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Spherically symmetric transient responses of functionally graded magneto-electro-elastic hollow sphere

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Abstract. On the basis of equilibrium equations for static electric and magnetic fields, two unknown functions related to electric and magnetic fields were firstly introduced to rewrite the governing equations, boundary conditions and initial conditions for mechanical field. Then by introducing a dependent variable and a special function satisfying the inhomogeneous mechanical boundary conditions, the governing equation for a new variable with homogeneous mechanical boundary conditions is obtained. By using the separation of variables technique as well as the electric and magnetic boundary conditions, the dynamic problem of a functionally graded magneto-electro-elastic hollow sphere under spherically symmetric deformation is transformed to two Volterra integral equations of the second kind about two unknown functions of time. Cubic Hermite polynomials are adopted to approximate the two undetermined functions at each time subinterval and the recursive formula for solving the integral equations is derived. Transient responses of displacements, stresses, electric and magnetic potentials are completely determined at the end. Numerical results are presented and discussed.

Keywords: hollow sphere; magneto-electro-elastic; functionally graded materials; transient response.

1. Introduction

The emergence of functionally graded materials (FGMs) is a great progress in materials science. Due to their distinguishing feature that material properties vary continuously with location, FGMs possess many advantages and have been used in many areas including: superheat resistance (furnace liners and space structures), biomedical (dental and bone implants), military (vehicle and body armor), and dielectric materials (wave guides and radar avoidance).

The investigations on static and free vibrations for FGM hollow cylinders and spheres have been reported extensively. Among them, many achievements have been obtained for the special cases that the material constants have a power law dependence on the radial coordinate (Jabbari *et al*. 2002, 2003, Tarn 2001, Horgan and Chan 1999, Eslami *et al*. 2005, Chen 1999a, 2000, Chen *et al*.

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1999b). It is well known that FGM structures are often used in severe environments, such as thermal shock, dynamic loads and impact pressures etc. The responses under dynamic loads are always larger than those under static loads. So to know the transient responses of FGM structures exactly is very important. But the investigations in this field are relatively scarce and really difficult and complex. Recently, Ding *et al.* (2002, 2003a,b) studied the dynamic responses of functionally graded elastic, piezoelectric and pyroelectric hollow spheres, respectively.

The investigations for interactions of multi-fields (mechanical/electric/magnetic) are relatively new. Pan and Heyliger (2002) studied the free vibrations of simply supported and multilayered magneto-electro-elastic plates. Chen *et al.* (2005) investigated the free vibrations of non-homogeneous transversely isotropic magneto-electro-elastic plates. Buchanan (2003) obtained the free vibration of an infinite magneto-electro-elastic cylinder. Hou and Leung (2004) further studied the transient responses of magneto-electric-elastic hollow cylinders. While to the authors' knowledge, the transient responses of FGM magneto-electric-elastic hollow sphere have not been reported yet.

In this paper, we first present the basic equations and their non-dimensional forms. Then by virtue of the motion equations of electric and magnetic fields, two unknown functions of time are introduced to rewrite the governing equations and boundary conditions as well as the initial conditions for mechanical field. The solution for displacement involving two unknown functions of time is obtained firstly. Then by utilizing the electric and magnetic boundary conditions, two Volterra integral equations about two unknown functions of time are derived. The integral equations are solved successfully by means of the interpolation method. The displacements, stresses, as well as all electric and magnetic quantities are determined completely at the end.

2. Problem statements

In this study, we assume that the under-considering material possesses magneto-electro-elastic coupling effect. Factually, such property can be observed in piezoelectric/piezomagnetic composites (Nan 1994, Wan *et al.* 2003).

Consider a magneto-electro-elastic hollow sphere with the inner and outer radii a and b, respectively, subjected to complex loads (Fig. 1).

For spherically symmetric problem, with the spherical coordinate system (r, θ, φ), the nonzero components of displacement, electric potential and magnetic potential can be denoted, respectively,

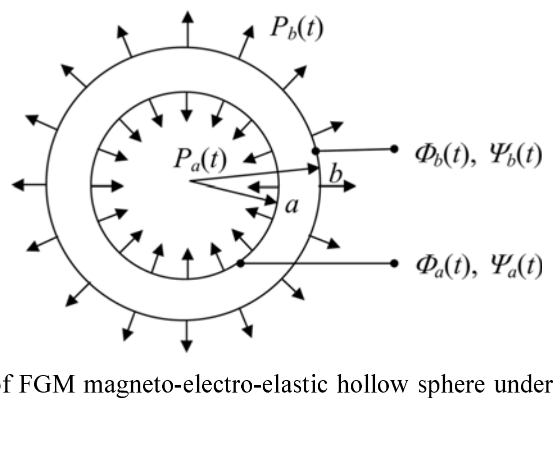

Fig. 1 Sketch of FGM magneto-electro-elastic hollow sphere under complex loads

as $u_r = u_r(r, t)$, $\Phi = \Phi(r, t)$, and $\Psi = \Psi(r, t)$. Then, the constitutive relations for radially polarized, hexagonal crystal media are read as (Buchanan 2003)

$$\begin{aligned}
\sigma_{\theta\theta} &= \sigma_{\varphi\varphi} = (c_{11} + c_{12})\frac{u_r}{r} + c_{13}\frac{\partial u_r}{\partial r} + e_{31}\frac{\partial \Phi}{\partial r} + q_{31}\frac{\partial \Psi}{\partial r} \\
\sigma_{rr} &= 2c_{13}\frac{u_r}{r} + c_{33}\frac{\partial u_r}{\partial r} + e_{33}\frac{\partial \Phi}{\partial r} + q_{33}\frac{\partial \Psi}{\partial r} \\
D_{rr} &= 2e_{31}\frac{u_r}{r} + e_{33}\frac{\partial u_r}{\partial r} - \varepsilon_{33}\frac{\partial \Phi}{\partial r} - m_{33}\frac{\partial \Psi}{\partial r} \\
B_{rr} &= 2q_{31}\frac{u_r}{r} + q_{33}\frac{\partial u_r}{\partial r} - m_{33}\frac{\partial \Phi}{\partial r} - \mu_{33}\frac{\partial \Psi}{\partial r}
\end{aligned} \tag{1}$$

where σ_{ij}, D_{rr} and B_{rr} are the components of stress, electric displacement and magnetic induction, respectively. c_{ij}, e_{3j}, q_{3j}, ε_{33}, m_{33}, and μ_{33} are elastic, piezoelectric, piezomagnetic, dielectric, electromagnetic and magnetic constants, respectively. In the absence of body force, electric charge density and electric current density, the equations of motion are

$$\frac{\partial \sigma_{rr}}{\partial r} + 2\frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = \rho\frac{\partial^2 u_r}{\partial t^2} \tag{2}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r^2 D_{rr}) = 0 \tag{3}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r^2 B_{rr}) = 0 \tag{4}$$

where ρ is the mass density. In this paper, we consider the hollow sphere formed by a special non-homogeneous medium. Suppose the material constants vary as power-law in r-direction as

$$\begin{aligned}
c_{ij} &= \xi^{2N}C_{ij},\ \ e_{3i} = \xi^{2N}E_{3i},\ \ q_{3i} = \xi^{2N}Q_{3i},\ \ \varepsilon_{33} = \xi^{2N}\Lambda_{33}, \\
m_{33} &= \xi^{2N}M_{33},\ \ \mu_{33} = \xi^{2N}K_{33},\ \ \rho = \xi^{2N}\rho_0,\ \ \xi = r/b
\end{aligned} \tag{5}$$

where C_{ij}, E_{3i}, Q_{3i}, Λ_{33}, M_{33}, K_{33} and ρ_0 are known constants, and N can be an arbitrary real number. Substituting Eqs. (5) into Eqs. (1)-(4) and introducing a series of non-dimensional quantities, we obtain

$$\begin{aligned}
\sigma_\theta &= \sigma_\varphi = \xi^{2N}\left[(C_{11\mathrm{P}} + C_{12\mathrm{P}})\frac{u}{\xi} + C_{13\mathrm{P}}\frac{\partial u}{\partial \xi} + E_1\frac{\partial \phi}{\partial \xi} + Q_1\frac{\partial \psi}{\partial \xi}\right] \\
\sigma_r &= \xi^{2N}\left[2C_{13\mathrm{P}}\frac{u}{\xi} + C_{33\mathrm{P}}\frac{\partial u}{\partial \xi} + E_3\frac{\partial \phi}{\partial \xi} + Q_3\frac{\partial \psi}{\partial \xi}\right] \\
D_r &= \xi^{2N}\left[2E_1\frac{u}{\xi} + E_3\frac{\partial u}{\partial \xi} - \frac{\partial \phi}{\partial \xi} - M_3\frac{\partial \psi}{\partial \xi}\right] \\
B_r &= \xi^{2N}\left[2Q_1\frac{u}{\xi} + Q_3\frac{\partial u}{\partial \xi} - M_3\frac{\partial \phi}{\partial \xi} - \frac{\partial \psi}{\partial \xi}\right]
\end{aligned} \tag{6}$$

$$\frac{\partial \sigma_r}{\partial \xi} + 2\frac{\sigma_r - \sigma_\theta}{\xi} = \xi^{2N}\frac{\partial^2 u}{\partial \tau^2} \tag{7}$$

$$\frac{1}{\xi}\frac{\partial}{\partial \xi}(\xi^2 D_r) = 0 \tag{8}$$

$$\frac{1}{\xi}\frac{\partial}{\partial \xi}(\xi^2 B_r) = 0 \tag{9}$$

where

$$\begin{gathered} C_{ij\mathrm{P}} = \frac{C_{ij}}{C_{33}}, \quad E_i = \frac{E_{3i}}{\sqrt{C_{33}\Lambda_{33}}}, \quad Q_i = \frac{Q_{3i}}{\sqrt{C_{33}K_{33}}}, \quad M_3 = \frac{M_{33}}{\sqrt{\Lambda_{33}K_{33}}} \\ \sigma_i = \frac{\sigma_{ii}}{C_{33}}(i = r, \theta, \varphi), \quad \phi = \sqrt{\frac{\Lambda_{33}}{C_{33}}}\frac{\Phi}{b}, \quad \psi = \sqrt{\frac{K_{33}}{C_{33}}}\frac{\Psi}{b}, \quad D_r = \frac{D_{rr}}{\sqrt{C_{33}\Lambda_{33}}} \\ B_r = \frac{B_{rr}}{\sqrt{C_{33}K_{33}}}, \quad u = \frac{u_r}{b}, \quad s = \frac{a}{b}, \quad \tau = \frac{c_v}{b}t, \quad c_v = \sqrt{\frac{C_{33}}{\rho_0}} \end{gathered} \tag{10a}$$

If the hollow sphere is subjected to dynamic pressures $P_a(t)$ and $P_b(t)$, respectively, at the interior and exterior surfaces. Also, we suppose the electric potentials and magnetic potentials imposed on the interior and exterior surfaces are, respectively, $\Phi_a(t), \Phi_b(t)$ and $\Psi_a(t), \Psi_b(t)$. For dynamic problem, we further suppose the initial displacement and initial velocity are $U_0(r)$ and $V_0(r)$, respectively. The above-mentioned quantities can be expressed in non-dimensional forms as

$$\begin{gathered} p_a = \frac{P_a}{C_{33}}, \quad p_b = \frac{P_b}{C_{33}}, \quad \phi_a = \sqrt{\frac{\Lambda_{33}}{C_{33}}}\frac{\Phi_a}{b}, \quad \phi_b = \sqrt{\frac{\Lambda_{33}}{C_{33}}}\frac{\Phi_b}{b} \\ \psi_a = \sqrt{\frac{K_{33}}{C_{33}}}\frac{\Psi_a}{b}, \quad \psi_b = \sqrt{\frac{K_{33}}{C_{33}}}\frac{\Psi_b}{b}, \quad u_0 = \frac{U_0}{b}, \quad v_0 = \frac{V_0}{c_v} \end{gathered} \tag{10b}$$

By means of Eqs. (10b), the boundary conditions and initial conditions are denoted in non-dimensional forms as

$$\sigma_r(s, \tau) = p_a(\tau), \ \sigma_r(1, \tau) = p_b(\tau) \tag{11}$$

$$\phi(s, \tau) = \phi_a(\tau), \ \phi(1, \tau) = \phi_b(\tau) \tag{12}$$

$$\psi(s, \tau) = \psi_a(\tau), \ \psi(1, \tau) = \psi_b(\tau) \tag{13}$$

$$u(\xi, 0) = u_0(\xi), \ \dot{u}(\xi, 0) = v_0(\xi) \tag{14}$$

In Eq. (14) and thereafter, a dot over a quantity denotes its partial derivative with respect to non-dimensional time τ.

3. Solution approach for mechanical field

3.1 Governing equations for mechanical field

The solutions of Eqs. (8) and (9) are

$$D_r(\xi,\,\tau) \;=\; \frac{1}{\xi^2}\eta(\tau), \quad B_r(\xi,\,\tau) = \frac{1}{\xi^2}\chi(\tau) \tag{15}$$

where $\eta(\tau)$ and $\chi(\tau)$ are unknown time functions of non-dimensional time τ. By virtue of Eq. (15), the following equations can be derived from the last two of Eqs. (6) as

$$\begin{aligned}\frac{\partial\phi}{\partial\xi} &= 2A_{11}\frac{u}{\xi} + A_{12}\frac{\partial u}{\partial\xi} - A_3\frac{1}{\xi^{2N+2}}\eta(\tau) + A_4\frac{1}{\xi^{2N+2}}\chi(\tau)\\ \frac{\partial\psi}{\partial\xi} &= 2A_{21}\frac{u}{\xi} + A_{22}\frac{\partial u}{\partial\xi} + A_4\frac{1}{\xi^{2N+2}}\eta(\tau) - A_3\frac{1}{\xi^{2N+2}}\chi(\tau)\end{aligned} \tag{16}$$

where

$$\begin{aligned}A_{11} &= (E_1 - Q_1M_3)/X,\;\; A_{12} = (E_3 - Q_3M_3)/X,\;\; A_3 = 1/X,\;\; A_4 = M_3/X\\ A_{21} &= (Q_1 - E_1M_3)/X,\;\; A_{22} = (Q_3 - E_3M_3)/X,\;\; X = 1 - M_3M_3\end{aligned} \tag{17}$$

The substitutions of Eqs. (16) into the first two equations in Eqs. (6) give

$$\begin{aligned}\sigma_\theta = \sigma_\varphi &= \xi^{2N}\left[(C_{11\mathrm{D}} + C_{12\mathrm{D}})\frac{u}{\xi} + C_{13\mathrm{D}}\frac{\partial u}{\partial\xi}\right] + E_{1\mathrm{D}}\frac{\eta(\tau)}{\xi^2} + Q_{1\mathrm{D}}\frac{\chi(\tau)}{\xi^2}\\ \sigma_r &= \xi^{2N}\left[2C_{13\mathrm{D}}\frac{u}{\xi} + C_{33\mathrm{D}}\frac{\partial u}{\partial\xi}\right] + E_{3\mathrm{D}}\frac{\eta(\tau)}{\xi^2} + Q_{3\mathrm{D}}\frac{\chi(\tau)}{\xi^2}\end{aligned} \tag{18}$$

where

$$\begin{aligned}C_{11\mathrm{D}} &= C_{11\mathrm{P}} + E_1A_{11} + Q_1A_{21}, \quad C_{12\mathrm{D}} = C_{12\mathrm{P}} + E_1A_{11} + Q_1A_{21}\\ C_{13\mathrm{D}} &= C_{13\mathrm{P}} + E_3A_{11} + Q_3A_{21}, \quad C_{33\mathrm{D}} = C_{33\mathrm{P}} + E_3A_{12} + Q_3A_{22}\\ E_{1\mathrm{D}} &= Q_1A_4 - E_1A_3, \quad E_{3\mathrm{D}} = Q_3A_4 - E_3A_3\\ Q_{1\mathrm{D}} &= E_1A_4 - Q_1A_3, \quad Q_{3\mathrm{D}} = E_3A_4 - Q_3A_3\end{aligned} \tag{19}$$

Substituting Eqs. (18) into Eq. (7), we obtain

$$\frac{\partial^2 u}{\partial\xi^2} + (2N+2)\frac{1}{\xi}\frac{\partial u}{\partial\xi} - \frac{\mu_1^2}{\xi^2}u \;=\; \frac{1}{c_L^2}\frac{\partial^2 u}{\partial\tau^2} + X_1(\xi)\,\eta(\tau) + X_2(\xi)\,\eta(\tau) \tag{20}$$

where

$$\begin{aligned}\mu_1 &= \sqrt{2\frac{C_{11\mathrm{D}} + C_{12\mathrm{D}} - (2N+1)C_{13\mathrm{D}}}{C_{33\mathrm{D}}}}, \quad c_L = \sqrt{C_{33\mathrm{D}}}\\ X_1(\xi) &= \frac{2E_{1\mathrm{D}}}{C_{33\mathrm{D}}}\frac{1}{\xi^{2N+3}}, \quad X_2(\xi) = \frac{2Q_{1\mathrm{D}}}{C_{33\mathrm{D}}}\frac{1}{\xi^{2N+3}}\end{aligned} \tag{21}$$

By virtue of the expression for σ_r, as given by the second of Eqs. (18), the mechanical boundary conditions (11) can be rewritten as

$$\begin{aligned}\xi = s:\quad & C_{33\mathrm{D}}\frac{\partial u}{\partial \xi}+2C_{13\mathrm{D}}\frac{u}{\xi} = s^{-(2N+2)}[s^2 p_a(\tau)-E_{3\mathrm{D}}\eta(\tau)-Q_{3\mathrm{D}}\chi(\tau)] \\ \xi = 1:\quad & C_{33\mathrm{D}}\frac{\partial u}{\partial \xi}+2C_{13\mathrm{D}}\frac{u}{\xi} = p_b(\tau)-E_{3\mathrm{D}}\eta(\tau)-Q_{3\mathrm{D}}\chi(\tau)\end{aligned} \tag{22}$$

In order to simplify Eq. (20), the following dependent variable $w(\xi, \tau)$ is introduced as

$$u(\xi, \tau) = \xi^{-\left(N+\frac{1}{2}\right)} w(\xi, \tau) \tag{23}$$

Then, the substitutions of Eq. (23) into Eqs. (20), (22) and (14) yield

$$\frac{\partial^2 w}{\partial \xi^2}+\frac{1}{\xi}\frac{\partial w}{\partial \xi}-\frac{\mu^2}{\xi^2}w = \frac{1}{c_L^2}\frac{\partial^2 w}{\partial \tau^2}+X_3(\xi)\eta(\tau)+X_4(\xi)\chi(\tau) \tag{24}$$

$$\xi = s:\quad \frac{\partial w}{\partial \xi}+h\frac{w}{\xi} = p_1(\tau);\quad \xi = 1:\quad \frac{\partial w}{\partial \xi}+h\frac{w}{\xi} = p_2(\tau) \tag{25}$$

$$w(\xi, 0) = u_1(\xi),\quad \dot{w}(\xi, 0) = v_1(\xi) \tag{26}$$

where

$$\begin{aligned} & h = 2C_{13\mathrm{D}}/C_{33\mathrm{D}}-\left(N+\frac{1}{2}\right),\quad \mu = \sqrt{\mu_1^2+\left(N+\frac{1}{2}\right)^2} \\ & p_1(\tau) = s^{-\left(N+\frac{3}{2}\right)}[s^2 p_a(\tau)-E_{3\mathrm{D}}\eta(\tau)-Q_{3\mathrm{D}}\chi(\tau)]/C_{33\mathrm{D}} \\ & p_2(\tau) = [p_b(\tau)-E_{3\mathrm{D}}\eta(\tau)-Q_{3\mathrm{D}}\chi(\tau)]/C_{33\mathrm{D}} \\ & X_3(\xi) = \xi^{N+\frac{1}{2}}X_1(\xi),\quad X_4(\xi) = \xi^{N+\frac{1}{2}}X_2(\xi) \\ & u_1(\xi) = \xi^{N+\frac{1}{2}}u_0(\xi),\quad v_1(\xi) = \xi^{N+\frac{1}{2}}v_0(\xi)\end{aligned} \tag{27}$$

3.2 Homogenization for mechanical boundary conditions

We first transform the inhomogeneous mechanical boundary conditions into homogeneous ones by assuming

$$w(\xi, \tau) = w_1(\xi, \tau)+w_2(\xi, \tau) \tag{28}$$

where $w_2(\xi, \tau)$, which is necessary to satisfy the inhomogeneous boundary conditions (25) only. By observing Eqs. (25), $w_2(\xi, \tau)$ can be taken the following form as

$$w_2(\xi, \tau) = \alpha_1(\xi-1)^2 p_1(\tau)+\alpha_2(\xi-s)^2 p_2(\tau) \tag{29a}$$

in which α_1 and α_2 can be determined by substituting Eq. (29a) into Eqs. (25). Then Eq. (29a) can be rewritten as

$$w_2(\xi,\tau) = f_0(\xi,\tau) + f_1(\xi)\eta(\tau) + f_2(\xi)\chi(\tau) \tag{29b}$$

where

$$\begin{gathered} f_0(\xi,\tau) = \beta_1(\xi)p_a(\tau) + \beta_2(\xi)p_b(\tau) \\ f_1(\xi) = -E_{3\mathrm{D}}[\beta_1(\xi)/s^2 + \beta_2(\xi)], \quad f_2(\xi) = -Q_{3\mathrm{D}}[\beta_1(\xi)/s^2 + \beta_2(\xi)] \\ \beta_1(\xi) = s^{-N+\frac{1}{2}}\alpha_1(\xi-1)^2/C_{33\mathrm{D}}, \quad \beta_2(\xi) = \alpha_2(\xi-s)^2/C_{33\mathrm{D}} \\ \alpha_1 = [2(s-1) + h(s-1)^2/s]^{-1}, \quad \alpha_2 = [2(1-s) + h(1-s)^2]^{-1} \end{gathered} \tag{30}$$

Substituting Eq. (28) into Eqs. (24)-(26) and utilizing Eq. (29b), we obtain

$$\frac{\partial^2 w_1(\xi,\tau)}{\partial\xi^2} + \frac{1}{\xi}\frac{\partial w_1(\xi,\tau)}{\partial\xi} - \frac{\mu^2}{\xi^2}w_1(\xi,\tau) = \frac{1}{c_L^2}\frac{\partial^2 w_1(\xi,\tau)}{\partial\tau^2} + g(\xi,\tau) \tag{31}$$

$$\frac{\partial w_1(\xi,\tau)}{\partial\xi} + h\frac{w_1(\xi,\tau)}{\xi} = 0 \ (\xi = s, 1) \tag{32}$$

$$\begin{aligned} w_1(\xi,0) &= u_2(\xi) - f_1(\xi)\eta(0) - f_2(\xi)\chi(0) \\ \dot{w}_1(\xi,0) &= v_2(\xi) - f_1(\xi)\dot{\eta}(0) - f_2(\xi)\dot{\chi}(0) \end{aligned} \tag{33}$$

where

$$\begin{gathered} g(\xi,\tau) = g_0(\xi,\tau) + g_1(\xi)\eta(\tau) + g_2(\xi)\chi(\tau) + [f_1(\xi)\ddot{\eta}(\tau) + f_2(\xi)\ddot{\chi}(\tau)]/c_L^2 \\ u_2(\xi) = u_1(\xi) - f_0(\xi,0), \quad v_2(\xi) = v_1(\xi) - \dot{f}_0(\xi,0) \\ g_0(\xi,\tau) = \left(\frac{\mu^2}{\xi^2} - \frac{1}{\xi}\frac{\mathrm{d}}{\mathrm{d}\xi} - \frac{\mathrm{d}^2}{\mathrm{d}\xi^2} + \frac{\partial^2}{c_L^2\partial\tau^2}\right)f_0(\xi,\tau) \\ g_j(\xi) = \left[\frac{\mu^2}{\xi^2} - \frac{1}{\xi}\frac{\mathrm{d}}{\mathrm{d}\xi} - \frac{\mathrm{d}^2}{\mathrm{d}\xi^2}\right]f_j(\xi) + X_{j+2}(\xi) \quad (j = 1, 2) \\ \dot{f}_0(\xi,0) = \left.\frac{\partial f_0(\xi,\tau)}{\partial\tau}\right|_{\tau=0}, \quad \dot{\eta}(0) = \left.\frac{\mathrm{d}\eta(\tau)}{\mathrm{d}\tau}\right|_{\tau=0}, \quad \dot{\chi}(0) = \left.\frac{\mathrm{d}\chi(\tau)}{\mathrm{d}\tau}\right|_{\tau=0} \end{gathered} \tag{34}$$

3.3 The orthogonal expansion technique

By means of the separation of variables technique, the solution of Eq. (31) can be assumed as

$$w_1(\xi,\tau) = \textstyle\sum_{i=1}^{\infty} R_i(\xi)\Omega_i(\tau) \tag{35}$$

where $\Omega_i(\tau)$ is an unknown function of τ, and $R_i(\xi)$ is a known function of ξ as

$$R_i(\xi) = J_\mu(k_i\xi)P_Y(\mu, k_i, s) - Y_\mu(k_i\xi)P_J(\mu, k_i, s) \tag{36}$$

in which $J_\mu(k_i\xi)$ and $Y_\mu(k_i\xi)$ are Bessel functions of the first and second kind, and of order μ, respectively. k_i, arranged in an ascending order, are a series of positive roots of the following eigenequation:

$$P_J(\mu, k_i, s)P_Y(\mu, k_i, 1) - P_J(\mu, k_i, 1)P_Y(\mu, k_i, s) = 0 \tag{37}$$

where

$$P_J(\mu, k_i, \xi) = \frac{\mathrm{d}J_\mu(k_i\xi)}{\mathrm{d}\xi} + h\frac{J_\mu(k_i\xi)}{\xi}, \quad P_Y(\mu, k_i, \xi) = \frac{\mathrm{d}Y_\mu(k_i\xi)}{\mathrm{d}\xi} + h\frac{Y_\mu(k_i\xi)}{\xi} \tag{38}$$

By virtue of the orthogonal property of Bessel function, we have

$$\int_s^1 \xi R_i(\xi)R_j(\xi)\mathrm{d}\xi = N_i\delta_{ij} \tag{39}$$

where δ_{ij} is the Kronecker delta, and

$$\begin{gathered} N_i = \frac{1}{2k_i^2}\left\{\left[\frac{dR_i(1)}{\mathrm{d}\xi}\right]^2 - s^2\left[\frac{\mathrm{d}R_i(s)}{\mathrm{d}\xi}\right]^2 + k_i^2[R_i^2(1) - s^2R_i^2(s)] - \mu^2[R_i^2(1) - R_i^2(s)]\right\} \\ \frac{\mathrm{d}R_i(s)}{\mathrm{d}\xi} = \left.\frac{\mathrm{d}R_i(\xi)}{\mathrm{d}\xi}\right|_{\xi=s}, \quad \frac{\mathrm{d}R_i(1)}{\mathrm{d}\xi} = \left.\frac{\mathrm{d}R_i(\xi)}{\mathrm{d}\xi}\right|_{\xi=1} \end{gathered} \tag{40}$$

Substituting Eq. (35) into Eq. (31) and utilizing Eq. (39), we obtain

$$\frac{\mathrm{d}^2\Omega_i(\tau)}{\mathrm{d}\tau^2} + \omega_i^2\Omega_i(\tau) = q_i(\tau) \tag{41}$$

where

$$\begin{gathered} q_i(\tau) = h_{0i}(\tau) + h_{1i}\eta(\tau) + h_{2i}\chi(\tau) + h_{3i}\ddot{\eta}(\tau) + h_{4i}\ddot{\chi}(\tau) \\ \omega_i = k_ic_L, \quad h_{0i}(\tau) = -\frac{c_L^2}{N_i}\int_s^1 \xi g_0(\xi, \tau)R_i(\xi)\mathrm{d}\xi \\ h_{1i} = -\frac{c_L^2}{N_i}\int_s^1 \xi g_1(\xi)R_i(\xi)\mathrm{d}\xi, \quad h_{2i} = -\frac{c_L^2}{N_i}\int_s^1 \xi g_2(\xi)R_i(\xi)\mathrm{d}\xi \\ h_{3i} = -\frac{1}{N_i}\int_s^1 \xi f_1(\xi)R_i(\xi)\mathrm{d}\xi, \quad h_{4i} = -\frac{1}{N_i}\int_s^1 \xi f_2(\xi)R_i(\xi)\mathrm{d}\xi \end{gathered} \tag{42}$$

The solution of Eq. (41) is

$$\Omega_i(\tau) = \Omega_i(0)\cos\omega_i\tau + \frac{\dot{\Omega}_i(0)}{\omega_i}\sin\omega_i\tau + \frac{1}{\omega_i}\int_0^\tau q_i(p)\sin\omega_i(\tau - p)\mathrm{d}p \tag{43}$$

And $\dot{\Omega}_i(\tau)$ can be expressed as

$$\dot{\Omega}_i(\tau) = -\omega_i\Omega_i(0)\sin\omega_i\tau + \dot{\Omega}_i(0)\cos\omega_i\tau + \int_0^\tau q_i(p)\cos\omega_i(\tau - p)\mathrm{d}p \tag{44}$$

Utilizing Eqs. (35) and (39), $\Omega_i(0)$ and $\dot{\Omega}_i(0)$ can be obtained from Eqs. (33) as

$$\Omega_i(0) = h_{3i}\eta(0) + h_{4i}\chi(0) + h_{5i}, \quad \dot{\Omega}_i(0) = h_{3i}\dot{\eta}(0) + h_{4i}\dot{\chi}(0) + h_{6i} \tag{45}$$

where

$$h_{5i} = \frac{1}{N_i}\int_s^1 \xi u_2(\xi)R_i(\xi)\mathrm{d}\xi, \quad h_{6i} = \frac{1}{N_i}\int_s^1 \xi v_2(\xi)R_i(\xi)\mathrm{d}\xi \tag{46}$$

It is noticed that $q_i(\tau)$ involves $\ddot{\eta}(\tau)$ and $\ddot{\chi}(\tau)$ as denoted in the first of Eqs. (42). Utilizing the integration-by-parts method, Eq. (43) can be rewritten as

$$\Omega_i(\tau) = \Omega_{1i}(\tau) + h_{3i}\eta(\tau) + h_{4i}\chi(\tau) + I_{1i}\int_0^\tau \eta(p)\sin\omega_i(\tau-p)\mathrm{d}p + I_{2i}\int_0^\tau \chi(p)\sin\omega_i(\tau-p)\mathrm{d}p \tag{47}$$

where

$$\begin{aligned} \Omega_{1i}(\tau) &= \Omega_i(0)\cos\omega_i\tau + \frac{\dot{\Omega}_i(0)}{\omega_i}\sin\omega_i\tau + \frac{1}{\omega_i}\int_0^\tau h_{0i}(p)\sin\omega_i(\tau-p)\mathrm{d}p \\ &-\frac{h_{3i}}{\omega_i}[\dot{\eta}(0)\sin\omega_i\tau + \eta(0)\omega_i\cos\omega_i\tau] - \frac{h_{4i}}{\omega_i}[\dot{\chi}(0)\sin\omega_i\tau + \chi(0)\omega_i\cos\omega_i\tau] \\ I_{1i} &= \frac{h_{1i}}{\omega_i} - h_{3i}\omega_i, \quad I_{2i} = \frac{h_{2i}}{\omega_i} - h_{4i}\omega_i \end{aligned} \tag{48}$$

4. The second kind Volterra integral equations

Substituting Eq. (28) into Eq. (23) and utilizing Eqs. (29b) and (35), we obtain

$$u(\xi, \tau) = \xi^{-\left(N+\frac{1}{2}\right)}\left[\textstyle\sum_{i=1}^{\infty} R_i(\xi)\Omega_i(\tau) + f_0(\xi, \tau) + f_1(\xi)\eta(\tau) + f_2(\xi)\chi(\tau)\right] \tag{49}$$

Integrating Eqs. (16) over the space interval $[s, \xi]$ and utilizing Eq. (49) as well as the Eqs. (12) and (13), we obtain

$$\begin{aligned} \phi(\xi, \tau) &= \phi_a(\tau) + \phi_0(\xi, \tau) + \phi_1(\xi)\eta(\tau) + \phi_2(\xi)\chi(\tau) + \textstyle\sum_{i=1}^{\infty}\phi_{3i}(\xi)\Omega_i(\tau) \\ \psi(\xi, \tau) &= \psi_a(\tau) + \psi_0(\xi, \tau) + \psi_1(\xi)\eta(\tau) + \psi_2(\xi)\chi(\tau) + \textstyle\sum_{i=1}^{\infty}\psi_{3i}(\xi)\Omega_i(\tau) \end{aligned} \tag{50}$$

where $\phi_0(\xi, \tau)$, $\phi_1(\xi)$, $\phi_2(\xi)$, $\phi_{3i}(\xi)$ and $\psi_0(\xi, \tau)$, $\psi_1(\xi)$, $\psi_2(\xi)$, $\psi_{3i}(\xi)$ are known functions and the specified expressions are presented in Appendix A.
If $\xi = 1$, Eqs. (50) then reads as

$$\begin{aligned} \phi_b(\tau) &= \phi_a(\tau) + \phi_0(1, \tau) + \phi_1(1)\eta(\tau) + \phi_2(1)\chi(\tau) + \textstyle\sum_{i=1}^{\infty}\phi_{3i}(1)\Omega_i(\tau) \\ \psi_b(\tau) &= \psi_a(\tau) + \psi_0(1, \tau) + \psi_1(1)\eta(\tau) + \psi_2(1)\chi(\tau) + \textstyle\sum_{i=1}^{\infty}\psi_{3i}(1)\Omega_i(\tau) \end{aligned} \tag{51}$$

From Eqs. (51), we have

$$\dot{\phi}_b(\tau) = \dot{\phi}_a(\tau) + \dot{\phi}_0(1, \tau) + \phi_1(1)\dot{\eta}(\tau) + \phi_2(1)\dot{\chi}(\tau) + \sum_{i=1}^{\infty}\phi_{3i}(1)\dot{\Omega}_i(\tau)$$
$$\dot{\psi}_b(\tau) = \dot{\psi}_a(\tau) + \dot{\psi}_0(1, \tau) + \psi_1(1)\dot{\eta}(\tau) + \psi_2(1)\dot{\chi}(\tau) + \sum_{i=1}^{\infty}\psi_{3i}(1)\dot{\Omega}_i(\tau) \tag{52}$$

Setting $\tau = 0$ and utilizing Eqs. (45), $\eta(0), \chi(0), \dot{\eta}(0)$ and $\dot{\chi}(0)$ then can be determined from Eqs. (51) and (52).

$$\eta(0) = (b_1a_{22} - b_2a_{12})/W, \quad \chi(0) = (b_2a_{11} - b_1a_{21})/W$$
$$\dot{\eta}(0) = (b_3a_{22} - b_4a_{12})/W, \quad \dot{\chi}(0) = (b_4a_{11} - b_3a_{21})/W \tag{53}$$

where

$$a_{11} = \phi_1(1) + \sum_{i=1}^{\infty}h_{3i}\phi_{3i}(1), \quad a_{12} = \phi_1(1) + \sum_{i=1}^{\infty}h_{4i}\phi_{3i}(1)$$
$$a_{21} = \psi_1(1) + \sum_{i=1}^{\infty}h_{3i}\psi_{3i}(1), \quad a_{22} = \psi_1(1) + \sum_{i=1}^{\infty}h_{4i}\psi_{3i}(1)$$
$$b_1 = \phi_b(0) - \phi_a(0) - \phi_0(1, 0) - \sum_{i=1}^{\infty}h_{5i}\phi_{3i}(1)$$
$$b_2 = \psi_b(0) - \psi_a(0) - \psi_0(1, 0) - \sum_{i=1}^{\infty}h_{5i}\psi_{3i}(1) \tag{54}$$
$$b_3 = \dot{\phi}_b(0) - \dot{\phi}_a(0) - \dot{\phi}_0(1, 0) - \sum_{i=1}^{\infty}h_{6i}\phi_{3i}(1)$$
$$b_4 = \dot{\psi}_b(0) - \dot{\psi}_a(0) - \dot{\psi}_0(1, 0) - \sum_{i=1}^{\infty}h_{6i}\psi_{3i}(1), \quad W = a_{11}a_{22} - a_{12}a_{21}$$

Then $\Omega_{1i}(\tau)$ become a known function by substituting Eqs. (53) into the first of Eqs. (48). Substituting Eq. (47) into Eqs. (51), we have

$$Z_{11}\eta(\tau) + Z_{12}\chi(\tau) + \sum_{i=1}^{\infty}\int_0^{\tau}[Z_{13i}\eta(p) + Z_{14i}\chi(p)]\sin\omega_i(\tau - p)\mathrm{d}p = F_1(\tau)$$
$$Z_{21}\eta(\tau) + Z_{22}\chi(\tau) + \sum_{i=1}^{\infty}\int_0^{\tau}[Z_{23i}\eta(p) + Z_{24i}\chi(p)]\sin\omega_i(\tau - p)\mathrm{d}p = F_2(\tau) \tag{55}$$

where

$$Z_{11} = \phi_1(1) + \sum_{i=1}^{\infty}\phi_{3i}(1)h_{3i}, \quad Z_{12} = \phi_2(1) + \sum_{i=1}^{\infty}\phi_{3i}(1)h_{4i}$$
$$Z_{21} = \psi_1(1) + \sum_{i=1}^{\infty}\psi_{3i}(1)h_{3i}, \quad Z_{22} = \psi_2(1) + \sum_{i=1}^{\infty}\psi_{3i}(1)h_{4i}$$
$$Z_{13i} = \phi_{3i}(1)I_{1i}, \quad Z_{14i} = \phi_{3i}(1)I_{2i}, \quad Z_{23i} = \psi_{3i}(1)I_{1i}, \quad Z_{24i} = \psi_{3i}(1)I_{2i} \tag{56}$$
$$F_1(\tau) = \phi_b(\tau) - \phi_a(\tau) - \phi_0(1, \tau) - \sum_{i=1}^{\infty}\phi_{3i}(1)\Omega_{1i}(\tau)$$
$$F_2(\tau) = \psi_b(\tau) - \psi_a(\tau) - \psi_0(1, \tau) - \sum_{i=1}^{\infty}\psi_{3i}(1)\Omega_{1i}(\tau)$$

It is noted that Eqs. (55) is a system of second kind Volterra integral equations. The effective numerical approach for Eqs. (55) has been discussed detailedly in Ding and Wang (2005). The detailed solving procedure is presented in Appendix B.

5. Some special practical degeneration cases

The present solution can be directly degenerated into those for some special practical cases as:

(1) If we set $q_{31} = q_{32} = q_{32} = m_{33} = 0$ in Eq. (1) and omit the equations for magnetic fields, the present solution becomes that for non-homogeneous radially polarized, spherically isotropic piezoelectric hollow sphere;

(2) If we set $e_{31} = e_{32} = e_{32} = \varepsilon_{33} = 0$ in Eq. (1) and omit the equations for electric fields, the present solution becomes that for non-homogeneous radially polarized, spherically isotropic piezomagnetic hollow sphere;

(3) If we set both $q_{31} = q_{32} = q_{32} = m_{33} = 0$ and $e_{31} = e_{32} = e_{32} = \varepsilon_{33} = 0$ in Eq. (1) and omit the equations for magnetic and electric fields, the present solution becomes that for non-homogeneous purely elastic, spherically isotropic hollow sphere.

(4) If we set $N = 0$, the present solution becomes that for homogeneous radially polarized, spherically isotropic magneto-electro-elastic hollow sphere. Also, in cases (1)-(3), if we further set $N = 0$, the solutions then degenerated into those for corresponding homogeneous cases.

6. Numerical results and analysis

The dynamic responses of a functionally graded magneto-electro-elastic hollow sphere subjected to a dynamic pressure on the interior surface are considered in this section. The material constants are listed in Table 1 (Buchanan 2003).

The boundary conditions are

$$\begin{aligned} p_a(\tau) &= H(\tau), \quad p_b(\tau) = 0 \\ \phi_a(\tau) &= 0, \quad \phi_b(\tau) = 0 \\ \psi_a(\tau) &= 0, \quad \psi_b(\tau) = 0 \end{aligned} \tag{57}$$

where $H(\tau)$ is Heaviside function. In the demonstration, we set $s = a/b = 0.5$ and suppose the hollow sphere is motionless at $\tau = 0$, i.e., $u_0(\xi) = 0$ and $v_0(\xi) = 0$. Also, the time step length $\Delta\tau = 0.02$ and 40 terms in Eq. (35) are adopted in the calculation.

In the following numerical experiments, the material constants of purely elastic and piezoelectric hollow spheres are taken the corresponding values of the magneto-electro-elastic one.

Table 1 Material constants

C_{11}	C_{12}	C_{13}	C_{33}	E_{31}	E_{33}
218×10^9	120×10^9	120×10^9	215×10^9	-2.5	7.5
Q_{31}	Q_{33}	Λ_{33}	M_{33}	K_{33}	
265	345	5.8×10^{-9}	2.82×10^{-9}	95×10^{-6}	

Units: C_{ij}-Pa; E_{3i}-C/m^2; Q_{3i}-N/(Am); Λ_{33}-C^2/(Nm^2); M_{33}-Ns/(VC); K_{33}-Ns^2/C^2

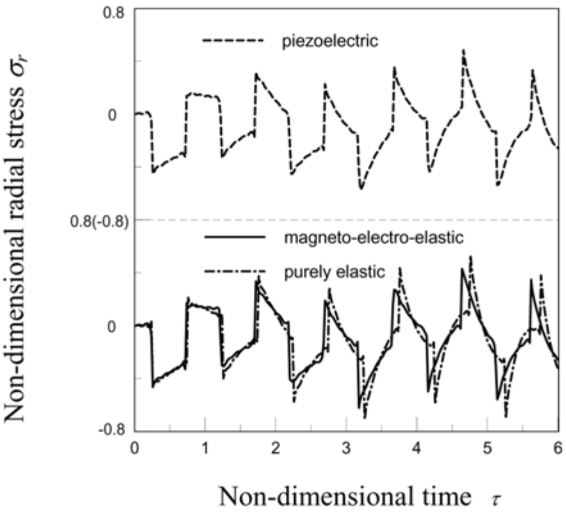

Fig. 2(a) Histories of radial stress σ_r of purely elastic, piezoelectric and magneto-electro-elastic hollow spheres at $\xi = 0.75$ for $N = -1$

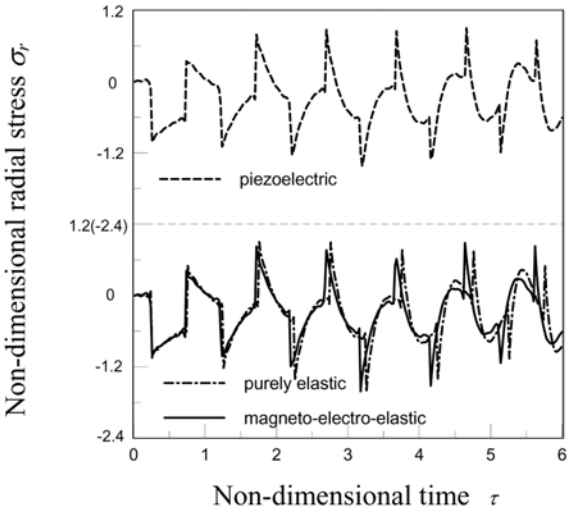

Fig. 2(b) Histories of radial stress σ_r of purely elastic, piezoelectric and magneto-electro-elastic hollow spheres at $\xi = 0.75$ for $N = 1$

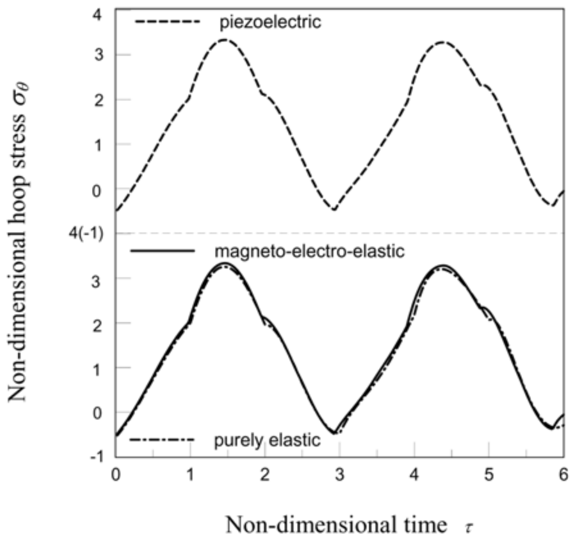

Fig. 3(a) Histories of hoop stress σ_θ of purely elastic, piezoelectric and magneto-electro-elastic hollow spheres at $\xi = 0.5$ for $N = -1$

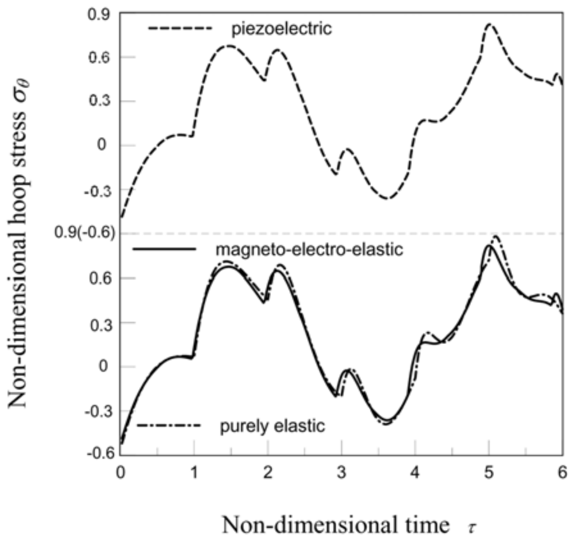

Fig. 3(b) Histories of hoop stress σ_θ of purely elastic, piezoelectric and magneto-electro-elastic hollow spheres at $\xi = 0.5$ for $N = 1$

Figs. 2(a) and 2(b) show the responses of radial stresses σ_r at the middle surface ($\xi = 0.75$) in functionally graded magneto-electro-elastic and purely elastic as well as piezoelectric hollow spheres for $N = -1$ and 1, respectively, subjected to a sudden constant pressure at the inner surface. From the curves, we can see that the radial stresses peaks periodically and at $\xi = 0.75$, the amplitudes of σ_r for $N = 1$ are larger than those for $N = -1$. For the same N, we find that the

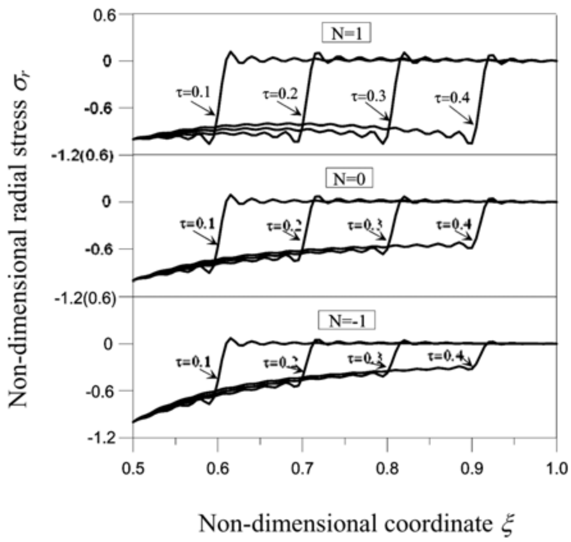

Fig. 4(a) Distributions of radial stress σ_r in magneto-electro-elastic hollow sphere for different *N*s at the very beginning stage I

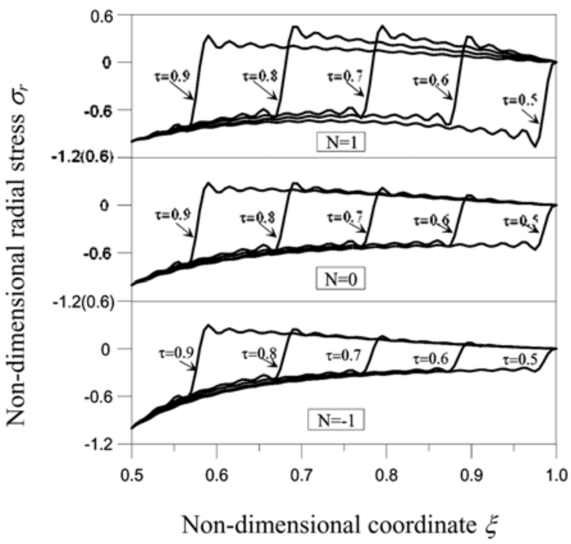

Fig. 4(b) Distributions of radial stress σ_r in magneto-electro-elastic hollow sphere for different *N*s at the very beginning stage II

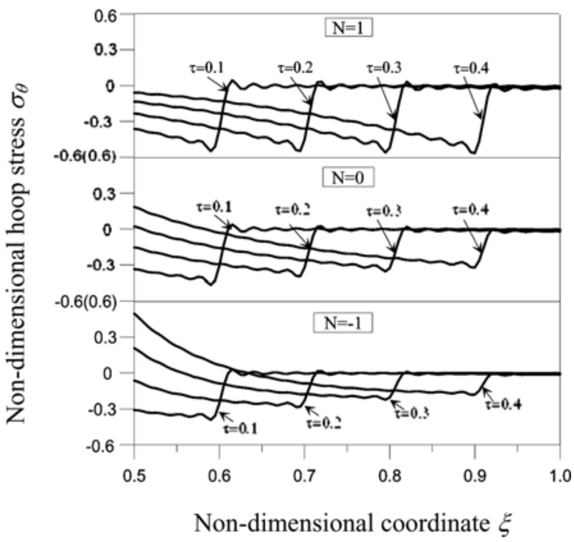

Fig. 5(a) Distributions of hoop stress σ_θ in magneto-electro-elastic hollow sphere for different *N*s at the very beginning stage I

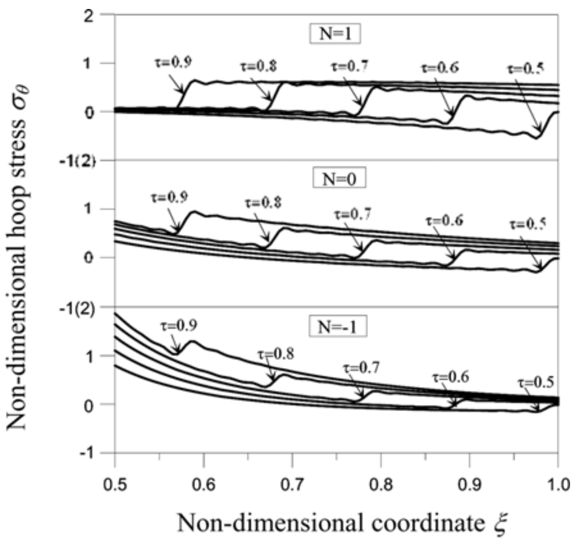

Fig. 5(b) Distributions of hoop stress σ_θ in magneto-electro-elastic hollow sphere for different *N*s at the very beginning stage II

responses of σ_r in piezoelectric hollow sphere are almost same as those in magneto-electro-elastic one. That is to say, the induced magnetic field has little influences on the responses of radial stresses. In order to show the results clearly, we present the responses of piezoelectric hollow sphere separately.

Figs. 3(a) and 3(b) depict the responses of hoop stresses σ_θ at the interior surface ($\xi = 0.5$) for $N = -1$ and 1, respectively. From the numerical results, we find that at the location $\xi = 0.5$, the

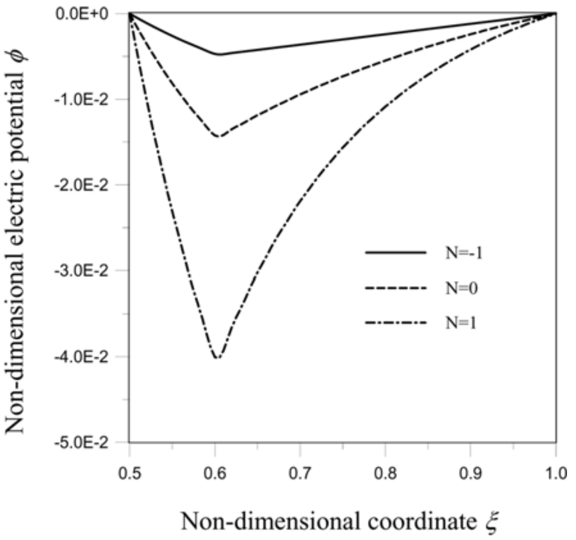

Fig. 6(a) Distributions of electric potential ϕ in magneto-electro-elastic hollow sphere for different Ns at $\tau = 0.1$

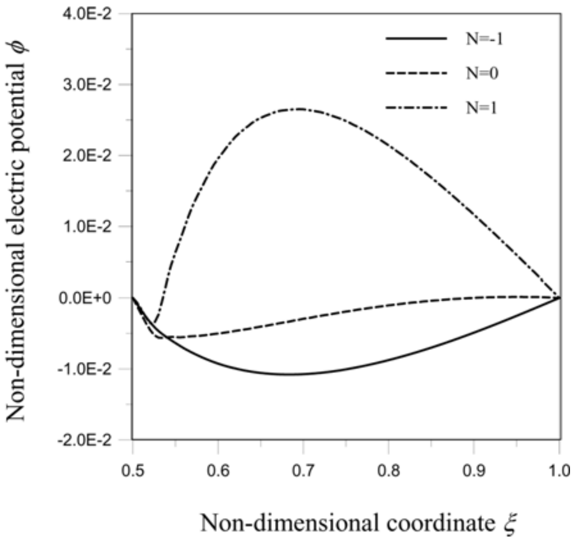

Fig. 6(b) Distributions of electric potential ϕ in magneto-electro-elastic hollow sphere for different Ns at $\tau = 1.0$

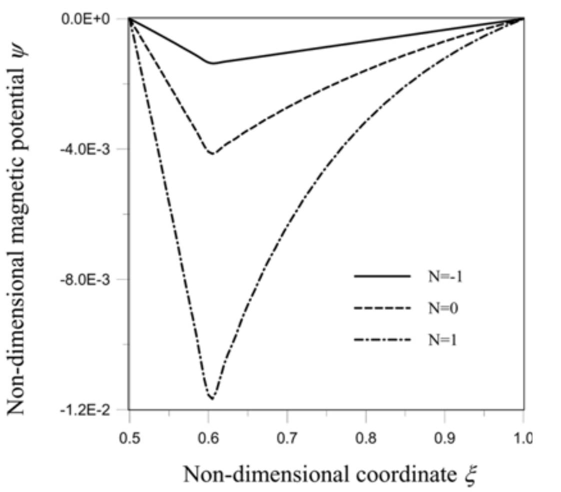

Fig. 7(a) Distributions of magnetic potential ψ in magneto-electro-elastic hollow sphere for different Ns at $\tau = 0.1$

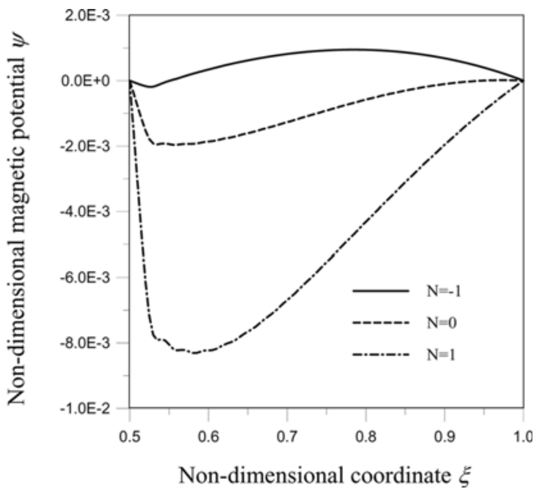

Fig. 7(b) Distributions of magnetic potential in magneto-electro-elastic hollow sphere ψ for different Ns at $\tau = 1.0$

maximum tensile stress of σ_θ for $N = -1$ is about 4 times larger than that for $N = 1$. Similarly, as noted for σ_r, the responses of σ_θ in piezoelectric hollow sphere are almost same as those in magneto-electro-elastic one. Also, we can conclude that the induced magnetic field has little influences on the responses of hoop stresses.

At the very beginning, the distributions of non-dimensional radial stresses and hoop stresses in magneto-electro-elastic hollow sphere for different Ns are shown, respectively, in Figs. 4(a) and 5(a) (stage I: $\tau = 0.1$, 0.2, 0.3 and 0.4) and Figs. 4(b) and 5(b) (stage II: $\tau = 0.5$, 0.6, 0.7, 0.8 and 0.9).

From the figures, we can find that the stress waves generate instantly after a sudden constant pressure acting on the interior surface and then propagate from the inner to outer. When the wavefronts reach the outer surface, the stress waves are then reflected and propagate along the opposite direction. We also find an interesting phenomenon that in spite of different values of N, the stress waves propagate with the same velocity.

Figs. 6(a),(b) and Figs. 7(a),(b) illustrate the distributions of non-dimensional electric potential ϕ and magnetic potential ψ at $\tau = 0.1$ and 1.0 in magneto-electro-elastic hollow spheres. We catch that at the very beginning ($\tau = 0.1$), the distribution form of electric potential and magnetic potential is similar with each other for different Ns and the amplitude of ϕ and ψ increases dramatically with the increasing of N. In Figs. 6(a) and 7(a), a sharp kink is clearly observed at the location around $\xi = 0.6$. Synchronously, in Figs. 4(a) and 5(a), we notice that the stress wavefronts arrive at the location around $\xi = 0.6$ at the time $\tau = 0.1$. So we can deduce that: at the very beginning stage, due to the special magneto-electro-elastic coupling effect, the electric potential ϕ and the magnetic potential ψ will reach the extremum values at the location when the stress wave wavefront arrives. While with the time processing, the distribution form of electric potential and magnetic potential for different Ns becomes really different, see Figs. 6(b) and 7(b).

7. Conclusions

The dynamic problem of a functionally graded magneto-electro-elastic hollow sphere in the state of spherically symmetric case is successfully transformed into two Volterra integral equations of the second kind with respect to two time functions. Interpolation method is introduced to solve the Volterra integral equations efficiently.

Numerical tests for functionally graded hollow sphere made of three different materials (magneto-electro-elastic, piezoelectric and purely elastic) subjected to a sudden constant pressure at the interior surface are presented. It is noted that the responses in functionally graded magneto-electro-elastic hollow sphere are almost same as those in functionally graded piezoelectric one. Then we can deduce that the induced magnetic field has little influences on the responses of stress. Furthermore, the stress wave in the functionally graded magneto-electro-elastic hollow sphere propagates with the same velocity in spite of the variation of N.

From the definition Eq. (5), we know that the material constants increasing gradually from the inner to outer for $N > 0$ and it is the contrary for $N < 0$. Also, the special case for $N = 0$ denote that the material is homogeneous. Numerical experiments indicate that the parameter N has important effect on elastic, electric and magnetic fields in functionally graded magneto-electro-elastic hollow sphere. In other words, it is feasible to design an optimal model according to practical application by adjusting the parameter N.

The present method is suitable for a functionally graded magneto-electro-elastic hollow sphere subjected to arbitrary spherically symmetric mechanical, electric and magnetic loads.

Acknowledgments

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Appendix A

The specified expressions of $\phi_0(\xi, \tau)$, $\phi_1(\xi)$, $\phi_2(\xi)$, $\phi_{3i}(\xi)$ and $\psi_0(\xi, \tau)$, $\psi_1(\xi)$, $\psi_2(\xi)$, $\psi_{3i}(\xi)$ are

$$\phi_0(\xi, \tau) = 2A_{11}H_{01}(\xi, \tau) + A_{12}H_{02}(\xi, \tau), \quad \psi_0(\xi, \tau) = 2A_{21}H_{01}(\xi, \tau) + A_{22}H_{02}(\xi, \tau)$$

$$\phi_1(\xi) = 2A_{11}H_{11}(\xi) + A_{12}H_{12}(\xi) - A_3H(\xi), \quad \psi_1(\xi) = 2A_{21}H_{11}(\xi) + A_{22}H_{12}(\xi) + A_4H(\xi)$$

$$\phi_2(\xi) = 2A_{11}H_{21}(\xi) + A_{12}H_{22}(\xi) + A_4H(\xi), \quad \psi_2(\xi) = 2A_{21}H_{21}(\xi) + A_{22}H_{22}(\xi) - A_3H(\xi)$$

$$\phi_{3i}(\xi) = 2A_{11}H_{31i}(\xi) + A_{12}H_{32i}(\xi), \quad \psi_{3i}(\xi) = 2A_{21}H_{31i}(\xi) + A_{22}H_{32i}(\xi)$$

$$H_{01}(\xi,\tau) = \int_s^{\xi} \xi^{-\left(N+\frac{3}{2}\right)} f_0(\xi,\tau)\mathrm{d}\xi, \quad H_{02}(\xi) = \xi^{-\left(N+\frac{1}{2}\right)} f_0(\xi,\tau) - s^{-\left(N+\frac{1}{2}\right)} f_0(s,\tau)$$

$$H_{11}(\xi,\tau) = \int_s^{\xi} \xi^{-\left(N+\frac{3}{2}\right)} f_1(\xi)\mathrm{d}\xi, \quad H_{12}(\xi) = \xi^{-\left(N+\frac{1}{2}\right)} f_1(\xi) - s^{-\left(N+\frac{1}{2}\right)} f_1(s)$$

$$H_{21}(\xi,\tau) = \int_s^{\xi} \xi^{-\left(N+\frac{3}{2}\right)} f_2(\xi)\mathrm{d}\xi, \quad H_{22}(\xi) = \xi^{-\left(N+\frac{1}{2}\right)} f_2(\xi) - s^{-\left(N+\frac{1}{2}\right)} f_2(s)$$

$$H_{31i}(\xi,\tau) = \int_s^{\xi} \xi^{-\left(N+\frac{3}{2}\right)} R_i(\xi)\mathrm{d}\xi, \quad H_{32i}(\xi) = \xi^{-\left(N+\frac{1}{2}\right)} R_i(\xi) - s^{-\left(N+\frac{1}{2}\right)} R_i(s)$$

$$H(\xi) = \begin{cases} [s^{-(2N+1)} - \xi^{-(2N+1)}]/(2N+1) & N \neq -1/2 \\ \ln(\xi/s) & N = -1/2 \end{cases} \tag{58}$$

Appendix B

The detailed procedure for solving Eqs. (55) is presented in this Appendix. The time interval $[0, \tau_n]$ is firstly divided into n subintervals with discrete time points $\tau_0 = 0, \tau_1, \tau_2, \ldots, \tau_n$. Then at each time interval $[\tau_{j-1}, \tau_j]$, a cubic Hermite polynomial is introduced to approximate the unknown functions $\eta(\tau)$ and $\chi(\tau)$ as

$$\begin{aligned} \eta(\tau) &= H_{0j}(\tau)\eta(\tau_{j-1}) + H_{1j}(\tau)\eta(\tau_j) + H_{2j}(\tau)\dot{\eta}(\tau_{j-1}) + H_{3j}(\tau)\dot{\eta}(\tau_j) \\ \chi(\tau) &= H_{0j}(\tau)\chi(\tau_{j-1}) + H_{1j}(\tau)\chi(\tau_j) + H_{2j}(\tau)\dot{\chi}(\tau_{j-1}) + H_{3j}(\tau)\dot{\chi}(\tau_j) \end{aligned} \quad (j = 1, 2, \ldots, n) \tag{59}$$

where

$$H_{0j}(\tau) = \left(1 + 2\frac{\tau - \tau_{j-1}}{\tau_j - \tau_{j-1}}\right)\left(\frac{\tau - \tau_j}{\tau_j - \tau_{j-1}}\right)^2, \quad H_{1j}(\tau) = \left(1 + 2\frac{\tau_j - \tau}{\tau_j - \tau_{j-1}}\right)\left(\frac{\tau - \tau_{j-1}}{\tau_j - \tau_{j-1}}\right)^2$$

$$H_{2j}(\tau) = (\tau - \tau_{j-1})\left(\frac{\tau - \tau_j}{\tau_j - \tau_{j-1}}\right)^2, \quad H_{3j}(\tau) = (\tau - \tau_j)\left(\frac{\tau - \tau_{j-1}}{\tau_j - \tau_{j-1}}\right)^2 \quad (j = 1, 2, \ldots, n) \tag{60}$$

The derivatives of Eqs. (55) are

$$\begin{aligned} Z_{11}\dot{\eta}(\tau) + Z_{12}\dot{\chi}(\tau) + \textstyle\sum_{i=1}^{\infty}\int_0^{\tau}[Z_{13i}\eta(p) + Z_{14i}\chi(p)]\omega_i\cos\omega_i(\tau-p)\mathrm{d}p &= \dot{F}_1(\tau) \\ Z_{21}\dot{\eta}(\tau) + Z_{22}\dot{\chi}(\tau) + \textstyle\sum_{i=1}^{\infty}\int_0^{\tau}[Z_{23i}\eta(p) + Z_{24i}\eta(p)]\omega_i\cos\omega_i(\tau-p)\mathrm{d}p &= \dot{F}_2(\tau) \end{aligned} \tag{61}$$

Setting $\tau = \tau_j$ and utilizing Eqs. (59), the following equations can be obtained form Eqs. (55) and (61).

$$\begin{aligned} F_1(\tau_j) = {} & Z_{11}\eta(\tau_j) + \textstyle\sum_{i=1}^{\infty} Z_{13i}\sum_{k=1}^{j}[L_{0ijk}\eta(\tau_{k-1}) + L_{1ijk}\eta(\tau_k) + L_{2ijk}\dot{\eta}(\tau_{k-1}) + L_{3ijk}\dot{\eta}(\tau_k)] \\ & + Z_{12}\chi(\tau_j) + \textstyle\sum_{i=1}^{\infty} Z_{14i}\sum_{k=1}^{j}[L_{0ijk}\chi(\tau_{k-1}) + L_{1ijk}\chi(\tau_k) + L_{2ijk}\dot{\chi}(\tau_{k-1}) + L_{3ijk}\dot{\chi}(\tau_k)] \\ F_2(\tau_j) = {} & Z_{21}\eta(\tau_j) + \textstyle\sum_{i=1}^{\infty} Z_{23i}\sum_{k=1}^{j}[L_{0ijk}\eta(\tau_{k-1}) + L_{1ijk}\eta(\tau_k) + L_{2ijk}\dot{\eta}(\tau_{k-1}) + L_{3ijk}\dot{\eta}(\tau_k)] \\ & + Z_{22}\chi(\tau_j) + \textstyle\sum_{i=1}^{\infty} Z_{24i}\sum_{k=1}^{j}[L_{0ijk}\chi(\tau_{k-1}) + L_{1ijk}\chi(\tau_k) + L_{2ijk}\dot{\chi}(\tau_{k-1}) + L_{3ijk}\dot{\chi}(\tau_k)] \end{aligned} \tag{62}$$

$$\begin{aligned}\dot{F}_1(\tau_j) &= Z_{11}\dot{\eta}(\tau_j) + \sum_{i=1}^{\infty} Z_{13i}\sum_{k=1}^{j}[K_{0ijk}\eta(\tau_{k-1}) + K_{1ijk}\eta(\tau_k) + K_{2ijk}\dot{\eta}(\tau_{k-1}) + K_{3ijk}\dot{\eta}(\tau_k)]\\ &\quad + Z_{12}\dot{\chi}(\tau_j) + \sum_{i=1}^{\infty} Z_{14i}\sum_{k=1}^{j}[K_{0ijk}\chi(\tau_{k-1}) + K_{1ijk}\chi(\tau_k) + K_{2ijk}\dot{\chi}(\tau_{k-1}) + K_{3ijk}\dot{\chi}(\tau_k)]\\ \dot{F}_2(\tau_j) &= Z_{21}\dot{\eta}(\tau_j) + \sum_{i=1}^{\infty} Z_{23i}\sum_{k=1}^{j}[K_{0ijk}\eta(\tau_{k-1}) + K_{1ijk}\eta(\tau_k) + K_{2ijk}\dot{\eta}(\tau_{k-1}) + K_{3ijk}\dot{\eta}(\tau_k)]\\ &\quad + Z_{22}\dot{\chi}(\tau_j) + \sum_{i=1}^{\infty} Z_{24i}\sum_{k=1}^{j}[K_{0ijk}\chi(\tau_{k-1}) + K_{1ijk}\chi(\tau_k) + K_{2ijk}\dot{\chi}(\tau_{k-1}) + K_{3ijk}\dot{\chi}(\tau_k)]\end{aligned} \tag{63}$$

where

$$\begin{gathered} L_{lijk} = \int_{\tau_{k-1}}^{\tau_k} H_{lk}(p)\sin\omega_i(\tau_j - p)\mathrm{d}p, \quad K_{lijk} = \int_{\tau_{k-1}}^{\tau_k} \omega_i H_{lk}(p)\cos\omega_i(\tau_j - p)\mathrm{d}p \\ (l = 0, 1, 2, 3;\ k = 1, 2, \ldots, j;\ j = 1, 2, \ldots, n)\end{gathered} \tag{64}$$

Eqs. (62) and (63) can be reorganized in a matrix form as

$$\begin{bmatrix} d_{11}^j & d_{12}^j & d_{13}^j & d_{14}^j \\ d_{21}^j & d_{22}^j & d_{23}^j & d_{24}^j \\ d_{31}^j & d_{32}^j & d_{33}^j & d_{34}^j \\ d_{41}^j & d_{42}^j & d_{43}^j & d_{44}^j \end{bmatrix} \begin{Bmatrix} \eta(\tau_j) \\ \chi(\tau_j) \\ \dot{\eta}(\tau_j) \\ \dot{\chi}(\tau_j) \end{Bmatrix} = \begin{Bmatrix} B_1^j \\ B_2^j \\ B_3^j \\ B_4^j \end{Bmatrix} \quad (j = 1, 2, \ldots, n) \tag{65}$$

where

$$\begin{aligned} &d_{11}^j = Z_{11} + \sum_{i=1}^{\infty} Z_{13i}L_{1ijj}, \quad d_{12}^j = Z_{12} + \sum_{i=1}^{\infty} Z_{14i}L_{1ijj}, \quad d_{13}^j = \sum_{i=1}^{\infty} Z_{13i}L_{3ijj}, \quad d_{14}^j = \sum_{i=1}^{\infty} Z_{14i}L_{3ijj}\\ &d_{21}^j = Z_{21} + \sum_{i=1}^{\infty} Z_{23i}L_{1ijj}, \quad d_{22}^j = Z_{22} + \sum_{i=1}^{\infty} Z_{24i}L_{1ijj}, \quad d_{23}^j = \sum_{i=1}^{\infty} Z_{23i}L_{3ijj}, \quad d_{24}^j = \sum_{i=1}^{\infty} Z_{24i}L_{3ijj}\\ &d_{31}^j = \sum_{i=1}^{\infty} Z_{13i}K_{1ijj}, \quad d_{32}^j = \sum_{i=1}^{\infty} Z_{14i}K_{1ijj}, \quad d_{33}^j = Z_{11} + \sum_{i=1}^{\infty} Z_{13i}K_{3ijj}, \quad d_{34}^j = Z_{12} + \sum_{i=1}^{\infty} Z_{14i}K_{3ijj}\\ &d_{41}^j = \sum_{i=1}^{\infty} Z_{23i}K_{1ijj}, \quad d_{42}^j = \sum_{i=1}^{\infty} Z_{24i}K_{1ijj}, \quad d_{43}^j = Z_{21} + \sum_{i=1}^{\infty} Z_{23i}K_{3ijj}, \quad d_{44}^j = Z_{22} + \sum_{i=1}^{\infty} Z_{24i}K_{3ijj}\\ B_1^j &= F_1(\tau_j) - \sum_{i=1}^{\infty} Z_{13i}\Big(\sum_{k=1}^{j}[L_{0ijk}\eta(\tau_{k-1}) + L_{2ijk}\dot{\eta}(\tau_{k-1})] + \sum_{k=1}^{j-1}[L_{1ijk}\eta(\tau_k) + L_{3ijk}\dot{\eta}(\tau_k)]\Big)\\ &\quad - \sum_{i=1}^{\infty} Z_{14i}\Big(\sum_{k=1}^{j}[L_{0ijk}\chi(\tau_{k-1}) + L_{2ijk}\dot{\chi}(\tau_{k-1})] + \sum_{k=1}^{j-1}[L_{1ijk}\chi(\tau_k) + L_{3ijk}\dot{\chi}(\tau_k)]\Big)\\ B_2^j &= F_2(\tau_j) - \sum_{i=1}^{\infty} Z_{23i}\Big(\sum_{k=1}^{j}[L_{0ijk}\eta(\tau_{k-1}) + L_{2ijk}\dot{\eta}(\tau_{k-1})] + \sum_{k=1}^{j-1}[L_{1ijk}\eta(\tau_k) + L_{3ijk}\dot{\eta}(\tau_k)]\Big)\\ &\quad - \sum_{i=1}^{\infty} Z_{24i}\Big(\sum_{k=1}^{j}[L_{0ijk}\chi(\tau_{k-1}) + L_{2ijk}\dot{\chi}(\tau_{k-1})] + \sum_{k=1}^{j-1}[L_{1ijk}\chi(\tau_k) + L_{3ijk}\dot{\chi}(\tau_k)]\Big)\\ B_3^j &= \dot{F}_1(\tau_j) - \sum_{i=1}^{\infty} Z_{13i}\Big(\sum_{k=1}^{j}[K_{0ijk}\eta(\tau_{k-1}) + K_{2ijk}\dot{\eta}(\tau_{k-1})] + \sum_{k=1}^{j-1}[K_{1ijk}\eta(\tau_k) + K_{3ijk}\dot{\eta}(\tau_k)]\Big)\\ &\quad - \sum_{i=1}^{\infty} Z_{14i}\Big(\sum_{k=1}^{j}[K_{0ijk}\chi(\tau_{k-1}) + K_{2ijk}\dot{\chi}(\tau_{k-1})] + \sum_{k=1}^{j-1}[K_{1ijk}\chi(\tau_k) + K_{3ijk}\dot{\chi}(\tau_k)]\Big)\\ B_4^j &= \dot{F}_2(\tau_j) - \sum_{i=1}^{\infty} Z_{23i}\Big(\sum_{k=1}^{j}[K_{0ijk}\eta(\tau_{k-1}) + K_{2ijk}\dot{\eta}(\tau_{k-1})] + \sum_{k=1}^{j-1}[K_{1ijk}\eta(\tau_k) + K_{3ijk}\dot{\eta}(\tau_k)]\Big)\\ &\quad - \sum_{i=1}^{\infty} Z_{24i}\Big(\sum_{k=1}^{j}[K_{0ijk}\chi(\tau_{k-1}) + K_{2ijk}\dot{\chi}(\tau_{k-1})] + \sum_{k=1}^{j-1}[K_{1ijk}\chi(\tau_k) + K_{3ijk}\dot{\chi}(\tau_k)]\Big)\end{aligned} \tag{66}$$

It is noted here that the terms involving $\sum_{k=1}^{j-1}(\ )$ in $B_l^j (l = 1, 2, 3, 4)$ for $j = 1$ are treated as zero in the computation. Since $\eta(0)$, $\dot{\eta}(0)$, $\chi(0)$ and $\dot{\chi}(0)$ have been determined in Eqs. (53), then $\eta(\tau_j)$, $\dot{\eta}(\tau_j)$, $\chi(\tau_j)$ and $\dot{\chi}(\tau_j)$ $(j = 1, 2, \ldots, n)$ can be obtained step by step from Eqs. (65).