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Technical Note

Adaptive finite element buckling analysis of folded plate structures

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1. Introduction

In this study, the finite element linear buckling analysis of folded plate structures using adaptive *h*-refinement methods is presented. The variable-node flat shell element used in this study possesses a drilling D.O.F. which, in addition to improvement of the element behavior, permits an easy connection to other elements with six degrees of freedom per node (CLS element, Choi and Lee 1996). Accordingly, the folded plate structures, for which it is hard to find the analytical solutions, can be analyzed with a relative ease using the developed flat shell elements. Using the adaptive *h*-refinement procedure, the convergent buckling modes and the critical loads of these modes can be obtained by the buckling analyses of those structures.

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2. Variable-node flat shell element

CLS element was established by combining CLM element (Choi and Lee 1996) with a drilling D.O.F. and Mindlin plate bending element. For the variable-node membrane elements with a drilling D.O.F., the Allman-type interpolations are used for the basic behavior of the element (Allman 1984, 1988). Variable-node plate bending element is formulated based on Reissner-Mindlin plate theory to account for the shear deformation and has the substitute shear strain fields (Choi and Park 1992). In the plate bending element based on Reissner-Mindlin plate theory, to account for the shear deformation the vertical displacement field and rotation field of an element are assumed separately and only C^o continuity is needed to be satisfied. To avoid excessive shear strains in the element stiffness formulation, the substitute shear strain polynomials are used, maintaining the bending part of the element stiffness unchanged. For the out-of-plane D.O.F. in the plate bending element with respect to the local coordinate system, Green-Lagrangian strains defined in the plate bending element can be expressed as the linear and nonlinear parts. As a result, the stress stiffness matrix $[\mathbf{K}_\sigma]$ in each element can be derived.

3. Finite element buckling analysis

First, a reference level of loading $[\mathbf{R}]_{ref}$ is applied to the structure and through the static analysis, and the membrane stresses in all the elements are evaluated. By assembling the stress stiffness matrices of elements, the stress stiffness matrix of the entire structure is composed. Then, the incremental form of the equilibrium equation results in the eigenvalue problem. In the numerical solution of the eigenvalue problem, the subspace iteration method is used, which is particularly suited for the calculation of a few eigenvalues and eigenvectors of large finite element systems.

4. Adaptive *h*-refinement method

The adaptive mesh refinement is normally performed after an initial solution has already been made available, and regions of the solution domain where the accuracy is not satisfactory have been identified according to a prescribed set of criteria. This process can be iterated using the last solution as the new initial solution. Two criteria are essential for adaptive mesh refinement, i.e., stopping criterion and refinement criterion. The stopping criterion is used to decide if the obtained error is within the prescribed maximum error tolerance or not. The adaptive mesh refinement process will stop if the requirement is satisfied (Zienkiewicz and Zhu 1989).

5. Numerical analysis

As examples of numerical analysis, a box section beam with holes and an octagon box section beam are chosen. Analyses are performed to show the results by the present analysis schemes.

A box section beam with holes under two opposite concentrated loads at the centers as shown in Fig. 1 is tested. Using the symmetry, a one-eighth segment is actually analyzed. The following

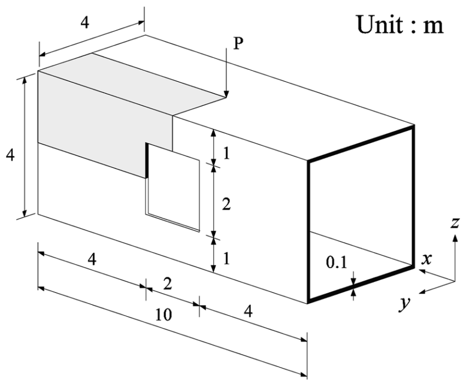
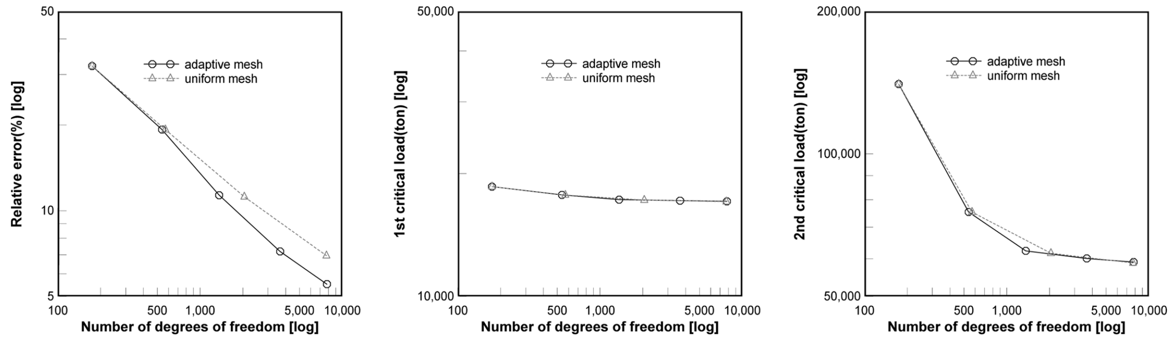
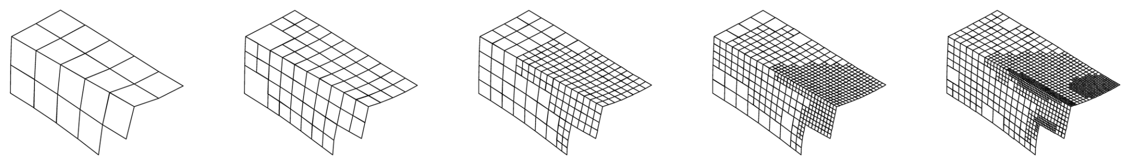


Fig. 1 A box section beam with holes



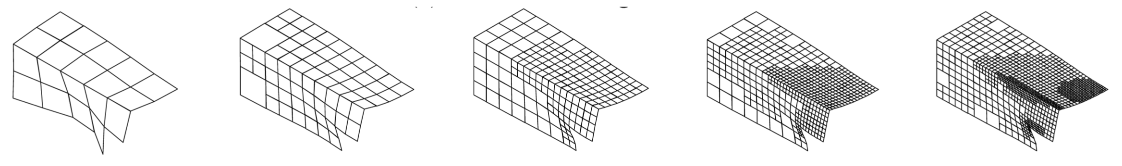
(a) Relative percentage error(μ) (b) The 1st critical load (c) The 2nd critical load

Fig. 2 Analysis of the box section beam with holes



NDF=174, η=32.17% NDF=540, η=19.26% NDF=1362, η=11.32% NDF=3654, η=7.19% NDF=7800, η=5.52%

(a) The 1st buckling mode



(b) The 2nd buckling mode

Fig. 3 Buckling modes of the box section beam with holes

material properties are used: the elastic modulus $E = 2{,}100{,}000$ kgf/cm^2 and Poisson's ratio $\nu = 0.18$. The two opposite concentrated load of the intensity $P = 1$ tonf is applied.

As the meshes are locally refined, the changes of the relative percentage error, the 1st critical load, and the 2nd critical load are shown in Fig. 2, where analysis results are compared with those using uniformly refined meshes. In addition, the buckling modes of adaptively refined meshes are shown in Fig. 3. The accuracy of the predicted critical loads are dependent on the number of degrees of freedom irrespective of the pattern of refined mesh. However, the relative percentage

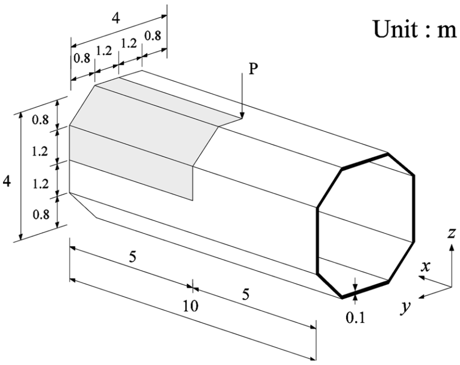
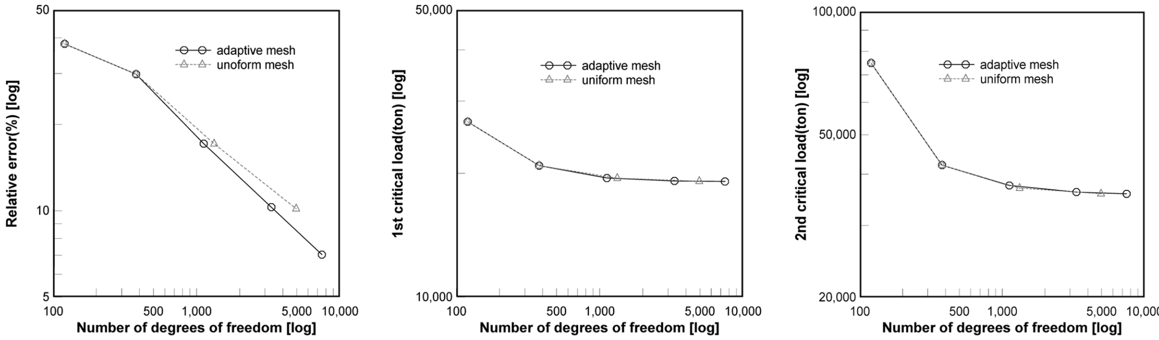
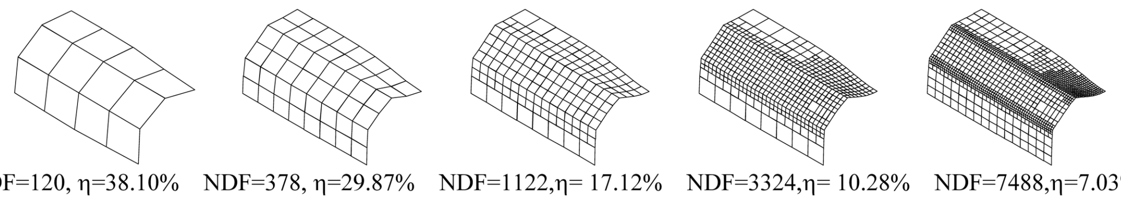


Fig. 4 An octagon box section beam



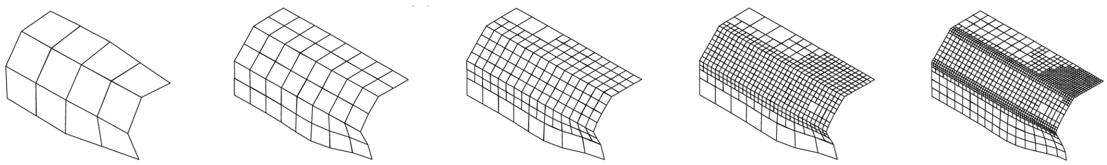
(a) Relative percentage error(μ) (b) The 1st critical load (c) The 2nd critical load

Fig. 5 Analysis of the octagon box section beam



NDF=120, η=38.10% NDF=378, η=29.87% NDF=1122,η= 17.12% NDF=3324,η= 10.28% NDF=7488,η=7.03%

(a) The 1st buckling mode



(b) The 2nd buckling mode

Fig. 6 Buckling modes of the octagon box section beam error becomes smaller when adaptive meshes are used than uniform meshes. Also, when the adaptive meshes are used, the positions of nodes can be located more efficiently in the view point of practical use and the more accurate buckling modes can be predicted.

An octagon box section beam under two opposite concentrated loads at the centers as shown in Fig. 4 is tested. Using a symmetry, a one-eighth segment is actually analyzed. The two opposite concentrated loads of the intensity $P = 1$ tonf is applied. As the meshes are refined, the changes of the relative percentage error, the 1st critical load, and the 2nd critical load are shown in Fig. 5, where analyses results are compared with those using uniformly refined meshes. In addition, the buckling modes of adaptively refined meshes are shown in Fig. 6.

6. Conclusions

In this study, the finite element linear buckling analysis of folded plate structures using adaptive h-refinement methods was presented. CLS element, which possesses a drilling D.O.F., was found effective for the modeling of folded plate structures. In addition, the formulation of the stress stiffness matrix of CLS element was presented for the finite element buckling analysis. Accordingly, the folded plate structures, for which it is hard to find the analytical solutions, can be analyzed using the CLS elements. Using the adaptive h-refinement procedure, the finite element linear buckling analyses for those structures are found to be more accurate and efficient. As a result of analysis, the convergent buckling modes and the critical loads of these respective modes could be obtained.

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