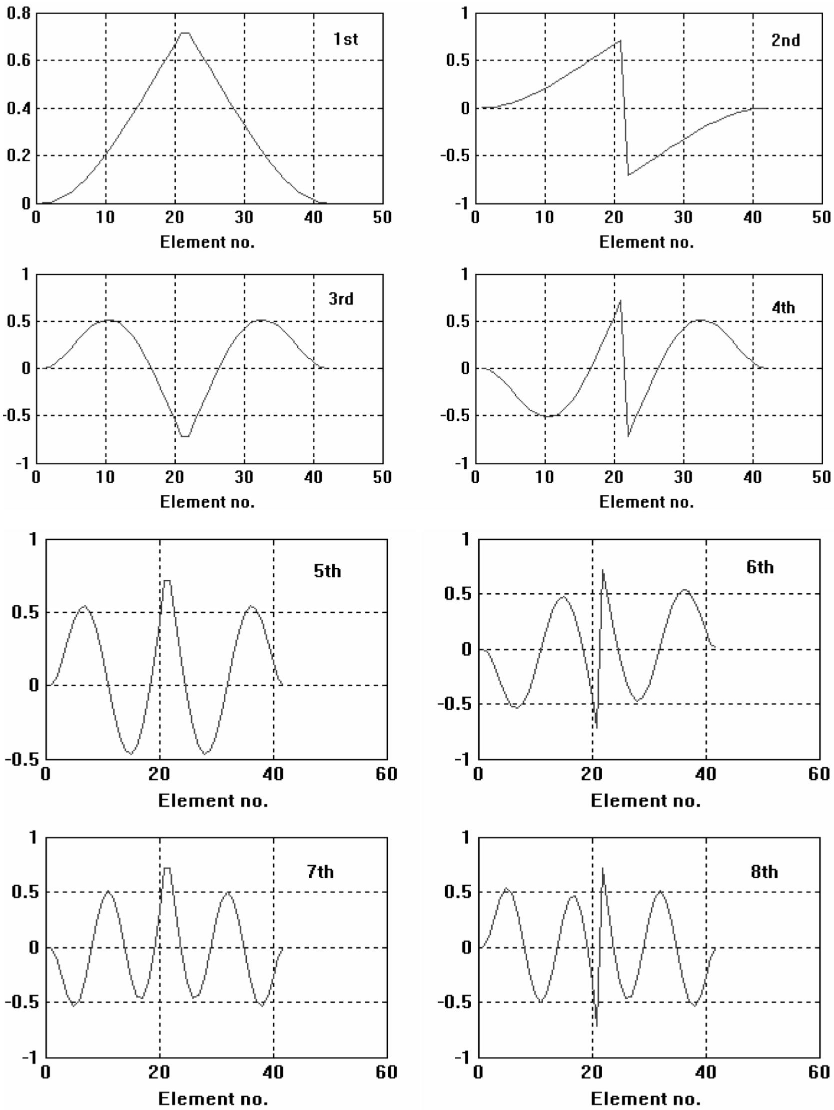

Fig. 2 The first 8 mode shapes of the original system

Table 1 The first 8 natural frequencies of the dual-span system

Mode no.	The original system		The disordered system ($\varepsilon = -0.04$)			
	Clamped	Elastic support	Disorder in EI	Disorder in ρA	Disorder in length	Disorder in support
1	2.1075	0.5273	2.0826	2.1050	2.1480	0.5270
2	2.1199	0.7988	2.1034	2.2471	2.3566	0.7985
3	13.2036	4.2543	12.9453	13.2118	13.2120	4.1922
4	13.2205	4.2756	13.2123	13.4848	14.3349	4.2665
5	36.9678	14.4877	36.2235	36.9703	36.9703	14.4614
6	36.9728	14.5018	36.9703	37.7327	40.1149	14.4962
7	72.4439	37.5620	70.9812	72.4448	72.4448	37.5526
8	72.4456	37.5668	72.4448	73.9386	78.6073	37.5648

3. Numerical example

3.1 A dual-span coupled clamped beam system

As shown in Fig. 1, a dual-span coupled clamped beam system is studied. The physical parameters of the original system are: $EI_{01} = EI_{02} = 175$ N·m^2, $m_{01} = m_{02} = 0.78$ kg/m, $l_{01} = l_{02} = 5.0$ m, $k_{s0} = 0.001EI_{01}$ N/m, $k_{r0} = 0.0001EI_{01}$ Nm/rad. In the finite element model of the system, each beam is discretized into 20 elements and altogether 40 elements. Table 1 shows the first 8 natural frequencies and Fig. 2 shows the first 8 mode shapes of the system. From this figure, one can see, without disorder, mode localization phenomenon does not occur in the coupled system. To investigate the phenomena of mode localization and frequency loci veering, the following study cases are carried out with the system.

Case 1 Disorder due to the Young's modulus of the right beam

In this case, the disorder in the Young's modulus of the right beam is considered. There is a 4% reduction in the Young's modulus of this beam, and the other parameters of the system remain unchanged. The first 8 natural frequencies are also shown in Table 1. From this table, one can see, the changes in the natural frequencies are small. Fig. 3 shows the first 8 mode shapes of the disordered system. Obviously, the mode localization phenomenon occurs from the first mode and the higher the mode, the stronger the phenomenon. The first 8 frequency loci of the disordered system are shown in Fig. 4. From this figure, one can see in subplot (a)-subplot (d), as the perturbation ε increases from -0.1 to 0, each pair of two loci approach progressively, it seems that the two curves cross at $\varepsilon = 0$, however, a closer view will find they do not cross but veer away from each other as ε increases further from 0 to 0.1. This shows the mode localization and frequency loci veering phenomena occur at the same time.

Case 2 Disorder due to the mass density of the left beam

In this case, the disorder in the mass density of the left beam is simulated. It is assumed that there is a 4% reduction in the Young's modulus of this beam, and the other parameters of the system remain unchanged. The first 8 natural frequencies are also shown in Table 1. Fig. 5 shows the first

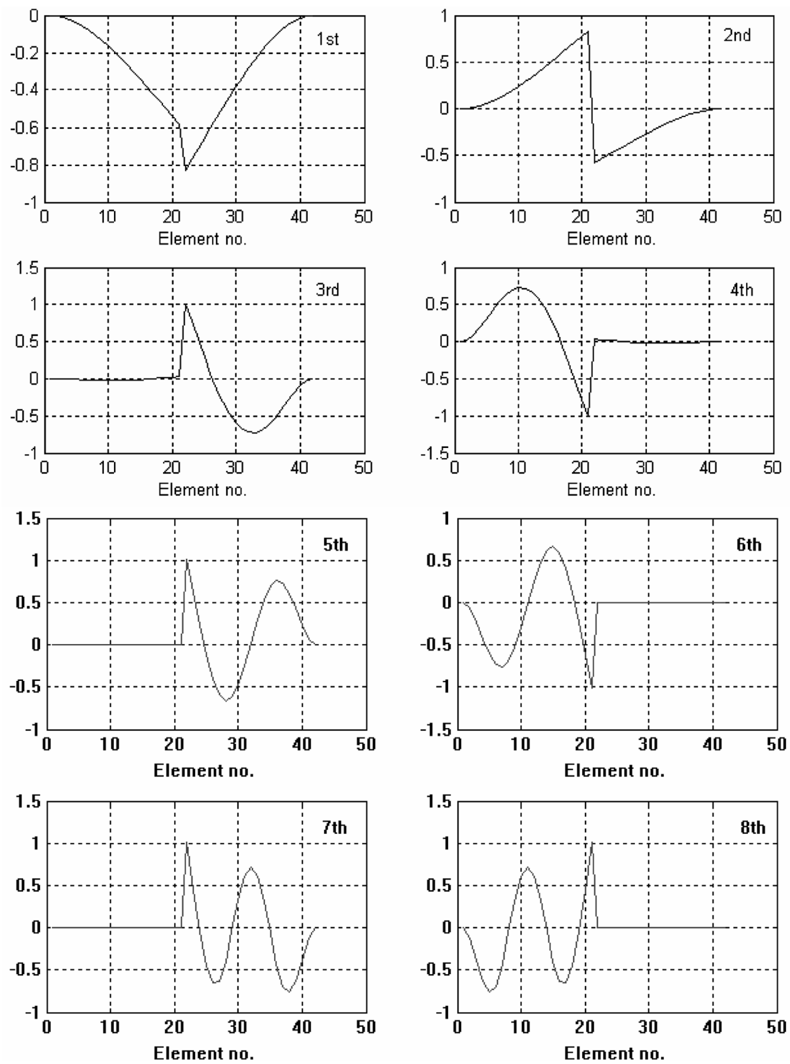

Fig. 3 The first 8 mode shapes of the disordered system with perturbation in *EI*

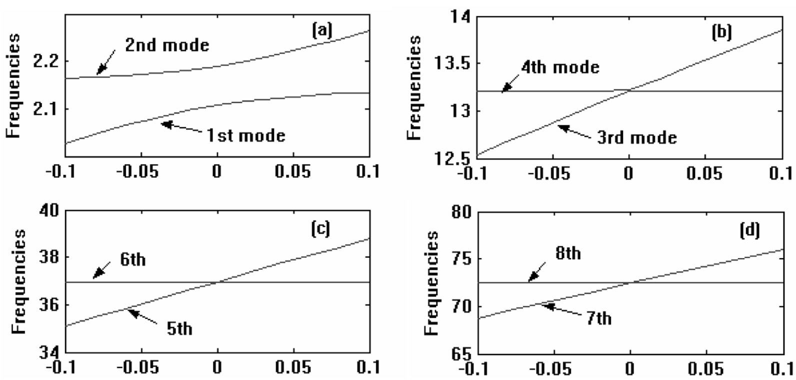

Fig. 4 The first 8 frequency loci veering of the disordered system with perturbation in *EI*

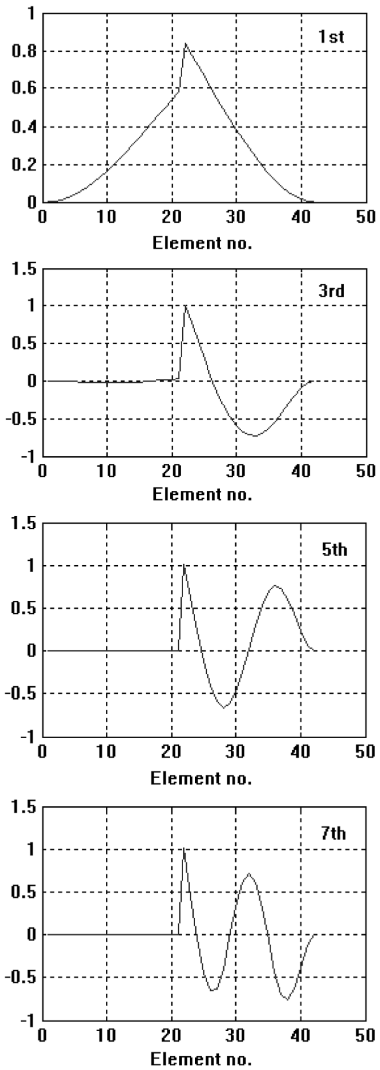

Fig. 5 The first 8 mode shapes of the disordered system with perturbation in mass density

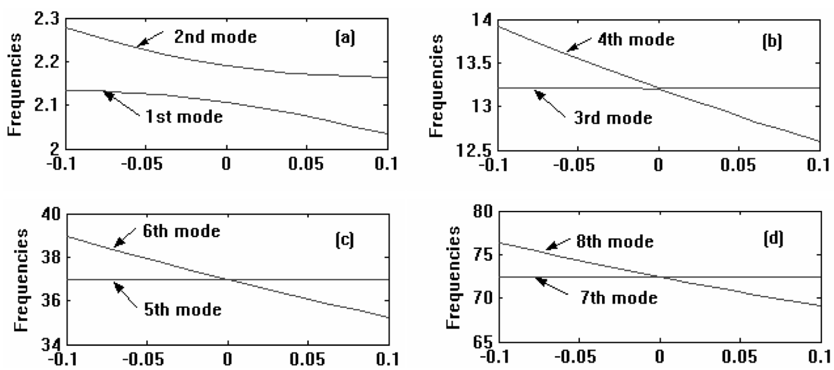

Fig. 6 The first 8 frequency loci veering of the disordered system with perturbation in mass density

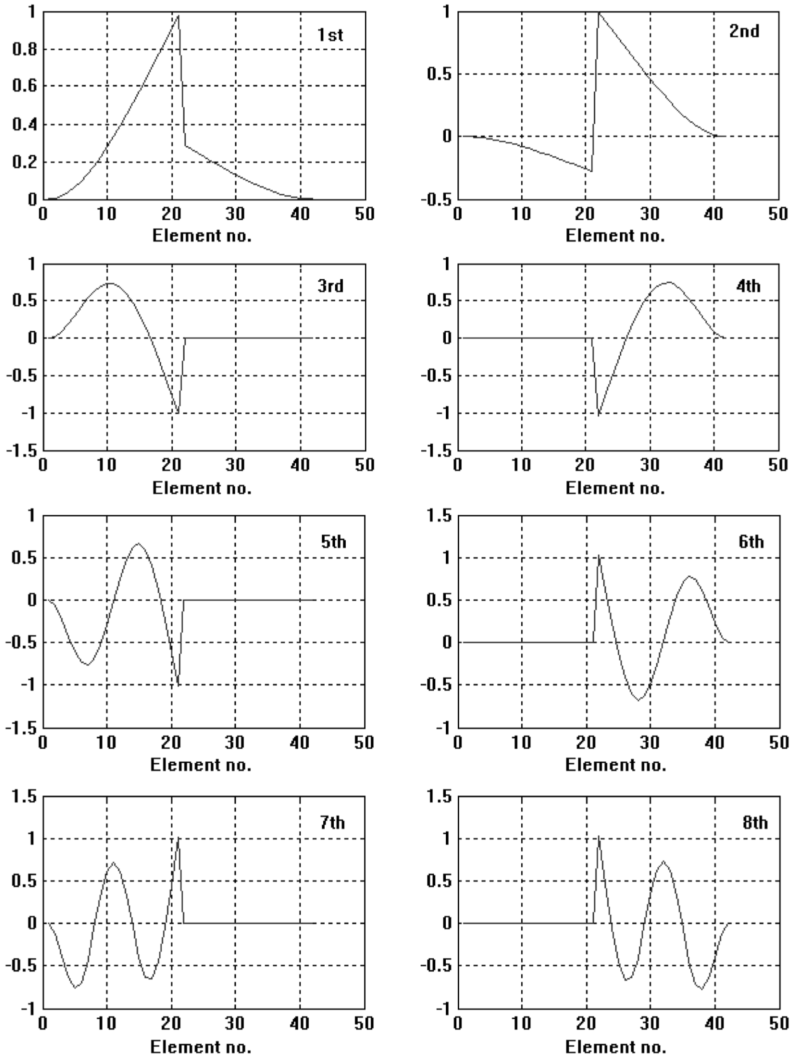

Fig. 7 The first 8 mode shapes of the disordered system with perturbation in length of substructure

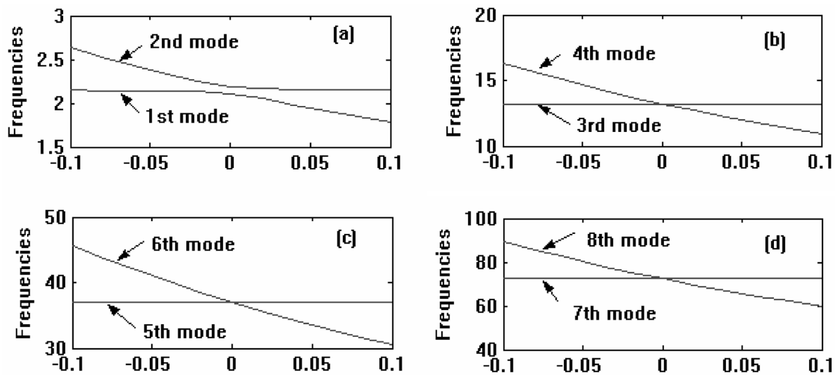

Fig. 8 The first 8 frequency loci veering of the disordered system with perturbation in length of substructure

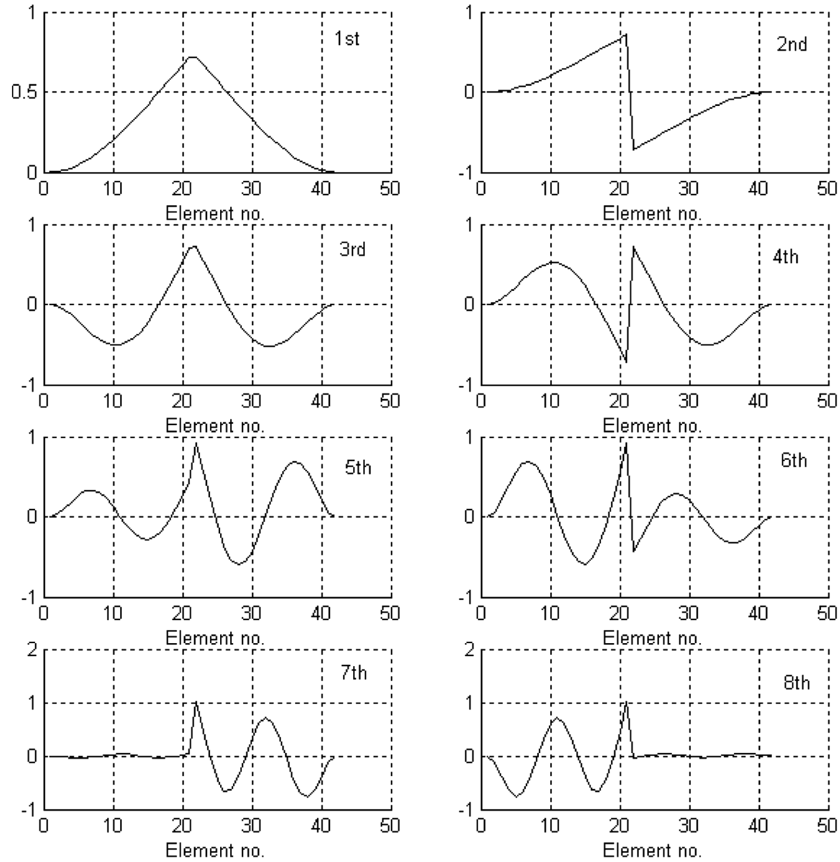

Fig. 9 The first 8 mode shapes of the disordered system with one percent perturbation in flexural rigidity

8 mode shapes of the disordered system. The first 8 frequency loci of the disordered system are shown in Fig. 6. Again, the mode localization phenomenon and loci veering occur from the first mode.

Case 3 Disorder due to the length of the right beam

This case discusses the effect of disorder in the span length of substructure beam on the mode localization. It is assumed there is a 4% reduction in the length of right beam, and the other parameters of the system remain unchanged. The first 8 natural frequencies are also shown in Table 1. Figs. 7 and 8 show the first 8 mode shapes and the first 8 frequency loci curves respectively. Comparing the figures of mode shapes and the loci veering curves, one can see, for the weakly coupled system, the stronger the localization phenomenon, the closer the two frequency curves at point $\varepsilon = 0$.

Case 4 Disorder due to a small disorder in one element, an extreme case

An extreme case is studied here; for the given coupled beam system, a disorder in a single element is simulated. It is assumed that there is only one percent reduction in the flexural rigidity in the 3rd element of the first beam. This can be used to simulate a small local damage in the beam.

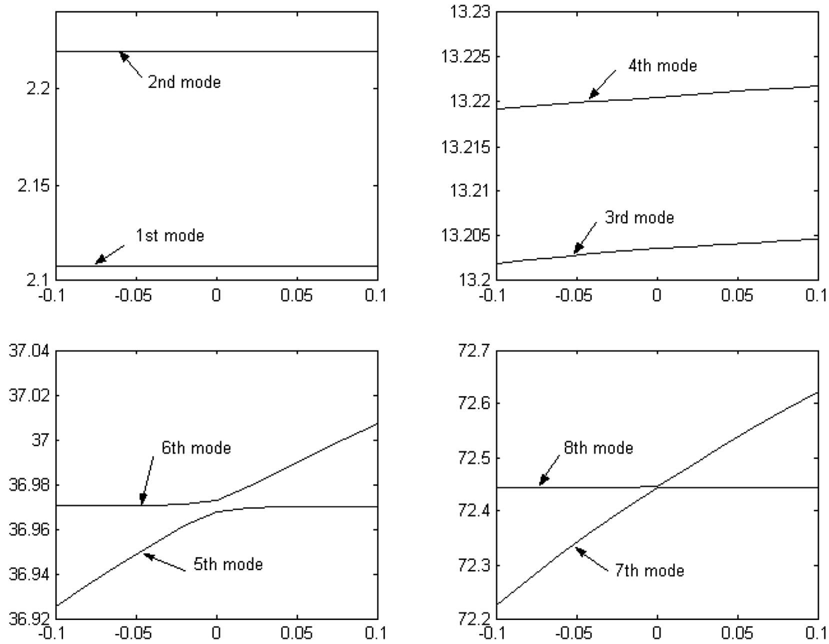

Fig. 10 The first 8 frequency loci veering of the disordered system with one percent perturbation in flexural rigidity

The other system parameters remain unchanged. Figs. 9 and 10 show the first 8 mode shapes and the first 8 frequency loci curves respectively. In this case, mode localization phenomenon also occurs from the third mode when there is such a small disorder in the system. From this study, one can see that the local damage (it often causes the local reduction in the flexural rigidity of the structure) will sometime lead to mode localization.

From study Cases 1 to 4, a conclusion can be drawn that an asymmetric disorder in the weakly coupled system will lead to the occurrence of mode localization and frequency loci phenomena.

3.2 A dual-spam coupled beam system on elastic supports

As shown in Fig. 11, the system studied above is extended to a case with more general boundary condition, the coupled beam with elastic support boundaries. The physical parameters of the original

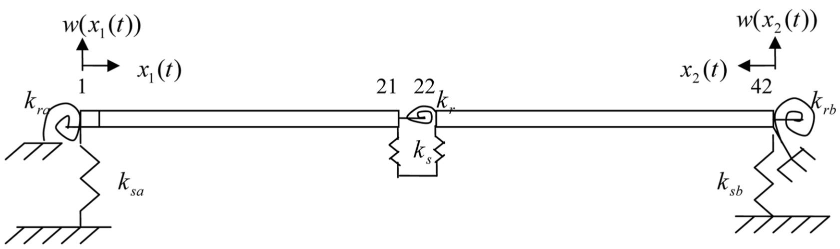

Fig. 11 The dual-span coupled beam system with elastic support

system are: $EI_{01} = EI_{02} = 175$ N·m^2, $m_{01} = m_{02} = 0.78$ kg/m, $l_{01} = l_{02} = 5.0$ m, $k_{s0} = 0.001EI_{01}$ N/m, $k_{r0} = 0.0001EI_{01}$ Nm/rad, $k_{sa0} = k_{sb0} = 20$ N/m, $k_{ra0} = k_{rb0} = 10$ Nm/rad. In the finite element model of the system, each beam is discretized into 20 elements. The first 8 natural frequencies are shown in Table 1.

Case 5 Disorder due to the Young's modulus of the right beam

A disorder in the Young's modulus of the right beam is simulated with a 4% reduction, and the other parameters of the system remain unchanged. Fig. 12 shows the first 8 mode shapes of the disordered system. From this figure, one can see the mode localization phenomenon occurs from the

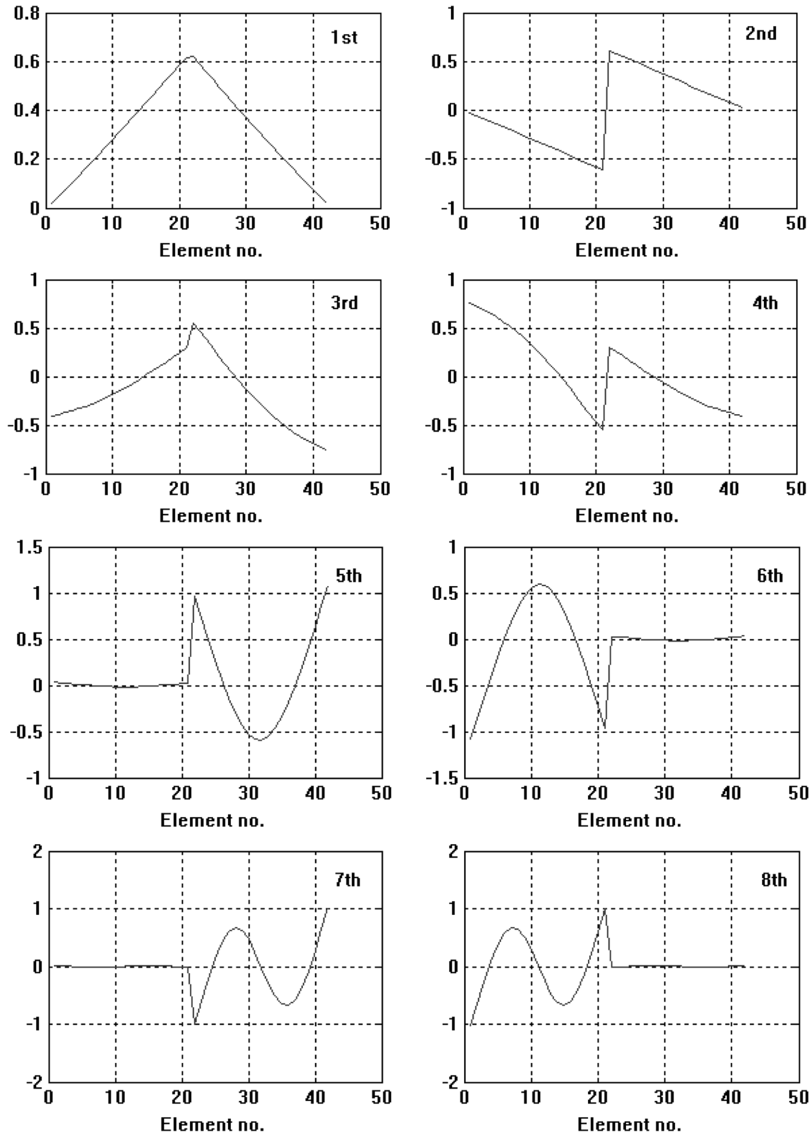

Fig. 12 The first 8 mode shapes of the disordered elastic support system with perturbation in *EI*

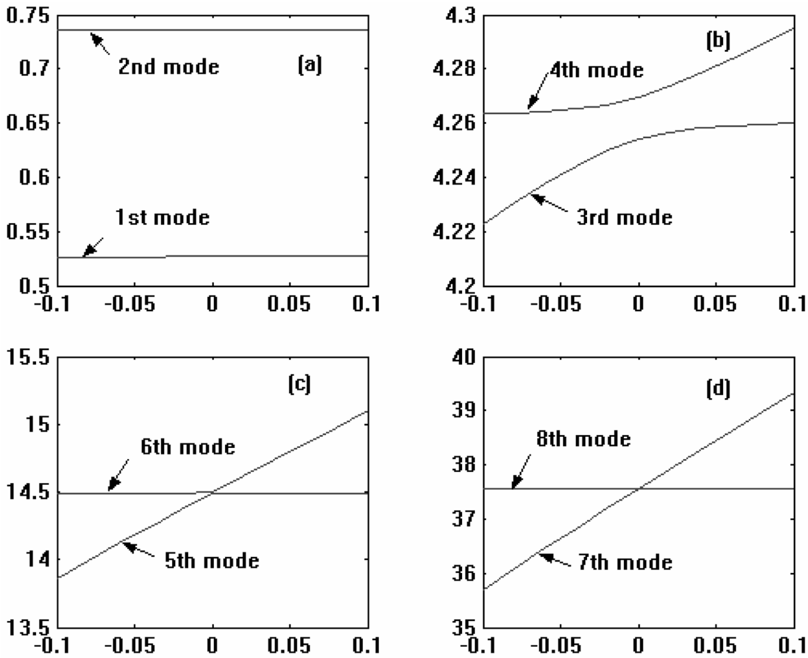

Fig. 13 The first 8 frequency loci veering of the disordered support system with perturbation in *EI*

third mode and the higher the mode, the stronger the phenomenon. The first 8 frequency loci of the disordered system are shown in Fig. 13. As shown in subplot (b)-subplot (d), when the perturbation ε increases from -0.1 to 0, frequency loci veering phenomenon occurs. However, in subplot (a), this phenomenon does not take place, the two frequency curves seems parallel to each other. This means that the mode localization phenomenon does not take place in these two modes, and this is verified from Fig. 12. Comparing this with Case 1, one can see, different boundary conditions may have effect on the mode localization, in Case 1, the mode localization occurs from the first mode, however, in this case, the mode localization occurs from the third mode.

Case 6 Disorder due to the coefficients of the elastic support

Many studies have been carried out on mode localization due to disorder in the material, geometry of the structure; in this case, the mode localization due to the disorder of the boundary spring is studied. It is assumed that there is a 4% reduction in the transverse spring of the left boundary, and the other parameters of the system remain unchanged. The first 8 natural frequencies are also shown in Table 1. And Figs. 14 and 15 show the first 8 mode shapes and first 8 frequency loci curves of the disordered system. Again, the mode localization phenomenon occurs only from the third mode.

Case 7 Sensitivity analysis of different types of disorder

Several studies are carried out above to investigate the effect of different types of disorder on the mode localization. But which physical parameter is the mode localization most sensivitive to? Here, the sensitivities of different types of disorder to the mode localization are analyzed. First of all, the norm of mode shape difference in two substructures is introduced, taking the *i*th mode as example

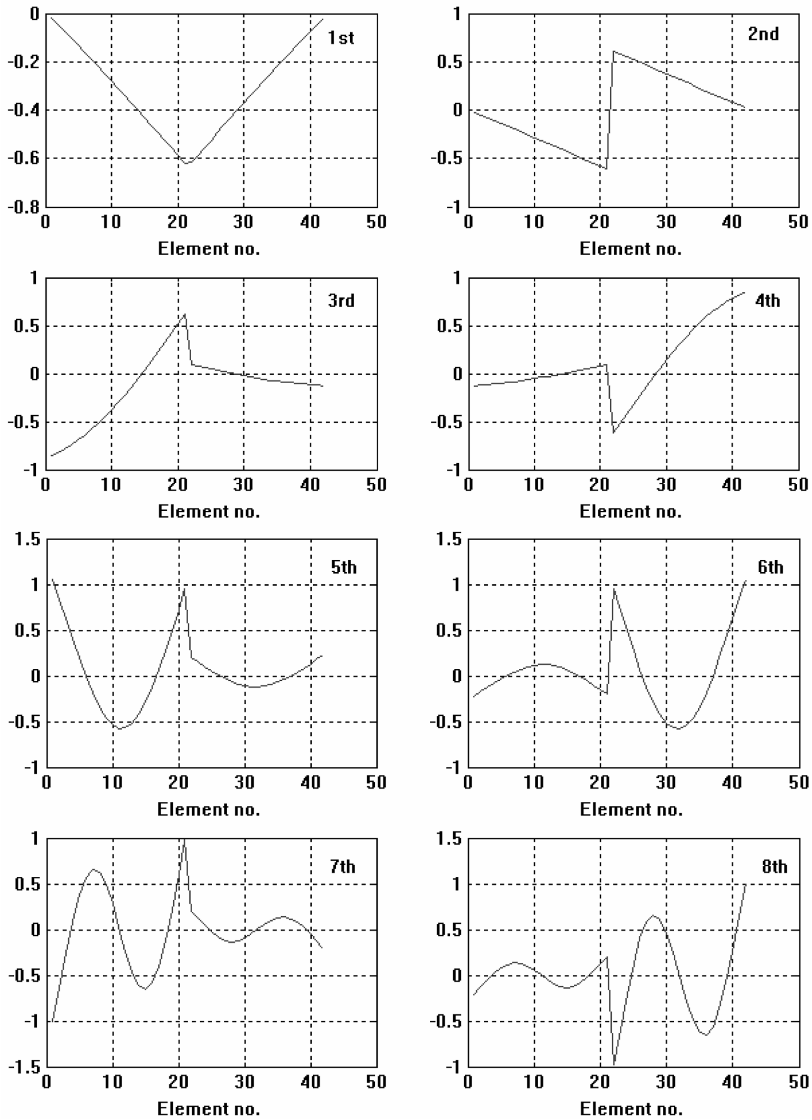

Fig. 14 The first 8 mode shapes of the disordered elastic support system with perturbation in elastic support

$$Norm(\Delta u)_i = \|u_{i1} - u_{i2}\| \tag{11}$$

where u_{i1} and u_{i2} are the normalized ith mode shapes of two substructures respectively. Figs. 16(a) and 16(b) show the norm of mode shape difference vs. the perturbation parameter ε in different physical parameters for the first two modes. From this figure, one can see, as the perturbation parameter ε increase from -0.1 to 0, the mode localization becomes weaker and as ε increase from 0 to 0.1, the mode localization becomes stronger. In addition, the mode localization seems more sensitive to the disorder in the length than to the disorder in mass density and Young's modulus.

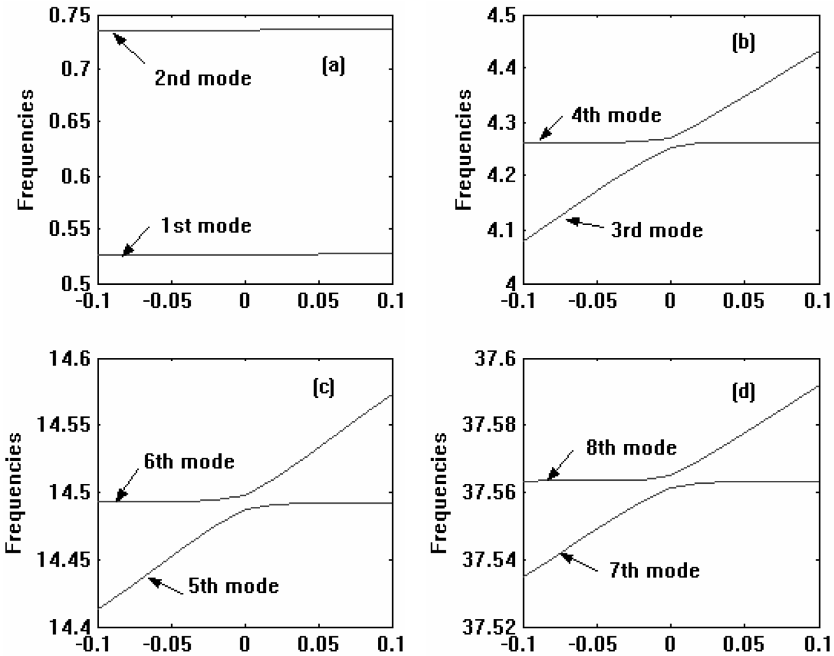

Fig. 15 The first 8 frequency loci veering of the disordered support system with perturbation in elastic support

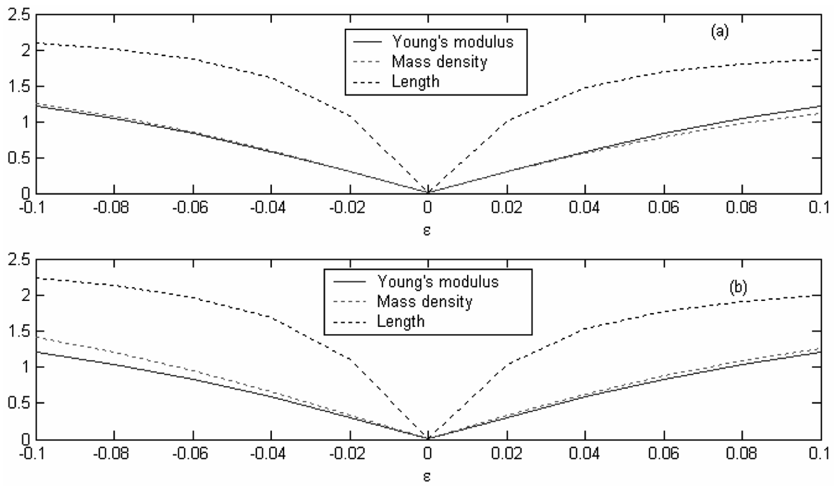

Fig. 16 Comparison on mode localization due to perturbations in different parameters (a) clamped boundary condition, (b) elastic support

4. Conclusions

In the present study, a coupled beam system is studied to investigate the mode localization and frequency loci veering phenomena using finite element analysis. The system is modeled by Euler-Bernoulli beam element and both the free and forced vibration analysis is conducted. The following conclusions can be drawn from the study cases carried out in this paper:

1. When there is some small disorder due to material (for instance, Young's modulus, mass density ect.) or geometry (e.g. The length of the substructure) in the symmetric structure system, the structure will undergo mode localization and frequency loci veering. Engineers should pay more attention to symmetric and weakly coupled structures in designing such structures because vibration mode localization will cause dramatic effects on these structures.
2. The occurrence of mode localization and frequency loci veering indicates that the dynamic system is very sensitive to the disordered parameters. Attention should be paid to the significance of the sensitivity for it affects the dynamic modes dramatically.
3. For the weakly coupled system, sometimes, local structural damage may also lead to the occurrence of mode localization phenomenon. One should pay more attention to such system in damage detection when mode localization occurs.
4. Attention should also be paid to the perturbation in the elastic supports of the weakly coupled structures. As shown in the study, the disorders in these factors will also lead to mode localization and frequency loci veering.

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