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Formulation for seismic response of a ship-block system

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Abstract. This paper presents a complete and consistent formulation to study the seismic response of a free-standing ship supported by an arrangement of *n* keel blocks which are all located in a dry dock. It is considered that the foundation of the system is subjected to both horizontal and vertical in plane excitation. The motion of the system is classified in eight different modes which are Rest (relative), Sliding of keel blocks, Rocking of keel blocks, Sliding of the ship, Sliding of both keel blocks and the ship, Sliding and rocking of keel blocks, Rocking of keel blocks with sliding of the ship, and finally Sliding and rocking of keel blocks accompanied with sliding of the ship. For each mode of motion the governing equations are derived, and transition conditions between different modes are also defined. This formulation is based on a number of fundamental assumptions which are 2D idealization for motion of the system, considering keel blocks as the rigid ones and the ship as a massive rigid block too, allowing the similar motion for all keel blocks, and supposing frictional nature for transmitted forces between contacted parts. Also, the rocking of the ship is not likely to take place, and the complete ship separation from keel blocks or separation of keel blocks from the base is considered as one of the failure mode in the system. The formulation presented in this paper can be used in its entirety or in part, and they are suitable for investigation of generalized response using suitable analytical, or conducting a time-history sensitivity analysis.

Keywords: free-standing ship; keel blocks; coulomb friction; horizontal and vertical base excitation; non-linear dynamic analysis; seismic response.

1. Introduction

The motion of rigid bodies lying on a rigid base subjected to horizontal and vertical excitation has been studied by various researchers. This is a complicated problem because of frictional nature of forces transferring between base and rigid bodies. Housner (1963) has proposed the minimum acceleration required for overturning of a rigid block standing over a rigid base under horizontal motion of the base. Lee (1975) developed a method for analyzing the nonlinear dynamic response of a column consisting of stacked elements excited by boundary motions. He also presented numerical results for the free vibrations of short columns and the boundary-excited response of tall columns. Ikushima (1979) presented an analytical method for predicting the behavior of a prismatic high-temperature gas-cooled reactor core under seismic excitation. In his model blocks were allowed to have vertical and rocking motions, but their relative horizontal movement was restricted.

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Yim *et al.* (1980) derived the equation of motion of an overturning block subjected to horizontal and vertical ground motion. Ishiyama (1982), Shenton *et al.* (1991) categorized the response of a rigid block under horizontal and vertical ground motion. Allen (1986) also investigates the dynamic behavior of rigid body assemblies. Shenton (1996) derived the criteria for initiation of slide, overturning and slide-overturning combined modes. Pompei *et al.* (1998) and Zahng (2001) have proposed conditions required to prevent sliding of blocks when overturning takes place. Lopez Garcia and Soong (2003) have presented a sliding failure of block-type components. In all of these investigations, the behavior of a block on a rigid base affected by ground motion either horizontal or vertical has been studied.

In order to inspect behavior of a real problem which can be modeled similar to a dual sliding-rocking system, a massive block mounted over an arrangement of n keel blocks. Authors attempted to study all kinds of responses probable for the system under horizontal and vertical base excitation. One of these real problems is a ship standing freely over an arrangement of n keel blocks in a dry dock. Dry docks adjacent to the sea are used for building, repairing and maintenance of ships. There, the ship is supported by some keel blocks which are not fixed and can move. For high seismic zones, it is of significant importance to study behavior of the system to probable earthquakes that may occur while the ship is located on keel blocks.

To study the seismic response of the system after regarding some fundamental assumptions, eight modes of motion are considered here which are:

1. Rest (relative)
2. Sliding of keel blocks
3. Rocking of keel blocks
4. Sliding of the ship
5. Sliding of both keel blocks and the ship
6. Sliding and rocking of keel blocks
7. Rocking of keel blocks with sliding of the ship
8. Sliding and rocking of keel blocks accompanied with sliding of the ship

For each mode of motion, by applying a Lagrangian formulation, equations of motion are derived. By solution of these equations, horizontal and vertical acceleration of blocks and ship, and rotational acceleration of blocks are calculated. In addition to equations of motion, the conditions of transition between different modes of motion are obtained. These conditions are achieved by comparison of inertia forces and friction forces in each mode, and by considering the geometry of the problem in hand. By having acceleration of keel blocks and the ship in each mode of motion and translation conditions between each mode, a time-history response analysis is possible under any base excitation.

2. Modeling

To model a dual sliding-rocking system subjected to horizontal and vertical base excitation some simplifying assumptions are made: the motion of the system is restricted to two dimensions, the ship is considered as a simple rigid block with mass M, lying on n similar keel blocks, and each keel block is assumed as a rigid rectangular block of width 2B and height 2H, [$\theta_c = \tan^{-1}(B/H)$]. The mass m and mass moment of inertia I are the other characteristics of each keel block. The system under investigation is shown in Fig. 1 and Fig. 2.

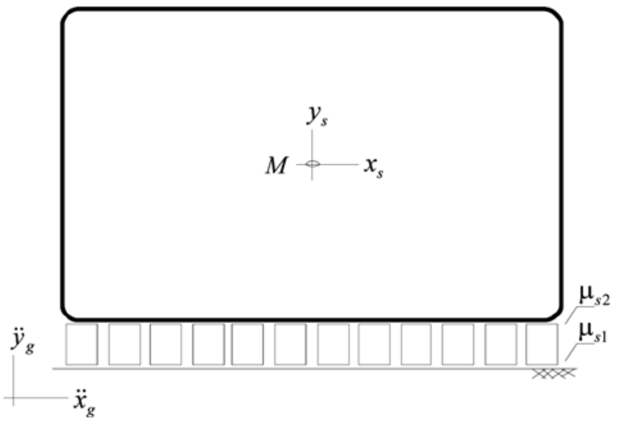

Fig. 1 A simple model of a ship over n similar keel blocks

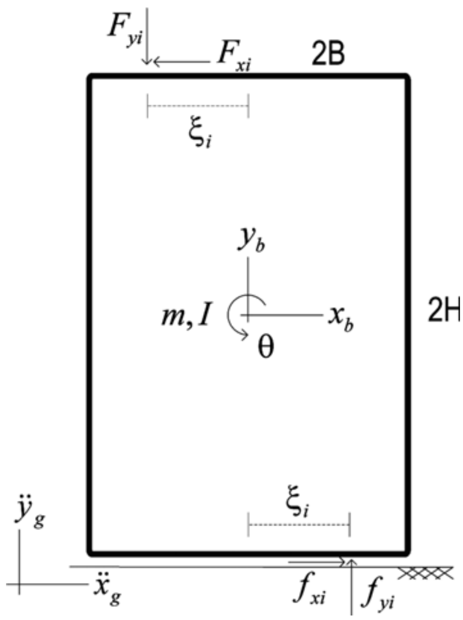

Fig. 2 Specifications of a sample block

The motion of the ship is considered in two directions relative to keel blocks and the rocking of the ship is neglected. x_s and y_s stand for translation in X-direction and Y-direction, respectively. The degrees of freedom for each keel block are translation relative to the ground in X-direction x_b, in Y-direction y_b and the counter clockwise rotation θ relative to the ground. Due to similarity of all keel blocks in dimensions, and similarity of friction coefficients between keel blocks-ground and between keel blocks-ship, uniform motion for all keel blocks can be assumed. Because of this uniform motion, horizontal forces between keel blocks-ground and between keel blocks-ship are identical for all keel blocks. In other words, if F_x, F_y stand for total horizontal and vertical forces transmitted between keel blocks-ship, and f_x, f_y stand for total horizontal and vertical transmitted force between keel blocks-ground, forces acting on each keel block are:

$$F_{xi} = \frac{F_x}{n}, \quad F_{yi} = \frac{F_y}{n}, \quad f_{xi} = \frac{f_x}{n}, \quad f_{yi} = \frac{f_y}{n} \tag{1}$$

Fig. 2 shows a sample 2B*2H keel block, its degrees of freedom and forces acting on it. The only mechanism of force transfer between keel blocks-ground and keel blocks-ship is the coulomb friction force between these surfaces. μ_s and μ_k represents the static and dynamic coefficients of friction, respectively. In order to account rotation of each keel block, points of applied forces are considered to be symmetrical relative to the center of blocks. Another fundamental concept in formulation of initial condition of motion is that the motion of the system starts from keel blocks and then transfers to the ship. After that, the motion of the ship affects the motion of keel blocks as a feedback mechanism.

3. Study of modes of motion

3.1 Rest (relative)

This mode of motion represents no displacement of the ship relative to keel blocks, and also no movement of keel blocks relative to the ground. Using mentioned simplification, all keel blocks can be replaced by a single block with mass $n \times m$, and ship can be considered as a rigid block with mass M. In complete relative rest state, using equilibrium equations, forces transmitted between different parts of the system can be simply found out as:

$$F_y = M(\ddot{y}_g + g) \tag{2}$$

$$F_x = M\ddot{x}_g \tag{3}$$

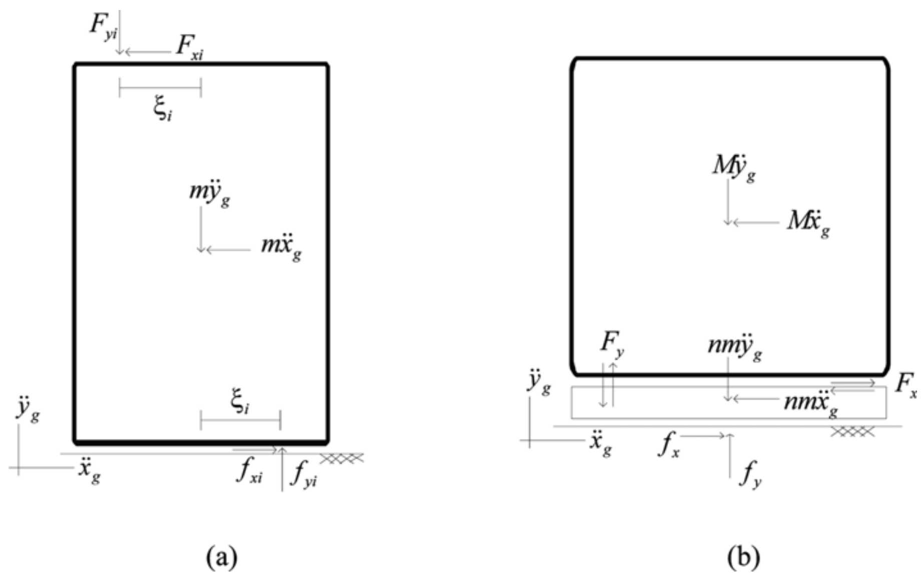

Fig. 3 Mode 1-Rest, (a) one keel block at rest, (b) the system at rest

$$f_y = (nm + M)(\ddot{y}_g + g) = M(M_n + 1) \cdot (\ddot{y}_g + g) \tag{4}$$

$$f_x = (nm + M)\ddot{x}_g = M(M_n + 1) \cdot \ddot{x}_g \tag{5}$$

Where M_n is defined as nm/M. The moment equilibrium equation can also be used to determine points on which applied forces are considered. These points are located in distance of ξ_i from vertical axe of the keel block.

$$(f_{xi} + F_{xi})H + (f_{yi} + F_{yi})\xi_i = 0 \tag{6}$$

Using Eqs. (2) to (5), Eq. (6) can be rewritten as:

$$\xi_i = \left(\frac{f_{xi} + F_{xi}}{f_{yi} + F_{yi}}\right)H = \left(\frac{f_x + F_x}{f_y + F_y}\right)H \tag{7}$$

For complete relative rest state, no motion should occur in the system, so some necessary conditions are expected. These conditions can be stated as follow:

- *No separation of keel blocks from the base*. This is possible if we have $f_y > 0$ (or $f_{yi} > 0$). By f_y provided in Eq. (4) this condition leads to:

$$\ddot{y}_g > -g \tag{8}$$

- *No keel blocks sliding*. To establish this condition, inertia forces applied on each keel block shall be less than friction forces between that block and the ground or $|f_{xi}| \le \mu_{s1} \cdot f_{yi}$. This may be expresses as:

$$\frac{|\ddot{x}_g|}{\ddot{y}_g + g} \le \mu_{S1} \tag{9}$$

In which μ_{S1} and μ_{S2} are defined as the static friction coefficients between keel blocks-ground and between keel blocks-ship, respectively.

- *No keel blocks overturning*. To satisfy this condition, ξ_i is the controlling parameter. As long as $|\xi_i| < B$ there is no any liftoff. Using Eqs. (2) to (5) and Eq. (7), this condition results to:

$$\left|\frac{\ddot{x}_g}{\ddot{y}_g + g}\right| \le \alpha \tag{10}$$

Where α stands for (B/H).

- *No ship sliding*. In this case, the inertia forces applied to the ship must be less than friction forces between keel blocks and the ship, in other words $|F_x|/F_y \le \mu_{s2}$. Using Eqs. (2) and (3), this condition can be rewritten in the following form:

$$\frac{|\ddot{x}_g|}{\ddot{y}_g + g} \le \mu_{S2} \tag{11}$$

- *No separation of the ship from keel blocks.* The result of this condition is similar to Eq. (8).

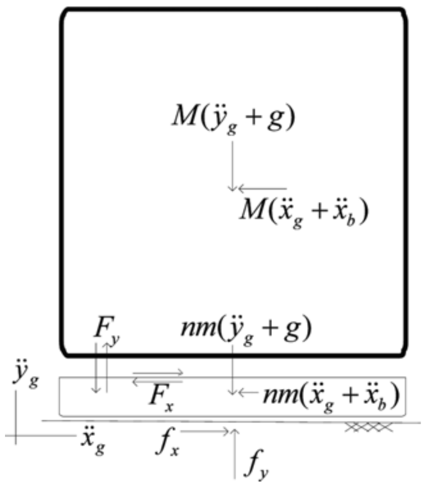

Fig. 4 Mode 2-Sliding of keel blocks

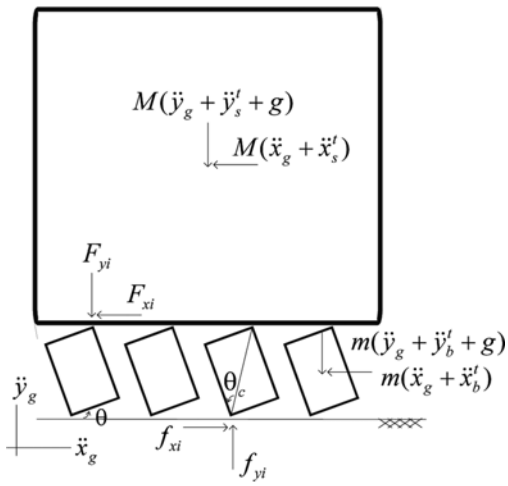

Fig. 5 Mode 3-Rocking of keel blocks around left bottom edge

3.2 Sliding of keel blocks

In this mode of motion, it is assumed that keel blocks slide, and the ship remains at rest relative to keel blocks. Fig. 4 shows this mode of motion. The dominating degree of freedom for this mode is the horizontal motion of keel blocks relative to the ground which is denoted by x_b. Applied forces on keel blocks and the ship can be obtained by the use of equilibrium equation along both X and Y direction. To calculate f_y and F_y, equilibrium equations along Y-direction are used which give results similar to Eqs. (2) and (4). By equilibrium equation along X-direction F_x is calculated as:

$$F_x = M(\ddot{x}_g + \ddot{x}_b) \tag{12}$$

Once sliding of keel blocks, the transmitted force between keel blocks-ground may be expressed as:

$$f_x = -S(\dot{x}_b) \cdot \mu_{k1} \cdot f_y = -S(\dot{x}_b) \cdot \mu_{k1} \cdot M(M_n + 1)(\ddot{y}_g + g) \tag{13}$$

Where $S(\dot{x}_b)$ is defined to account for the direction of motion as:

$$S(\dot{x}_b) = \begin{cases} 1 & \dot{x}_b > 0 \\ -1 & \dot{x}_b < 0 \end{cases}$$

The governing equation of motion for this mode is given by:

$$\ddot{x}_b = -S(\dot{x}_b) \cdot \mu_{k1} \cdot (\ddot{y}_g + g) - \ddot{x}_g \tag{14}$$

After initiation of this mode of motion the conditions needed for the ship sliding or keel blocks rocking are probable to be satisfied, and a new mode of motion will occur. According to Eqs. (2) and (12) the ship sliding condition in this case ($|F_x|/F_y > \mu_{s2}$), can be expressed as:

$$\frac{|-S(\dot{x}_b)\cdot\mu_{k1}\cdot(\ddot{y}_g+g)|}{\ddot{y}_g+g} > \mu_{s2} \tag{15}$$

As $(\ddot{y}_g+g)>0$ and $|-S(\dot{x}_b)| = 1$, this condition reduces to $\mu_{k1}>\mu_{s2}$. To establish the condition for incoming to keel blocks rocking mode ξ_i must be equal or greater than α; hence, it can be said that:

$$\left|\frac{-S(\dot{x}_b)\cdot\mu_{k1}\cdot M(M_n+1)(\ddot{y}_g+g)-S(\dot{x}_b)\cdot\mu_{k1}\cdot(\ddot{y}_g+g)}{(M_n+2)(\ddot{y}_g+g)}\right| \geq \alpha \tag{16}$$

By simplifying Eq. (16), the initiation condition of keel blocks rocking is $|\mu_{k1}| \geq \alpha$.

3.3 Rocking of keel blocks

In this case, keel blocks can rock to left or right about the corner points due to the base excitation. The sliding motions of the ship and keel blocks are restricted. This mode of motion is shown in Fig. 5. It is assumed that the contact between the ship and keel blocks occur in a single point on each keel block. Geometry of the problem does not allow the ship to rock. To obtain applied forces transmitted between keel blocks and the ship, the coordinates of keel block's mass center x_b^t, y_b^t, and the ship's mass center x_s^t, y_s^t are defined due to rotational degree of freedom θ, as given in below:

$$x_b^t = -S(\theta)\cdot R[\sin\theta_c - \sin(\theta_c - |\theta|)] \tag{17}$$

$$y_b^t = R[\cos(\theta_c - |\theta|) - \cos\theta_c] \tag{18}$$

$$x_s^t = -2S(\theta)R[\sin\theta_c - \sin(\theta_c - |\theta|)] \tag{19}$$

$$y_s^t = 2R[\cos(\theta_c - |\theta|) - \cos\theta_c] \tag{20}$$

Where R is defined as $\sqrt{(B^2+H^2)}$ and the superscript t is used for total displacement.
$S(\theta)$ is also defined to account for the direction of rotation of keel blocks as:

$$S(\theta) = \begin{cases} 1 & \theta > 0 \\ -1 & \theta < 0 \end{cases}$$

If equilibrium equation in the rotational state is used, the applied forces can be calculated as follows:

$$F_y = M\{(\ddot{y}_g+g)+2R[S(\theta)\ddot{\theta}\sin(\theta_c-|\theta|)-\dot{\theta}^2\cos(\theta_c-|\theta|)]\} \tag{21}$$

$$F_x = M\{\ddot{x}_g+2R[\ddot{\theta}\cos(\theta_c-|\theta|)+S(\theta)\dot{\theta}^2\sin(\theta_c-|\theta|)]\} \tag{22}$$

$$f_y = M\{(M_n+1)(\ddot{y}_g+g)+R(M_n+2)[S(\theta)\ddot{\theta}\sin(\theta_c-|\theta|)-\dot{\theta}^2\cos(\theta_c-|\theta|)]\} \tag{23}$$

$$f_x = M\{(M_n+1)\ddot{x}_g-R(M_n+2)[\ddot{\theta}\cos(\theta_c-|\theta|)+S(\theta)\dot{\theta}^2\sin(\theta_c-|\theta|)]\} \tag{24}$$

To obtain the equation of motion a Lagrangian formulation is used. Therefore, the governing equation of motion is derived as:

$$\ddot{\theta} = \frac{3}{4R}\left(\frac{M_n+2}{M_n+3}\right)[\ddot{x}_g\cos(\theta_c-|\theta|)-S(\theta)(\ddot{y}_g+g)\sin(\theta_c-|\theta|)] \tag{25}$$

In this mode of motion, if the conditions needed for sliding of keel blocks or the ship are satisfied a new mode of motion will occur. To establish sliding of keel blocks, based on the equation $|f_x/f_y| > \mu_{s1}$ we obtain:

$$\left|\frac{\left(\frac{M_n+1}{M_n+2}\right)\ddot{x}_g - R[\ddot{\theta}\cos(\theta_c-|\theta|)+S(\theta)\dot{\theta}^2\sin(\theta_c-|\theta|)]}{\left(\frac{M_n+1}{M_n+2}\right)(\ddot{y}_g+g)+R[S(\theta)\ddot{\theta}\sin(\theta_c-|\theta|)-\dot{\theta}^2\cos(\theta_c-|\theta|)]}\right| > \mu_{s1} \tag{26}$$

Based on the equation $|f_x/f_y| > \mu_{s2}$, sliding of the ship may be probable. This condition can be expressed as:

$$\left|\frac{\ddot{x}_g - 2R[\ddot{\theta}\cos(\theta_c-|\theta|)+S(\theta)\dot{\theta}^2\sin(\theta_c-|\theta|)]}{(\ddot{y}_g+g)+2R[S(\theta)\ddot{\theta}\sin(\theta_c-|\theta|)-\dot{\theta}^2\cos(\theta_c-|\theta|)]}\right| > \mu_{s2} \tag{27}$$

3.4 Sliding of the ship

Since only the ship slides in this mode of motion, the degree of freedom is horizontal motion of the ship relative to keel blocks which is denoted by x_s. To obtain f_y and F_y, equilibrium equations along Y-direction are used which give results similar to Eqs. (2) and (4). According to Eq. (2) and considering the ship sliding, F_x is obtained as:

$$F_x = -S(\dot{x}_s)\cdot\mu_{k2}\cdot F_y = -S(\dot{x}_s)\cdot\mu_{k2}\cdot M\cdot(\ddot{y}_g+g) \tag{28}$$

To calculate f_x, F_x given in Eq. (28) should be inserted in keel blocks equilibrium equation along X-direction. The following relationship will be resulted:

$$f_x = M[M_n\ddot{x}_g - S(\dot{x}_s)\cdot\mu_{k2}\cdot(\ddot{y}_g+g)] \tag{29}$$

The equation of motion can be derived by the use of Lagrangian formulation as:

$$\ddot{x}_s = -S(\dot{x}_s)\cdot\mu_{k2}\cdot(\ddot{y}_g+g)-\ddot{x}_g \tag{30}$$

If $|f_x|/f_y > \mu_{s1}$, keel blocks sliding will occur. According to Eqs. (29) and (2) this condition can be simplified as:

$$\left|\left(\frac{M_n}{M_n+1}\right)\frac{\ddot{x}_g}{\ddot{y}_g+g} - S(\dot{x}_s)\cdot\mu_{k2}\cdot\frac{1}{M_n+1}\right| > \mu_{s1} \tag{31}$$

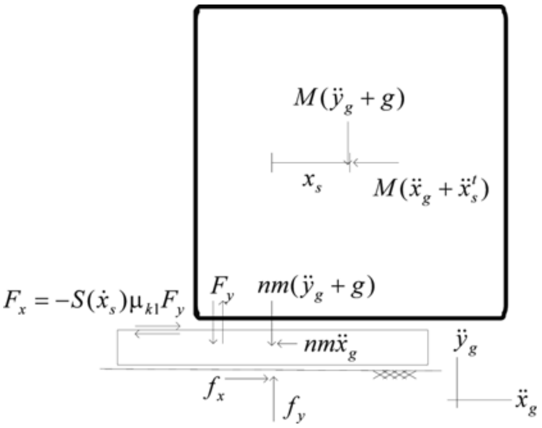

Fig. 6 Mode 4-Sliding of the ship

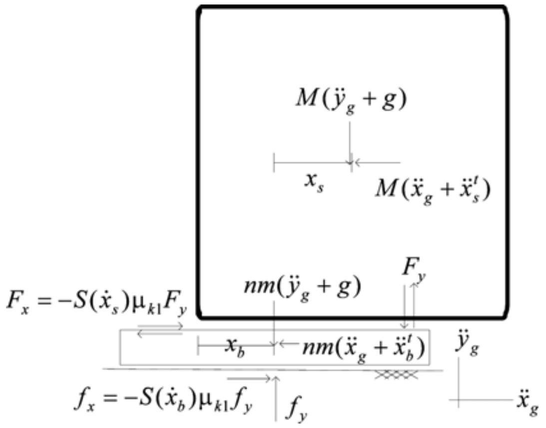

Fig. 7 Mode 5-Sliding of both keel blocks and the ship

To consider the rocking of keel blocks and knowing the rocking initiation condition $|f_x+F_x|/|f_y+F_y| \geq (B/H)$ the following equation is obtained:

$$\left|\frac{M_n}{M_n+2}\frac{\ddot{x}_g}{\ddot{y}_g+g} - S(\dot{x}_s)\cdot\mu_{k2}\cdot\frac{2}{M_n+2}\right| \geq \alpha \tag{32}$$

3.5 Sliding of both keel blocks and the ship

Sliding of keel blocks and the ship occurs simultaneously, in this mode of motion. Degrees of freedom in this mode are the horizontal translation of keel blocks relative to the ground, x_b and the horizontal translation of the ship relative to the blocks, x_s. Such as previous cases, Values for f_y and F_y are same as Eqs. (2) and (4).

Satisfying the condition corresponding to the ship sliding F_x can be expressed as follows:

$$F_x = -S(\dot{x}_s)\cdot\mu_{k2}\cdot M\cdot(\ddot{y}_g+g) \tag{33}$$

and while keel blocks also slide, f_x is equal to:

$$f_x = -S(\dot{x}_b)\cdot\mu_{k1}\cdot M(M_n+1)(\ddot{y}_g+g) \tag{34}$$

To drive equations of motion in this mode two coupled Lagrangian equations should be solved. After that, the governing equations can be presented as:

$$\ddot{x}_b = \frac{1}{M_n}(\ddot{y}_g+g)[-S(\dot{x}_b)\cdot\mu_{k1}\cdot(M_n+1)+S(\dot{x}_s)\cdot\mu_{k2}]-\ddot{x}_g \tag{35}$$

$$\ddot{x}_s = \frac{M_n+1}{M_n}(\ddot{y}_g+g)[S(\dot{x}_b)\cdot\mu_{k1}+S(\dot{x}_s)\cdot\mu_{k2}] \tag{36}$$

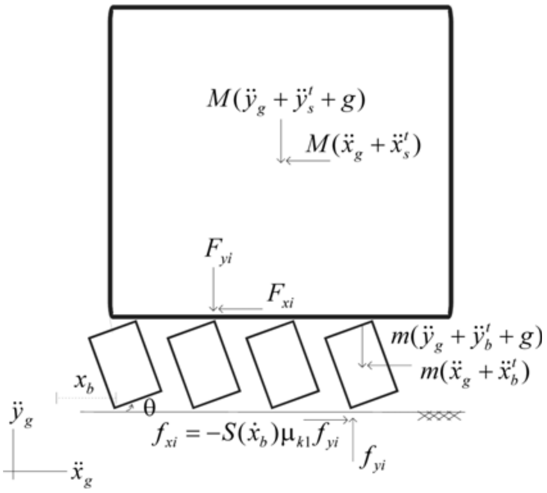

Fig. 8 Mode 6-Sliding and rocking of keel blocks

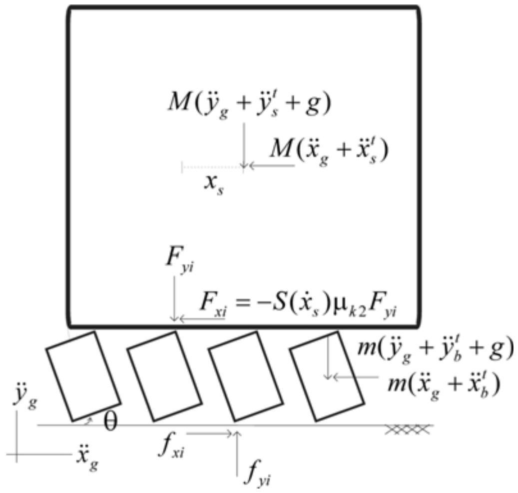

Fig. 9 Mode 7-Rocking of keel blocks with sliding of the ship

Satisfaction of rocking condition for keel blocks in this mode $|f_x+F_x|/|f_y+F_y| \geq (B/H)$ make a new mode of motion. This condition can be rewritten as:

$$\left| S(\dot{x}_b)\cdot\mu_{k1}\cdot\left(\frac{M_n+1}{M_n+2}\right)+S(\dot{x}_s)\cdot\mu_{k2}\cdot\frac{1}{M_n+2}\right| \geq \alpha \tag{37}$$

3.6 Sliding and rocking of keel blocks

While this mode of motion occurs, keel blocks experience rocking and sliding together. Degrees of freedom are defined as horizontal translation of keel blocks relative to the ground, x_b, and counter clockwise rotation of keel blocks, θ. The coordinates of keel blocks' mass center and the ship's mass center can be defined in the following formulation:

$$x_b^t = x_b - S(\theta)\cdot R\cdot[\sin\theta_c - \sin(\theta_c - |\theta|)] \tag{38}$$

$$x_s^t = x_b - 2S(\theta)R[\sin\theta_c - \sin(\theta_c - |\theta|)] \tag{39}$$

Definitions of y_s^t, y_b^t are similar to Eqs. (18) and (20).

The applied forces on keel blocks and ship, when this mode of motion happens are as follows:

$$F_x = M\{\ddot{x}_g + \ddot{x}_b - 2R[\ddot{\theta}\cos(\theta_c - |\theta|) + S(\theta)\dot{\theta}^2\sin(\theta_c - |\theta|)]\} \tag{40}$$

$$f_x = -S(\dot{x}_s)\mu_{k1}M\{(M_n+1)(\ddot{y}_g+g)+(M_n+2)R[S(\theta)\ddot{\theta}\sin(\theta_c-|\theta|)-\dot{\theta}^2\cos(\theta_c|\theta|)]\} \tag{41}$$

Values of f_y and F_y are similar to Eqs. (2) and (4).

By applying a Lagrangian formulation and solving derived set of equations, the solution is obtained as:

$$\ddot{x}_b = \frac{\left[A_1 + A_2 + \frac{4R}{3M_2}\left(\frac{A_3}{M_1} - R\dot{\theta}^2 A_4\right)\right]}{\frac{4}{3}\frac{M_1}{M_2} - A_5} \tag{42}$$

$$\ddot{\theta} = \frac{6(2 + M_n)(A_6 + A_7)(A_4)}{R[12(-1 + A_8) + M_n(-5 + 3A_8)(M_n + 4)]} \tag{43}$$

The coefficients in these equations are defined as:

$$A_1 = -\ddot{x}_g \cos(\theta_c - |\theta|) + S(\theta)(\ddot{y}_g + g)\sin(\theta_c - |\theta|)$$

$$A_2 = \cos(\theta_c - |\theta|) - S(\dot{x}_g)S(\theta)\mu_{k1}\sin(\theta_c - |\theta|)$$

$$A_3 = S(\dot{x}_b)S(\ddot{y}_g + g)\mu_{k1}\ddot{x}_g$$

$$A_4 = S(\theta)\sin(\theta_c - |\theta|) + S(\dot{x}_b)\mu_{k1}\cos(\theta_c - |\theta|)$$

$$A_5 = [\cos(\theta_c - |\theta|) - S(\dot{x}_b)S(\theta)\mu_{k1}\sin(\theta_c - |\theta|)]\cos(\theta_c - |\theta|)$$

$$A_6 = (\ddot{y}_g + g) - 2R\cos(\theta_c - |\theta|)\dot{\theta}^2$$

$$A_7 = M_n[(\ddot{y}_g + g) - R\cos(\theta_c - |\theta|)\dot{\theta}^2]$$

$$A_8 = \cos 2(\theta_c - |\theta|) - S(\dot{x}_b)S(\theta)\mu_{k1}\sin 2(\theta_c - |\theta|)$$

$$M_1 = \frac{M_n + 1}{M_n + 2}$$

$$M_2 = \frac{M_n + 2}{M_n + 3}$$

To establish the mode of ship sliding, the condition $|F_x|/F_y > \mu_{s2}$ should be satisfied. So we have:

$$\frac{\left|\{\ddot{x}_g + \ddot{x}_b - 2R[\ddot{\theta}\cos(\theta_c - |\theta|) + S(\theta)\dot{\theta}^2\sin(\theta_c - |\theta|)]\}\right|}{\{\ddot{y}_g + g + 2R[S(\theta)\ddot{\theta}\sin(\theta_c - |\theta|) - \dot{\theta}^2\cos(\theta_c - |\theta|)]\}} > \mu_{s2} \tag{44}$$

3.7 Rocking of keel blocks with sliding of the ship

This mode occurs when rocking of keel blocks and sliding of the ship happen simultaneously. Fig. 9 shows this mode of motion. Degrees of freedom for in this case are counter clockwise rotation of keel blocks θ and horizontal translation of the ship, x_s. Similar to the other modes coordinates of the mass centers can be defined as:

$$x_b^t = -S(\theta) \cdot R \cdot [\sin\theta_c - \sin(\theta_c - |\theta|)] \tag{45}$$

$$x_s^t = x_s - 2S(\theta)R[\sin\theta_c - \sin(\theta_c - |\theta|)] \tag{46}$$

Definition of y_s^t, y_b^t are same as Eqs. (18) and (20). Forces along X-direction can be evaluated as:

$$F_x = -S(\dot{x}_s) \cdot \mu_{k2} \cdot M\{(\ddot{y}_g + g) + 2R[\ddot{\theta}S(\theta)\sin(\theta_c - |\theta|) - \dot{\theta}^2\cos(\theta_c|\theta|)]\} \tag{46}$$

$$f_x = nm\{\ddot{x}_g - R[\ddot{\theta}\cos(\theta_c - |\theta|) - S(\theta)\dot{\theta}^2\sin(\theta_c - |\theta|)]\}$$

$$-S(\dot{x}_s)\mu_{k2}M \cdot \{(\ddot{y}_g + g) + 2R[S(\theta)\ddot{\theta} \cdot \sin(\theta_c - |\theta|) - \dot{\theta}^2 \cdot \cos(\theta_c - |\theta|)]\} \tag{47}$$

Values of f_y and F_y are same as those given by Eqs. (2) and (4). Applying a Lagrangian formulation and solving the obtained set of equations, governing equations of motion become:

$$\ddot{x}_s = -\ddot{x}_g + A_1 - \frac{3A_2[A_3 + 2(A_4 + A_5)]}{-2M_n + 3A_6} \tag{48}$$

$$\ddot{\theta} = \frac{3[A_3 + 2(A_4 + A_5)]}{2R(2M_n - 3A_6)} \tag{49}$$

Where the coefficients in Eqs. (48) and (49) are:

$$A_1 = 2R[S(\theta) \cdot \sin(\theta_c - |\theta|) + S(\dot{x}_s)\mu_{k2} \cdot \cos(\theta_c - |\theta|)]\dot{\theta}^2$$

$$A_2 = \cos(\theta_c - |\theta|) - S(\dot{x}_s)S(\theta)\mu_{k2} \cdot \sin(\theta_c - |\theta|)$$

$$A_3 = M_n[\ddot{x}_g\cos(\theta_c - |\theta|) - S(\theta)(\ddot{y}_g + g)\sin(\theta_c - |\theta|)]$$

$$A_4 = -S(\theta)(\ddot{y}_g + g)\sin(\theta_c - |\theta|)$$

$$A_5 = [S(\theta) \cdot R \cdot \sin 2(\theta_c - |\theta|) + 2R\mu_{k2} \cdot S(\dot{x}_s)\cos^2(\theta_c - |\theta|)]\dot{\theta}^2$$

$$A_6 = \cos 2(\theta_c - |\theta|) - S(\dot{x}_s)S(\theta)\mu_{k2} \cdot \sin 2(\theta_c - |\theta|) - 1$$

If $|f_x/f_{yi}| > \mu_{s1}$ sliding of keel blocks will occur. That leads to the following formulation:

$$\left|\frac{M_n\ddot{x}_g - S(\dot{x}_s) \cdot \mu_{k2}(\ddot{y}_g + g) - R[\ddot{\theta}(M_n\cos(\theta_c - |\theta|) + 2S(\dot{x}_s)\mu_{k2} \cdot S(\theta)\sin(\theta_c - |\theta|)) - \dot{\theta}^2[S(\theta)\sin(\theta_c - |\theta|) + 2S(\dot{x}_s)\mu_{k2}\cos(\theta_c - |\theta|)]]}{(M_n + 1)(\ddot{y}_g + g) + (M_n + 2)R[S(\theta)\ddot{\theta}\sin(\theta_c - |\theta|) - \dot{\theta}^2\cos(\theta_c - |\theta|)]}\right| > \mu_{s1} \tag{50}$$

3.8 Sliding and rocking of keel blocks accompanied with sliding of the ship

Since keel blocks experience sliding and rocking, sliding of the ship happens, also. Degrees of freedom in this mode of motion are horizontal translation of the ship relative to the blocks, x_s, horizontal translation of keel blocks relative to the ground, x_b, and counter clockwise rotating of blocks, θ. This mode of motion is shown in Fig. 10. The coordinates of mass centers are:

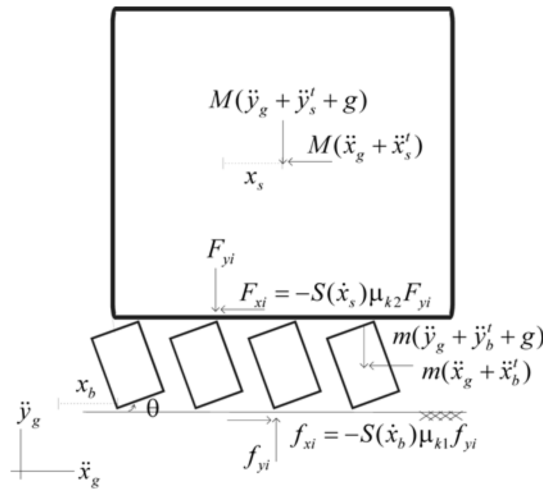

Fig. 10 Mode 8-Sliding and rocking of keel blocks accompanied with sliding of the ship

$$x_b^t = x_b - S(\theta)R[\sin\theta_c - \sin(\theta_c - |\theta|)] \tag{51}$$

$$x_s^t = x_b + x_s - 2S(\theta)R[\sin\theta_c - \sin(\theta_c - |\theta|)] \tag{52}$$

Definition of y_s^t, y_b^t are same as Eqs. (18) and (20). Considering the slide of keel blocks and the ship, forces along X-direction can be expressed as:

$$F_x = -S(\dot{x}_s) \cdot \mu_{k2} \cdot M\{(\ddot{y}_g + g) + 2R[S(\theta) \cdot \ddot{\theta} \cdot \sin(\theta_c - |\theta|) - \dot{\theta}^2\cos(\theta_c - |\theta|)]\} \tag{53}$$

$$f_x = -S(\dot{x}_b)\mu_{k1}\{(nm + M)(\ddot{y}_g + g) + (nm + 2M)R[S(\theta)\ddot{\theta}\sin(\theta_c - |\theta|) - \dot{\theta}^2\cos(\theta_c - |\theta|)]\} \tag{54}$$

Values of f_y and F_y are same as those given by Eqs. (2) and (4).

Applying the Lagrangian formulation and solving the obtained set of equations, the following resulted equations of motion are obtained:

$$\ddot{x}_s = \frac{1}{A_7 + A_8} \cdot \{2[A_1 + (A_2 + A_3)(\ddot{y}_g + g)] + M_n[A_4 + A_5(\ddot{y}_g + g)] + A_6 \tag{55}$$

$$\ddot{x}_b = -\frac{M_n + 2}{M_n\cos(\theta_c - |\theta|)} \cdot \left[\frac{A_9 + A_{10}(A_{11} - A_{12} - A_4\cos(\theta_c - |\theta|))}{(M_n + 2)(A_7 + A_8)}\right] \tag{56}$$

$$\ddot{\theta} = \frac{3[A_{11} + A_{12} + A_4\cos(\theta_c - |\theta|)]}{R(A_7 + A_8)} \tag{57}$$

The coefficients of Eqs. (55), (56) and (57) are:

$$A_1 = -2R\dot{\theta}^2[3S(\theta)\sin(\theta_c - |\theta|) + (S(\dot{x}_b)\mu_{k1} + 2S(\dot{x}_s)\mu_{k2})\cos(\theta_c - |\theta|)]$$

$$A_2 = 3S(\theta)[\cos(\theta_c - |\theta|) - 3(\theta)(S(\dot{x}_b)\mu_{k2}S(\dot{x}_s)\mu_{k2}\sin(\theta_c - |\theta|)]\sin(\theta_c - |\theta|)$$

$$A_3 = S(\dot{x}_b)\mu_{k1}[-2 + 3\cos^2(\theta_c - |\theta|) - 9S(\theta)S(\dot{x}_s)\mu_{k2}\cos(\theta_c - |\theta|)\sin(\theta_c - |\theta|)]$$

$$A_4 = -2R\dot{\theta}^2[2S(\theta)\sin(\theta_c - |\theta|) + (S(\dot{x}_b)\mu_{k1} + S(\dot{x}_s)\mu_{k2})\cos(\theta_c - |\theta|)]$$

$$A_5 = 3S(\theta)\sin(\theta_c - |\theta|)[\cos(\theta_c - |\theta|) - 2S(\dot{x}_s)S(\theta)\mu_{k2}\sin(\theta_c - |\theta|)] + S(\dot{x}_b)\mu_{k1}[-1 + 3\cos^2(\theta_c - |\theta|) - 6S(\dot{x}_s)S(\theta)\mu_{k2}\cos(\theta_c - |\theta|)\sin(\theta_c - |\theta|)]$$

$$A_6 = 12[-S(\dot{x}_s)\mu_{k2}\sin^2(\theta_c - |\theta|)(\ddot{y}_g + g) - S(\theta)S(\dot{x}_b)S(\dot{x}_s)\mu_{k1}\mu_{k2}\cos(\theta_c - |\theta|)(\ddot{y}_g + g)]$$

$$A_7 = M_n[4 + 3\cos^2(\theta_c - |\theta|) - 3S(\theta)S(\dot{x}_b)\mu_{k1}\cos(\theta_c - |\theta|)\sin(\theta_c - |\theta|)]$$

$$A_8 = 6[-2 + 2\cos^2(\theta_c - |\theta|) - S(\theta)\cos(\theta_c - |\theta|)\sin(\theta_c - |\theta|)[S(\dot{x}_b)\mu_{k1} + S(\dot{x}_s)\mu_{k2}]]$$

$$A_9 = \ddot{x}_g\cos(\theta_c - |\theta|) - S(\theta)(\ddot{y}_g + g)\sin(\theta_c - |\theta|) - 2\frac{\cos(\theta_c - |\theta|)}{M_n + 2}[\ddot{x}_g - 2R\dot{\theta}^2[S(\theta)\sin(\theta_c - |\theta|) + S(\dot{x}_s)\mu_{k2}\cos(\theta_c - |\theta|)]$$

$$A_{10} = 4[3\sin^2(\theta_c - |\theta|) + M_n + 3S(\theta)S(\dot{X}_s)\mu_{k2}\cos(\theta_c - |\theta|)\sin(\theta_c - |\theta|)]$$

$$A_{11} = -M_n[[\ddot{y}_g + g] - R\dot{\theta}^2\cos(\theta_c - |\theta|)[S(\theta)\sin(\theta_c - |\theta|) + S(\dot{x}_b)\mu_{k1}\cos(\theta_c - |\theta|)]$$

$$A_{12} = [2S(\theta)\sin(\theta_c - |\theta|) + S(\dot{x}_b)\mu_{k1}\cos(\theta_c - |\theta|)](\ddot{y}_g + g)$$

4. Impact induced by rocking of keel blocks

For the case when rocking of keel blocks takes place, changing in rotation direction causes an impact between keel blocks-ground and between keel blocks-ship. This impact results to some energy dissipation which causes reduction in the angular velocity of keel blocks after the impact. The governing impact model is from classical impact theory. This model assumes a point impact, nonzero coefficient of restitution, and finite value of friction. This model can be expressed as the following condition considering the law of conservation of momentum:

$$\dot{\theta}(t^+) = 2e\dot{\theta}(t^-) \qquad 0 \le e \le 1$$

In which e is the coefficient of restitution; t^+ is the time right after the impact; t^- is the time right before the impact. The duration of impact is ignored and changes in angular velocity are considered to occur instantaneously.

5. Conclusions

The complete, two-dimensional formulation of all probable modes of motion for a ship-block system which is located in a dry dock, and excited in both horizontal and vertical direction, has been presented. Some assumptions are considered throughout modeling: the ship is modeled as a rigid block, and coulomb friction is applied between contacted surfaces, the supposed degrees of freedom are horizontal translation of keel blocks and ship, and counter clockwise rotation of the blocks. The motion of all keel blocks is considered to be the same. The formulation has been described in terms of eight mode of motion which is the combination between sliding of the ship, sliding of keel blocks, and rocking of keel blocks. By applying Lagrangian formulation, the horizontal acceleration of keel blocks and the ship, and angular acceleration of keel blocks are derived. The complete separations of the ship from keel blocks and keel blocks from ground are regarded as failure. Vertical motions for the ship and keel blocks are defined in terms of keel block rotational degree of freedom. Equations of motion for each mode as well as applied forces acting on keel blocks and ship are formulated. Furthermore, using the interface forces and geometric relationships, transition conditions between various modes are investigated. A classical impact theory was used to describe the impact happened when the rotation direction of keel blocks changes. A point-impact, nonzero coefficient of restitution and finite value of friction are the basic characters of this model.

In the accompanying paper a complete sensitivity analysis of the system is presented. In such cases of properties, conducting a time-history analysis, responses of the system are studied.

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