Structural Engineering and Mechanics, Vol. 24, No. 1 (2006) 51-62
DOI: http://dx.doi.org/10.12989/sem.2006.24.1.051

Analysis of free vibration of beam on elastic soil using differential transform method

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(Received November 11, 2005, Accepted April 28, 2006)

Abstract. Differential transform method (DTM) for free vibration analysis of both ends simply supported beam resting on elastic foundation is suggested. The fourth order partial differential equation for free vibration of the beam resting on elastic foundation subjected to bending moment, shear and axial compressive load is obtained by using Winkler hypothesis and small displacement theory. It is assumed that the material is linear-elastic, and that axial load and modulus of subgrade reaction to be constant. In the analysis, shear and axial load effects are considered. The frequency factors of the beam are calculated by using DTM due to the values of relative stiffness; the results are presented in graphs and tables.

Keywords: differential transformation method; partial differential equation; motion equation; free motion; elastic soil.

1. Introduction

Differential equations are widely used to describe continuous time physical problems. In most cases, these problems may be too complicated to solve analytically. Alternatively, the numerical methods can provide approximate solutions rather than the analytical solutions of problems. The equation of free vibration of the beam resting on elastic foundation subjected to bending moment shear and axial load is a forth order partial differential equation. Many researchers have solved the forth order partial differential equation of motion by using different methods in the past. Doyle and Pavlovic (1982) have solved motion equation of Euler beam partially resting on elastic foundation by using separation of variables. West and Mafi (1984) have obtained the eigenvalues for free vibration of column-beam systems on elastic soil using an initial-value numerical method. Çatal (2002) has obtained the free vibration circular frequencies of the piles partially embedded in the soil due to supporting conditions of top and bottom ends of the pile by using separation of variables. Chen and Ho (1996, 1999), using differential transform technique have proposed a method to solve eigenvalue problems for the free and transverse vibration problems of a rotating twisted Timeshenko beam under axial loading Özdemir and Kaya (2006), flapwise bending vibration of a rotating tapered cantilever Bernoulli-Euler beam has considered by using differential transform technique. Jang and Chen (1997), the differential transformation method has applied to solve a second order

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non-linear differential equation that describes the under damped and over damped motion of a system subject to external excitation. Chen and Liu (1998) have considered first order both the linear and non-linear two-point boundary value problems using the differential transform method, Jang *et al.* (2000) have interested in the first order linear and non-linear initial-value problems have solved by differential transformation method with fixed grid size. Köksal and Herdem (2002) have proposed for the analysis of the electrical circuits using differential Taylor transformation. Hassan (2002) has studied the solution of Sturm-Lioville eigenvalue problem by the help of differential transform method). Ayaz (2004) has obtained numerical solution of linear differential equations by using differential transform method. In this study, a new transformation called differential transform has introduced to solve the equation of motion of the beam on elastic soil. The concept of differential transform was first proposed by Zhou in 1986 and was applied to solve linear and non-linear initial value problems in electric circuit analysis (Zhou 1986).

2. Problem formulation

A beam resting on elastic foundation, internal forces and deformation of differential beam segment are presented in Fig. 1(a) and Fig. 1(b), respectively. It is assumed that the elastic soil that the beam is on behaves due to Winkler hypothesis. The relation between displacement function $y(x, t)$ of the beam on elastic soil and the distributed force $q(x, t)$ existing at the elastic soil under the beam can be written by $q(x, t) = Cs\ y(x, t)$. Where $Cs = C_0\ b$, C_0 is the modulus of subgrade reaction, b is beam width.

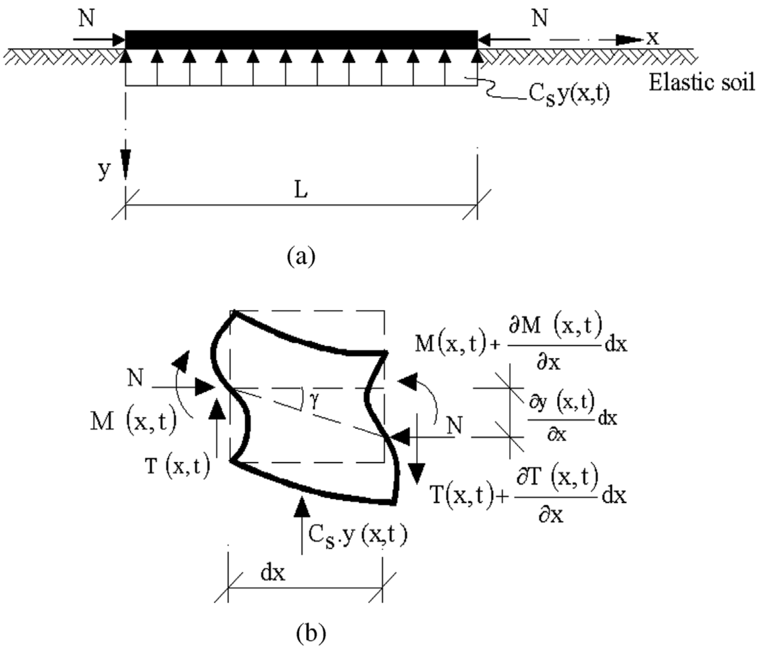


Fig. 1(a) A beam on elastic soil, (b) Internal forces and deformations of differential beam segment on elastic soil

The equation of lateral motion of the beam resting on an elastic foundation is written using the equilibrium equations of the internal forces acting on the differential beam segment (Fig. 1b) and neglecting the second order terms as follows (Çatal 2002)

$$\frac{\partial^4 y(x,t)}{\partial x^4} + \left[\frac{N}{EI} - \frac{\bar{k}C_s}{AG}\right]\frac{\partial^2 y(x,t)}{\partial x^2} - \frac{\bar{m}\bar{k}}{AG}\frac{\partial^4 y(x,t)}{\partial x^2 \partial t^2} + \frac{\bar{m}}{EI}\frac{\partial^2 y(x,t)}{\partial t^2} + \frac{C_s}{EI}y(x,t) = 0 \tag{1}$$

The lateral displacement function of the beam may be written for harmonic motion as follows:

$$Y(x,t) = Y(x) \cdot \sin(\omega t + \theta) \tag{2}$$

Where $Y(x)$ is the mod shape, ω is beam circular frequency, θ is phase angle, t is time variable.

If the position variable (x) and the mode shape $Y(x)$ are nondimensionalized by defining new variables $\phi(z) = Y(x)/L$, $z = x/L$, the equation of lateral motion of the beam is written as follows:

$$\left\{\phi^{1V}(z) + \left[\pi^2 N_r + \frac{(\bar{m}\omega^2 - Cs)\bar{k}L^2}{AG}\right]\phi^{11}(z) + \frac{(Cs - \bar{m}\omega^2)L^4}{EI}\phi(z)\right\}\sin(\omega t + \theta) = 0 \tag{3}$$

Where $N_r = NL^2/(\pi^2 EI)$ is ratio of compressive axial load N acting on the beam to Euler buckling load, L is beam length, $\phi^{11}(z) = \dfrac{d^2\phi_1(z)}{dz^2}$, $\phi^{IV}(z) = \dfrac{d^4\phi_1(z)}{dz^4}$

In the case that axial and shear force effects are neglected $\left[\dfrac{N}{EI} - \dfrac{\bar{k}C_s}{AG}\right]$ and $\dfrac{\bar{m}\bar{k}}{AG}$ terms are taken being zero (Tuma and Cheng 1983).

If both side of Eq. (3) is divided by Sin$(\omega t + \theta)$ following equations are obtained.

$$\phi^{1V}(z) + \left[\pi^2 N_r + \frac{(\bar{m}\omega^2 - Cs)\bar{k}L^2}{AG}\right]\phi^{11}(z) + \frac{(Cs - \bar{m}\omega^2)L^4}{EI}\phi(z) = 0 \tag{4}$$

3. Differential transformation

The differential transformation technique, which was first proposed by Zhou in 1986, is one of the numerical methods for ordinary and partial differential equations that use the form of polynomials as the approximation to the exact solutions that are sufficiently differentiable. $f(z, t)$ function that will be solved and the calculation of following derivatives necessary in the solution become more difficult when the order increases. This is in contrast with the traditional high-order Taylor series method. Instead, the differential transform technique provides an iterative procedure to obtain higher-order series; therefore, it can be applied to the case high order.

Basic definitions and operations of differential transformation are introduced as follows. Differential transformation of the function $\phi(z)$ is defined as follows:

$$\Phi(k) = \frac{1}{k!}\left[\frac{d^k\phi(z)}{dz^k}\right]_{z=z_0} \tag{5}$$

In Eq. (5), $\phi(z)$ is the original function and $\Phi(k)$ is transformed function which is called the T-function (it is also called the spectrum of the $\phi(z)$ at $z = z_0$, in the K domain). The differential inverse transformation of $\Phi(k)$ is defined as:

$$\phi(z) = \sum_{k=0}^{\infty}(z-z_0)^k\Phi(k) \tag{6}$$

from Eq. (5) and Eq. (6) we get

$$\phi(k) = \sum_{k=0}^{\infty}\frac{(z-z_0)^k}{k!}\left[\frac{d^k\phi(z)}{dz^k}\right]_{z=z_0} \tag{7}$$

Eq. (7) implies that the concept of the differential transformation is derived from Taylor's series expansion, but the method does not evaluate the derivatives symbolically. However, relative derivative are calculated by iterative procedure that are described by the transformed equations of the original functions.

From the definitions of Eq. (5) and Eq. (6), it is easily proven that the transformed functions comply with the basic mathematical operations shown in below. In real applications, the function ϕ(z) in Eq. (6) is expressed by a finite series and can be written as

$$\phi(z) = \sum_{k=0}^{\overline{N}}(z-z_0)^k\Phi(k) \tag{8}$$

Eq. (8) implies that $\sum_{k=\overline{N}+1}^{\infty}(z-z_0)^k\Phi(k)$ is negligibly small and $\overline{N}$ is decided by the converge of the eigenvalues. Where $\overline{N}$ is series size.

3.1 Some basic mathematical operations of the differential transformation:

The fundamental mathematical operations performed by differential transformation are listed, where the transformed functions $\Phi(k)$ are related with the known original function $\phi(z)$, as follows:

Original function $\phi(z)$	Transformed function $\Phi(k)$	
$a\phi(z)$	$a\Phi(k)$	(9)
$\phi_1(z) \pm \phi_2(z)$	$\Phi_1(k) \pm \Phi_2(k)$	(10)
$d\phi(z)/dz$	$(k+1)\,\Phi(k+1)$	(11)
$d^2\phi(z)/dz^2$	$(k+1)(k+2)\,\Phi(k+2)$	(12)
$d^3\phi(\mathrm{z})/dz^3$	$(k+1)(k+2)(k+3)\,\Phi(k+3)$	(13)
$d^4\phi(\mathrm{z})/dz^4$	$(k+1)(k+2)(k+3)(k+4)\,\Phi(k+4)$	(14)

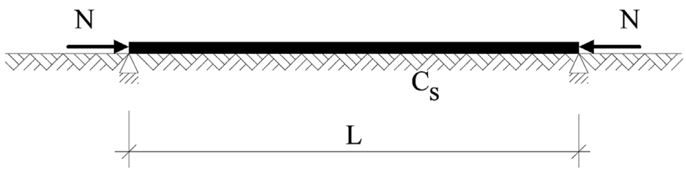


Fig. 2 Both ends simply supported beam on elastic soil

3.2 Using differential transformation to solve motion equations:

The boundary conditions of the beam resting on elastic foundation and both ends simply supported shown in Fig. 2 are given in Eqs. (15)-(18).

$$\phi(z=0) = 0 \tag{15}$$

$$\left.\frac{d^2\phi(z)}{dz^2}\right|_{z=0} = 0 \tag{16}$$

$$\phi(z=1) = 0 \tag{17}$$

$$\left.\frac{d^2\phi(z)}{dz^2}\right|_{z=1} = 0 \tag{18}$$

If transform values of Eqs. (9)-(14) are substituted into dimensionless displacement functions $\phi(z)$ and its derivative in equation of motion (4) for the case that axial and shear force effects are taken into consideration, the following equation is obtained.

$$\Phi(k+4) = -C\frac{\Phi(k+2)}{(k+3)(k+4)} - D\frac{\Phi(k)}{(k+1)(k+2)(k+3)(k+4)} \tag{19}$$

where $\gamma^4 = \dfrac{\overline{m}\,\omega^2 L^4}{EI}$, $\lambda = \dfrac{CsL^4}{EI}$, $C = \pi^2 N_r + \dfrac{(\overline{m}\,\omega^2 - Cs)\overline{k}L^2}{AG}$, $D = \lambda - \gamma^4$

Boundary conditions (15) and (17) are written using Eq. (8) for near the $z_0 = 0$ point as in the following respectively:

$$\text{for } z = 0; \qquad \Phi(0) = 0 \tag{20}$$

$$\text{for } z = 1; \qquad \sum_{k=0}^{\overline{N}} \Phi(k) = 0 \tag{21}$$

Boundary conditions (16) and (18) are written using the function obtained by derivating Eq. (8) twice as in the following:

$$\text{for } z = 0; \qquad \Phi(2) = 0 \tag{22}$$

$$\text{for } z = 1; \qquad \sum_{k=2}^{\overline{N}} k(k-1)\Phi(k) = 0 \tag{23}$$

Put $\Phi(1) = \alpha_1$, $\Phi(3) = \beta_1$ from Eq. (19) we find that

$$
\begin{array}{lll}
\text{for} & k=0 & \Phi(4) = 0 \\
 & k=1 & \Phi(5) = (-3!\beta_1 C - D\alpha_1)/5! \\
 & k=2 & \Phi(6) = 0 \\
 & k=3 & \Phi(7) = (3!\beta_1 C^2 + CD\alpha_1 - 3!\beta_1 D)/7! \\
 & k=4 & \Phi(8) = 0 \\
 & k=5 & \Phi(9) = (-3!\beta_1 C^3 - C^2 D\alpha_1 + 2*3!\beta_1 CD + D^2\alpha_1)/9! \\
 & k=6 & \Phi(10) = 0 \\
 & k=7 & \Phi(11) = (3!\beta_1 C^4 + C^3 D\alpha_1 - 3*3!\beta_1 C^2 D - 2CD^2\alpha_1 + 3!\beta_1 D^2)/11! \\
 & k=8 & \Phi(12) = 0 \\
 & k=9 & \Phi(13) = (-3!\beta_1 C^5 - C^4 D\alpha_1 + 4*3!\beta_1 C^3 D + 3C^2 D^2\alpha_1 - 3*3!\beta_1 CD^2 - D^3\alpha_1)/13! \\
 & & \vdots
\end{array}
\tag{24}
$$

We calculate up to the $\bar{N}$th term $\Phi(\bar{N})$ and substituting from $\Phi(1)$ to $\Phi(\bar{N})$ into Eq. (21) and (23) we obtain system,

$$
\begin{bmatrix}
1 + \sum_{k=2}^{\bar{N}} \frac{(-1)^k}{(2k+1)!} \sum_{m=1}^{k \ge 2m} \binom{k-m-1}{m-1} C^{k-2m} D^m (-1)^m & \sum_{k=1}^{\bar{N}} \frac{(-1)^k}{(2k+1)!} \sum_{m=1}^{k \ge m} \binom{k-m}{m-1} C^{k-2m+1} D^{m-1} (-1)^m \\
\sum_{k=0}^{\bar{N}} \frac{(-1)^k}{(2k-1)!} \sum_{m=1}^{k \ge 2m} \binom{k-m-1}{m-1} C^{k-2m} D^m (-1)^m & \sum_{k=0}^{\bar{N}} \frac{(-1)^k}{(2k-1)!} \sum_{m=1}^{k \ge m} \binom{k-m}{m-1} C^{k-2m+1} D^{m-1} (-1)^m
\end{bmatrix}
* \begin{Bmatrix} \alpha_1 \\ 3!\beta_1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}
\tag{25}
$$

Determinant of the coefficients matrices of equations system (25) is obtained as in the following.

$$
\begin{aligned}
f_1^{(\bar{N})} = &\left[1 + \sum_{k=2}^{\bar{N}} \frac{(-1)^k}{(2k+1)!} \sum_{m=1}^{k \ge 2m} \binom{k-m-1}{m-1} C^{k-2m} D^m (-1)^m \right] * \left[\sum_{k=0}^{\bar{N}} \frac{(-1)^k}{(2k-1)!} \sum_{m=1}^{k \ge m} \binom{k-m}{m-1} C^{k-2m+1} D^{m-1} (-1)^m \right] \\
&- \left[\sum_{k=1}^{\bar{N}} \frac{(-1)^k}{(2k+1)!} \sum_{m=1}^{k \ge m} \binom{k-m}{m-1} C^{k-2m+1} D^{m-1} (-1)^m \right] * \left[\sum_{k=0}^{\bar{N}} \frac{(-1)^k}{(2k-1)!} \sum_{m=1}^{k \ge 2m} \binom{k-m-1}{m-1} C^{k-2m} D^m (-1)^m \right] = 0
\end{aligned}
\tag{26}
$$

Also the series expansions of Eq. (26) is

$$
\begin{aligned}
f_1^{(\bar{N})} = &\left\{ 1 - \frac{D}{5!} + \frac{C.D}{7!} + \frac{(D^2 - C^2 D)}{9!} + \dots \right\} * \left\{ 1 - \frac{C}{3!} + \frac{(C^2 - D)}{5!} + \dots \right\} \\
&- \left\{ \frac{1}{3!} - \frac{C}{5!} + \frac{(C^2 - D)}{7!} + \dots \right\} * \left\{ -\frac{D}{3!} + \frac{C.D}{5!} + \frac{(D^2 - C^2 D)}{7!} + \dots \right\} = 0
\end{aligned}
\tag{27}
$$

In the case that both axial and shear force effects are neglected the value of C is taken being zero in Eq. (27). In the case that only axial force effect is neglected the value of C is taken being $(\overline{m}\,\omega^2 - C_s)\,\overline{k}\,L^2/AG$ in Eq. (27). In the case that only shear force effect is neglected the value of C is taken being $N_r\,\pi^2$ in Eq. (27).

Solving (26) we get $\omega = \omega_i^{(\overline{N})}$, $i = 1, 2, 3, \ldots$ where $\omega_i^{(\overline{N})}$ is the $\overline{N}$th estimated ω circular frequency corresponding to $\overline{N}$, and $\overline{N}$ is indicated by

$$\left|\omega_i^{(\overline{N})} - \omega_i^{(\overline{N}-1)}\right| \le \varepsilon \tag{28}$$

where $\omega_i^{(\overline{N}-1)}$ is the ith estimated circular frequency corresponding to $\overline{N} - 1$ and ε is a positive and small value.

4. Numerical examples

Both ends simply supported beam made by IPB 700 steel profile is resting on elastic foundation having modulus of subgrade reaction of 60.000 kN/m^2. Natural circular frequencies for the first three modes of the beam are calculated using DTM for the case that bending moment, axial and shear force effects are taken into consideration and for the case that axial, shear force effects are neglected. The characteristics of IPB 700 steel profile are presented as in the following:

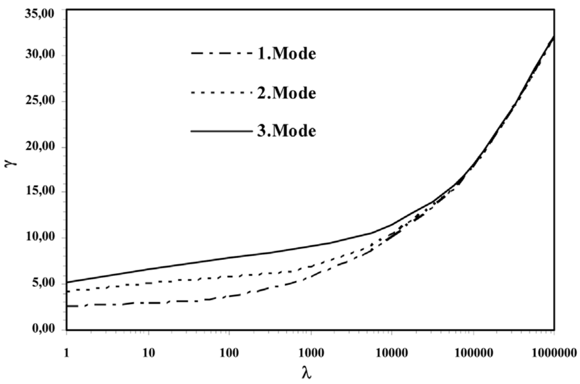
$I = 256.9 * 10^{-5}$ m^4; $A = 3.06 * 10^{-2}$ m^2; $\overline{m} = 0.24$ kNsec^2/m; $\overline{k} = 1.54$;
$E = 2.1 * 10^8$ kN/m^2; $G = 8.1 * 10^7$ kN/m^2

Lengths of the beam (L) on elastic soil are calculated due to relative stiffness values (λ) and are presented in Table 1 in numerical example:

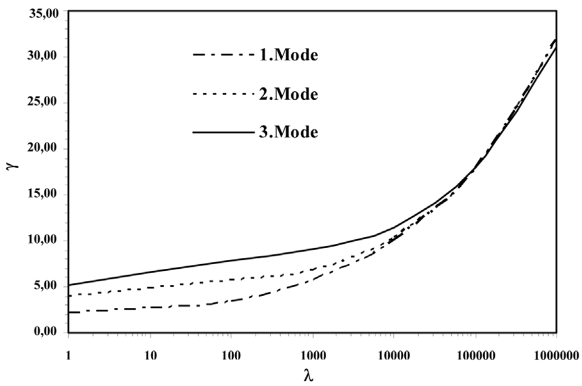
Variation due to relative stiffness of frequency factors $\gamma = \sqrt[4]{\dfrac{\overline{m}\,\omega^2 L^4}{EI}}$ calculated due to circular frequencies of the beam on elastic foundation that bending moment, shear and axial force effects are taken into consideration and that, shear, axial force effects are neglected are presented respectively in Fig. 3 and Fig. 4 for $N_r = 0.25$; $N_r = 0.50$; $N_r = 0.75$.

Table 1 The beam lengths (L) due to relative stiffness (λ)

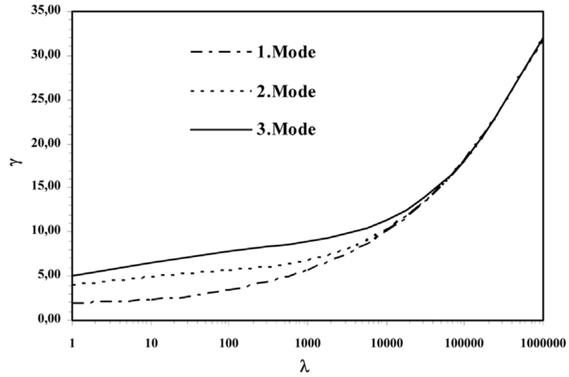
$\lambda = Cs\,L^4/EI$	L (m)
1	1.731
10	3.079
100	5.476
1000	9.738
10000	17.316
100000	30.793
1000000	54.769



(a)



(b)



(c)

Fig. 3(a) Variation of frequency factors due to relative stiffness for N_r = 0.25, (b) Variation of frequency factors due to relative stiffness for N_r = 0.50, (c) Variation of frequency factors due to relative stiffness for N_r = 0.75

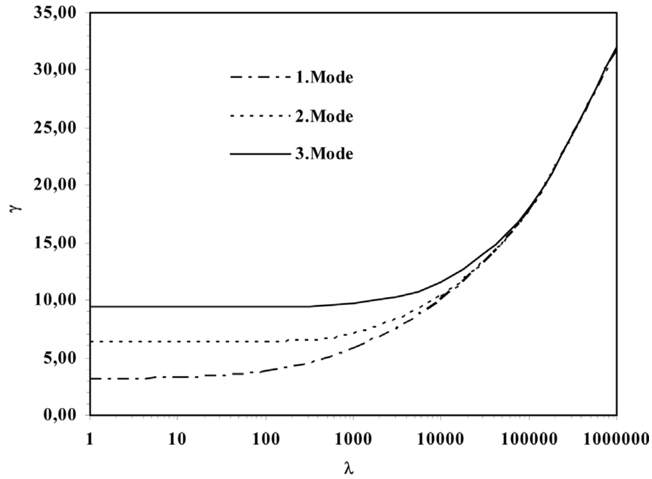


Fig. 4 Variation of frequency factors due to relative stiffness of the beam that axial and shear force effects are neglected.

Table 2 The frequency factors due to relative stiffness of the beam that axial and shear force effects are neglected.

$\lambda = C_s L^4 / EI$	1st. Mode		2nd. Mode		3rd. Mode	
	Present	Doyle *et al.*	Present	Doyle *et al.*	Present	Doyle *et al.*
1	3.15	3.15	6.28	6.28	9.42	9.42
10	3.22	3.20	6.29	6.28	9.43	9.43
100	3.75	3.75	6.38	6.38	9.45	9.45
1000	5.76	5.75	7.11	7.10	9.71	9.70
10000	10.02	10.03	10.35	10.35	11.56	11.56
100000	17.79	17.75	17.85	17.85	18.12	18.10
1000000	31.69	-	31.84	-	32.09	-

To demonstrate rate of convergence and accuracy of the presented DTM both ends simply supported beams resting on elastic foundation having modulus of subgrade reaction of 60.000 kN/m^2 and made by IPB700 steel profile are considered neglecting shear and axial force effects. The frequency factors of these beams calculated due to relative stiffness by using DTM. The results of DTM and the results which are obtained from graphs of Doyle and Pavlovic (1982) are presented in Table 2.

Neglecting axial force effects, the natural vibration frequencies and frequency factors of both end simply supported beam not resting on elastic foundation can be calculated by using $\omega_r = \dfrac{r^2\pi^2}{L^2}\sqrt{\dfrac{EI}{\overline{m}}}$ and, respectively (Chopra 1995) where r is mode number. Due to lengths of the beam and series size $(\overline{N})$, frequency factors of both ends simply supported beams which are made of IPB700 steel profiles are calculated by using DTM and assuming $C_s = 0$. The results of DTM and the results of Euler beam are presented in Table 3.

5. Discussions

Fig. 3 indicate that, for all values of N_r, the curves of frequency factor for the first three modes of the beam having relative stiffness between 100000 and 1000000 cluster together.

Frequency factor values for lower modes decreases as the magnitudes of the axial force increases for relative stiffness value between 1 and 10000.

The comparison between Figs. 3 and 4 indicates that the shear and axial force effects are significant on frequency factors especially for the first two modes of the beam having relative stiffness between 1 and 100000. The frequency factors with shear and axial force effects of the beam having relative stiffness between 1 and 100000 are lower than the values without shear and axial force effects of the beam having same relative stiffness. Fig. 4 shows that the curves of frequency factor for the first three modes of the beam having relative stiffness between 1000000 and 1000000 cluster for the case that axial and shear force effects are neglected.

The frequency factors with and without shear and axial force effects for the first three modes of the beam having relative stiffness between 100000 and 1000000 are very close together. This result

Table 3 The frequency factors of both ends simply supported Euler beams

Method	$\overline{N}$	L = 1.731 m			L = 3.079 m			L = 5.476 m		
		γ_1	γ_2	γ_3	γ_1	γ_2	γ_3	γ_1	γ_2	γ_3
DTM	8	3.14159265	6.32966375	6.32966518	3.14159265	6.32967997	7.61925411	3.14159265	6.32968044	7.61926842
	10	3.14159265	6.28365898	6.28366518	3.14159265	6.28365183	8.84833813	3.14159265	6.28368330	8.84844017
	12	3.14159265	6.28319931	9.37460423	3.14159265	6.28328531	9.3741798	3.14159265	6.28328531	9.37489605
	14	3.14159265	6.28318531	9.42348003	3.14159265	6.28318531	9.42350674	3.14159265	6.28318531	9.42362565
	16	3.14159265	6.28318531	9.42477795	3.14159265	6.28318531	9.42477795	3.14159265	6.28318531	9.42477795
Exact solution		3.14159265	6.28318531	9.42477795	3.14159265	6.28318531	9.42477795	3.14159265	6.28318531	9.42477795

	$\overline{N}$	L = 9.738 m			L = 17.316 m			L = 30.793 m		
		γ_1	γ_2	γ_3	γ_1	γ_2	γ_3	γ_1	γ_2	γ_3
DTM	8	3.14159265	6.32968044	7.61927366	3.14159265	6.32976341	7.61929178	3.14159265	6.32995558	7.61961699
	10	3.14159265	6.28365183	8.84844875	3.14159265	6.28380871	8.84841061	3.14159265	6.28383589	8.84842205
	12	3.14159265	6.28319931	9.37488747	3.14159265	6.28328531	9.37514210	3.14159265	6.28328531	9.37539577
	14	3.14159265	6.28318531	9.42352009	3.14159265	6.28318531	9.42359447	3.14159265	6.28318531	9.42381382
	16	3.14159265	6.28318531	9.42477795	3.14159265	6.28318531	9.42477795	3.14159265	6.28318531	9.42477795
Exact solution		3.14159265	6.28318531	9.42477795	3.14159265	6.28318531	9.42477795	3.14159265	6.28318531	9.42477795

	$\overline{N}$	L = 54.769 m		
		γ_1	γ_2	γ_3
DTM	8	3.14159265	6.33047342	7.61982059
	10	3.14159265	6.28447914	8.84909916
	12	3.14159265	6.28319931	9.37594509
	14	3.14159265	6.28318531	9.42488766
	16	3.14159265	6.28318531	9.42477795
Exact solution		3.14159265	6.28318531	9.42477795

indicates that the effects of shear and axial force on frequency factor for the beam having relative stiffness between 100000 and 1000000 are negligible.

In application of DTM, frequency factor values of the beam are calculated by increasing series size $\overline{N}$. It is determined that change in the results of frequency factors with shear and axial force effects are negligible for series size $\overline{N} > 12$ and change in results of frequency factors without shear and axial force effects are negligible for series size $\overline{N} > 5$. The series size is taken respectively 15 and 7 in the numerical example for the case that shear, axial force effects are taken into consideration and are neglected.

The numerical values of frequency factors obtained for the first three modes of the beam using DTM become fixed for the case that the series size is taken higher than a definite value. Table 2 is achieved that the results of DTM agree with the results of Doyle *et al.* (1982). The results in Table 3 indicate that the results of Euler beam obtained by using DTM and assuming $C_s = 0$ are rapidly converging on the results of exact solution and the results of DTM are accurate.

6. Conclusions

DTM was employed for free vibration analysis of both ends simply supported beam resting on elastic foundation. Bending moment, axial and shear force effects are taken into consideration in this analysis. If axial, shear force effects are neglected and it is assumed that $C_s = 0$ in equations obtained by DTM, the equations are obtained for free vibration analysis of Euler beam having same boundary conditions. It is seen from the results of DTM and results in the references that rate of convergence and accuracy of DTM is very good and that the frequency factors of the beam having relative stiffness values between 1 and 1000 increase as the values of axial compressive loads acting on the beam decrease.

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