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A simplified procedure to incorporate soil non-linearity in missile penetration problems

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Abstract. In this paper, a simplified mathematical procedure is presented to incorporate nonlinearity in soil material to predict the deceleration time history, penetration depth and other relevant parameters for normal impact of missiles into soil targets. Numerical method is employed for these predictions. The results of the study are compared with experimental observations and predictions available in the literature. A good agreement is found with experimental observations and an improvement is observed with existing predictions. A comparison is also made with linear soil model. Some parametric studies are also carried out to obtain the results of practical interest.

Keywords: missile penetration; projectiles; missiles; soil targets; soil penetration.

1. Introduction

Underground structures such as bunkers, buried nuclear containment etc. are the target of enemy during wartime. For hitting these structures missiles have to penetrate the overlying soil. The safety or destruction of such targets depends on correct estimation of depth of penetration, force and stresses in the surrounding or overlying soil medium. It is due to these reasons that the problem of missile penetration into soil targets achieves great significance. Though studies are available pertaining to the impact of missiles on targets such as metallic plates, shells, concrete barriers etc. (Goldsmith 1999, Corbett *et al.* 1996, Abbas *et al.* 1995, 1996, Chaudhury *et al.* 2002, Siddiqui *et al.* 2003, Khan *et al.* 2003, Siddiqui 2003) the work on soil targets is scanty. This is perhaps due to the difficulty in soil modeling and difficulties involved in the development of experimental set up for such studies.

It is almost well established that when a missile impacts and penetrates a soil target, it creates a cavity. This cavity expands under the action of stress waves generated in the target medium. To study the expansion phenomenon of soil target, three models have been reported in the literature viz. spherical cavity expansion model, cylindrical cavity expansion model and model of orthogonal layers (Backman and Goldsmith 1978, Goldsmith 1999, Forrestal *et al.* 1981, 1992, Siddiqui and Abbas 2002). Using these models, past investigators have proposed various formulae for the

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prediction of possible deceleration-time history, penetration depth, forces at the missile nose etc. These expressions, however, are generally based on linear soil model and are analytical in nature. Predictions considering nonlinear soil model and numerical approach are, however, scanty. Keeping these points in view, in the present study, a simplified mathematical procedure is presented to incorporate nonlinearity in soil material to predict the deceleration time history, penetration depth and other relevant parameters for normal impact of missiles into soil targets. Numerical method is employed for these predictions. The results of the study are compared with experimental observations and predictions available in the literature. A good agreement is found with experimental observations and considerable improvement is observed with existing predictions. A comparison is also made with linear soil. In addition to this, some parametric studies are also carried out to obtain the results of practical interest.

2. Problem formulation

To formulate the present problem following assumptions and idealizations have been made.

- Missile is a rigid projectile and its impact on soil target is normal and axi-symmetric.
- Deformation of missile is negligible and only soil deforms during penetration.
- Wave propagation is one dimensional and in the radial direction.
- The missile does not carry any warhead and, therefore, there is no explosion.
- The total energy of the missile is used to penetrate the soil and the loss of its energy in the form of heat and sound is assumed to be negligible.

2.1 Material model

In the present study, the target soil medium has been modeled using Mohr-Coulomb yield criterion. This criterion can be mathematically represented in a general form as:

$$\sigma_r - \sigma_\theta = \tau_0 + \lambda p \tag{1}$$

where, σ_r, σ_θ are the radial and tangential components of Cauchy stress (positive in compression); p is the hydrostatic pressure and τ_0, λ define the yield condition.

In Eq. (1) $\lambda = \lambda_1$ for linear soil model and $\lambda = \lambda_1 + \lambda_2 p$ for non-linear soil model. Here, τ_0, λ_1 and λ_2 are the parameters that have to be obtained from best-fit of experimental data plotted between shear strength and hydrostatic pressure. The hydrostatic pressure p is given by (Forrestal and Luk 1992):

$$p = (\sigma_r + 2\sigma_\theta)/3 \tag{2}$$

2.2 Stresses in the soil medium

When a rigid missile nose (Fig. 1) penetrates a uniform target medium with normal incidence, a spherically symmetric cavity is formed. This spherically symmetric cavity expands with constant velocity V under the action of stress waves. This expansion produces plastic and elastic response regions bounded by the radii Vt and ct, where t is the time and c is elastic-plastic interface velocity

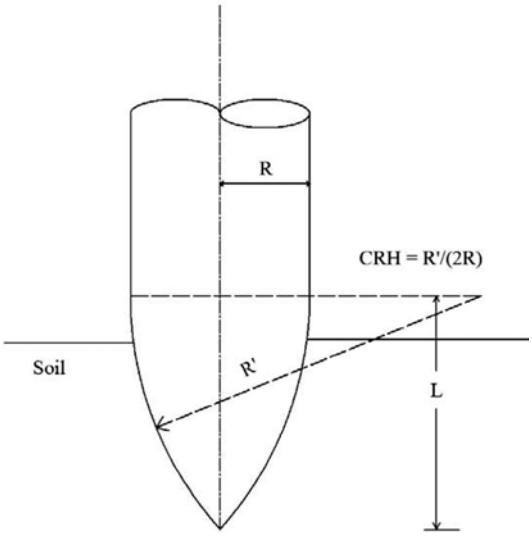

Fig. 1 A missile with ogival nose geometry

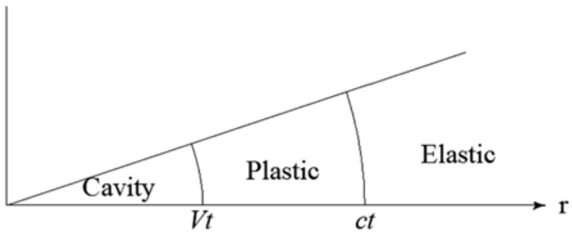

Fig. 2 Response regions

(Fig. 2). The element of such an expanded layer at a radial distance r from the axis of symmetry is subjected to shear stress $(\sigma_r - \sigma_\theta)$ and hydrostatic pressure p given by Eqs. (1) and (2).

Now using above material model, following equations of momentum and mass conservation in Langrangian coordinates have been derived (Forrestal and Luk 1992).

$$(r+u)^2\frac{\partial \sigma_r}{\partial r} + 2\left(1+\frac{\partial u}{\partial r}\right)(r+u)(\sigma_r - \sigma_\theta) + \rho_o r^2 \frac{\partial^2 u}{\partial t^2} = 0 \tag{3}$$

$$\frac{1}{3}\frac{\partial}{\partial r}[(r+u)^3] = \frac{\rho_o}{\rho} r^2 \tag{4}$$

where r is the Lagrangian coordinate, ρ is the current density and u is the radial displacement (positive outward) which satisfies the following boundary condition at the cavity interface:

$$u(r=0, t) = Vt \tag{5}$$

To reduce Eqs. (3) and (4) into ordinary differential equations, following similarity transformations may be used:

$$\xi = \frac{r}{ct} \tag{6}$$

$$\bar{u}(\xi) = \frac{u(r,t)}{ct}, \quad \text{and} \tag{7}$$

$$S = \frac{\sigma_r}{\tau_o} \tag{8}$$

Substitution of above transformations (Eqs. 6 to 8) in Eqs. (3) and (4) leads to

$$(\xi + \overline{u})\frac{dS}{d\xi} + 2\alpha\frac{d}{d\xi}(\xi + \overline{u})S = \frac{-2\alpha}{\lambda}\frac{d}{d\xi}(\xi + \overline{u}) - \frac{\rho_0 c^2}{\tau_0}\frac{\xi^4}{(\xi + \overline{u})}\frac{d^2\overline{u}}{d\xi^2} \tag{9}$$

and, $$\frac{1}{3}\frac{d}{d\xi}[(\xi + \overline{u})^3] = (1 - \eta^*)\xi^2 \tag{10}$$

in which $$\alpha = 3\lambda/(3 + 2\lambda) \tag{11}$$

$$\eta^* = \text{locked volumetric strain} = \left(1 - \frac{\rho_0}{\rho^*}\right) \tag{12}$$

ρ_0 = initial mass density; and
ρ^* = locked mass density;

The solution of above differential equations for $\xi = 0$ may be expressed as (Forrestal and Luk 1992):

$$S(\xi = 0) = A + B\rho_o V^2/\tau_o \tag{13}$$

where,

$$A = \frac{1}{\alpha}\left(\frac{1 + \tau_o/2E}{\gamma}\right)^{2\alpha} - \frac{1}{\lambda} \tag{14}$$

$$B = \frac{3}{(1 - \eta^*)(1 - 2\alpha)(2 - \alpha)} + \frac{1}{\gamma^2}\left(\frac{1 + \tau_o/2E}{\gamma}\right)^{2\alpha}$$

$$\left\{(3\tau_o/E) + \eta^*(1 - 3\tau/2E)^2 - \frac{\gamma^3[2(1 - \eta^*)(2 - \alpha) + 3\gamma^3]}{(1 - \eta^*)(1 - 2\alpha)(2 - \alpha)(1 + \tau_o/2E)^4}\right\} \tag{15}$$

in which $$\gamma = \left[\left(1 + \frac{\tau_0}{2E}\right)^3 - (1 - \eta^*)\right]^{1/3}; \tag{16}$$

E = modulus of elasticity; and
τ = shear stress

The above equation is indeterminate for $\lambda = 0$ and $\lambda = 3/4$. These cases therefore are separately analyzed by Forrestal and Luk (1992) and following equations for A and B are derived:
For $\lambda = 0$,

$$A = \frac{2}{3}\left\{1 - \ln\left[\frac{(1 + \tau_o/2E)^3 - (1 - \eta^*)}{(1 + \tau_o/2E)^3}\right]\right\} \tag{17}$$

$$B = \frac{3}{2(1 - \eta^*)} + \frac{(3\tau_o/E) + \eta^*(1 - 3\tau_o/2E)^2}{[(1 + \tau_o/2E)^3 - (1 - \eta^*)]^{2/3}}$$

$$-\left[\frac{(1+\tau_o/2E)^3-(1-\eta^*)}{2(1+\tau_o/2E)^4}\right]^{1/3}\left[1+\frac{3(1+\tau_o/2E)^3}{(1-\eta^*)}\right] \tag{18}$$

For $\lambda = 3/4$,

$$A = 2(1+\tau_o/2E)/\gamma - 4/3 \tag{19}$$

$$B = \frac{-2\ln\gamma}{(1-\eta^*)} + \frac{(1+\tau_o/2E)[3\tau_o/E+\eta^*(1-3\tau_o/2E)^2]}{\gamma^3} - \frac{2}{3}\left[\frac{1}{(1+\tau_o/2E)^3} - \frac{3\ln(1+\tau_o/2E)}{(1-\eta^*)}\right] \tag{20}$$

The cavity expansion velocity V as required in Eq. (13) may be obtained in terms of missile rigid body velocity V_z, nose length L, penetration depth z, radius of aft body R and CRH ψ as (Siddiqui and Abbas 2002)

$$V = V_Z\frac{(L-z)}{2R\psi} \tag{21}$$

Having known the value of S at $\xi = 0$ from Eq. (13), we can estimate the radial stress component σ_r at the missile nose using Eq. (8).

2.3 Forces on missile nose and deceleration

The penetration of a missile into the soil target results in the radial movement of the target material at the cavity interface which produces radial stress in the target material. Using cylindrical cavity expansion model the incremental radial force on the missile nose for a thin target thickness dz can be written as,

$$dF_r = 2\pi\sigma_r(0)R(z)dz \tag{22}$$

where $\sigma_r(0)$ is the radial stress in the target material at its interface with missile, and $R(z)$ is the radius of the missile nose at a distance z from its tip (Fig. 1). In the above equation, radial stress component can be obtained using the procedure discussed in section 2.2. It is to be noted that the radial stress component σ_r at the missile nose is assumed to be the same for Cylindrical and Spherical cavity expansion theories. The expression of $R(z)$ for an ogive nose is given by (Siddiqui and Abbas 2002):

$$R(z) = -a + \sqrt{a^2 - z^2 + 2Lz} \tag{23}$$

where,

a = $(R' - R)$;

R' = radius of the ogive nose (Fig. 1);

R = radius of the aft body of missile (Fig. 1); and

L = nose length of missile.

The vertical force at the nose of the missile due to the vertical stiffness of the target material of thickness dz is

$$dF_v = dF_r \tan\theta \tag{24}$$

where,
dF_v = incremental vertical force;
dF_r = incremental force in radial direction; and
θ = equivalent cone angle.

Another force acting at the nose is the drag force, which is tangential to the surface of the missile nose arising due to the friction between the target material and missile. The magnitude of incremental drag force dF_d for the elemental target thickness dz is equal to the product of coefficient of dynamic friction at the interface of missile surface and target material μ_d and force normal to the missile nose dF_n i.e.,

$$dF_d = \mu_d dF_n \tag{25}$$

where,
$$dF_n = dF_r \sec\theta \tag{26}$$

Therefore,
$$dF_d = \mu_d dF_r \sec\theta \tag{27}$$

Hence, the total incremental vertical upward component dF_z of the target reaction will be

$$dF_z = dF_v + dF_d \cos\theta$$

Substituting expressions for dF_v and dF_d from Eqs. (24) and (27) respectively, we get

$$\begin{aligned} dF_z &= dF_r \tan\theta + \mu_d dF_n \cos\theta \\ &= dF_r(\mu_d + \tan\theta) \end{aligned} \tag{28}$$

where, the radial force dF_r and vertical force dF_v are in fact the radial and vertical components of the normal force dF_n. The total vertical target reaction on the missile nose has been obtained by integrating Eq. (28) from 0 to penetration depth z (where, $z \le L$):

$$\begin{aligned} F_z &= \int_0^z (\mu_d + \tan\theta) dF_r \\ &= \int_0^z (\mu_d + \tan\theta) 2\pi\sigma_r R(z) dz \end{aligned} \tag{29}$$

If the depth of penetration of missile is greater than the nose length then the upper limit of integration will be up to L because we are getting reaction only from the nose. Eq. (29) has been applied for the estimation of total vertical target reaction F_z for ogive nose shaped missiles.

The substitution of $R(z)$ for ogive nose from Eq. (23) and the value of $\tan\theta$ at a distance z from the tip of the nose (Fig. 1) leads to

$$F_z = 2\pi\int_0^z \sigma_r R(z)\left(\frac{L-z}{R(z)+a} + \mu_d\right)dz \tag{30}$$

$$= 2\pi\int_0^z \sigma_r\left(-a + \sqrt{a^2 - z^2 + 2Lz}\right)\left(\frac{L-z}{\sqrt{a^2 - z^2 + 2Lz}} + \mu_d\right)dz \tag{31}$$

It is to be noted here that σ_r for ogive nose is a function of z. The force F_z given by Eq. (31) for the ogive nose can be obtained using any standard numerical integration scheme.

2.4 Response estimation

To obtain the response time histories of velocity, penetration depth, and the deceleration of missile, the dynamic equilibrium of missile has been considered that results in the following well-known equation:

$$m\frac{dV_z}{dt} = -F_z \tag{32}$$

The integration of above equation, using any standard numerical integration scheme, will yield time histories of velocity, penetration depth and the deceleration of missile. In the present study, the forward finite difference approach has been employed for its integration. Using this approach, the velocity V_z, deceleration a_z and the penetration depth z of missile at $(i+1)$th time step can be obtained by the following relations:

$$V_z^{i+1} = V_z^i - \frac{1}{m}(F_z^{i+1}\Delta t) \tag{33}$$

$$a_z^{i+1} = \frac{V_z^{i+1} - V_z^i}{\Delta t} \tag{34}$$

$$z^{i+1} = z^i + V_z^i\Delta t \tag{35}$$

3. Numerical study

The methodology for the response analysis of soil target under missile impact has been presented in above section. This methodology has now been applied to a numerical example and the results are presented in this section. The important numerical studies that are included here are comparison of predicted response for linear and nonlinear soil models with experimental observations; prediction of maximum penetration depth; and comparison of maximum estimated depth with published literature prediction and experimentally observed depth. In addition to these, some parametric studies have also been performed in this section to obtain the results of practical interest.

3.1 Soil material

To describe the soil material model completely we need three important parameters τ_0, λ_1 and λ_2

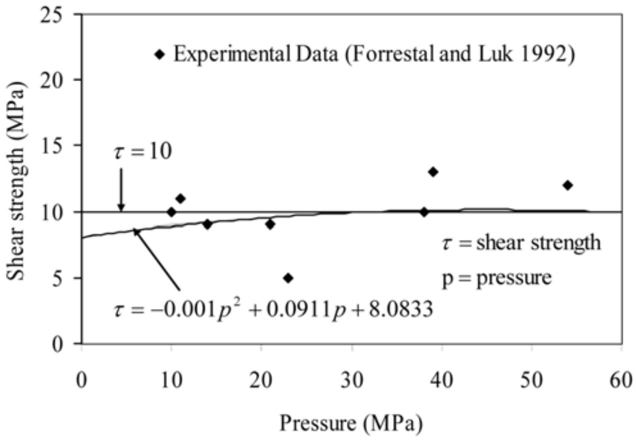

Fig. 3 Shear strength versus hydrostatic pressure for the soil target

(Eq. 1). These three parameters have been obtained by plotting graph between shear strength and hydrostatic pressure using the experimental data and then fitting linear and non-linear curves to these data points as shown in Fig. 3 for soil target in Forrestal and Luk (1992). We get following values of τ_0, λ_1 and λ_2 for linear and nonlinear materials:

τ_0 = 10.0 MPa ; λ_1 = 0.0 for linear model (λ_2 is zero for this case);

τ_0 = 8.083 MPa ; λ_1 = 0.091 and λ_2 = –0.001 for nonlinear material model.

3.2 Comparison with field tests

In the present study, the results of penetration depth obtained from the analysis have been compared with six tests that were conducted by Forrestal and Luk (1992) at the Sandia Test Range, Nevada, USA into a soil target at a site called Antelope Lake. The average water content of soil target was about 20% (in a depth of 6.1 m). Further details about the target soil may be found in Ehrgott and Kinnebrew (1991).The soil target is having the following average properties:

Initial mass density, $\rho_o = 1.86 \times 10^3$ kg/m^3

Locked volumetric strain, $\eta^* = 0.13$

Modulus of elasticity, E = 160 MPa

Coefficient of dynamic friction, μ_d = 0.13 (Siddiqui and Abbas 2002).

To carry out the tests, Forrestal and Luk (1992) employed a mobile, 152 mm diameter, smooth bore gas gun. Using this gun they fired six missiles (i.e., projectiles) with caliber radius head (CRH) 3.0 and mass 23.1 kg. The missiles were carrying an onboard recording package and accelerometers attached to its walls. The average depth of penetration was observed through these tests as 5.04 m in the soil target. The dispersion, measured in terms of coefficient of variation, in this average depth was about 11%.

Fig. 4 shows the predicted missile decelerations of linear as well as nonlinear models along with the experimental data for six experiments. Neglecting the low amplitude oscillations in the data that are caused by structural vibrations and non-homogeneous soil target, deceleration measurements and predictions show a monotonic decay to a nearly constant value followed by a jump at end of the trajectory. From behavior point of view, however, the missiles are stopping in our studies earlier

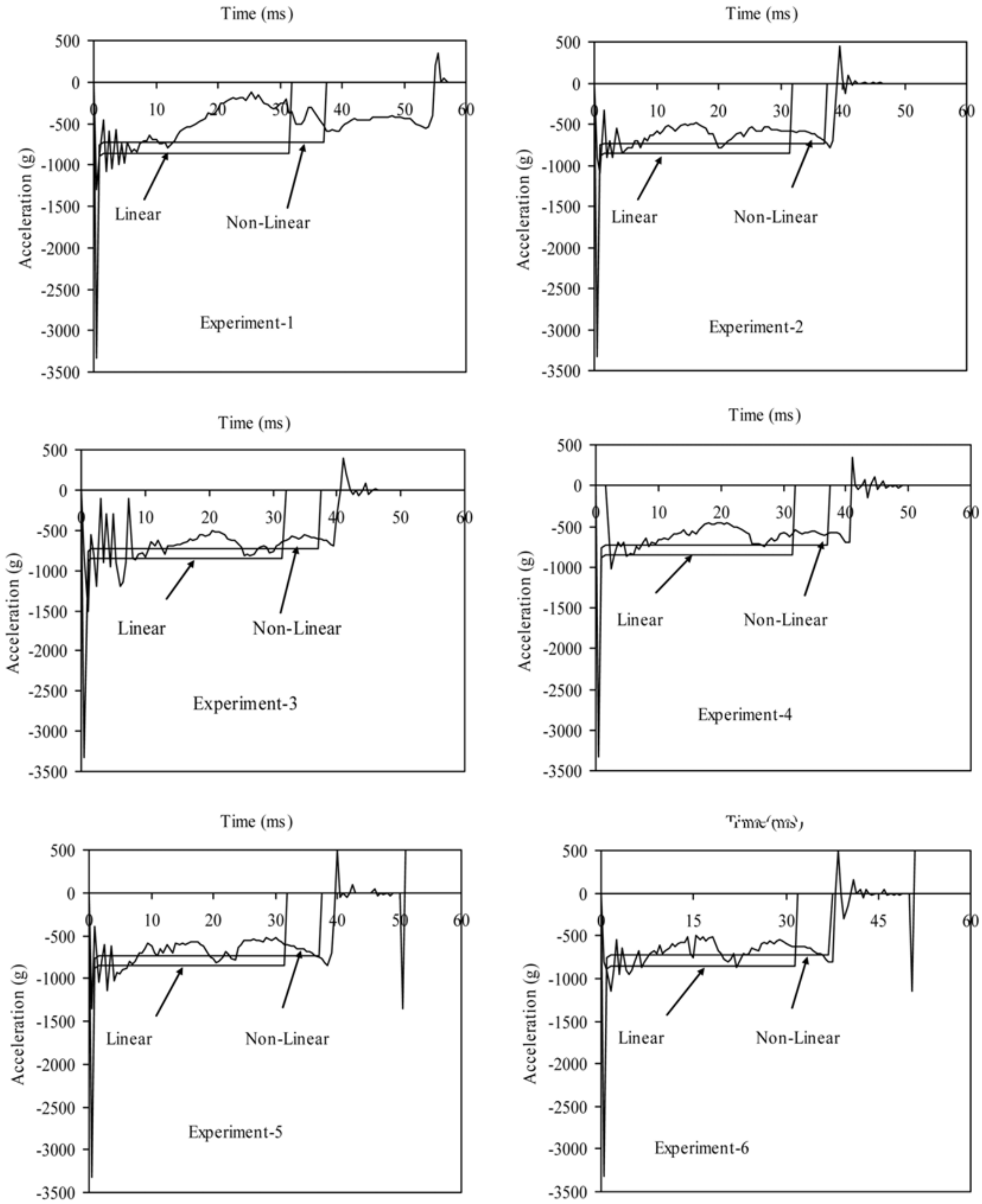

Fig. 4 Predicted and experimentally observed deceleration time histories

than experiments. This may be due to the fact that in our analysis we have assumed perhaps a little higher value of coefficient of friction than actual that exists between the missile nose and surrounding soil medium. Further the results show that the predictions of nonlinear model are better than the linear model.

Table 1 Comparison with Forrestal and Luk (1992) prediction

	Friction neglected	Friction considered
Forrestal and Luk (1992)	4.98 m	3.16 m
Present Study (Linear)	6.23 m	4.34 m
Present Study (Non-linear)	7.24 m	5.08 m
Average Experimentally observed depth:	-	5.04 m*

*with coefficient of variation = 11%

3.3 Comparison with Forrestal and Luk prediction

Table 1 shows that if friction force on missile nose is neglected, the prediction of Forrestal and Luk (1992) is close to the experimental depth of penetration, however, if friction is considered, their prediction underestimates the depth of penetration. But the sliding friction on the nose of the missile cannot be ignored in the present problem. Moreover, in the present study, though the depth of penetration neglecting friction is quite high but the consideration of friction gives the magnitude which is reasonably close to the actual depth of penetration particularly when nonlinear material model has been considered (difference is about 5%). This shows that the present study is a good improvement of Forrestal and Luk (1992) model.

3.4 Penetration depth and impact velocity with time

Fig. 5 shows the variation of penetration depth and velocity with time for linear as well as nonlinear models. The nonlinear model predicts the higher depth of penetration and higher velocity as compared to the linear model. This is due to the fact that in the nonlinear material model, strength of the material at lower values of hydrostatic pressure is less than the linear model (Fig. 3). For the data considered in the present study, the marginal increase in the shear strength of the nonlinear model over the linear model could not affect the trend.

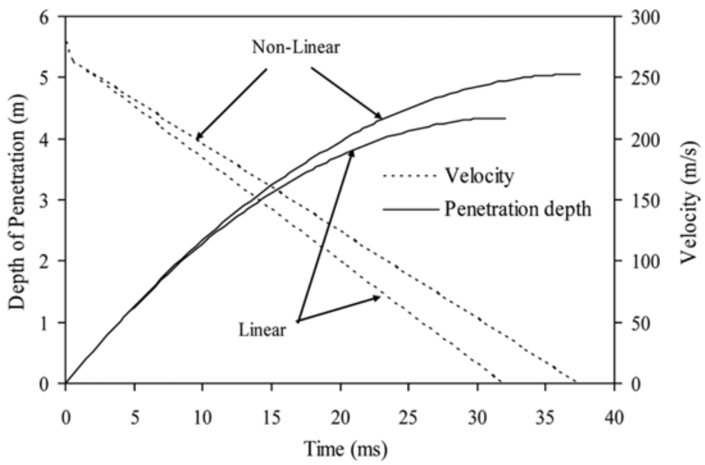

Fig. 5 Variation of penetration depth and velocity with time

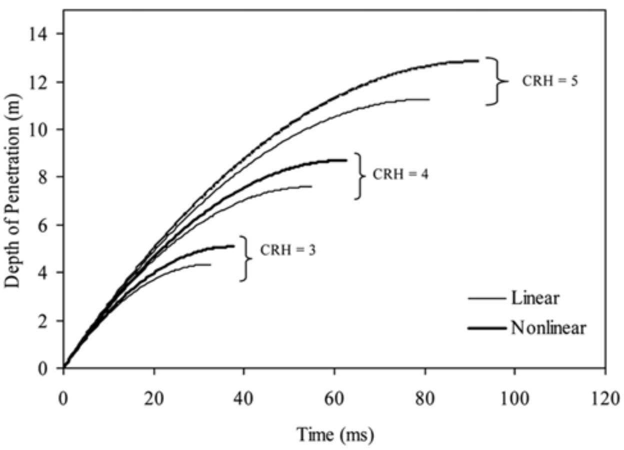

Fig. 6 Effect of CRH on penetration depth

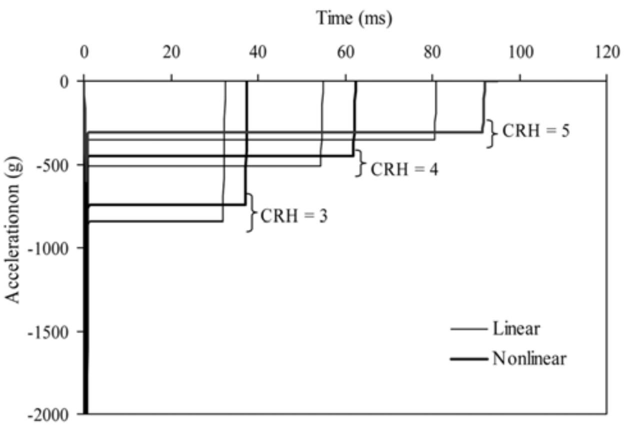

Fig. 7 Effect of CRH on deceleration of missile

3.5 Parametric studies

To obtain the results of academic and field interest some parametric studies have also been conducted and discussed in the following section:

3.5.1 Effect of CRH

Figs. 6 and 7 show the variation of depth of penetration and missile deceleration with time for different CRH by keeping all other parameters same. These figures show that as we increase the CRH of missile nose, the depth of penetration increases, whereas deceleration decreases. It is due to the fact that as CRH increases the nose length increases (nose length = 252, 294 and 331 mm for CRH = 3, 4, and 5 respectively) and shape of the nose becomes more pointed that makes the penetration easier and, therefore, penetration depth increases whereas deceleration decreases. This pattern is same for the linear as well as the nonlinear material models.

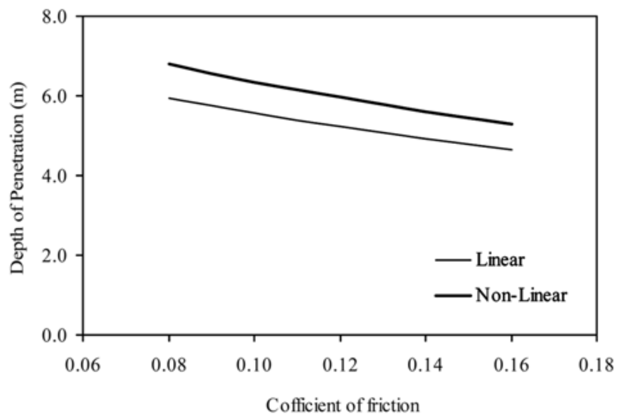

Fig. 8 Effect of coefficient of friction on penetration depth

3.5.2 Effect of coefficient of friction

Coefficient of friction is an uncertain parameter that directly governs the force of resistance offered by the material to the penetration of missile. It is expected that as the coefficient of friction increases the depth of penetration should decrease. Fig. 8 shows the same trend and the variation is almost linear for practical purposes. A decrease in the coefficient of friction of 10% results in an increase in the depth of penetration by about 15%. Again for the same coefficient of friction, nonlinear model offers lesser resistance, therefore, we observe greater depth of penetration for this model in comparison to the linear material model.

3.5.3 Effect of missile mass

For given velocity, mass is a direct measure of kinetic energy of a missile. Fig. 9 shows that as we are increasing the mass, keeping velocity and all other parameters constant, penetration depth increases. This is an expected trend. However, this should be kept in mind that practically it is not always feasible to increase the mass dramatically without affecting its velocity. Keeping this in view this parametric study has been conducted within a small variation in mass (22 to 24 kg). An increase of 10% in the mass of missile causes about 5% increase in the depth of penetration.

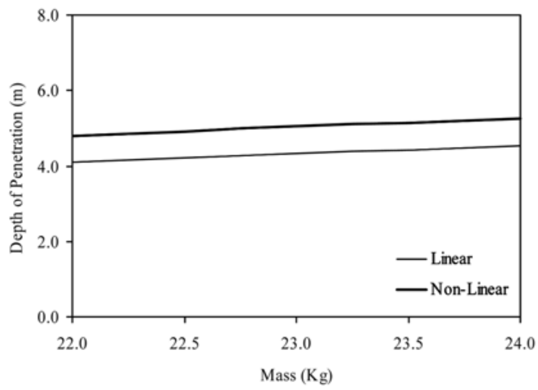

Fig. 9 Effect of missile mass on penetration depth

Table 2 Effect of modulus of elasticity on various parameters

Parameter		Modulus of elasticity (MPa)		
		120	160	210
Maximum deceleration (g)	Linear	3746	3763	3775
	Non-linear	2841	2979	2761
Minimum deceleration (g)	Linear	799	842	878
	Non-linear	713	741	763
t_f (ms)	Linear	34	32.5	31
	Non-linear	39	37.5	36.5
Penetration depth (m)	Linear	4.57	4.34	4.17
	Non-linear	5.26	5.08	4.95

3.5.4 Effect of modulus of elasticity

The modulus of elasticity is not a simple parameter to obtain for any soil, for it varies with soil type, state, confinement, and depth. Therefore, there may be a large variation in its estimation. As the soil modulus of elasticity of soil increases it makes the soil stiffer which consequently makes the missile penetration difficult into the soil. It is due to this reason, increase in maximum deceleration; decrease in depth of penetration; and decrease in stopping time has been observed in Table 2 with increase in the modulus of elasticity of soil.

4. Conclusions

In the present study, a mathematical procedure has been presented to predict deceleration-time history, penetration depth and other relevant parameters for normal impact of missile into nonlinear soil targets. The results of the study have been compared with experimental observations and analytical predictions of Forrestal and Luk (1992). A good agreement has been found with experimental observations and considerable improvement has been observed with existing predictions. A comparison has been made with linear soil model and it has been observed that the linear soil model underestimates the penetration depth. Influence of CRH on penetration depth and deceleration has been studied and it has been observed that CRH affects significantly the deceleration and penetration depth both. Effect of sliding friction on deceleration and depth of penetration has been studied and it has been observed that the sliding friction should be carefully determined for true estimation of any of the design parameters. Increase in the modulus of elasticity of soil causes, increase in maximum deceleration; decrease in depth of penetration; and decrease in stopping time.

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Notation

a, λ_1 and λ_2 = parameters
c = elastic-plastic interface velocity
dF_n = force normal to the missile nose
dF_r = incremental force in radial direction
dF_v = incremental vertical force
E = modulus of elasticity
L = nose length of missile
p = hydrostatic pressure
r = Lagrangian coordinate
R = radius of the aft body of missile
$R(z)$ = radius of the missile nose at a distance z from its tip
R' = radius of the ogive nose
t = time
u = radial displacement (positive outward)
z = penetration depth
σ_r, σ_θ = radial and tangential components of Cauchy stress (positive in compression)
τ = shear stress
ρ = current density
θ = equivalent cone angle
τ_0, λ = define the yield condition
μ_d = coefficient of dynamic friction at the interface of missile surface and target