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**Technical Note**

# Numerical method for the undamped forced dynamics of steel cable network structures

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## 1. Introduction

Steel cable network structures are characterized by strong geometric nonlinearities and high flexibility, which are the main causes of their unexpected behavior, especially under dynamic loading conditions (Levy and Spillers 1995). Excluding cable-stayed and cable-suspended structural systems as well as low-tension cable nets, the majority of static and dynamic analysis techniques used are approximate and complex in nature, while their approaches differ significantly. There are five widely accepted methodologies dealing with the foregoing structural systems, with their salient features summarized in the work of Kwan (1998), while combinations, variations and extensions of these, taking into account numerous parameters, such as slackening, friction, pre-stress etc., have also been reported (Goslin and Korban 2001, Kanno *et al.* 2002, Kanno and Ohsaki 2005, Talvik 2001, Volokh *et al.* 2003, and others not cited herein). To this end, in the present study a numerical technique for the undamped dynamics of cable networks is proposed, in which pretension of cable elements, uniform time dependent temperature change as well as flexibility and temporal dependence of boundary conditions are accounted for. Applications of the method produce results in very good agreement with existing ones and important conclusions for structural design purposes are drawn.

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## 2. Description of the method

### 2.1 General considerations and basic equations

We consider a steel cable network structure with given initial geometry, each member of which can transfer only tensile forces, may undergo large displacements combined with small strains, while the whole system is considered simply supported, with generally time-varying boundary conditions. Each member is discretized into $n$ straight pinned jointed elements, while the dynamic loading is also discretized in a similar manner. A uniform temperature change may also act on the system, which is considered initially at rest, i.e. in static equilibrium due only to self-weight and possible pre-tension. Denoting as $u_i(t)$, $v_i(t)$, $w_i(t)$ the dynamic displacement components of joint $i$ (along the $x$, $y$ and $z$ direction), the physical and geometrical equations valid for element $ik$ are given by:

$$\varepsilon_{ik}(t) = \frac{\ell_{ik} - \overset{0}{\ell}_{ik}}{\overset{0}{\ell}_{ik}} + \alpha_{ik}T_{ik} + \overset{\Delta}{\varepsilon}_{ik}, \quad N_{ik} = E_{ik}A_{ik}\varepsilon_{ik}, \quad i, k = 1, 2, \ldots, n, \quad t \in [0, \infty) \qquad \text{(1a,b)}$$

In the above, $a_{ik}$ is the coefficient of linear thermal expansion, $T_{ik}$ the increase of temperature, $\overset{\Delta}{\varepsilon}_{ik}$ preliminary (initial strains), $N_{ik}$ the axial force developing on element $ik$, $A_{ik}$ its cross-sectional area and $E_{ik}$ Young's modulus of elasticity, while $\overset{0}{\ell}_{ik}$, $\ell_{ik}$ represent the element's length before and after deformation. Under a dynamic external loading $P_i(t)$, acting on the $i$-th joint, after setting as $m_i$ discretized concentrated masses on every joint $i$, the undamped motion is governed by the following set of differential equations:

$$m_i\ddot{u}_i = \sum_{k=1}^{m} N_{ik}\frac{x_k - x_i}{\ell_{ik}} + P_{x_i}, \quad m_i\ddot{v}_i = \sum_{k=1}^{m} N_{ik}\frac{y_k - y_i}{\ell_{ik}} + P_{y_i}, \quad m_i\ddot{w}_i = \sum_{k=1}^{m} N_{ik}\frac{z_k - z_i}{\ell_{ik}} + P_{z_i} \qquad (2)$$

The corresponding boundary conditions, evaluated at the $j$ bearing joints, can be written in the form of:

$$\tilde{x}_j(t) = \tilde{x}_j^0 + \tilde{u}_j^0(t) + S_{xj}(t)F_{xj}(t), \;\; \tilde{y}_j(t) = \tilde{y}_j^0 + \tilde{v}_j^0(t) + S_{yj}(t)F_{yj}(t), \;\; \tilde{z}_j(t) = \tilde{y}_j^0 + \tilde{w}_j^0(t) + S_{zj}(t)F_{zj}(t) \qquad (3)$$

where $\tilde{x}_j^0, \tilde{y}_j^0, \tilde{z}_j^0$ are the known co-ordinates of the supporting joints in the initial configuration, $\tilde{u}_j^0$, $\tilde{v}_j^0$, $\tilde{w}_j^0$ also known displacements of these joints, varying in time, $S_{xj}$, $S_{yj}$, $S_{zj}$ components of the reaction forces and $F_{xj}$, $F_{yj}$, $F_{zj}$ flexibility of the supports. Furthermore, the most general case of initial conditions is considered and prescribed below:

$$u_i(0) = \overset{0}{u}_i, \;\; \dot{u}_i(0) = \overset{0}{\dot{u}}_i, \;\; v_i(0) = \overset{0}{v}_i, \;\; \dot{v}_i(0) = \overset{0}{\dot{v}}_i, \;\; w_i(0) = \overset{0}{w}_i, \;\; \dot{w}_i(0) = \overset{0}{\dot{w}}_i \qquad (4)$$

### 2.2 Numerical procedure

Eliminating the inertia terms from Eqs. (2), we obtain the static equilibrium equations which are treated numerically through an iterative process, in each consecutive step of which the new co-ordinates $x_i^k, y_i^k, z_i^k$ are computed as $x_i^k = x_i^{k-1} + \Delta x_i\xi$, $y_i^k = y_i^{k-1} + \Delta y_i\xi$, $z_i^k = z_i^{k-1} + \Delta z_i\xi$, where $k$ denotes the number of consecutive iterations and $\xi$ may be thought of as a *super-relaxation* coefficient; for the requirements of the proposed method a value of $\xi = 1$ is used, which has been proven quite adequate for rapid convergence and less computer time. Performing a direct numerical integration of the motion equations (Hoffman 2001), their final form for the $i$-th joint, where $m$ elements meet, is

$$\sum_{k=1}^{m} E_{ik}A_{ik}\left[\frac{\ell_{ik}^{t}-\ell_{ik}^{0}}{\ell_{ik}^{0}}-\alpha_{ik}T_{ik}-\varepsilon_{ik}^{\Delta}\right]\frac{s_{k}^{t}-s_{i}^{t}}{\ell_{ik}^{t}}+P_{x_i}^{t} = m_i\ddot{s}_i^{t}, \quad s = x, y, z \tag{5}$$

Setting as $g_i$ the dynamic displacement vector of joint $i$, its time derivatives are approximated by:

$$\dot{g}_j^{t} = \frac{c_1}{c_2 h}(g_j^{t}-g_j^{t-1})+\left(1-\frac{c_1}{c_2}\right)\dot{g}_j^{t-1}+h\left(1-\frac{c_1}{2c_2}\right)\ddot{g}_j^{t-1}$$

$$\ddot{g}_j^{t} = \frac{1}{c_2 h^2}\left(g_j^{t}-g_j^{t-1}-\dot{g}_j^{t-1}h+h^2\left(\frac{1}{2}-c_2\right)\ddot{g}_j^{t-1}\right), \quad j = i, k \tag{6a,b}$$

where $h$ is the integration step and $c_1$, $c_2$ parameters of the method.

## 3. Applications of the proposed method

### 3.1 Saddle cable network

The first application deals with a steel cable network having the shape of a saddle, for which a

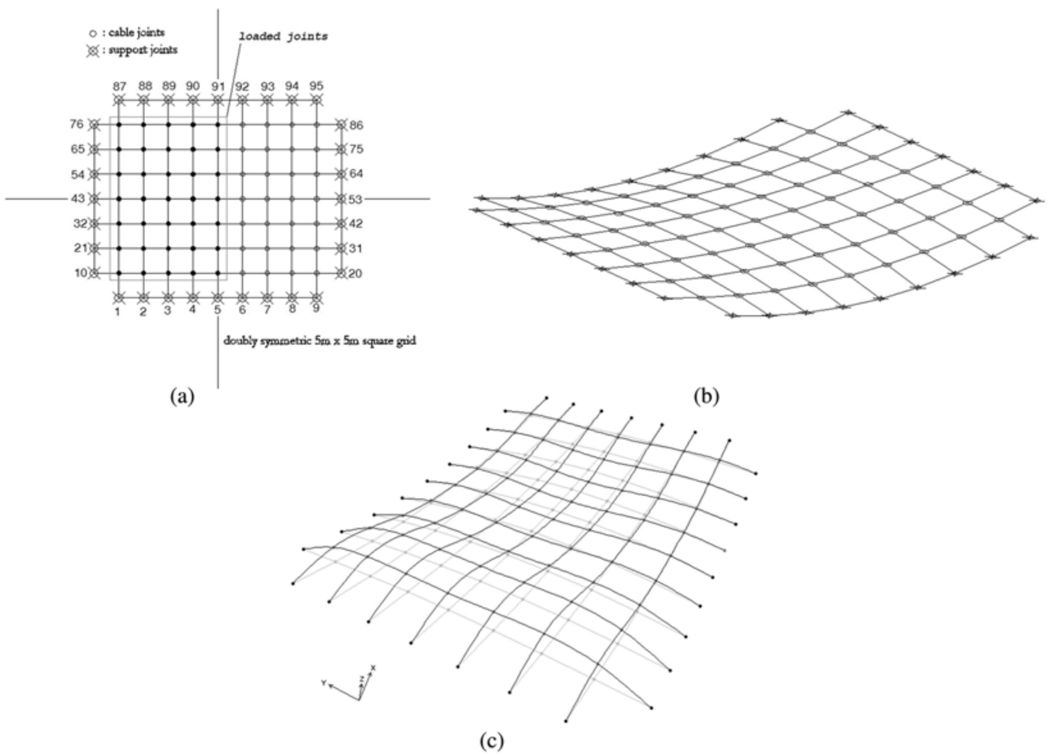
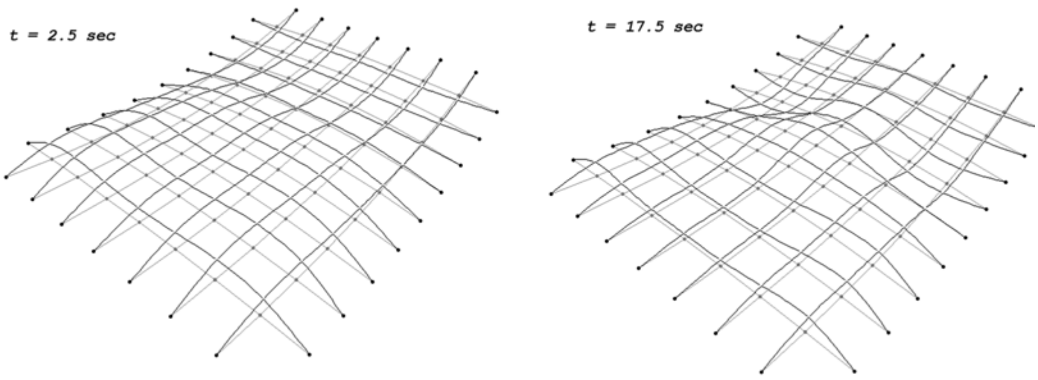
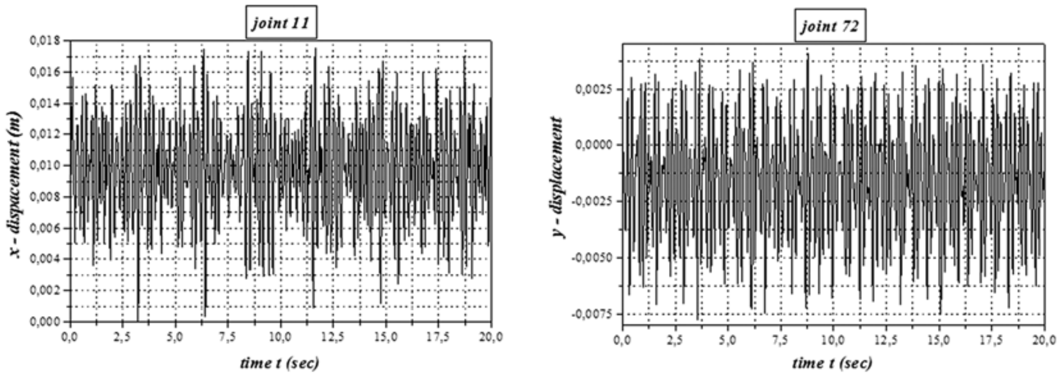



Fig. 1 Saddle cable network configuration (a,b) and corresponding static response (c)




(a)




(b)

Fig. 2 Dynamic response of the saddle net

variety of theoretical results have been presented (Kwan 1998). As depicted in Figs. 1(a), (b), it consists of a total of 95 nodes (25 supporting ones). All cable segments have $EA$ = 44.982 MN and possess an initial pretension of 60 kN in magnitude. The structure is acted upon by a step point conservative loading of infinite duration at joints 11-15, 22-26, 33-37, 44-48, 55-59, 66-70 and 77-81 with $P_x$ = 1 kN, $P_y$ = 0.00 kN and $P_z$ = 1 kN. The static response results obtained are in excellent agreement with previously reported ones (Kwan 1998) for the displacements and for the cable tension forces after loading, with the static configuration of the network shown in Fig. 1(c). The method also captures the undamped dynamic response of the structure; in Fig. 2(a), the dynamic displacement of the network at two time incidents is shown, while in Fig. 2(b) the time series plots of displacement components for joints 11 and 72 are depicted. Periodic motions around the static equilibria are reported and it is anticipated that if damping is also accounted for, the transients would finally decay to the expected point attractor of the static case.

### 3.2 Spider - web like cable network

The 2nd application refers to an initially plane steel cable network having a spider-web form. As

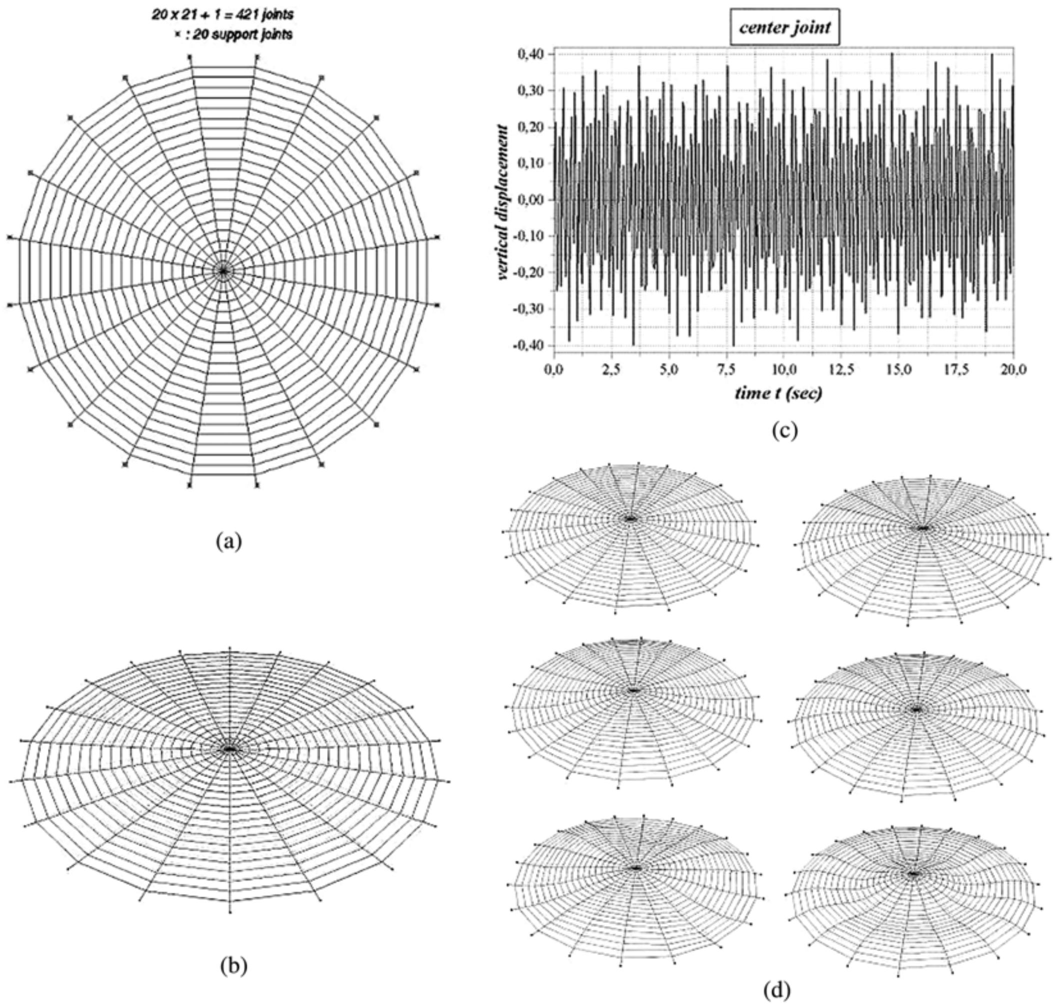



Fig. 3 Spider-web cable net geometry and corresponding static and dynamic response

shown in Fig. 3(a), the 20 perimeter nodes coincide with the vertexes of a canonical icosaplevron inscribed in a 10.5 m diameter circle, while the inner icosaplevra are inscribed in corresponding circles increasing with a 0.5 m ratio in diameter to the center. The longitudinal rigidity of the 20 radial members is equal to 224.4 MN while the rigidity of the parallel members is equal to 149.06 MN. Preliminary tension by progressive shortening of all parallel cable segments is realized in steps as follows: by 0.1% of their length on the 1st external layer, 0.2% for the 2nd interior layer and so on up to 2% for the middle cables. The structure is acted upon on the center node by a suddenly applied constant directional loading (with gravitational direction) of infinite duration with magnitude of $P$ = 2500 kN. The static configuration was determined without significant difficulties, yielding a center node vertical deformation of about 0.40 m, as qualitatively shown in Fig. 3(b). This node exhibits closed vertical vibrations around its static equilibrium, illustrated in the time series plot of Fig. 3(c), while all nodes also undergo in plane vibrations along $x$ and $y$ axes, of rather small amplitude. This is due to the geometric nonlinearity arising from the differential action of the parallel and radial cables and shown in Fig. 3(d), where dynamic deformation patterns of the spider-web network are given. A nonlinear radial increase (starting from the center node) of the joints' vertical deformation was established, while the maximum cable tensile forces appear in radial cable segments, significantly larger than the ones developing on the parallel members.

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