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Sensitivity analysis for seismic response of a ship-block system

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Abstract. In this paper, seismic response of a free-standing ship located in a dry dock and supported by an arrangement of n keel blocks due to base excitation is addressed. Formulation of the problem including derivation of governing equations in various modes of motion as well as transition conditions from one mode to another is given in Moghaddasi and Bargi (2006) by same authors. On the base of numerical solution for presented formulation, several numbers of analyses are conducted to study sensitivity of system's responses to some major contributing parameters. These parameters include friction coefficients between contacting surfaces, block dimensions, peak ground acceleration, and the magnitude of vertical ground acceleration. Finally, performance of a system with usual parameters normally encountered in design is investigated.

Keywords: free-standing ship; keel blocks; coulomb friction; seismic response; sensitivity analysis.

1. Introduction

Free-standing bodies can slide or rock during an earthquake. A very important application and usual example of such a system is a free-standing massive ship supported by an arrangement of n keel blocks which are located in a dry dock: a ship-block system. An earthquake motion may cause movement and instability in this system, and consequently can cause significant damage to the dry dock structure and to the ship itself. Therefore, it is of major importance to study in depth behavior of such systems during an earthquake motion.

Behavior of a single block on a rigid base is studied by several authors. Ishiyama (1982) and Shenton *et al.* (1991) have classified the response of a single block to horizontal and vertical earthquake excitation. In another work, Taniguchi (2002) has studied nonlinear response of the same problem with describing response calculation methods, and time-history response evaluation of that system. In another attempt, Soong and Garcia (2003) have investigated sliding of free-standing bocks. In that presentation, they investigated a sliding failure of those blocks.

In this paper, the authors study the behavior of a ship-block system in depth. Studying the behavior of a ship-block system excited in base is generally complicated and some simplifying assumptions are to be incorporated in formulation. Major simplifying assumptions made throughout

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this work are: analysis is limited in a two dimension, motion of the system is considered by eight different modes of motion which are: Rest (relative), Sliding of keel blocks, Rocking of keel blocks, Sliding of the ship, Sliding of both keel blocks and the ship, Sliding and rocking of keel blocks, Rocking of keel blocks with sliding of the ship, Sliding and rocking of keel block accompanied with sliding of the ship. Also, it is assumed that the ship behaves as a rigid block, and rocking of the ship is not likely to take place. Totally separation of each part of the system from one another is considered as a failure.

In Moghaddasi and Bargi (2006), governing equations in each mode of motion were formulated, and in this paper a numerical scheme is employed to solve them for various assumptions. Time integrated scheme is the foundation of this numerical analysis. All analyses are aimed to determine the response of a ship-block system to a prescribed earthquake excitation. Influence of friction coefficients between keel blocks-ground and keel blocks-ship, blocks dimensions, PGA, and the magnitude of vertical ground motion on the response of a ship-block system are studied, also.

Furthermore, behavior of a system for usual design ranges of aforementioned parameters is examined.

2. Time integration scheme to solve governing equations

As presented in advance, a main variable of q as well as its first and second derivations can appear in governing equations of motion. To solve these equations numerically, a time integration scheme is used. In a numerical solution, $q^{t+\Delta t}, \dot{q}^{t+\Delta t}, \ddot{q}^{t+\Delta t}$ are undetermined variables at time step $t+\Delta t$, and they should be calculated due to $q^t, \dot{q}^t, \ddot{q}^t$ as determined variables at time step t. In order to compute $\ddot{q}^{t+\Delta t}$ in an equation of motion, expressions of $q^{t+\Delta t}$ and $\dot{q}^{t+\Delta t}$ should be replaced by some other determined parameters. Following equations make it practical.

$$q^{t+\Delta t} = q^t + \frac{\Delta t}{2}(\dot{q}^t + \dot{q}^{t+\Delta t}) \tag{1}$$

$$\dot{q}^{t+\Delta t} = \dot{q}^t + \frac{\Delta t}{2}(\ddot{q}^t + \ddot{q}^{t+\Delta t}) \tag{2}$$

Using these equations $q^{t+\Delta t}$ can be evaluated due to $\ddot{q}^{t+\Delta t}$ and $\ddot{q}^t$

$$q^{t+\Delta t} = q^t + \Delta t(\dot{q}^t) + \frac{\Delta t^2}{4}(\ddot{q}^t + \ddot{q}^{t+\Delta t}) \tag{3}$$

Replacing $q^{t+\Delta t}$ and $\dot{q}^{t+\Delta t}$ in the main equation of motion expresses. This equation in term of $\ddot{q}^{t+\Delta t}$; hence, $\ddot{q}^{t+\Delta t}$ can be computed easily. After all, with a linear interpolation accompanied by a numerical integration of acceleration and velocity functions, $q^{t+\Delta t}$ can be determined.

3. Numerical assumptions considered in the solution

Considering three subsequent time instances t_{i-1}, t_i and t_{i+1} two following assumptions can be studied. If it is supposed that condition needed to initiation of motion is launched at a time between t_{i-1} and t_i, in the employed numerical scheme, this time is approximated by the instance t_i which

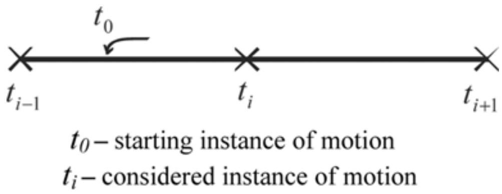

Fig. 1 Selection the initial instant of motion

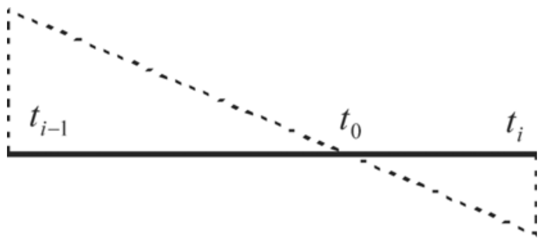

Fig. 2 Degradation numerical error in selecting the stop instance

means that at time t_i, as the start time of motion, initial condition is prescribed. Of course, this results to enter a numerical error in the solution process. Knowing the initial conditions of the motion at t_i, the instance t_{i+1} with unknown motion parameters will be the first step for numerical integration of motion equations. Fig. 1 depicts the represented assumption.

The instance at which the system will be at rest can be determined by more accurate approach. If t_0 is considered as the unknown time for ending of the motion and it is bounded between two time instances t_{i-1} and t_i, by moving t_0 to the nearest time instance numerical error can be reduced.

4. Solution algorithm

The algorithm used to solve the time-history problem can be summarized as:

1. Knowing the initial conditions of the motion and integrating equations of the motion, $\ddot{x}_b$, $\ddot{x}_s$ and $\ddot{\theta}$ at the end of the first time step is calculated.
2. Linear interpolation is used for the acceleration between the start and the end time of the current time step.
3. Velocity at the end of the current time step is equal to the velocity at the start time plus the area under the interpolated acceleration function.
4. Linear interpolation is used for the velocity between the start and the end time of the current time step.
5. Displacement at the end of the current time step is equal to the displacement at the start time plus the area under the interpolated velocity function.
6. Conditions needed for continuation of the motion is checked and if the motion is expected to continue, time integration of the next time step begins with repeating stages two to seven above, elsewhere it should be started from stage 1.

The algorithm is implemented in a computer code by "Mathematica" and is used for further analyses given herein.

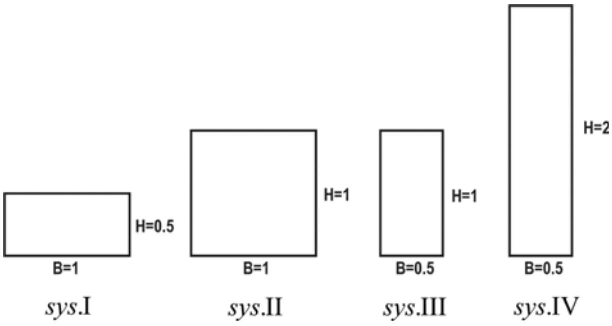

Fig. 3 Different sizes of keel blocks used for systems I through IV

Table 1 Different cases for relationship between friction coefficients in a ship-block system

Case No.	Relationship between μ_{s1} and μ_{s2}
1	$\mu_{s1} = \mu_{s2}$ for small values
2	$\mu_{s1} = \mu_{s2}$ for intermediate values
3	$\mu_{s1} = \mu_{s2}$ for large values
4	$\mu_{s1} < \mu_{s2}$
5	$\mu_{s1} > \mu_{s2}$

5. Parametric study for sensitivity analysis

Using abovementioned numerical scheme, complete time-history response of the system can be calculated. To study the behavior of a general system and sensitivity of the responses to various contributing physical parameters, several kinds of parametric study are conducted. These parametric studies indicate sensitivity of responses to friction coefficients, blocks dimensions, PGA, and the magnitude of vertical ground acceleration. Sensitivity analyses for four different systems with keel blocks in different sizes are performed. The block sizes are summarized in Fig. 3.

To study the effect of friction coefficients on the behavior of the system, three different values of static friction coefficients are considered, which are 0.2 (small), 0.5 (intermediate) and 0.8 (large). These coefficients can be used between keel blocks-ground (μ_{s1}) and between keel bocks-ship (μ_{s2}). Dynamic friction coefficient is taken to be the static value divided by 1.05. For combination of friction coefficients, five different cases are considered that are summarized in Table 1. Although some of these cases may not be of practical value, however, they offer remarkable insight to the ship-block system behavior. A practical system is discussed in Section 6. In all presented analyses, the coefficient of restitution (e) is taken to be 0.8, and the ship mass is 20,000,000 kg·s^2/m.

5.1 The effects of variation in friction coefficients

Friction coefficients between keel blocks-ground and keel blocks-ship are taken as variables for parametric study in this part. The imposed excitation is taken as horizontal and vertical Northridge Earthquake time-history. The horizontal component is scaled for PGA = 0.6 g. Same factor is used to scale the vertical component of the earthquake. After analyses for mentioned cases, the results are

Table 2 Maximum responses calculated by variation in friction coefficients

System	Response	Friction coefficients (μ_{s1}, μ_{s2})				
		(0.2,0.2)	(0.2,0.8)	(0.5,0.5)	(0.8,02)	(0.8,0.8)
I	x_b (m)	0.073	0.073	0.002		
	x_s (m)				0.073	
	θ (rad)					
II	x_b (m)	0.073	0.073	0.002		
	x_s (m)				0.073	
	θ (rad)					
III	x_b (m)	0.073	0.073	Failure		
	x_s (m)			0.036	0.073	
	θ (rad)					0.001
IV	x_b (m)	0.073	0.073			
	x_s (m)				0.171	
	θ (rad)			0.049	Failure	0.049

summarized in Table 2.

From results given in Table 2, it can be observed that:

For small equal values of friction coefficients (case 1 in Table 1), similar sliding occurs for all systems, and there is no sensitivity to the size of keel blocks.

For intermediate equal friction coefficients (case 2 in Table 1), systems I and II show only sliding behavior and amount of sliding is significantly less than the former case; which means that for these systems, increase of friction coefficients reduces sliding values. System IV shows rocking motion. And in system III (with not thin or thick keel block) sliding of keel blocks and sliding of the ship occur, simultaneously. This condition leads to extra sliding of keel blocks and though failure of the system.

For large equal values of friction coefficient (case 3 in Table 1), systems I and II (with thicker keel blocks) experience no motion. However, systems III and IV (with thinner blocks) undergo

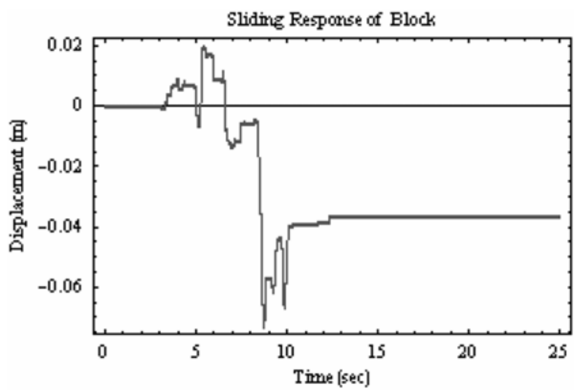

Fig. 4 Sliding of keel blocks in System I with (μ_{s1} = 0.2, μ_{s2} = 0.2)

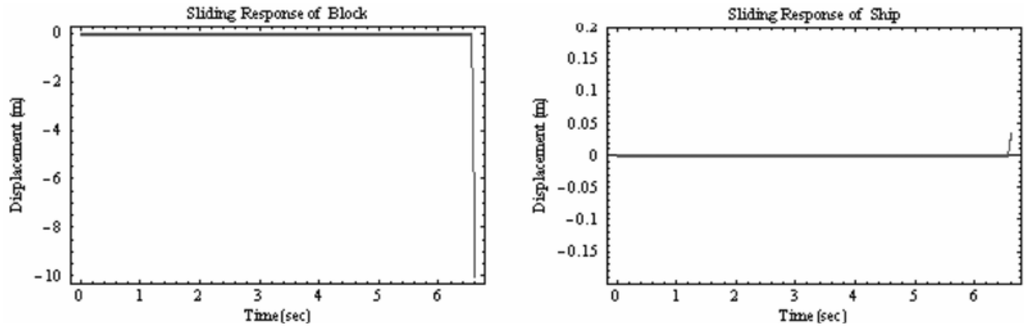

Fig. 5 Failure in System III with (μ_{s1} = 0.5, μ_{s2} = 0.5) due to extra sliding of keel blocks

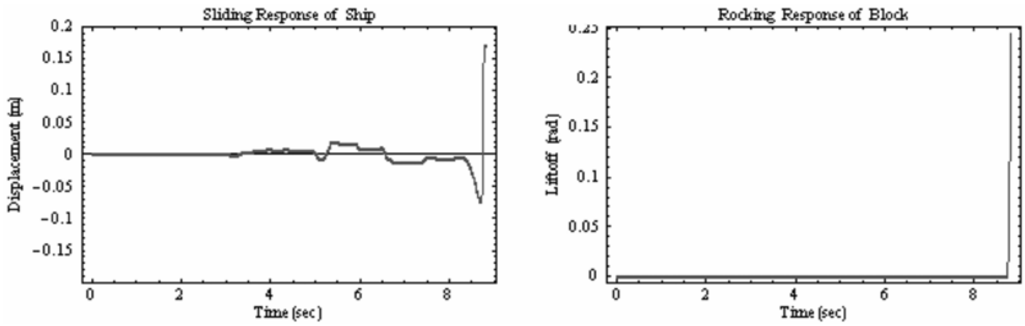

Fig. 6 Failure in System IV with (μ_{s1} = 0.8, μ_{s2} = 0.2) due to extra rocking of keel blocks

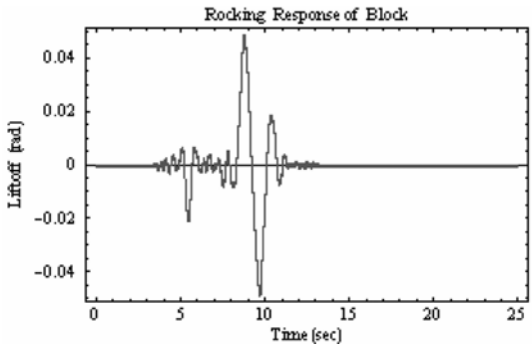

Fig. 7 Rocking of keel blocks in System IV with (μ_{s1} = 0.8, μ_{s2} = 0.8)

rocking motion which is increased for thinner keel blocks.

For case 4 in Table 1 ($\mu_{s1} < \mu_{s2}$), all systems experience identical sliding, and the sliding amount is independent of keel block size.

For case 5 in Table 1 ($\mu_{s1} > \mu_{s2}$) and systems I, II and III, sliding of the ship occurs and this sliding amount is identical for all mentioned systems. For system IV with thin keel blocks, in addition of sliding of the ship, rocking motion of keel blocks is observed which results to collapse of the system.

In Fig. 4 through Fig. 7 some of the results are illustrated.

Table 3 Maximum responses calculated by variation in PGA

System	Response	Friction coefficients (μ_{s1}, μ_{s2})				
		(0.2,0.2)	(0.2,0.8)	(0.5,0.5)	(0.8,02)	(0.8,0.8)
I	x_b (m)	0.351	0.351	0.073	Failure	0.019
	x_s (m)				Failure	
	θ (rad)					
II	x_b (m)	0.351	0.351	0.073	Failure	0.019
	x_s (m)				Failure	
	θ (rad)					
III	x_b (m)	0.351	0.351	Failure		2.492
	x_s (m)			0.189	0.710	
	θ (rad)			0.110	Failure	0.058
IV	x_b (m)	0.351	0.351			
	x_s (m)				0.975	
	θ (rad)			0.138	Failure	0.138

5.2 The effects of variation in earthquake PGA

Sensitivity of system response to PGA value of excitation is studied in this section. The imposed excitation is taken as Northridge Earthquake time-history scaled to PGA = 1.2 g. The results associated with the above analysis are given in Table 3.

It can be observed that:

For small equal values of friction coefficients, the response is same as the former loading (PGA = 0.6 g), however, sliding is increased.

For intermediate equal friction coefficients, sliding amount is increased for systems I and II. System IV undergoes larger rocking. And system III experiences rocking in addition to sliding of keel blocks and the ship. However, same as the former loading collapse takes place due to excessive sliding of keel blocks.

For large equal values of friction coefficient, systems I and II show sliding behavior in keel blocks. And for system III, sliding of keel blocks occurs and rocking motion is significantly increased. Same as system III, increase of rocking motion happens for system IV.

For $\mu_{s1} < \mu_{s2}$, the behavior is same as former loading with increased sliding amount.

For $\mu_{s1} > \mu_{s2}$, changes are more pronounced compared former loading. Systems I and II show excessive sliding both in keel blocks and the ship, which result to collapse of the system. Sliding of the ship as well as rocking of keel blocks takes place in systems III and IV. Excessive rocking of keel blocks results to collapse of the system.

It can be concluded that compared to the former loading, case 5 in Table 1 indicates maximum changes where collapse occurs for every systems. In Fig. 8 through Fig. 10 some of the mentioned results are depicted.

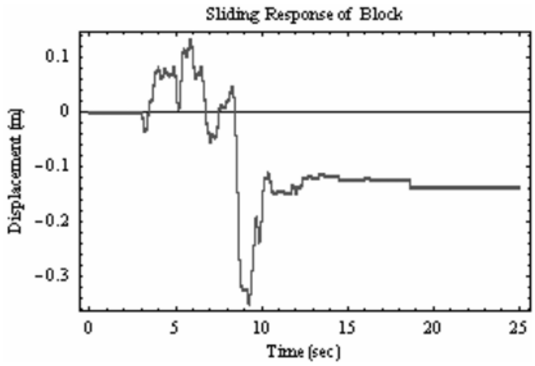

Fig. 8 Sliding of keel blocks in System I with (μ_{s1} = 0.2, μ_{s2} = 0.2)

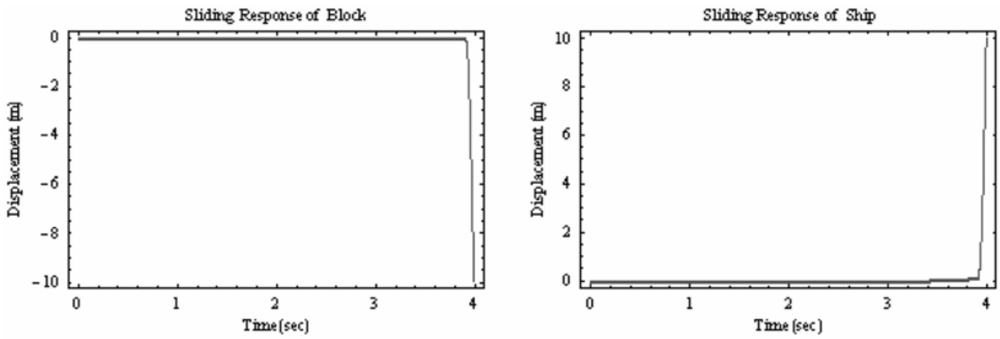

Fig. 9 Failure in System I with (μ_{s1} = 0.8, μ_{s2} = 0.2) due to extra sliding of keel blocks and the ship

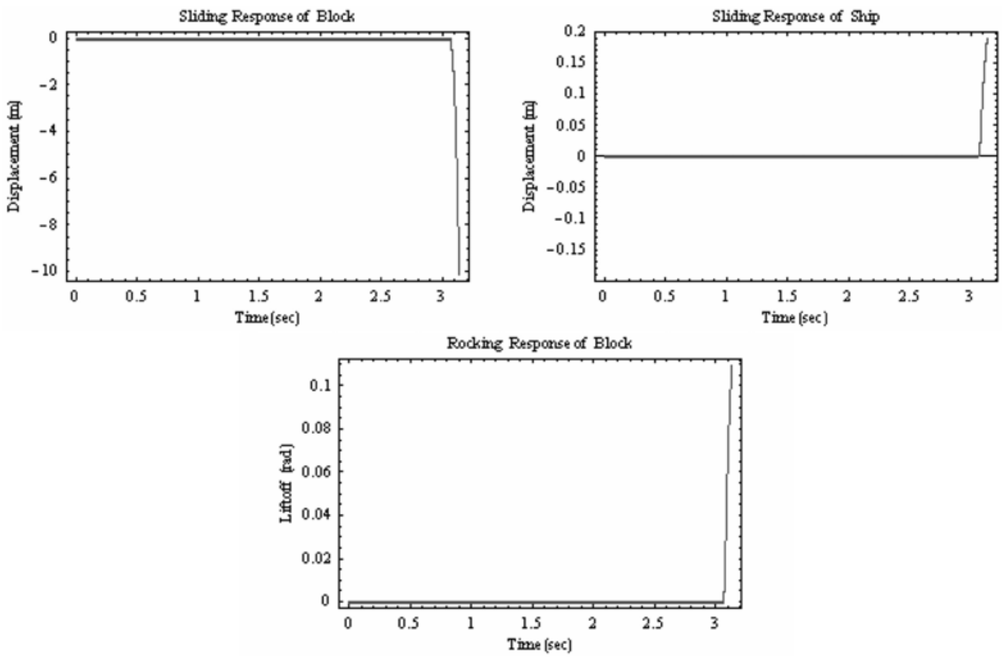

Fig. 10 Failure in System III with (μ_{s1} = 0.5, μ_{s2} = 0.5) due to extra sliding of keel blocks

Table 4 Maximum responses calculated by variation in the magnitude of vertical ground acceleration

System	Response	Friction coefficients (μ_{s1}, μ_{s2})				
		(0.2,0.2)	(0.2,0.8)	(0.5,0.5)	(0.8,02)	(0.8,0.8)
I	x_b (m)	0.070	0.070	0.006		0.003
	x_s (m)				0.070	
	θ (rad)					
II	x_b (m)	0.070	0.070	0.006	0.017	0.003
	x_s (m)				0.053	
	θ (rad)					
III	x_b (m)	0.070	0.070			
	x_s (m)			Failure	0.070	
	θ (rad)			0.018		Failure
IV	x_b (m)	0.070	0.070			
	x_s (m)				Failure	
	θ (rad)			0.018	0.001	0.018

5.3 The effects of variation in the magnitude of vertical ground acceleration

To study the effect of the vertical component of earthquake excitation, the vertical component is doubled, and its influence on the response is observed. The results in this situation are summarized in Table 4.

The following conclusions can be observed from the Table 4:

For small equal values of friction coefficients, no significant change in response is seen.

For intermediate equal friction coefficients, sliding of keel blocks for systems I and II is slightly increased. Mode of motion changes in system III. Rocking motion and sliding of the ship happen together. And collapse mechanism changes which is due to excessive sliding of the ship.

For large equal values of friction coefficient, sliding of keel blocks occurs for systems I and II and for system III excessive rocking of keel blocks results to collapse of the system. In reverse of other systems, Rocking is decreased for system IV.

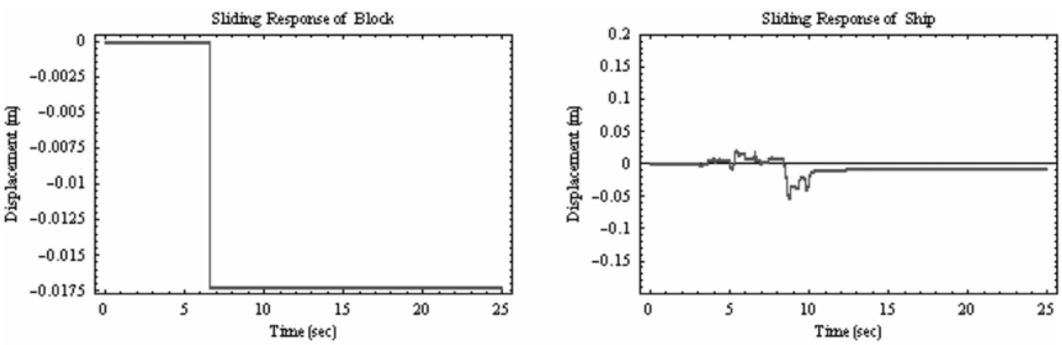

Fig. 11 Sliding of keel blocks and the ship in System 2 with (μ_{s1} = 0.8, μ_{s2} = 0.2)

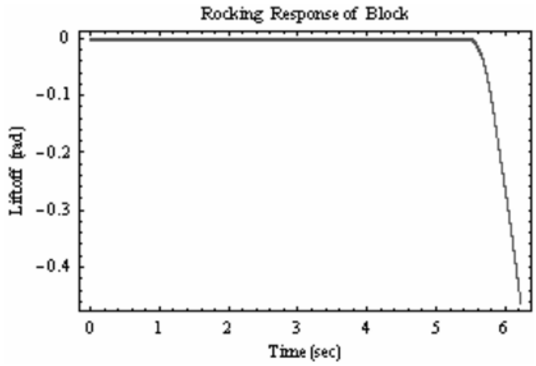

Fig. 12 Failure in System 3 with (μ_{s1} = 0.8, μ_{s2} = 0.8) due to extra rocking of keel blocks

No significant change is observed for $\mu_{s1} < \mu_{s2}$.

For $\mu_{s1} > \mu_{s2}$, no remarkable change is seen in the behavior of systems I and III. However, for system II, in addition to sliding of keel blocks, sliding of the ship occurs as well. For system IV, mode of collapse changes which is due to excessive sliding of the ship.

It is concluded that cases 2 and 3 in Table 1 and system III suffer maximum changes due to increase of the magnitude of vertical excitation acceleration. In Fig. 11 and Fig. 12 some of the mentioned results are depicted.

6. Assessment the behavior of common ship-block systems

In this section, sensitivity of a practical system to imposed excitation is studied. Four types of keel blocks normally used in reality are considered. The size of theses keel blocks are 1 m width and 1.5 m height. Keel blocks are made of concrete and a wooden cap which may be used on them or not. Referring to values given in the practical engineering literature, a friction coefficient between 0.2 and 0.6 for wood-steel interface can be considered. For concrete-steel interface, the friction coefficient is between 0.57 and 0.7, as well. For concrete-concrete interfaces tests are required to achieve the exact friction coefficient. However, referring to the value given by ACI a range of 0.4 to 0.8 is acceptable. Due to these mentioned friction coefficients, four systems can be considered by different kinds of keel blocks. These cases are System I with keel blocks made of hard concrete without cap (μ_{s1} == 0.5, μ_{s2} = 0.62), System II with keel blocks made of mild concrete without cap (μ_{s1} == 0.7, μ_{s2} = 0.62), System III with keel blocks made of hard concrete with wooden cap (μ_{s1} == 0.5, μ_{s2} = 0.2), and System IV with keel blocks made of mild concrete with wooden cap (μ_{s1} == 0.7, μ_{s2} = 0.2), where μ_{s1} is friction coefficient between keel blocks-ground and μ_{s2} is the friction coefficient between keel blocks-ship.

6.1 Studding the behavior of a common ship-block system

The introduced system in this section is subjected to Northridge Earthquake time-history scaled to PGA=0.6 g; vertical components is also scaled with the same factor as the horizontal component. Analyses results are given in Table 5.

Table 5 Maximum responses of a common ship-block system to earthquake excitation

Response	Type of system			
	I	II	III	IV
x_b (m)	0.001			
x_s (m)			0.073	0.073
θ (rad)				

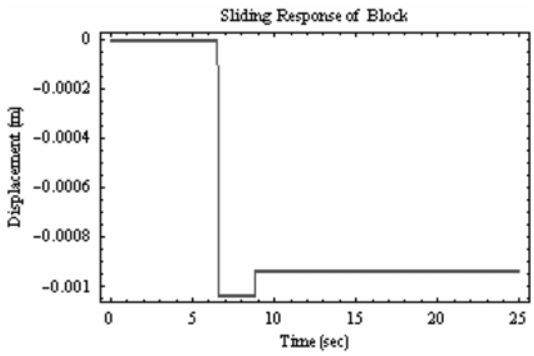

Fig. 13 Sliding of keel blocks in System I

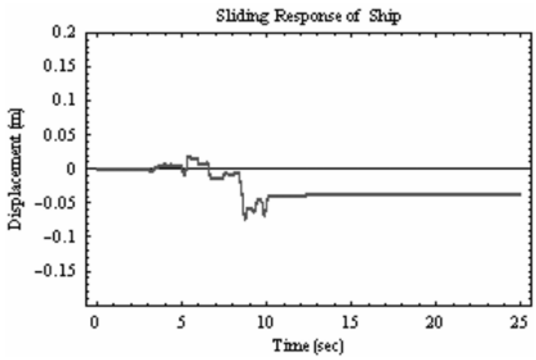

Fig. 14 Sliding of the ship in System III

It can be concluded that:

Sliding of keel blocks in system I takes place. No motion occurs for system II due to the loading and for systems III and IV sliding occurs for the ship which is equal for both cases. Fig. 13 and Fig. 14 demonstrate some of results in this situation.

6.2 The effects of PGA on the common ship-block system

The loading is increased for this stage. The horizontal component of Northridge Earthquake time-history is scaled to PGA =1.2 g and the vertical component is scaled with the same factor used for horizontal scaling. The results are summarized in Table 6.

As it can be seen, sliding of system I is increased. For system II rocking of keel blocks accompanied with sliding of the ship occurs; in this case the system collapses due to excessive sliding of the ship. For system III sliding of both keel blocks and the ship happens together; due to

Table 6 Maximum responses of a common ship-block system to variation in PGA

Response	Type of system			
	I	II	III	IV
x_b (m)	0.047		Failure	0.477
x_s (m)		Failure	Failure	2.704
θ (rad)		0.001		Failure

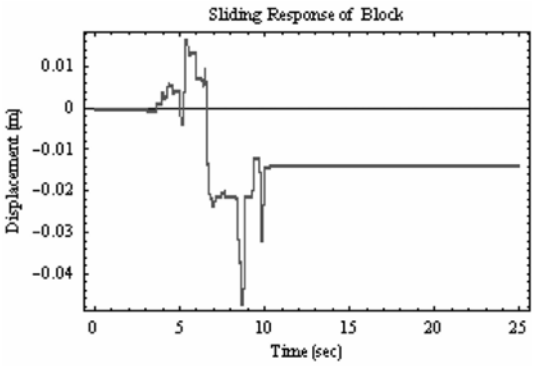

Fig. 15 Sliding of keel blocks in System I

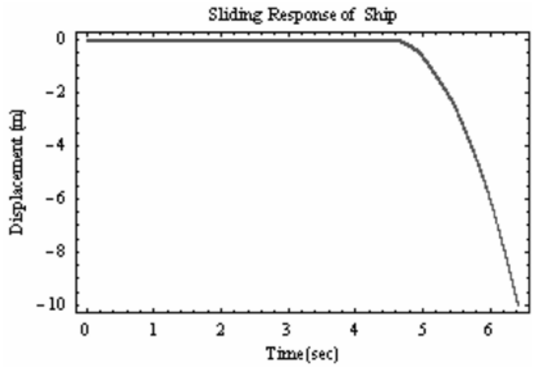

Fig. 16 Failure in System II due to extra sliding of the ship

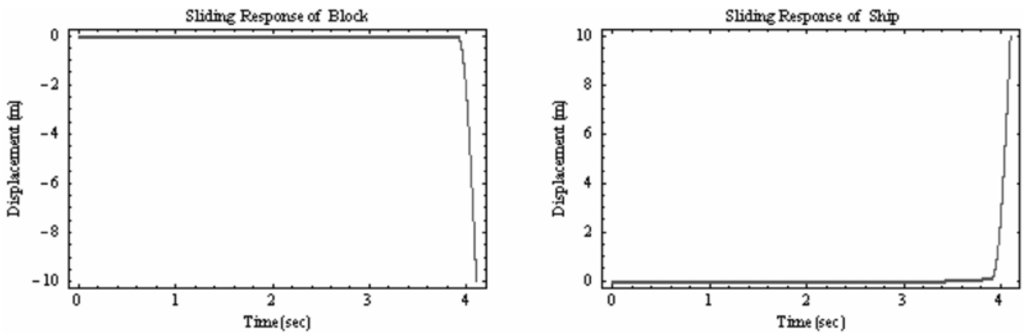

Fig. 17 Failure in System III due to extra sliding of the ship and keel blocks

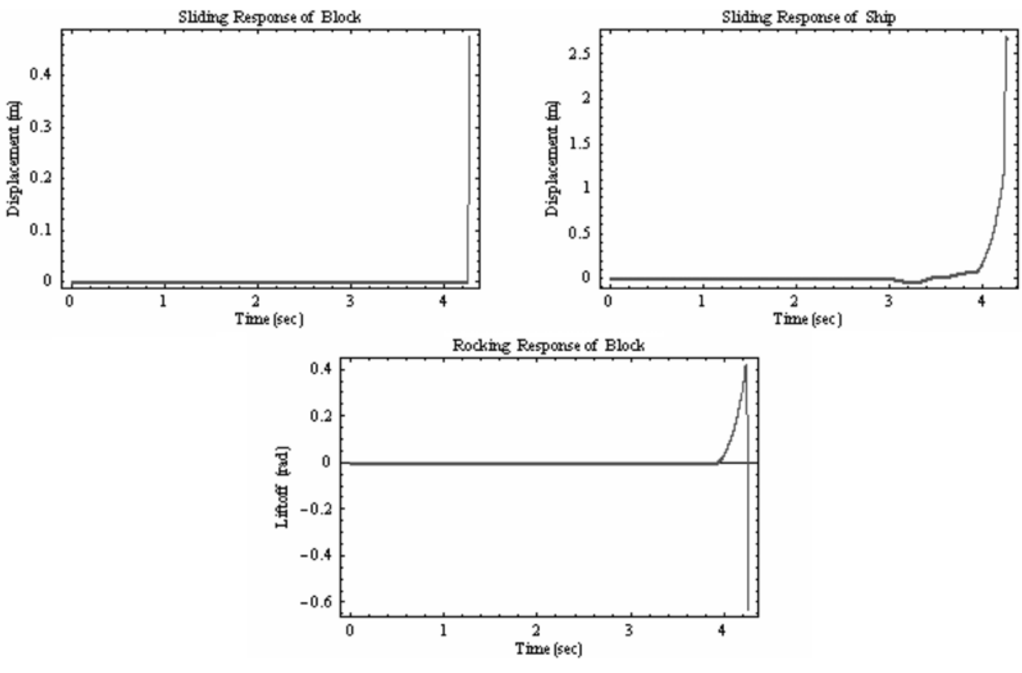

Fig. 18 Failure in System IV due to extra rocking of keel blocks

excessive sliding in both keel blocks and the ship, system experiences the collapse. For system IV, Mode 8th of the motion emerges and excessive rocking of keel blocks result to collapse of the system.

Fig. 15 through Fig. 18 depicts the responses of all four systems considered.

6.3 The effects of vertical ground acceleration on the common ship-block system

In the next study, the effects of vertical component of earthquake are investigated. The horizontal component is scaled to PGA = 0.6 g with using same factor for the vertical component. Then the vertical component is double to observe the influence on the results. These results are summarized in Table 7.

As it can be seen, no significant change happens for system I except minor changes in sliding amounts. In system II, by increase of the magnitude of vertical ground acceleration, sliding of the ship and rocking of keel blocks take place which leads to collapse of the system. For system types III and IV, the type of behavior changes and sliding of keel blocks and sliding of the ship occur. Furthermore, excessive sliding of the ship leads to collapse of the system.

As a conclusion, increase of PGA and vertical magnitude of earthquake increase potential of collapse of the presented practical system for systems type II, III and IV. Fig. 19 shows the response of system III.

Table 7 Maximum responses of a common ship-block system to variation in VGA

Response	Type of system			
	I	II	III	IV
x_b (m)	0.005		0.002	0.001
x_s (m)		Failure	Failure	Failure
θ (rad)		0.001		

*VGA-Vertical Ground Acceleration

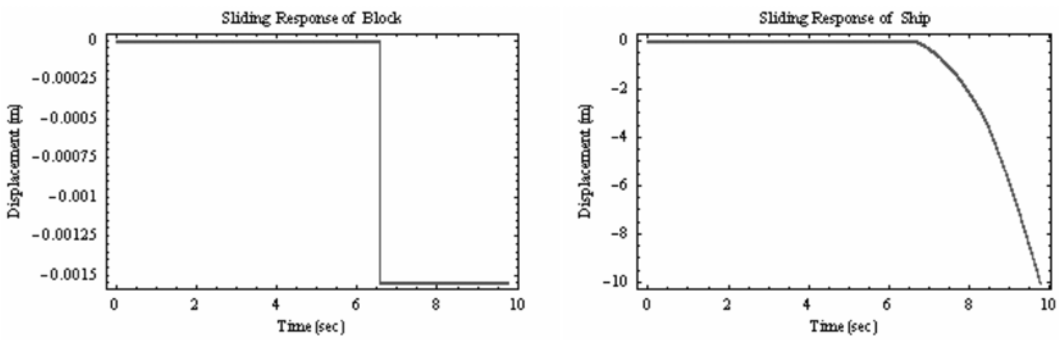

Fig. 19 Failure in System III due to extra sliding of the ship

7. Conclusions

Using the introduced formulation and knowing keel block's specifications, earthquake excitation and mechanical friction properties on different contact surfaces, time-history response of the system can be completely calculated and behavior of the system can be studied. Various cases were analyzed to indicate sensitivity of responses on various contributing parameters. Theses parameters consist of friction coefficients between surfaces, block dimensions, PGA, and the magnitude of vertical ground acceleration. Finally a typical ship-block system was excited by Northridge Earthquake. The responses were studied, and the effects of PGA, the magnitude of vertical ground acceleration are investigated. The major conclusions can be summarized as:

As a key parameter, friction coefficients play a very important role in response of the system and they may significantly influence the mode of motion and amplitude of displacement. Increasing of friction coefficients in a system with thick keel blocks reduces the amount of sliding and in a system with thin blocks changes the mode of motion from sliding into the rocking of keel blocks. Sliding of the ship is probable only if friction coefficient between keel blocks-ship (μ_{s2}) is less than friction coefficient between keel blocks-ground (μ_{s1}). In a system with no thin or thick keel blocks (sys. III in Fig. 3), using of intermediate equal friction coefficients causes failure in the system. A system with thick keel blocks can experiences failure when $\mu_{s1} > \mu_{s2}$, and this failure is due to excessive rocking of keel blocks.

In all systems with condition of $\mu_{s1} > \mu_{s2}$, increasing of PGA makes failure. The reason of this failure in a system with thick keel blocks is establishment of an unsteady condition producing excessive sliding; and in a system with thin keel blocks, excessive rocking of keel blocks can make a failure. As a case with ordinary exciting, a system with no thin or thick keel blocks fails, but in this case all expecting motions for system happen. In other situation, increasing of the PGA can only raise the amount of sliding or rocking.

The excitation with higher magnitude of vertical ground acceleration causes failure in a system with no thin or thick keel blocks. In this case, if a system with thin keel blocks accompanied with the condition of $\mu_{s1} > \mu_{s2}$, failure take place. In other circumstances only a little change in sliding or rocking of the system is expected.

A common system does not experience any failure during the typical excitation. A system with keel blocks and a wooden cap slides, and a system with keel blocks without wooden cap has the mode of rocking in keel blocks. If PGA increases all systems except one that contains of hard concrete keel blocks will fail. In a system with hard concrete keel blocks, $\mu_{s1} < \mu_{s2}$ and the difference between them is noticeable. So this behavior is predictable as mentioned before. When an excitement with higher magnitude of vertical ground acceleration occurs, all systems experience failure except a system with hard concrete keel blocks. Excessive sliding of the ship causes this failure.

To prevent failure mechanism in a common system, friction coefficients are to be kept an appropriate value. In a conclusion, using of hard concrete keel blocks without a cap is the safest way for mounting a ship in dry docks.

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