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Shear strength of steel fiber reinforced concrete beams with stirrups

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Abstract. The present paper proposes a semi-empirical analytical expression that is capable of determining the shear strength of reinforced concrete beams with longitudinal bars, in the presence of reinforcing fibers and transverse stirrups. The expression is based on an evaluation of the strength contribution of beam and arch actions and it makes it possible to take their interaction with the fibers into account. For the strength contribution of stirrups, the effective stress reached at beam failure was considered by introducing an effectiveness function. This function shows the share of beam action strength contribution on the global strength of the beam calculated including the effect of fibers. The expression is calibrated on the basis of experimental data available in literature referring to fibrous reinforced concrete beams with steel fibers and recently obtained by the authors. It can also include the following variables in the strength previsions: - geometrical ratio of longitudinal bars in tension; - shear span to depth ratio; - strength of materials and fiber characteristics; - size effects. Finally, some of the more recent analytical expressions that are capable of predicting the shear strength of fibrous concrete beams, also in the presence of stirrups, are mentioned and a comparison is made with experimental data and with the results obtained by the authors.

Keywords: shear-moment interaction; concrete; fibers; stirrups; shear strength.

1. Introduction

Several papers published in the last 25 years have focused on the effectiveness of fibers for shear reinforcement. Each one gives analytical expressions for the determination of the shear strength (Sharma 1986, Mansur *et al*. 1986, Narayanan and Darwish 1986, Al Ta'an and Al Feel 1990, Ashour *et al*. 1992, Swamy *et al*. 1993, Imam *et al*. 1995, Di Prisco and Romero 1996, Adebar *et al*. 1997, Casanova and Rossi 1997, Khuntia *et al*. 1999, Kim *et al*. 2002, Gettu *et al*. 2002, Dupont and Vandewalle 2003).

It is well known that the shear failure of reinforced concrete beams is brittle and occurs when the principal stress exceeds the tensile strength of the concrete and the principal cracks propagate diagonally, producing diagonal tension or shear compression failure modes, depending on the ratio

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between the shear span and the effective depth of the beams.

In both failure modes the addition of fibers in a suitable percentage and geometry produces a significant increase in shear strength and in some cases can also change the failure mode from shear to flexure. In these cases, fibers can partially substitute transverse stirrups and have the same effect in terms of shear strength. They can also make it very easy to manufacture critical portions of R.C. structures as well as allowing highly effective control of the concrete cracking process in particular low force levels.

In the case of using high strength concretes which are becoming very popular today and are useful for the realization of large span structures and high rise buildings the use of fibers is particularly suitable.

From a mechanical point of view, several studies present in literature, grouped in Khuntia *et al.* (1999), refer to the determination of the shear strength of fibrous reinforced concrete beams made of normal or high strength concrete, both normal and lightweight. The analytical expressions given are often of an empirical nature and they account for the increase in the flexural capacity of the beams, attributing these effects to the post-cracking tensile strength of fibrous concrete. They consider the strength contributions due to the different parameters governing shear failure separately.

Few experimental and very few analytical studies (Sharma 1986, Imam *et al.* 1995, Tompos and Frosh 2002, Cho and Kim 2003, Dupont and Vandewalle 2003) analyze the presence of fibers and stirrups and in these cases the shear contributions of beams due to stirrups and fibers are considered separately. What is more, many models consider the stirrups as having yielded at beam rupture. On the contrary, in the case of beams with stirrups it has been shown (Russo and Puleri 1997) that the stirrups are not always yielded at beam failure.

In addition to this, in the presence of stirrups and fibers a further reduction in the working stress of stirrups is observed experimentally at beam failure (Furlan and De Hanai 1997, Dupont and Vandewalle 2003, Campione *et al.* 2003, Cucchiara *et al.* 2004).

On the basis of the considerations made above, the focus of the present paper will regard the determination of the shear strength of reinforced concrete beams in the presence of stirrups and fibers. The classical approach for calculating shear strength will be used, through an evaluation of arch and beam effects. This method will also study the effect of fibers and the presence of stirrups. The effective stress in the stirrups will be evaluated by means of an effectiveness function.

2. The case study

The case study regards a prismatic reinforced concrete beam with rectangular cross-sections, base b and height h, which is cast with plain or fibrous concrete.

The static scheme adopted is constituted by simply supported beams under four-point bending tests, in which the beams are subjected to symmetrical vertical loads, V, acting at distance a from the support, as shown in Fig. 1.

The beam is reinforced on the bottom side with longitudinal deformed bars having area A_s and on the top side with steel bars having area A_s', which is assumed as being negligible in respect to A_s. In addition, transverse steel stirrups having area of one leg A_{st} are placed in the beam at pitch p.

The main longitudinal steel bars (bottom bars) are bent at the support with an adequate anchorage length so as to avoid any slippage or premature splitting failure. In the case of fibrous concrete, the fibers utilized had length L_f, equivalent diameter ϕ and volume percentage V_f.

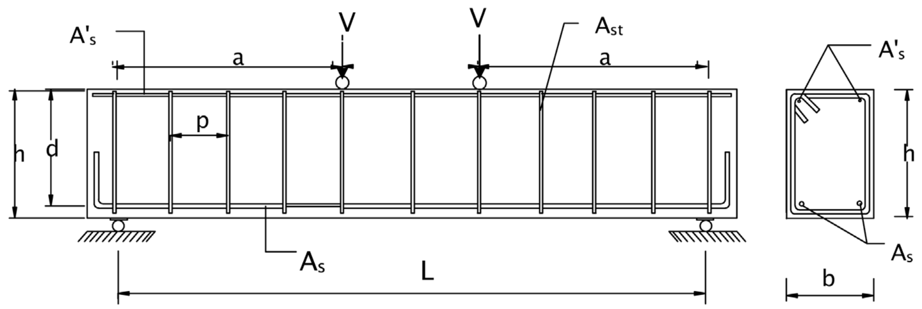

Fig. 1 Static scheme of the beam

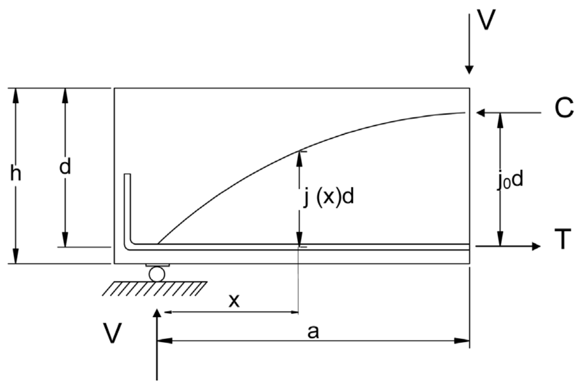

Fig. 2 Beam and arch actions

In the following sections, in order to take the fibers' characteristics into account, the fiber factor F will be introduced, defined as $F = V_f \cdot L_f/\phi \cdot \beta$, with β being the shape factor assumed as 1 and 0.5 for deformed and straight fibers, respectively. This assumption for β is in agreement with the results obtained in Banthia and Trottier (1994) from which it emerges that the pull-out resistance of straight fibers is less than for hooked or crimped steel fibers (having similar aspect of ratio) while the values of β are those suggested in Campione *et al.* (1999).

Bazant and Kim (1984) propose a mechanical model to calculate the flexural capacity in shear of reinforced concrete beams and take the sum of the strength contributions due to the beam and to the arch actions into consideration. These contributions are identified, as shown in Fig. 2, by imposing the equilibrium conditions of the beam enclosed between the support and the loaded section (shear span a).

With reference to the symbols utilized in Fig. 2, the bending moment M and the shear force V at the generic cross-section can be related to the axial force T in the longitudinal bar and to the internal arm jd by means of:

$$M = V \cdot x = T \cdot jd \tag{1}$$

Moreover, the shear force V is related to the variation in M by the relationship:

$$V = \frac{dM}{dx} = jd \cdot \frac{dT}{dx} + T \cdot \frac{d(jd)}{dx} \tag{2}$$

thus obtaining, by means of Eq. (2), the two fundamental strength contributions that are well known in literature as beam effect (*jd* constant) and arch effect (*jd* variable).

According to this model, which was used so effectively by Bazant and Kim (1984) and which is also used here for fibrous concrete beams, the following simplified hypothesis are assumed: beam and arch effects are considered separately; in the evaluation of arch effects, it is assumed that the tension force in the longitudinal bar remains constant across the shear span; in the evaluation of beam effects, the tension force is assumed to be variable across the shear span.

3. The effect of fibers

In the following sections there will be a separate examination of both the influence of fibers in beam and arch actions, as well as the action of fibers in bridging the principal cracks.

3.1 Beam actions

From Eq. (2) it emerges that the strength contribution made by the beam effect is defined by:

$$V_1 = j_0 d \cdot \frac{dT(x)}{dx} \tag{3}$$

in which j takes on a constant value j_0 (determined in the loaded section when subjected to maximum moment) while the variation in the moment M is due to a variation in the force T.

In reference to Fig. 3, which highlights the contribution of the beam effect, denoted as V_1, it should be observed that dT/dx can be expressed in terms of bond stresses q_b transmitted by the longitudinal bars. If we consider a beam portion of length dx and we consider the equilibrium of forces along the bars we have:

$$\frac{dT}{dx} = \pi \cdot q_b \cdot \sum_{i=1}^{n} D_i \tag{4}$$

where D_i is the diameter of the i-th bar constituting the main reinforcement of area A_s.

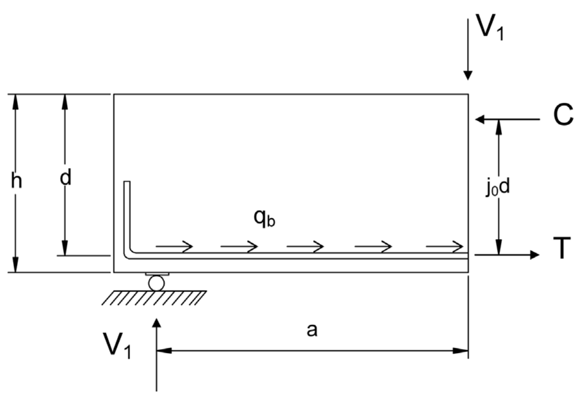

Fig. 3 Strength mechanism due to beam action

It should be observed that the term $\sum_{i=1}^{n} D_i$ is related to the geometrical ratio of longitudinal bars ρ, by the following expression:

$$\rho = \frac{A_s}{b \cdot d} = \frac{\pi \cdot \sum_{i=1}^{n} D_i^2}{4 \cdot b \cdot d} \tag{5}$$

where it can be assumed that

$$\sum_{i=1}^{n} D_i \propto \rho^{1/2} \tag{6}$$

In the case of ordinary concrete it was assumed that, as in ACI 318-02 (2002) and as suggested in the original work of Bazant and Kim (1984), the bond stress q_b is proportional to the tensile strength of the concrete, which in turn is proportional to the square root of the cylindrical compressive strength f_c'. In terms of symbols we have:

$$q_b \propto \sqrt{f_c'} \tag{7}$$

The choice of assuming the bond strength as being proportional to the square root of cylindrical compressive strength of fibrous concrete is also supported experimentally by Harajili *et al.* (1995).

It should be observed that this choice is not universally accepted. For instance, the ACI Committee 408 (1992) supports the fourth root of cylindrical compressive strength, while the European study (Eurocode 2, 1989) supports the third root.

By taking the relation in Eq. (4) into account, by means of Eq. (6) and Eq. (7), and by introducing a c_1 dimensional constant, as in Bazant and Kim (1988), it is possible to obtain:

$$\frac{dT}{dx} = c_1 \cdot \rho^{1/2} \cdot \sqrt{f_c'} \tag{8}$$

On the basis of Eq. (3) and Eq. (8) it results that:

$$V_1 = c_1 \cdot d \cdot j_0 \cdot \rho^{1/2} \cdot \sqrt{f_c'} \tag{9}$$

By dividing the shear forces V_1 by the area bd, it is possible to determine the nominal shear stress due to the beam action, which can be defined as:

$$v_1 = A \cdot j_0 \cdot \rho^{1/2} \cdot \sqrt{f_c'} \tag{10}$$

in which $A = c_1/b$ is a constant which will be calibrated on the basis of experimental data, and which is capable of giving the best fit with the experimental results (see next sections).

By using translation and rotational equilibrium conditions to define jd in the loaded section, when subjected to the maximum bending moment (see Fig. 4), it is possible to determine the position of the neutral axis x_c, and the ultimate moment M_{fl}. In the following sections, the stress block hypothesis for the equilibrium condition will be utilized, as also suggested by ACI 544 (1988) for fibrous concrete. The yielding of longitudinal bars is also assumed. The ordinate of the compressed

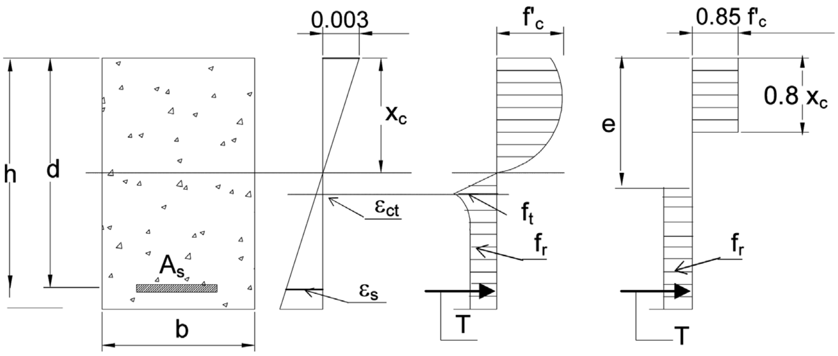

Fig. 4 Equilibrium of the transverse cross-section

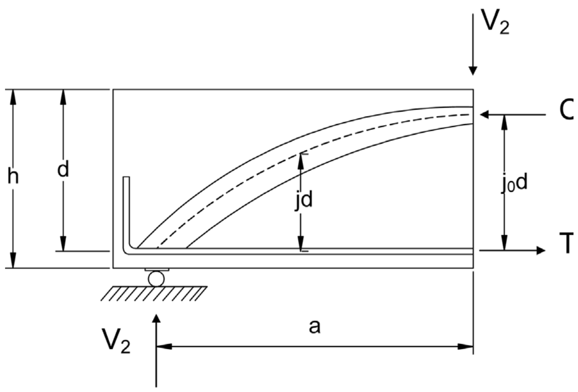

Fig. 5 Strength mechanism due to arch action

zone (see Fig. 4) is assumed as being equal to $0.85\,f_c'$, with a depth of $0.8\,x_c$.

With reference to the symbols in Fig. 5, if we consider that:

$$e = x_c \cdot \frac{\frac{f_t}{E_{ct}} + 0.003}{0.003} \tag{11}$$

from the translational equilibrium, we obtain:

$$x_c = \frac{d}{0.8} \cdot \frac{\rho \cdot f_y + f_r \cdot \frac{h}{d}}{0.85 \cdot f_c' + f_r \cdot \frac{f_t/E_{ct} + 0.003}{0.80 \cdot 0.003}} \tag{12}$$

and from the rotational equilibrium:

$$\frac{M_{fl}}{b \cdot d^2} = \rho \cdot f_y \cdot \left(1 - 0.5 \cdot \frac{0.8 \cdot x_c}{d}\right) + f_r \cdot \left(\frac{h}{d} - \frac{e}{d}\right) \cdot \left(\frac{h}{d} - \frac{h-e}{2 \cdot d} - 0.5 \cdot \frac{0.8 \cdot x_c}{d}\right) \tag{13}$$

Moreover, the arm of the internal forces is $j_0\,d$ in which j_0 can be expressed by the equilibrium across the compressive centroid in the following form:

$$j_0 = \frac{\rho \cdot f_y \cdot \left(1 - 0.4 \cdot \frac{x_c}{d}\right) + f_r \cdot \left(\frac{h}{d} - \frac{e}{d}\right) \cdot \left(\frac{h+e}{2 \cdot d} - 0.4 \cdot \frac{x_c}{d}\right)}{\rho \cdot f_y + f_r \cdot \left(\frac{h}{d} - \frac{e}{d}\right)} \tag{14}$$

In these equations, f_t and f_r indicate the maximum and the residual strength of fibrous concrete, f_y is the yielding stress of the longitudinal bars, while E_{ct} indicates the elastic modulus of fibrous concrete in tension (this can be estimated using mixture rules but in practice it may be assumed to be that of plain concrete). Its value can be assumed as being equal to half of the modulus in compression, this latter being assumed as $E_c \cong 5200 \cdot \sqrt{f_c'}$, with f_c' in MPa.

In reference to the maximum and residual tensile strengths, f_t and f_r, there are several analytical expressions for their prediction. According to Naaman (2003), the maximum strength for fibrous concrete (with a volume percentage of up to 2% and an aspect ratio L_f/ϕ of up to 100 (material with strain-softening behavior)) does not exhibit significant variation with respect to that of plain concrete. On the other hand, the residual strength takes on a significant value that can be related to F (fiber factor) and to the bond stress τ of a single fiber embedded in the concrete.

In regard to the τ values, it should be observed that the experimental research mentioned in the literature gives a wide range of values. Narayanan and Darwish (1986) suggest the adoption of an average value for fibers pulled out from the matrices with fiber direction parallel to the load equal to 4.15 MPa. Swamy *et al.* (1993), however, suggest an average value of 5.12 MPa. According to Zingone *et al.* (1999), for fibers of different geometric and material types (carbon, polyolefin, steel), and to Banthia and Trottier (1994), for steel fibers with different geometry (hooked, straight, crimped), the value of the ultimate shear stress τ depends on several factors, such as matrix characteristics, the length and inclination of the fiber, and the type of test utilized for pulling-out single fibers or group of fibers, etc.. For this reason it is difficult to extrapolate a representative value.

On the basis of extensive experimental pull-out tests carried out on steel fibers, Valle and Buyukozoturk (1993) suggest an empirical correlation of the bond strength and the compressive strength of matrices. They also suggest the assumption of the following relation for both normal and high strength concretes:

$$\tau = 0.66 \cdot \sqrt{f_c'} \tag{15}$$

To determine the value of the residual strength of fibrous, normal strength concrete, ACI 544 (1988) suggests the assumption of the following expression:

$$f_r = 0.772 \cdot F \tag{16}$$

Thus, by using the expression recently given by Naaman (2003) and introducing Eq. (15), it is possible to obtain:

$$f_r = 0.25 \cdot 1.2 \cdot \tau \cdot F = (0.25 \cdot 1.2 \cdot 0.66) \cdot \sqrt{f_c'} \cdot F \cong 0.2 \cdot \sqrt{f_c'} \cdot F \tag{17}$$

A comparison between Eq. (16) and Eq. (17) shows similar values for normal strength concrete, with the advantage that if Eq. (17) is utilized, it is possible to take different concrete grades into account.

3.2 Arch actions

The strength contribution made by arch action can be evaluated, with reference to the mechanism shown in Fig. 5, by relating the shear force, V_2, to the variation in j, where T is assumed to be constant, and by using the following relation:

$$V_2 = T \cdot d \cdot \frac{dj(x)}{dx} \tag{18}$$

To define Eq. (18) it is necessary to establish a possible law of j with variation in x. As originally suggested in Bazant and Kim (1984), it can be assumed to be:

$$j = j_0 \cdot \left(\frac{x}{a}\right)^{\alpha} \tag{19}$$

where x is measured starting from the support and α is a shape parameter defining the power of the curve describing the arch. Deriving Eq. (19) with respect to x, the following relation can be obtained:

$$\frac{dj(x)}{dx} = j_0 \cdot \frac{\alpha}{a} \cdot \left(\frac{x}{a}\right)^{\alpha - 1} \tag{20}$$

Moreover, it is well known that:

$$T = \sigma_s \cdot \rho \cdot b \cdot d \tag{21}$$

σ_s being the stress in the longitudinal bar.

In the case of fibrous concrete it appears appropriate, in addition to T, to consider a further force: the contribution made by the residual tensile strength expressed in terms of f_r in the direction parallel to T (see Fig. 4). The whole T_t force proves to be:

$$T_t = \sigma_s \cdot \rho \cdot b \cdot d + f_r \cdot (h - e) \cdot b \tag{22}$$

In Eq. (22), for simplicity, a fictitious geometrical ratio ρ_f is introduced, equivalent to a geometrical longitudinal bar ratio, thus giving:

$$T_t = \sigma_s \cdot (\rho + \rho_f) \cdot b \cdot d = \eta \cdot f_y \cdot (\rho + \rho_f) \cdot b \cdot d \tag{23}$$

with $\rho_t = \rho + \rho_f$ and $\rho_f = \frac{f_r}{\eta \cdot f_y} \cdot \frac{h}{d}\left(1 - \frac{e}{h}\right)$ (24)

in which $\eta = \sigma_s / f_y$.

Moreover, and as suggested by Bresler (1974) on the basis of extensive experimental research, it should be recalled that in the arch mechanism, the principal crack typically forms at distance d away from the support ($x = d$) and extends upward. By substituting Eq. (20) and Eq. (23) in Eq. (18) it is possible to obtain:

$$V_2 = \eta \cdot f_y \cdot \rho_t \cdot b \cdot d^2 \cdot j_0 \cdot \frac{\alpha}{a} \cdot \left(\frac{d}{a}\right)^{\alpha - 1} = \alpha \cdot \eta \cdot f_y \cdot \rho_t \cdot b \cdot d \cdot j_0 \cdot \frac{d}{a} \cdot \left(\frac{d}{a}\right)^{\alpha - 1} \tag{25}$$

By dividing V_2 by bd and introducing the B constant including $\alpha\ \eta$, we have:

$$v_2 = B \cdot f_y \cdot \rho_t \cdot j_0 \cdot \left(\frac{d}{a}\right)^{\alpha} \tag{26}$$

The B and α constants will be assumed according to experimental data (see next section).

The v_2 term, as suggested by Zsutty (1971), can be further modified by introducing a corrective factor ε that depends on the a/d ratio and which is able to take the case of deep beams into account. According to Zsutty (1971), this corrective coefficient is equal to $\varepsilon = 2.5\ d/a$ when $a/d \leq 2.5$ and $\varepsilon = 1$ when $a/d > 2.5$. In this way, introducing ε in Eq. (26), we obtain:

$$v_2 = B \cdot \varepsilon \cdot f_y \cdot \rho_t \cdot j_0 \cdot \left(\frac{d}{a}\right)^{\alpha} \tag{27}$$

3.3 Action of fibers across the principal crack

To estimate the shear strength contribution made by fibers across the principal crack we refer, as shown in Fig. 6 and also as suggested in Narayanan and Darwish (1986), to the case in which the principal crack develops with inclination γ (generally assumed to be 45°) from the tip of the compressed zone down to the bottom portion of the beam. Interlocking actions and dowel effects in longitudinal bars are influenced positively by the presence of fibers (Swamy and Bahia 1979) but in this paper, to simplify the analytical expressions, they are neglected.

To estimate the vertical component of fiber action across the principal crack, it is necessary to determine the number (see Fig. 6) of fibers crossing a unit area, which according to Romualdi *et al.* (1964) is:

$$n_w = \frac{1.64 \cdot V_f}{\pi \cdot \phi^2} \tag{28}$$

By considering the length of the principal crack as approximately equal to $(jd)/\sin\gamma$, (see Fig. 6), the total number of fibers across the crack proves to be:

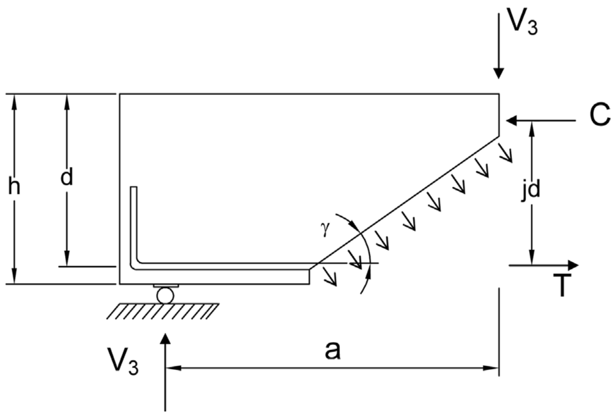

Fig. 6 Strength mechanism due to fiber across the principal crack

$$n = n_w \cdot b \cdot \frac{j \cdot d}{\sin\gamma} \tag{29}$$

and the total lateral resisting area of fibers crossing the principal crack is:

$$A_b = n \cdot \frac{\pi \cdot \phi \cdot L_f}{4} \tag{30}$$

in which $L/4$ was chosen as the average length of fibers across the crack (to take fiber orientation and embedded length into account, as proposed in Narayanan and Darwish 1986).

The resultant F_b of resisting forces exercised by the fibers is equal to:

$$F_b = A_b \cdot \tau = \frac{b \cdot j \cdot d}{\sin\gamma} \cdot \frac{1.64 \cdot V_f}{\pi \cdot \phi^2} \cdot \frac{\pi\phi \cdot L_f}{4} \cdot \tau \tag{31}$$

Therefore, the shear contribution made by fibers is $V_3 = F_b \cdot \cos\gamma$ and the nominal strength due to the fibers proves to be:

$$v_3 = \frac{F_b \cdot \cos\gamma}{b \cdot j \cdot d} = \frac{A_b \cdot \tau \cdot \cos\gamma}{b \cdot j \cdot d} = 0.41 \cdot \frac{\tau \cdot F}{\tan\gamma} \tag{32}$$

A recent study (Tompos and Frosch 2002), referring to beams with stirrups in shear, showed that the inclination of the principal crack depends on the reinforcing level exercised by the stirrups and varies between 30° and 60°, with a smaller angle for a high confinement level. In the opinion of the authors it appears reasonable to suppose that the inclination angle of the principal crack in the presence of fibers depends on the volume percentage and on the type of fiber, although there is not experimental confirmation of this. This supposition could find support in a recent study referring to the diagonal compression field theory, which, if applied to fibrous reinforced concrete characterized by high residual strength in tension and by a confinement effect in the compression zone, should show that the direction of the compressed zone changes beneficially, increasing both the tensile strength of the material and, as a consequence, the fracture angle, with respect to the case of plain concrete.

In this context and in the absence of available experimental data confirming the above mentioned hypothesis, it is assumerd that for fibrous concrete beams, the inclination of the principal crack at rupture is 45°. Consequently if in Eq. (32) we assume $\gamma = 45°$ we have:

$$v_3 = 0.41 \cdot \tau \cdot F \tag{33}$$

By substituting Eq. (15) in Eq. (33) we obtain:

$$v_3 = 0.27 \cdot F \cdot \sqrt{f_c'} \tag{34}$$

It is interesting to observe that if the presence of fibers produces variation in the angle γ in the range 30-60°, Eq. (34) should give:

$$0.468 \le \frac{v_3}{F \cdot \sqrt{f_c'}} \le 0.156 \tag{35}$$

The values 0.468 F and 0.156 F represents the upper and the lower limits for shear strength contribution of fibrous concrete dimensionless with respect to the square root of the compressive cylindrical strength.

3.4 Shear strength capacity

By considering the sum of the strength contributions due to the beam and arch actions and due to the fibers across the principal cracks we obtain:

$$v_u = v_1 + v_2 + v_3 \tag{36}$$

By substituting Eqs. (10), (27), (34) in Eq. (36) we obtain the expression of shear strength for fibrous concrete beams with hooked steel fibers:

$$v_u = A \cdot j_0 \cdot \rho^{1/2} \cdot \sqrt{f_c'} + B \cdot \varepsilon \cdot f_y \cdot \rho_t \cdot j_0 \cdot \left(\frac{d}{a}\right)^{\alpha} + 0.27 \cdot F \cdot \sqrt{f_c'} \tag{37}$$

To take the size effect into account, Bazant and Kim (1984) suggest introducing a ξ coefficient for beams made of ordinary concrete, defined as:

$$\xi = \frac{1}{\sqrt{1 + \dfrac{d}{25 \cdot d_a}}} \tag{38}$$

where d_a is the maximum aggregate size of the concrete.

Recently, RILEM TC 162-TDF (2003) has suggested an analytical expression which, more specifically, is capable of taking the size effect in fibrous concrete beams into account using the k coefficient, defined as:

$$k = 1 + \sqrt{\frac{200}{d}} \quad \text{(with } d \text{ in mm)} \tag{39}$$

In the following, the ξ coefficient is assumed as taking the size effect into account, also, if not specifically derived, for fibrous concrete. This choice, as it will show in the next section with the comparison between analytical and experimental results, allows one to derive accurate prediction of experimental data.

By introducing Eq. (38) in Eq. (37) we obtain:

$$v_u = \xi \left\{ j_0 \cdot \left[A \cdot \rho^{1/2} \cdot \sqrt{f_c'} + B \cdot f_y \cdot \varepsilon \cdot \rho_t \cdot \left(\frac{d}{a}\right)^{\alpha} \right] + 0.27 \cdot F \cdot \sqrt{f_c'} \right\} \tag{40}$$

Experimental data given in literature for the main characteristics of the beams tested (212 data are given in Table 1) were used to calibrate the A, B and α coefficients.

The assumed values of A, B and α were A = 1.3, B = 0.3 and α = 1.8. Therefore Eq. (40) becomes:

$$v_u = \xi \left\{ j_0 \cdot \left[1.3 \cdot \rho^{1/2} \cdot \sqrt{f_c'} + 0.3 \cdot f_y \cdot \varepsilon \cdot \rho_t \cdot \left(\frac{d}{a}\right)^{1.8} \right] + 0.27 \cdot F \cdot \sqrt{f_c'} \right\} \quad \text{(in MPa)} \tag{41}$$

Table 1(a) Experimental data of fibrous concrete beams without stirrups

Ref.	Geometry of beams			Steel characteristics		Fiber properties			Strength values		
	d (mm)	b (mm)	a/d	ρ_l (%)	f_y (MPa)	V_f (%)	L_f/D_f	Shape	f_c' (MPa)	f_r (MPa)	v_u (MPa)
Al- Ta'an and Al-Feel (1990)	197	150	2.80	1.34	-	0.75	60	hooked	20.60	-	1.52
"	197	150	2.80	2.00	-	0.75	60	"	20.60	-	2.03
"	197	150	2.00	1.34	-	0.50	60	"	29.10	-	2.54
"	197	150	2.80	1.34	-	0.50	60	"	29.10	-	1.78
"	197	150	3.60	1.34	-	0.50	60	"	29.10	-	1.52
"	135	75	2.50	1.55	-	0.75	83	"	29.20	-	2.72
"	197	150	2.00	1.34	-	0.75	60	"	29.90	-	2.88
"	197	150	2.80	1.34	-	0.75	60	"	29.90	-	2.03
"	197	150	2.80	2.00	-	0.75	60	"	29.90	-	2.20
"	135	75	2.50	1.55	-	0.75	63	"	30.60	-	2.37
"	135	75	2.50	1.55	-	0.75	100	"	31.20	-	2.70
"	135	75	2.50	1.55	-	0.75	50	"	31.40	-	2.15
"	127	102	4.80	3.10	-	0.22	19	Crimped	33.20	-	1.70
"	127	102	4.80	3.10	-	0.22	53	"	33.20	-	1.70
"	127	102	4.80	3.10	-	0.22	100	straingth	33.20	-	1.65
"	127	102	4.60	3.10	-	0.22	100	"	33.20	-	1.95
"	127	102	4.20	3.10	-	0.22	100	"	33.20	-	1.74
"	127	102	4.30	3.10	-	0.22	100	"	33.20	-	1.78
"	127	102	4.60	3.10	-	0.22	46	crimped	33.20	-	1.71
"	127	102	4.00	3.10	-	0.22	46	"	33.20	-	1.92
"	127	102	4.60	3.10	-	0.22	46	"	33.20	-	1.60
"	127	102	4.40	3.10	-	0.22	46	"	33.20	-	1.63
"	127	102	5.00	3.10	-	0.22	46	"	33.20	-	1.51
"	127	102	4.80	3.10	-	0.22	46	"	33.20	-	1.66
"	127	102	4.80	3.10	-	0.22	46	"	33.20	-	1.52
"	197	150	2.80	2.00	-	0.75	60	hooked	33.40	-	2.91
"	127	102	3.40	3.10	-	0.88	46	crimped	39.70	-	2.36
"	127	102	2.80	3.10	-	1.76	46	"	39.80	-	3.47
"	127	102	1.80	3.10	-	1.76	46	"	39.80	-	4.76
"	127	102	1.20	3.10	-	1.76	46	"	39.80	-	8.80
"	130	85	2.00	2.00	-	0.25	100	"	39.90	-	2.71

Table 1(b) Experimental data of fibrous concrete beams without stirrups

Ref.	Geometry of beams			Steel characteristics		Fiber properties			Strength values		
	d (mm)	b (mm)	a/d	ρ_l (%)	f_y (MPa)	V_f (%)	L_f/D_f	Shape	f_c' (MPa)	f_r (MPa)	v_u (MPa)
Al- Ta'an and Al-Feel (1990)	130	85	3.00	2.00	-	0.25	100	crimped	39.90	-	1.94
“	127	102	4.20	3.10	-	0.44	100	straight	40.20	-	2.04
“	127	102	4.00	3.10	-	0.44	100	“	40.20	-	2.01
“	127	102	4.00	3.10	-	0.44	46	crimped	40.20	-	1.94
“	127	102	4.20	3.10	-	0.44	46	“	40.20	-	1.98
“	127	102	3.20	3.10	-	0.44	46	“	40.20	-	2.27
“	127	102	3.40	3.10	-	0.44	46	“	40.20	-	2.12
“	130	85	3.00	2.00	-	1.00	100	“	41.40	-	2.97
“	130	85	3.00	2.00	-	0.50	133	“	42.30	-	1.97
“	128	85	3.00	3.69	-	0.50	133	“	42.30	-	2.24
“	126	85	3.10	5.72	-	0.50	133	“	42.30	-	2.33
“	130	100	2.00	1.16	-	0.75	53	“	48.00	-	2.39
“	182	100	1.50	2.20	-	0.75	53	“	48.00	-	4.55
“	182	100	2.00	2.20	-	0.75	53	“	48.00	-	3.15
“	182	100	2.50	2.20	-	0.75	53	“	48.00	-	2.53
“	182	100	3.00	2.20	-	0.75	53	“	48.00	-	2.30
“	182	100	3.50	2.20	-	0.75	53	“	48.00	-	2.02
“	280	100	2.00	2.00	-	0.75	53	“	48.00	-	2.45
Balàza and Kovacs (2000)	130	100	4.62	3.09	-	1.00	60	hooked	33.20	-	2.33
“	130	100	4.62	3.09	-	0.50	60	“	33.50	-	1.93
“	130	100	4.62	3.09	-	1.00	60	crimped	36.30	-	2.98
“	130	100	4.62	3.09	-	0.50	60	“	37.50	-	2.24
Furlan and Hanai (1997)	80	100	3.75	1.77	-	0.50	840	straight	48.00	3.45	3.50
“	80	100	3.75	1.77	-	1.00	100	crimped	49.30	3.85	3.70
Mansur *et al.* (1986)	200	150	2.80	0.79	-	0.75	60	hooked	20.60	2.17	1.04
“	197	150	2.80	1.34	-	0.75	60	“	20.60	2.17	1.33
“	197	150	2.80	2.00	-	0.75	60	“	20.60	2.17	1.78
“	197	150	2.00	1.34	-	0.50	60	“	29.10	2.67	2.22

Table 1(c) Experimental data of fibrous concrete beams without stirrups

Ref.	d (mm)	b (mm)	a/d	ρ_l (%)	f_y (MPa)	V_f (%)	L_f/D_f	Tipo fibre	f'_c (MPa)	f_r (MPa)	v_u (MPa)
Mansur *et al.* (1986)	197	150	2.80	1.34	-	0.50	60	hooked	29.10	2.67	1.56
"	197	150	3.60	1.34	-	0.50	60	"	29.10	2.67	1.33
"	197	150	4.40	1.34	-	0.50	60	"	29.10	2.67	1.13
"	197	150	2.00	1.34	-	0.75	60	"	29.90	2.88	2.52
"	197	150	2.80	1.34	-	0.75	60	"	29.90	2.88	1.78
"	197	150	3.60	1.34	-	0.75	60	"	29.90	2.88	1.41
"	197	150	4.40	1.34	-	0.75	60	"	29.90	2.88	1.21
"	200	150	2.80	0.79	-	0.75	60	"	29.90	2.88	1.11
"	197	150	2.80	2.00	-	0.75	60	"	29.90	2.88	1.93
"	197	150	2.00	1.34	-	1.00	60	"	30.00	3.36	2.76
"	197	150	2.80	1.34	-	1.00	60	"	30.00	3.36	1.93
"	197	150	3.60	1.34	-	1.00	60	"	30.00	3.36	1.50
"	197	150	4.40	1.34	-	1.00	60	"	30.00	3.36	1.30
"	200	150	2.80	0.79	-	0.75	60	"	33.40	3.63	1.39
"	197	150	2.80	1.34	-	0.75	60	"	33.40	3.63	2.22
"	197	150	2.80	2.00	-	0.75	60	"	33.40	3.63	2.55
Narayanan and Darwish (1987)	130	85	2.00	2.00	530	0.25	100	crimped	33.12	3.03	2.71
"	130	85	2.50	2.00	530	0.25	100	"	33.12	3.03	2.07
"	130	85	3.00	2.00	530	0.25	100	"	33.12	3.03	1.94
"	130	85	3.00	2.00	530	1.00	100	"	34.36	3.73	2.97
"	130	85	3.00	2.00	530	0.50	133	"	35.11	3.27	1.97
"	128	85	3.00	3.69	530	0.50	133	"	35.11	3.27	2.24
"	126	85	3.10	5.72	530	0.50	133	"	35.11	3.27	2.33
"	130	85	3.50	2.00	530	0.50	133	"	46.23	4.46	2.61
"	128	85	3.00	3.69	530	0.50	133	"	46.23	4.46	2.96
"	126	85	3.10	5.72	530	0.50	133	"	46.23	4.46	3.55
"	126	85	3.10	5.72	530	2.00	100	"	46.40	5.17	4.93
"	126	85	2.00	5.72	530	2.00	100	"	46.40	5.17	6.30
"	126	85	2.00	5.72	530	0.50	100	"	49.22	3.82	5.46
"	126	85	2.00	5.72	530	1.00	100	"	49.80	4.63	6.77
Sharma (1986)	263	150	1.90	1.53	-	0.90	83	hooked	48.60	6.50	3.03

Table 1(d) Experimental data of fibrous concrete beams without stirrups

Ref.	d (mm)	b (mm)	a/d	ρ_l (%)	ρ_l (%)	f_y (MPa)	V_f (%)	L_f/D_f	Tipo fibre	f_c' (MPa)	f_r (MPa)
Swamy (1986)	210	175	4.50	1.95	460	0.80	100	crimped	36.44	9.10	1.73
“	210	175	4.50	4.00	460	0.40	100	“	36.85	6.30	1.82
“	210	175	4.50	4.00	460	0.80	100	“	38.84	8.67	2.61
“	210	175	4.50	3.05	460	0.80	100	“	39.59	7.73	2.70
“	210	175	4.50	4.00	460	1.20	100	“	41.33	9.47	2.63
“	210	175	4.50	1.95	460	0.80	100	“	43.41	8.27	2.20
Swamy and Bahia (1979)	210	175	4.00	4.00	470	0.40	100	“	40.00	5.60	0.62
“	210	175	4.00	4.00	470	0.80	100	“	40.00	7.07	0.75
“	210	175	4.00	4.00	470	1.20	100	“	40.00	8.90	1.04
Swamy *et al.* (1993)	265	115	4.91	1.55	460	1.00	100	“	33.70	-	0.86
“	265	115	3.43	2.76	460	1.00	100	“	34.36	-	1.32
“	265	115	3.43	1.55	460	1.00	100	“	35.77	-	1.20
“	265	115	2.00	4.31	460	1.00	100	“	36.90	-	2.33
“	265	115	2.00	1.55	460	1.00	100	“	37.00	-	1.97
“	265	115	4.91	2.76	460	1.00	100	“	37.26	-	1.24
“	265	115	4.91	4.31	460	1.00	100	“	37.35	-	1.23
“	265	115	2.00	2.76	460	1.00	100	“	39.17	-	2.08
“	265	115	3.43	4.31	460	1.00	100	“	42.41	-	1.71

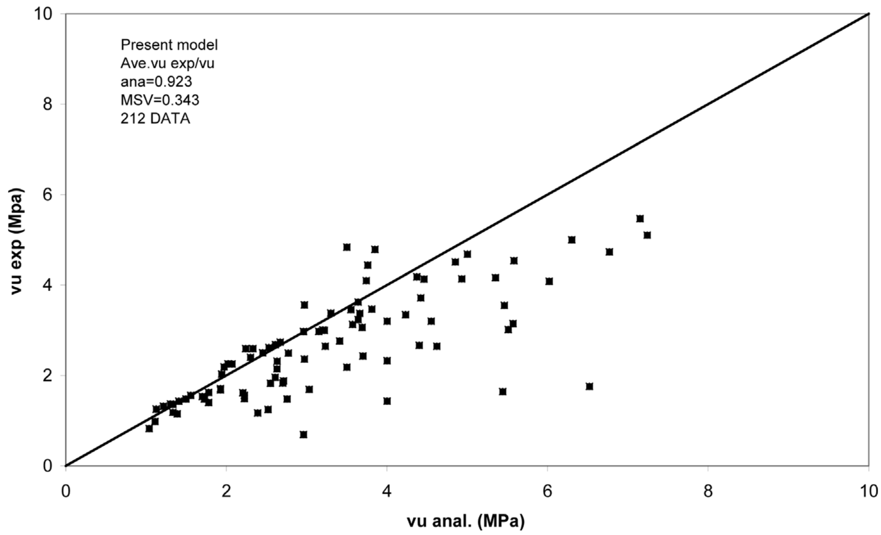

Fig. 7 Comparison between analytical and experimental results for fibrous concrete beam

Table 2 Analytical expressions for shear prediction of fibrous concrete beams

Ref.	Equation	Mean vu_{exp}/vu_{ans}	MSV
Sharma (1986)	$v_f = 0.66 \cdot f'_{sp} \cdot \left(\frac{d}{a}\right)^{0.25}$ f'_{sp} = split cylindrical strength	0.936	0.417
Mansur *et al.* (1986)	$v_u = 0.16 \cdot \sqrt{f'_c} + 17.2 \cdot \rho \cdot \frac{d}{a} + 0.41 \cdot \tau \cdot F$	1.153	0.512
Narayanan and Darwish (1987)	$v_u = \varepsilon \cdot \left[0.24 \cdot f'_{sp} + 80 \cdot \rho \cdot \frac{d}{a}\right] + 0.41 \cdot \tau \cdot F$ $\varepsilon = 1$ for $a/d > 2.8$; elsewhere $\varepsilon = 2.8\ a/d$	0.842	0.333
Al Ta'an and Al Feel (1990)	$v_u = \left(0.17 \cdot \sqrt{f'_c} + 106 \cdot \rho \cdot \frac{d}{a}\right) \cdot \varepsilon + 0.94 \cdot 1.2 \cdot F$ $\varepsilon = 1\ a/d > 2.5$; $\varepsilon = 2.5d/a$ for $a/d < 2.5$	1.040	0.389
Ashour *et al.* (1992)	$v_u = (0.7 \cdot \sqrt{f'_c} + 7 \cdot F) \cdot \frac{d}{a} + 17.2 \cdot \rho \cdot \frac{d}{a}$	0.853	0.324
Swamy *et al.* (1993)	$v_u = v_c + 0.34 \cdot \tau \cdot F$ With v_c strength contribution due to concrete	1.30	0.560
Shin *et al.* (1994)	$v_u = 0.22 \cdot f'_{sp} + 217 \cdot \rho \cdot \frac{d}{a} + 0.34 \cdot \tau \cdot F$ for $a/d < 3$ $v_u = 0.19 \cdot f'_{sp} + 93 \cdot \rho \cdot \frac{d}{a} + 0.34 \cdot \tau \cdot F$ for $a/d \geq 3$	0.809	0.543

Table 2 Continued

Ref.	Equation	Mean vu_{exp}/vu_{ans}	MSV
Imam *et al.* (1997)	$v_u = 0.6 \cdot \dfrac{1+\sqrt{\dfrac{5.08}{d_a}}}{\sqrt{1+\dfrac{d}{25 \cdot d_a}}} \cdot \sqrt[3]{\rho \cdot (1+4 \cdot F)} \cdot \left[(f_c')^{0.5} + 275 \cdot \sqrt{\dfrac{\rho \cdot (1+4 \cdot F)}{(a/d)^5}}\right]$	1.060	0.378
Khuntia *et al.* (1999)	$v_u = (0.167 \cdot \alpha + 0.25 \cdot F) \cdot \sqrt{f_c'}$ $\alpha = 2.5\ d/a < 3$ per $a/d < 2.5$	1.370	0.543
Kim *et al.* (2002)	$v_u = 3.7 \cdot \varepsilon \cdot (f_{sp}')^{2/3} \cdot \left(\rho \cdot \dfrac{d}{a}\right)^{1/3} + 0.8 \cdot (0.41 \cdot \tau \cdot F)$ $\varepsilon = 1.0$ for $a/d > 3.4$; $\varepsilon = 3.4d/a$ for $a/d < 3.4$	0.911	0.259

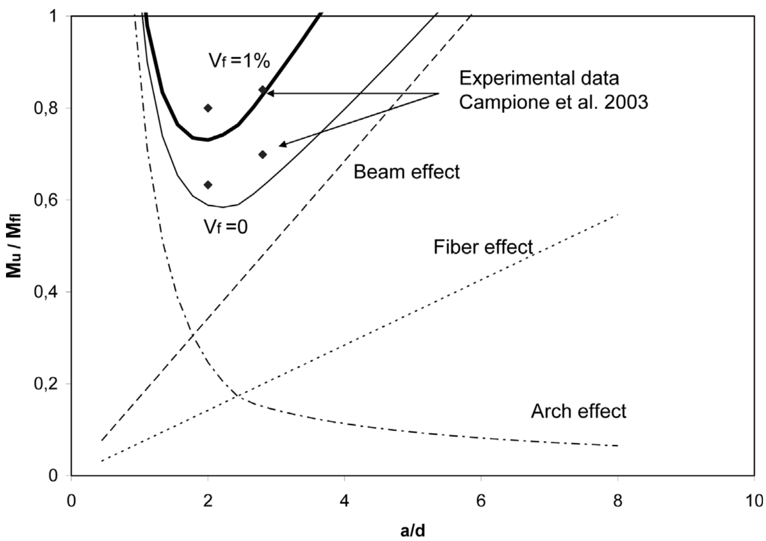

Fig. 8 Relative flexural capacity for fibrous concrete beam

Fig. 7 shows a comparison between the predicted values, using the presented model, and the experimental results given in literature.

In Fig. 7, the ordinate presents the experimental value and the abscissa, the analytical data.

Some of the existing analytical expressions given in the literature are shown in Table 2, using the symbols introduced by the authors. These expressions were also verified against the experimental data (shown in Fig. 7). Table 2 also presents the average value between experimental and analytical values. MSVs (mean square values) are given to show the performance of expressions in predicting the experimental values.

In Table 2, the symbol f_{sp}' denotes the splitting tensile strength measured by means of split cylinders.

An interesting graphical interpretation of Eq. (41) is shown in Fig. 8, referring to the Kani valley (1968). The ordinate of Fig. 8 represents the relative flexural capacity (the ratio between the ultimate moment ($M_u = v_u \cdot b \cdot d \cdot a$) and the complete flexural capacity M_{fl}, obtained by Eq. (13). Meanwhile, the abscissa shows the ratio *a*/*d*. In the graph, the shear contributions made by beam, arch and fiber actions are represented. The graph was obtained by using the data referring to the experimental investigation carried out by the authors (Campione *et al.* 2003). The concrete used to make the beams had compressive strength $f_c' = 41.2$ MPa, while the steel of the longitudinal bars was constituted by deformed bars with $f_y = 610$ MPa and geometrical ratio $\rho = 1.9\%$. In the same graph, experimental values referring to the research of Campione *et al.* (2003) are given, showing the capability of the proposed model to predict the experimental values.

It is interesting to observe that the presence of fibers produced a significant increase in the shear strength for different *a*/*d* values with a consequent narrowing of the Kani valley.

4. The effect of stirrups and fibers

Several studies show that it is possible to obtain the shear strength of reinforced concrete beams with stirrups by summing the contributions made by beam and arch actions with the strength contribution made by the stirrups bridging the principal crack. The interaction between the single mechanisms should not be considered. Moreover, the inclination of the principal cracks is assumed to be 45°. On this basis, ACI 318-02 (2002) and Eurocode 2 (1989) suggest using the following expressions:

$$v_u = 0.16 \cdot \sqrt{f_c'} + 17.2 \cdot \rho \cdot \frac{d}{a} + \rho_{sw} \cdot f_{yw} \quad \text{(in MPa) (ACI 318-02 2002)} \qquad (42)$$

$$v_u = v_c + \rho_{sw} \cdot f_{yw} \quad \text{(Eurocode 2 1989)} \qquad (43)$$

in which v_c in Eq. (43) is the shear strength contribution in the absence of stirrups and $\rho_{sw} f_{yw}$ is the shear contribution made by the stirrups, which are assumed to have yielded. The term ρ_{sw} is the

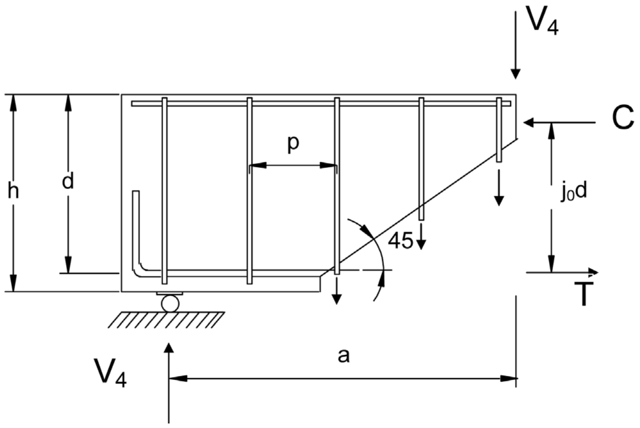

Fig. 9 Strength mechanisms due to stirrups across the principal crack

geometrical ratio of the transverse stirrups in spacing p ($\rho_{sw} = 2A_{st}/(bp)$) and f_{yw} is the yielding stress of the stirrups.

Russo and Puleri (1997) have shown that the stirrups do not always yield at beam rupture and the effective stress can be estimated if the contributions of beam action to the sum of beam and arch action are known. In this way they introduce an effectiveness coefficient that is capable of determining the share of yielding stress in the stirrups at beam rupture.

Fig. 9 shows the effect of the stirrups on the shear resistance mechanism at rupture, which is assumed to be characterized by a principal crack with angle 45°.

The effect of fibers may be included in the stirrup effectiveness function originally proposed by Russo and Puleri (1997), so as to determine the shear strength contribution made by stirrups. This is expressed as follows:

$$v_4 = \Phi_f \cdot \rho_{sw} \cdot f_{yw} \tag{44}$$

in which the yielding stress in the stirrups is reduced by Φ_f, determined as follows.

The influence of the stirrups on the strength of the beams depends on the influence of beam actions (also including the effect of fibers) on the whole strength contribution of the beam. This influence is estimated, according to Russo and Puleri (1997), by means of the index I_b expressed by $I_b = M_1/M_u$, M_1 and M_u being the moment contribution made by the beam effect and the ultimate moment.

In particular M_1 is the moment calculated by multiplying v_1 by bd and by a (a four point bending test), while M_u is obtained by multiplying v_u by bd and by a.

In the case of fibrous concrete beams, if in the calculus of M_1 and M_u we take the effect of fibers into account by means of Eq. (10), where $A = 1.3$, and Eq. (41) we obtain:

$$I_b = \frac{1.3 \cdot j_0 \cdot \rho^{1/2} \cdot \sqrt{f_c'}}{j_0 \cdot \left[1.3 \cdot \rho^{1/2} \cdot \sqrt{f_c'} + 0.3 \cdot \varepsilon \cdot f_y \cdot \rho_t \cdot \left(\frac{d}{a}\right)^{1.8}\right] + 0.27 \cdot F \cdot \sqrt{f_c'}} \tag{45}$$

To determine the Φ_f function, the same hypothesis assumed by Russo and Puleri (1997) is adopted, namely that I_b, and Φ_f are related by a linear relationship in the form:

$$\Phi_f = kI_b + q \tag{46}$$

in which k and q are two constants to be determined.

To determine the q value, it is necessary to analyze the case in which the whole load is supported by an arch mechanism, giving $V_1 = M_1 = 0$ and therefore $I_b = 0$, as in Russo and Puleri (1997). Thus, $q = 0$.

If shear compression failure occurs in the case of ordinary concrete beams, a value $I_b < 0.6$ is observed by Russo and Puleri (1997) and the working stress in the stirrup is lower than the yielding value. Therefore $\Phi_f < 1$. In contrast, if beam behavior governs the shear strength and failure is due to diagonal tension $I_b > 0.6$ (defined by Russo and Puleri 1997), the stirrups yield completely. Because of the continuity between I_b and Φ_f in the presence or absence of fibers, $\Phi_f = 1$ for $I_b = 0.6$ and therefore $k = 1.67$, as already shown in Russo and Puleri (1997). Finally $\Phi_f = 1.67I_b$.

From the considerations made, it follows that:

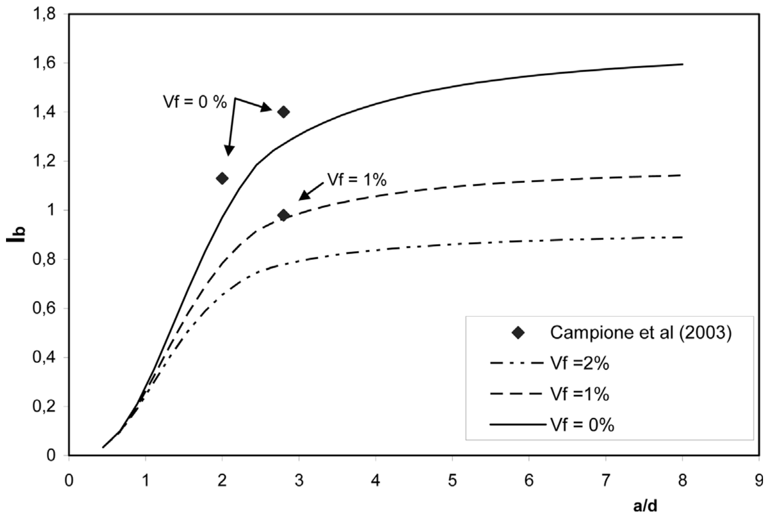

Fig. 10 Variation in the I_b with *a/d* for plain and fibrous concrete beams

$$\Phi_f = \frac{2.17 \cdot j_0 \cdot \rho^{1/2} \cdot \sqrt{f_c'}}{j_0 \cdot \left[1.3 \cdot \rho^{1/2} \cdot \sqrt{f_c'} + 0.3 \cdot \varepsilon \cdot f_y \cdot \rho_t \cdot \left(\dfrac{d}{a}\right)^{1.8}\right] + 0.27 \cdot F \cdot \sqrt{f_c'}} \tag{47}$$

Fig. 10 shows the variation of I_b with a/d for a fixed value of ρ (1.9%) and for three different volume percentages of hooked steel fibers (V_f = 0%, V_f = 1% and V_f = 2%) having an aspect ratio of 60 (this is the case for the investigation carried out by Campione *et al.* 2003). In the same graph we can also find values of I_b determined using Φ_f values obtained experimentally (as the ratio between the measured strain in the stirrups and the yielding value). They show good agreement. It should be observed that in measuring strains in stirrups the strain measured is often not the highest strain, unless it is right at the location of the cracks.

For this reason the experimental strain value referred to in Fig. 10 is a mean value measured at the mid point of legs in the stirrups located at different positions in the shear span and in particular in the stirrups that are nearest to and the furthest from the loaded section.

Finally, by using Eq. (41) and Eq. (44) it is possible to obtain the expression of the shear strength in the presence of stirrups:

$$v_u = \xi \cdot \left\{ j_0 \cdot \left[1.30 \cdot \rho^{1/2} \cdot \sqrt{f_c'} + 0.30 \cdot \varepsilon \cdot f_y \cdot \rho_t \cdot \left(\frac{d}{a}\right)^{1.8}\right] + 0.27 \cdot F \cdot \sqrt{f_c'} \right\} + \Phi_f \cdot \rho_{sw} \cdot f_{yw} \quad \text{(in MPa)} \tag{48}$$

This expression was supported using 27 experimental data found in literature dealing with the main characteristics of the tested beams (as shown in Table 3).

A comparison between the above mentioned experimental results and analytical predictions is shown in Fig. 11 with the same symbols of Fig. 8. The same expression was also verified for stirrups without fibers (see the aforementioned data in Russo and Puleri 1997).

Table 4 shows some of the available analytical expressions that are capable of calculating the shear strength of reinforced concrete beams in the presence of fibers and stirrups. In Table 4 the average value of experimental and analytical values and MSVs (mean square values) is given showing the aptitude of the expressions for predicting the experimental values.

Table 3 Experimental data of fibrous concrete beams with stirrups

Ref.	Geometry of beams				Steel characteristics						Fiber characteristis			Strength values		
	h	d	b	a/d	A_f	D	f_y	ϕ_{st}	p	f_{yst}	L_f	ϕ	V_f	f_c'	f_t'	v_u
Balazs and Kovacs (2000)	150	132	100	4.5	401.92	16	450	6	240	250	30	0.5	0	48	3.17	1.71
"	150	132	100	4,5	401.92	16	450	6	240	250	30	0.5	0.5	49	3.24	2.55
"	150	132	100	4.5	401.92	16	450	6	240	250	30	0.5	1	48	3.21	3.18
"	150	132	100	4.5	401.92	16	450	6	120	250	30	0.5	0	48	3.17	2.48
"	150	132	100	4.5	401.92	16	450	6	120	250	30	0.5	0.5	49	3.24	3.14
"	150	132	100	4.5	401.92	16	450	6	120	250	30	0.5	1	48	3.21	3.76
Kearsley and Moster (2004)	230	214	150	2.8	339.12	12	450	8	0	250	25	0.5	0	49	5.25	1.30
"	230	214	150	2.8	339.12	12	450	8	250	250	25	0.5	0	49	5.25	1.48
"	230	214	150	2.8	339.12	12	450	8	125	250	25	0.5	0	49	5.25	2.08
"	230	214	150	2.8	339.12	12	450	8	250	250	25	0.5	0.5	49	5.25	1.98
"	230	214	150	2.8	339.12	12	450	8	250	250	25	0.5	1	49	5.25	2.50
"	230	214	150	2.8	339.12	12	450	8	250	250	25	0.5	1.2	49	5.25	2.70
Campione and Mindess (1999)	125	105	100	2.2	628	20	300	6.35	180	550	30	0.5	2	70	4.66	8.09
"	125	105	100	2.2	628	20	300	6.35	98	550	30	0.5	2	70	4.66	9.25
"	125	105	100	2.2	628	20	300	6.35	180	550	30	0.5	0	70	4.66	4.85
"	125	105	100	2.2	628	20	300	6.35	98	550	30	0.5	0	70	4.66	6.05
Dupont and Vandewalle (2003)	300	260	200	3.5	1846.32	28	400	4	180	560	30	0.5	0	49	3.26	1.84
"	300	260	200	3.5	1846.32	28	400	4	90	560	30	0.5	0	50	3.33	2.22
"	300	262	200	2.5	602.88	16	400	6	120	561	30	0.5	0	40	2.64	2.03
"	300	260	200	2.5	602.88	16	400	6	120	562	30	0.5	0	40	2.64	2.05
"	350	305	200	2.5	628	20	400	10	250	600	0	0.5	0	36	2.39	2.43
"	350	305	200	2.5	628	20	400	10	250	600	65	0.5	0.38	35	2.29	3.27
"	350	305	200	2.5	628	20	400	10	250	600	65	0.5	0.58	35	2.29	3.72
"	350	305	200	2.5	628	20	400	10	250	600	80	0.5	0.38	29	1.95	3.30
"	350	305	200	2.2	628	20	400	10	250	600	80	0.5	0.58	31	2.08	3.90
"	350	305	200	3.0	628	20	400	8	200	600	0	0.5	0	34	2.29	1.85

Data in mm, MPa (all fibers are hooked steel ones)

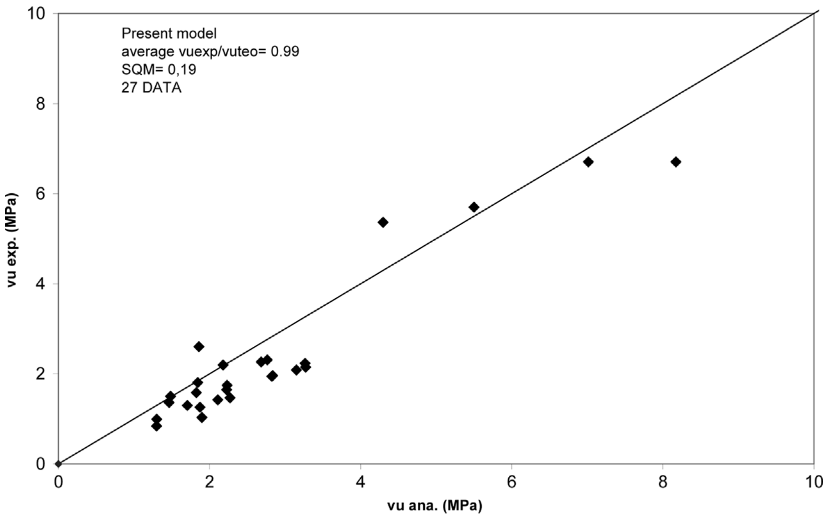

Fig. 11 Comparison between analytical and experimental results for fibrous concrete beams with stirrups

Table 4 Analytical expressions for shear prediction of fibrous concrete beams with stirrups

Ref.	Equation	Mean V_{exp}/V_{teo}	MSV
Sharma (1986)	$v = 0.66 \cdot f'_{sp} \cdot \left(\frac{d}{a}\right)^{0.25} + \rho_{sw} \cdot f_{yw}$	0.968	0.303
Imam *et al.* (1997)	$v_u = 0.6 \cdot \frac{1 + \sqrt{\frac{5.08}{d_a}}}{\sqrt{1 + \frac{d}{25 \cdot d_a}}} \cdot \sqrt[3]{\rho \cdot (1 + 4 \cdot F)} \cdot \left[(f'_c)^{0.44} + 275 \sqrt{\frac{\rho \cdot (1 + 4 \cdot F)}{(a/d)^5}}\right] + \rho_{sw} \cdot f_{yw}$	0.867	0.108
Tompos and Frosch (2002)	$v_{st} = A_v \cdot f_y \cdot INT\left(\frac{d - l_{dv}}{s}\right)$ with $l_{dv} = \frac{1}{1.25} \cdot \frac{1200 \cdot d_b}{\sqrt{f'_c}} \cdot \frac{f_y}{60000}$ (in psi)	/	/
Cho and Kim (2003)	$v_u = 1.05 \cdot \left(1 - 0.35 \cdot \frac{a}{h}\right) \cdot f'_{sp} + 225 \cdot \Sigma A_{st} \cdot \frac{d}{h} \cdot \sin^2\beta \cdot \frac{1}{b \cdot h}$ (in MPa) and β the angle of inclined strut (generally assumed 45°)	/	/
Dupont and Vandewalle (2003)	$v_u = 0.12 \cdot \left(1 + \sqrt{\frac{200}{d}}\right) \cdot (100 \cdot \rho \cdot f_{ck})^{1/3} + \left(\frac{1600 - d}{1000}\right) \cdot \tau_{fd} + 0.9 \cdot \rho_{sw} \cdot f_{yw}$ $\tau_{fd} = \frac{\frac{d}{a} \cdot 0.5 \cdot \frac{f_{eq,3}}{0.7}}{\gamma_c}$ (see Dupont and Vandewalle 2003)	/	/
CNR-DT 204 (2006)	$v_u = \frac{0.18}{\gamma_c} \cdot \left(1 + \sqrt{\frac{200}{d}}\right) \cdot \left[100 \cdot \rho \cdot \left(1 + 7.5 \cdot \frac{f_{Ftuk}}{f_{ctk}}\right) \cdot f_{ck}\right]^{1/3} + \rho_{sw} \cdot f_{yw}$ (see CNR-DT 204-2006)	/	/

v_{st} shear contribution due to stirrup

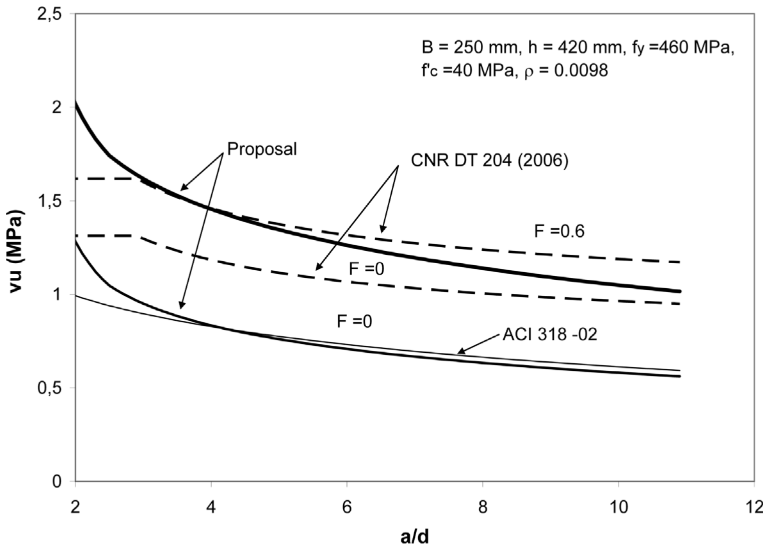

Fig. 12 Comparison between analytical expressions

It is interesting to observe that only the expression proposed by Dupont and Vandewlle (2003) considers a reduction of yielding stress in the stirrups in the presence of fibers.

Table 4 also gives some recent analytical expressions that are capable of calculating the shear strength of beams with stirrups but without fibers, including those of Tompos and Frosch (2002) and Cho and Kim (2003). These expressions are also of significant interest because they could lead to further developments in the field of fibrous concrete beams. The expression given by Tompos and Frosch (2002) mentions a case of failure in shear caused by the splitting of longitudinal bars. That of Cho and Kim (2003) considers the possibility of adopting a variable angle in the direction of the principal cracks at rupture. Finally, Table 4 also includes the analytical expression recently proposed by CNR-DT204 (2006), developed on the basis of RILEM TC 162-TF (this latter was itself based on the study of Dupon and Vandewalle 2003). Fig. 12 shows a comparison between the analytical expression proposed in the present paper (which has a similar structure to the ACI equation for shear strength) and the one proposed by CNR-DT 204 (2006). Also analytical predictions of shear strength according to the ACI 318-02 (2002) equation is given for a comparison of reinforced concrete beams. It has to be observed that in the equation derived by CNR-DT 204 (2006) we adopt for residual tensile strength and for comparison with Eq. (48) the expressions of f_r derived in the present paper (see Eq. (17)).

The comparison regarding shear strength versus *a*/*d* ratio refers to beams with and without fibers and shows good agreement despite the different origins of the two equations.

5. Experimental validation and comparison with analytical results

To verify the proposed analytical model, the experimental results obtained by the aforementioned authors (Campione *et al.* 2003, Cucchiara *et al.* 2004) regarding the flexural behavior of fibrous concrete beams in the presence of stirrups are utilized and briefly shown. The research refers to

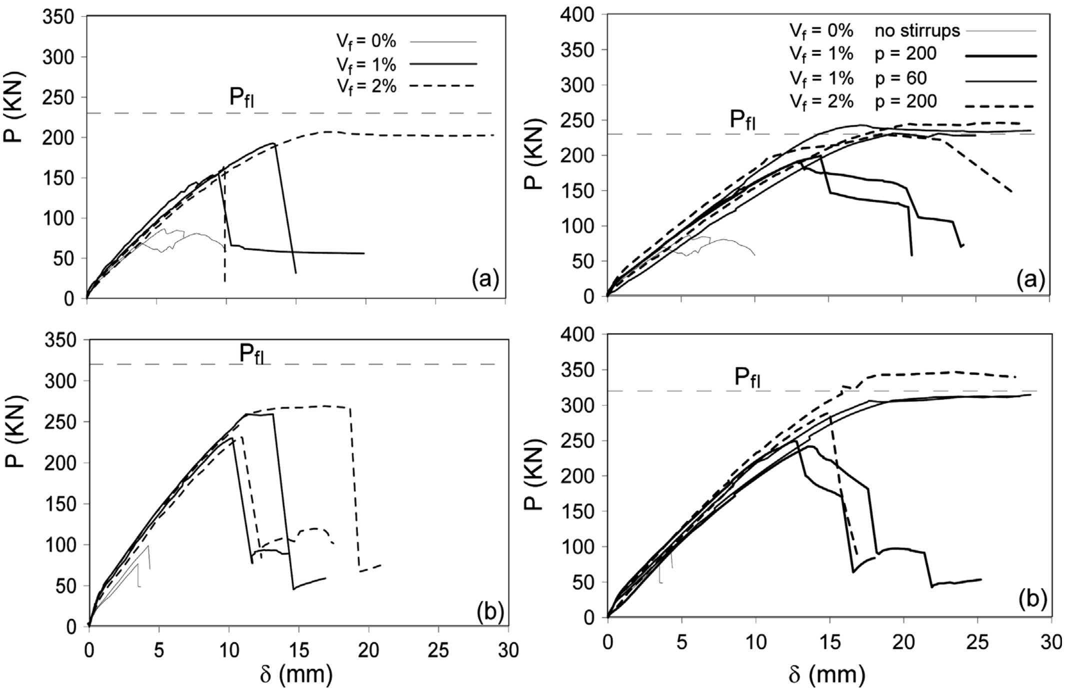

Fig. 13 Load-deflection curves for fibrous concrete beams with and without stirrups: (a) *a*/*d* = 2.8, (b) *a*/*d* = 2, (Campione *et al.* 2003)

four-point bending tests carried out on medium size beams that were 2500 mm long and which had a transversal cross-section depth and height of 150 mm and 250 mm respectively. The beams were reinforced with two deformed longitudinal bars, with a diameter of 20 mm, placed at the bottom portion (bent at the supports) and two deformed 10 mm bars placed at the top. These latter were able to sustain stirrups with a diameter of 6 mm and were placed at a pitch of 200 or 60 mm. The concrete used had a compressive cylindrical strength of 41.20 MPa at 28 days while the steel bars, with diameters of 20 mm and 6 mm respectively, had a yielding stress of 610 and 510 MPa respectively. The maximum aggregate size was 10 mm. In the case of fibrous concrete, hooked steel fibers, 30 mm long with a diameter of 0.5 mm, were added to fresh concrete in the volume percentages of 1 and 2%. The tests were carried out in controlled displacement. Two different *a*/*d* ratios equal to 2 and 2.8 were adopted respectively.

During the tests, strain gauges were installed on the legs of the stirrups to record their deformations.

Fig. 13 shows the load-deflection curves for some of the beams tested with variations in fiber volume percentages and stirrup pitch for the two different *a*/*d* values examined. The graphs clearly show the role of the fibers in shear reinforcement: they produce significant increases in shear strength that transform the failure mode from shear to flexure. Moreover, the results show that it is possible, for identical strength levels, to substitute the stirrups with an adequate percentage of fibers. The graphs also give the ultimate load value, P_{fl} (which corresponds to flexural failure) that was calculated with reference to M_{fl} (see Eq. (13)). The load given in the graph is $P = 2V$.

Fig. 14 Shear failure of a beam with stirrups at pitch 60 mm and for $a/d = 2.0$

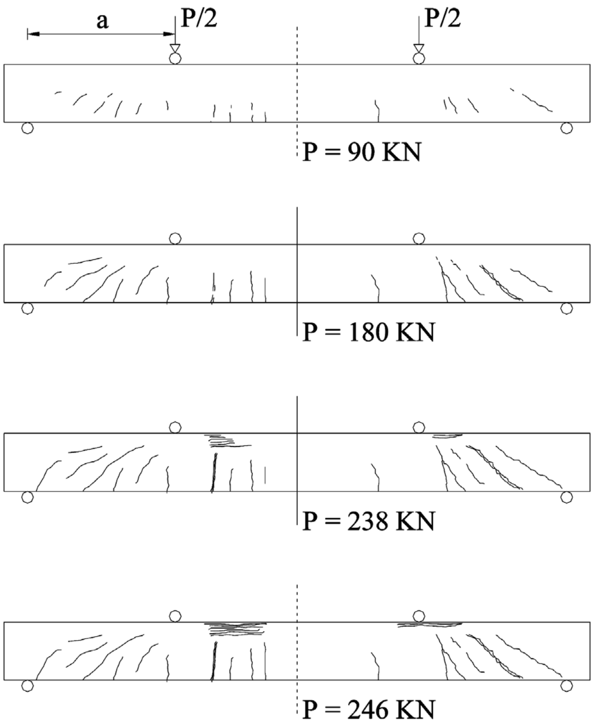

Fig. 15 Evolution of the cracks up to failure of a beam with V_f = 2%, p = 200 mm and a/d = 2.8 (Campione *et al*. 2003)

Fig. 14 shows the final condition for a beam with stirrups, and also highlights the marked inclination of the principal cracks (close to 45°) and the bridging action of the stirrups.

Fig. 15 shows the cracking pattern of a beam with stirrups at the end of the test. It shows clearly and in accordance with the proposed model that principal crack forms at rupture and its inclination is very closed to 45°.

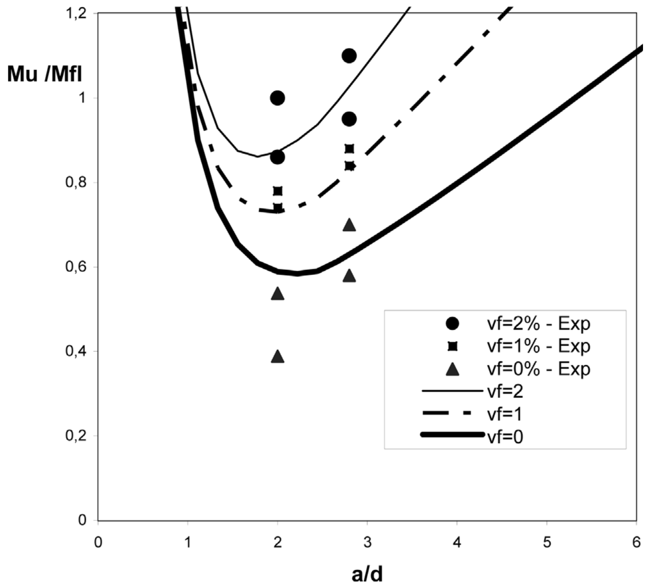

Fig. 16 Flexural relative capacity with the variation of *a/d* and $p = 200$ mm (Campione *et al.* 2003)

Fig. 16 shows the Kani valley, deduced using the proposed model for the case of $V_f = 0$, 1 and 2%, and for pitch 200 mm. It emerges that an increase in the volume percentage of fibers produces a significant increase in the shear strength for a wide range of *a/d.* The same graph also shows the experimental values expressed in terms of relative flexural capacity. The comparison demonstrates the good level of approximation and the capacity of the proposed model to take into account the interaction between the stirrups and the fibers in the beam and arch resistant mechanisms.

6. Conclusions

In the present paper, a semi-empirical analytical expression is proposed that has the aim of determining the maximum shear strength of reinforced concrete beams in the presence of fibers and stirrups.

The analytical expression is derived from a mechanical model based on the determination of strength contributions made by beam and arch effects. It also includes the effects of fibers and the effective stresses of the stirrups at rupture.

The analytical expression obtained for strength considers that:

- the presence of fibers increases the strength contribution made by beam actions, which are themselves related to the increase in the internal arm of the beam and caused by the residual strength in tension of the fibrous concrete. Moreover, better bond conditions are observed, thus increasing the shear strength;
- the presence of fibers produces a further strength contribution in the arch effect due to the horizontal component of residual strength. This is taken into account by means of a fictitious additional geometrical percentage of main bars that is capable of considering the effect of fibers;
- the strength contribution made by the fiber bridging the principal cracks produces an additional

strength with respect to the beam and arch effects. This is evaluated in the hypothesis of a 45^o inclination of the principal cracks and is related to the residual tensile strength of the concrete. The latter depends on the square root of the compressive cylindrical strength and on the fiber factor, which is able to take the geometry and percentage of fibers into account.

In the presence of fibers and stirrups and in the hypothesis of a resistance mechanism created by stirrups (including the principal cracks at an incline of 45^o), an effectiveness function is introduced that is capable of evaluating the effective stresses in the stirrups at rupture. This coefficient includes the effect of fibers and gives information about the effective influence of beam actions on the strength contribution made by beam and arch actions.

Finally, the model was calibrated on the basis of both experimental data given in literature and of results recently obtained by the authors in a recent experimental investigation. The result is a model that has demonstrated its ability to predict the shear strength of the beams with good approximation.

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Notation

A_b : total resisting area of fibers
A_s : area of longitudinal bars in tension
A_s' : area of longitudinal bars in compression
A_{st} : area of leg of transverse stirrups
b : base of the rectangular cross-section
c : cover thickness of bar in tension
d_a : maximum aggregate size of concrete
d : effective height of transverse cross-section of the beam
D_i : diameter of the i-th longitudinal bar in tension
e : depth of tensile zone in fibrous concrete beams
E_c : modulus of elasticity of concrete
E_{ct} : elastic modulus of elasticity of concrete in tension
F_b : resultant of resisting force exercised by the fibers across the principal cracks
f_c' : cylindrical compressive strength of concrete
f_{yw} : yielding stress of stirrups
f_{sp}' : split cylindrical strength of concrete
f_t : maximum tensile strength of concrete
f_r : residual tensile strength of fibrous concrete
f_y : yielding stress of longitudinal bar in tension
F : fiber factor
h : height of transverse cross-section of the beam
I_b : beam index
V_f : volume percentage of fibers
L_f : equivalent length of fiber
T : axial force in longitudinal bars
M : bending moment
n : total number of fibers across the principal cracks
n_w : number of fibers crossing a unit area
V : shear force
p : pitch of stirrups
q_b : bond stresses around the bar in tension
x_c : neutral axis depth

j_0 : dimensionless internal arm at the maximum bending moment section
j : dimensionless internal arm
β : shape factor of fiber
γ : angle of principal crack
ϕ : equivalent diameter of fiber
Φ_f : effectiveness function of fibers
σ_s : working stress in the stirrups
ξ : reduction factor to take into account size effect
τ : bond stress of single fiber embedded in concrete
ρ : geometrical ratio of longitudinal bars
ρ_f : fictitious geometrical ratio equivalent to fibers
ρ_{sw} : geometrical ratio of stirrups in the pitch