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The nonlocal theory solution for two collinear cracks in functionally graded materials subjected to the harmonic elastic anti-plane shear waves

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Abstract. In this paper, the scattering of harmonic elastic anti-plane shear waves by two collinear cracks in functionally graded materials is investigated by means of nonlocal theory. The traditional concepts of the non-local theory are extended to solve the fracture problem of functionally graded materials. To overcome the mathematical difficulties, a one-dimensional non-local kernel is used instead of a two-dimensional one for the anti-plane dynamic problem to obtain the stress field near the crack tips. To make the analysis tractable, it is assumed that the shear modulus and the material density vary exponentially with coordinate vertical to the crack. By use of the Fourier transform, the problem can be solved with the help of a pair of triple integral equations, in which the unknown variable is the displacement on the crack surfaces. To solve the triple integral equations, the displacement on the crack surfaces is expanded in a series of Jacobi polynomials. Unlike the classical elasticity solutions, it is found that no stress singularities are present at crack tips.

Keywords: crack; nonlocal theory; functionally graded materials; lattice parameter; stress waves.

1. Introduction

With the development of the functionally graded materials (FGMs), it has been widely introduced and applied to the development of thermal and structural components due to its ability to not only reduce the residual and thermal stresses but to increase the bonding strength and toughness as well. In recent years, many analytical and theoretical studies in fracture mechanics of FGMs have been widely done (Erdogan and Wu 1997, Delale and Erdogan 1988, Chen 1990, Ozturk and Erdogan 1996). Among them there are the solutions for a FGM strip containing an imbedded or an edge crack perpendicular to the surfaces (Erdogan and Wu 1997); for a crack in the non-homogeneous interlayer bounded by dissimilar homogeneous media (Delale and Erdogan 1988); and for a crack at

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the interface between homogeneous and non-homogeneous materials (Chen 1990, Ozturk and Erdogan 1996). Similar problems of delamination or an interface crack between a functionally graded coating and a substrate were considered in Jin and Batra (1996), Bao and Cai (1997), Shbeeb and Binienda (1999). The crack problem in FGM layers under thermal stresses was studied by Erdogan and Wu (1996). They considered an unconstrained elastic layer under statically self-equilibrating thermal or residual stresses. However, it is found that all the solutions in Erdogan and Wu (1997), Delale and Erdogan (1988), Chen (1990), Ozturk and Erdogan (1996), Jin and Batra (1996), Bao and Cai (1997), Shbeeb and Binienda (1999), Erdogan and Wu (1996) contain the stress singularities near the crack tips. This phenomenon is not reasonable according to the physical nature. As a result of this, beginning with Griffith, all fracture criteria in practice today based on other considerations, e.g., energy, the J-integral (Rice 1968) and the strain gradient theory (Xia and Hutchinson 1996).

To overcome the stress singularity in the classical elastic fracture theory, Eringen (1977, 1978, 1979) used nonlocal theory to discuss the stress near the tip of a sharp line crack in an isotropic elastic plate subject to uniform tension, shear and anti-plane shear, and the resulting solutions did not contain any stress singularities. This allows us to use the maximum stress as a fracture criterion. In contrast to these local approaches, of zero-range internal interactions, the modern nonlocal continuum mechanics originated and developed in the last four decades. Edelen (1976) contributed some mathematical formalism while Green and Rivilin (1965) simply enunciated some postulates for the non-local theory. On the other hand, Eringen (1976) contributed not just the complete physics and mathematics of the non-local theory but also, in addition, shaped the theory into a concrete form making it viable for practical applications to boundary value problems. According to nonlocal theory, the stress at a point X in a body depends not only on the strain at point X but also on that at all other points of the body. This is contrary to the classical theory that the stress at a point X in a body depends only on the strain at point X. In Pan and Takeda (1998), the basic theory of nonlocal elasticity was stated with emphasis on the difference between the nonlocal theory and classical continuum mechanics. The basic idea of nonlocal elasticity is to build a relationship between macroscopic mechanical quantities and microscopic physical quantities within the framework of continuum mechanics. The constitutive theory of nonlocal elasticity has been developed in Edelen (1976), in which the elastic modulus is influenced by the microstructure of the material. In Pan and Xing (1997), Pan and Takeda (1997), it has been found that the microstructure of the material not only affects the constitutive equation, but also the basic balance laws and boundary conditions. Other results have been given by the application of nonlocal elasticity to the fields such as a dislocation near a crack (Pan 1992, 1994), solid defects (Pan 1996, Pan and Fang 1996) and fracture mechanics problems (Pan 1995, Pan and Fang 1993). The literature on the fundamental aspects of nonlocal continuum mechanics is relatively extensive. The results of those concrete problems that were solved display a rather remarkable agreement with experimental evidence. This can be used to predict the cohesive stress for various materials and the results close to those obtained in atomic lattice dynamics (Eringen and Kim 1974, 1977). Likewise, a nonlocal study of the secondary flow of viscous fluid in a pipe furnishes a streamline pattern similar to that obtained experimentally by Eringen (1977). Other examples of the effectiveness of the nonlocal approach are: (i) prediction of the dispersive character of elastic waves demonstrated experimentally (and lacking in the classical theory) (Eringen 1972) and (ii) calculation of the velocity of short Love waves whose nonlocal estimates agree better with seismological observations than the local ones (Nowinski 1984). Several nonlocal theories have been formulated to address strain-gradient and size effects (Forest 1998). Recently, some fracture problems (Zhou *et al.* 1999, Zhou and Shen 1999, Zhou *et al.* 2003, Zhou and Wang 2003, Sun and Zhou 2004) in an isotropic elastic material and the piezoelectric material have been studied by use of nonlocal theory with a somewhat different method. However, to our knowledge, the scattering of harmonic elastic anti-plane shear waves by two collinear cracks in functionally graded materials has not been studied by use of the nonlocal theory. Thus, the present work is an attempt to fill this information needed. Here, we just attempt to give a theoretical solution for this problem.

In the present paper, the scattering of harmonic elastic anti-plane shear waves by two collinear cracks in functionally graded materials is investigated by use of nonlocal theory with Schmidt method (Morse and Feshbach 1958, Yan 1967). The traditional concepts of the non-local theory are extended to solve the dynamic fracture problem of functionally graded materials. The Fourier transform is applied and a mixed boundary value problem is reduced to a pair of triple integral equations. To solve the triple integral equations, the displacement on the crack surfaces is expanded in a series of Jacobi polynomials. Numerical solutions are obtained for the stress fields near the crack tips.

2. The crack model

It is assumed that there are two collinear symmetric cracks of length $1-b$ along the x-axis in functionally graded material plane as shown in Fig. 1. $2b$ is the distance between the two cracks (The solution of two collinear cracks of length $c-b$ in the functionally graded materials can easily be obtained by a simple change in the numerical values of the present paper for crack length $1-b/c.c>b>0$). A Cartesian coordinate system (x, y) is positioned as shown in Fig. 1. In this paper, the harmonic elastic stress wave is vertically incident. Let ω be the circular frequency of the incident wave. $w_0(x, y, t)$ is the mechanical displacement. $\tau_{zk0}(x, y, t)$ $(k = x, y)$ is the anti-plane shear stress field. Because the incident wave is the harmonic anti-plane shear stress wave, all field quantities of $w_0(x, y, t)$ and $\tau_{zk0}(x, y, t)$ can be assumed to be of the forms as follows:

$$[w_0(x, y, t),\ \tau_{zk0}(x, y, t)] = [w(x, y),\ \tau_{zk}(x, y)]e^{-i\omega t} \tag{1}$$

In what follows, the time dependence of $e^{-i\omega t}$ will be suppressed but understood. Here, the standard superposition technique was used in the present paper. As discussed in Eringen (1979),

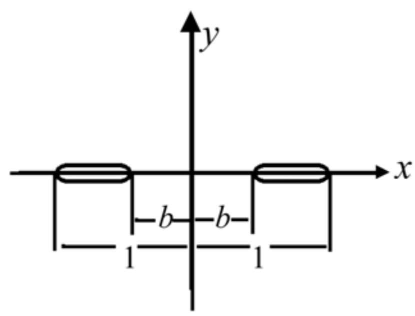

Fig. 1 Geometry and coordinate system for two collinear cracks

Zhou and Shen (1999) and Srivastava *et al.* (1983), the boundary conditions can be written as following (In this paper, we just consider the perturbation stress fields):

$$\tau_{yz}(x, 0^+) = \tau_{yz}(x, 0^-) = -\tau_0(x), \quad b \le |x| \le 1, \quad y = 0 \tag{2}$$

$$w(x, 0^+) = w(x, 0^-) = 0, \quad |x| < b, \; |x| > 1, \quad y = 0 \tag{3}$$

$$w(x, y) = 0, \quad \text{for} \quad (x^2 + y^2)^{1/2} \to \infty \tag{4}$$

where $\tau_0(x)$ is a magnitude of the incident wave.

3. Basic equation of non-local function graded materials

In the absence of body forces, the basic equations of anti-plane shear fracture problem in the functionally graded materials can be written as follows:

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} = -\rho(y)\,\omega^2 w(x, y) \tag{5}$$

$$\tau_{xz}(x, y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mu^*(|x' - x|, |y' - y|)\frac{\partial w(x', y')}{\partial x'}dx'dy' \tag{6}$$

$$\tau_{yz}(x, y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mu^*(|x' - x|, |y' - y|)\frac{\partial w(x', y')}{\partial y'}dx'dy' \tag{7}$$

where $-\rho(y)\omega^2 w(x,y)e^{-i\omega t} = \rho(y)\dfrac{\partial^2 w_0(x, y, t)}{\partial t^2} = \rho(y)\dfrac{\partial^2 (w(x,y)e^{-i\omega t})}{\partial t^2}$. $\rho(y)$ is the material density. The only difference from the classical elasticity is in the stress constitutive Eqs. (6)-(7), the stress $\tau_{xz}(x, y)$ and $\tau_{yz}(x, y)$, at a point (x, y) depend on the $\dfrac{\partial w(x, y)}{\partial x}$ and $\dfrac{\partial w(x, y)}{\partial y}$, at all points of the body. For the functionally graded materials anti-plane shear problem there exits only a materials parameter $\mu^*(|x' - x|, |y' - y|)$. In Eringen and Kim (1974, 1977), Eringen (1977), we obtained the form of $\mu^*(|x' - x|, |y' - y|)$ for which the dispersion curves of plane elastic waves coincide with those known in lattice dynamics. Among several possible curves the following has been found to be very useful

$$\mu^*(|x' - x|, |y' - y|) = \alpha(|x' - x|, |y' - y|)\mu(y') \tag{8}$$

Crack problems in the functionally graded materials do not appear to be analytically tractable for arbitrary variations of material properties. Usually, one tries to generate the forms of the functionally graded materials for which the problem becomes tractable. Similar to the treatment of the crack problem of the isotropic functionally graded materials in Erdogan and Wu (1997), Delale and Erdogan (1988), Chen (1990), Ozturk and Erdogan (1996), we assume the material properties are described by:

$$\mu(y) = \mu_0 e^{\lambda y}, \quad \rho(y) = \rho_0 e^{\lambda y} \tag{9}$$

where μ_0 and ρ_0 are the shear modulus and the mass density along $y = 0$, respectively. They are constants. λ is the functionally graded parameter. This is a purely mechanics assumption. $\lambda \neq 0$ is the case for the functionally graded materials. When $\lambda = 0$, it will return to the homogenous piezoelectric material case.

Substituting Eqs. (8)-(9) into Eqs. (6)-(7) yield

$$\tau_{kz}(X) = \int_V \alpha(|X' - X|)\sigma_{kz}(X')dV(X') \quad (k = x, y) \tag{10}$$

where
$$\sigma_{kz} = e^{\lambda y} c_{440} w_{,k} \quad (k = x, y) \tag{11}$$

The expression (11) is the classical constitutive equations for the functionally graded materials.

4. The dual integral equation

Substituting Eqs. (6)-(7) into Eq. (5) and using Green-Gauss theorem leads to

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \alpha(|x' - x|, |y' - y|)\mu(y')\left[\frac{\partial^2 w(x', y')}{\partial x'^2} + \frac{\partial^2 w(x', y')}{\partial y'^2} + \lambda\frac{\partial w(x', y')}{\partial y'}\right]dx'dy'$$
$$-\left[\int_{-1}^{-b} + \int_b^1\right]\alpha(|x' - x|, |y' - y|)[\sigma_{yz}(x', 0^+) - \sigma_{yz}(x', 0^-)]dx' = -\rho(y)\omega^2 w(x, y) \tag{12}$$

Here the surface integral may be dropped since the displacement field vanishes at infinity.

As mentioned in Eringen (1979), it can be obtained that $[\sigma_{yz}(x, 0^+) - \sigma_{yz}(x, 0^-)] = 0$. What now remains is to solve the integrodifferential Eq. (12) for the function $w(x, y)$. It is impossible to obtain a rigorous solution at the present stage. It seems obvious that in the solution of such a problem we encounter serious if not unsurmountable mathematical difficulties and will have to resort to an approximate procedure. In the given problem, as discussed in Nowinski (1984a,b), Zhou and Wang (2002), it is assumed that the non-local interaction in the y-direction is ignored. This is a purely assumption for mathematical tractability. In view of our assumptions, it can be given as

$$\alpha(|x' - x|, |y' - y|) = \alpha_0(|x' - x|)\delta(y' - y) \tag{13}$$

where $\alpha_0(|x' - x|) = \frac{1}{\sqrt{\pi}}(\beta/a)\exp[-(\beta/a)^2(x' - x)^2]$. β is a constant and can be obtained by experiment. a is the characteristic length. The characteristic length may be selected according to the range and sensitivity of the physical phenomena. For instance, for the perfect crystals, a may be taken as the lattice parameter. For granular materials, a may be considered to be the average granular distance and for fiber composites, the fiber distance, etc. In the present paper, a is taken as the lattice parameter. From Eq. (13), we have

$$\int_{-\infty}^{\infty} \alpha_0(|x'-x|)\left[\frac{\partial^2 w(x',y)}{\partial x'^2} + \lambda\frac{\partial w(x',y)}{\partial y} + \frac{\partial^2 w(x',y)}{\partial y^2}\right]dx' = -\frac{\rho_0}{\mu_0}\omega^2 w(x,y) \tag{14}$$

almost everywhere.

To solve the problem, the Fourier cosine transform of Eq. (14) with x can be given as follow:

$$\bar{\alpha}_0(|s|)\left[-s^2\bar{w}(s,y) + \lambda\frac{\partial\bar{w}(s,y)}{\partial y} + \frac{\partial^2 w(s,y)}{\partial y^2}\right] = -\frac{\rho_0}{\mu_0}\omega^2\bar{w}(s,y) \tag{15}$$

A superposed bar indicates the Fourier cosine transform through the paper.

From Eq. (13), we have

$$\bar{\alpha}_0(s) = \exp\left(-\frac{(sa)^2}{4\beta^2}\right) \tag{16}$$

Because of the symmetry, it suffices to consider the problem for $x \geq 0$, $|y| < \infty$. The solution of Eq. (15) satisfying Eq. (4) can be given as follow

$$w(x,y) = \begin{cases} \frac{2}{\pi}\int_0^{\infty} A(s)e^{-\gamma y}\cos(sx)ds, & y \geq 0 \\ -\frac{2}{\pi}\int_0^{\infty} A(s)e^{\gamma y}\cos(sx)ds, & y \leq 0 \end{cases} \tag{17}$$

where $\gamma = \dfrac{\lambda + \sqrt{\lambda^2 + 4[s^2 - \omega^2/c_1^2\bar{\alpha}_0(s)]}}{2}$, $c_1 = \sqrt{\mu_0/\rho_0}$. $A(s)$ is an unknown function to be determined from the boundary conditions.

Substituting Eq. (17) into Eq. (10), it can be obtained

$$\tau_{yz}(x,y) = -\frac{2\mu_0}{\pi}\int_0^{\infty}\gamma e^{-(\gamma-\lambda)y}A(s)ds\int_{-\infty}^{\infty}[\alpha_0(|x'-x|) + \alpha_0(|x'-x|)]\cos(sx')dx' \tag{18}$$

Substituting for α from Eq. (13), the integrations may be performed with respect to x' and y' by noting the integrals (Gradshteyn and Ryzhik 1980)

$$\int_{-\infty}^{\infty}\exp(-px'^2)\begin{Bmatrix}\sin\xi(x'+x)\\ \cos\xi(x'+x)\end{Bmatrix}dx' = (\pi/p)^{1/2}\exp\left(-\frac{\xi^2}{4p}\right)\begin{Bmatrix}\sin(\xi x)\\ \cos(\xi x)\end{Bmatrix} \tag{19}$$

Hence

$$\tau_{yz}(x,y) = -\frac{4\mu_0}{\pi}\int_0^{\infty}\gamma e^{-(\gamma-\lambda)y}e^{-\frac{s^2}{4p}}A(s)\cos(sx)ds \tag{20}$$

where $p = \left(\frac{\beta}{a}\right)^2$,

So the boundary conditions (2)-(3) can be expressed as:

$$\tau_{yz}(x,0) = -\frac{4\mu_0}{\pi}\int_0^{\infty}\gamma e^{-\frac{s^2}{4p}}A(s)\cos(sx)ds = -\tau_0, \quad b \leq x \leq 1 \tag{21}$$

$$\int_0^\infty A(s)\cos(sx)ds = 0, \quad x > 1, 0 < x < b \tag{22}$$

It can be obtained that $\operatorname{Lim}_{a \to 0} e^{-\frac{s^2}{4p}} = 1$. So Eqs. (21)-(22) will revert to the well-known triple integral equations of the classical theory for the limit $a \to 0$. To determine the unknown function $A(s)$, the previous pair of triple integral Eqs. (21)-(22) must be solved.

5. Solution of the triple integral equation

The only difference between the classical and nonlocal equations is in the influence function $e^{-s^2/4p}$, it is logical to utilize the classical solution to convert the system Eqs. (21)-(22) to an integral equation of the second kind, which is generally better behaved. For the lattice parameter $a \to 0$, then $e^{-s^2/4p}$ equals to a non-zero constant and Eqs. (21)-(22) reduce to a pair of triple integral equations for the same problem in classical elasticity. As discussed in Eringen *et al.* (1977), the triple integral Eqs. (21)-(22) cannot be transformed into a Fredholm integral equation of the second kind, because $e^{-s^2/4p}$ does not tend to a constant $C(C \neq 0)$ for $s \to \infty$. Of course, the triple Eqs. (21)-(22) can be considered to be a single integral equation of the first kind with discontinuous kernel. It is well-known in the literature that integral equations of the first kind are generally ill-posed in sense of Hadamard, i.e., small perturbations of the data can yield arbitrarily large changes in the solution. This makes the numerical solution of such equations quite difficult. To overcome the difficult, the Schmidt method (Morse and Feshbach 1958, Yan 1967) is used to solve the triple integral Eqs. (21)-(22). The displacement w on the crack surface can be represented by the following series:

$$w(x, 0) = \sum_{n=0}^{\infty} b_n P_n^{\left(\frac{1}{2}, \frac{1}{2}\right)}\left(\frac{x - \frac{1+b}{2}}{\frac{1-b}{2}}\right)\left(1 - \frac{\left(x - \frac{1+b}{2}\right)^2}{\left(\frac{1-b}{2}\right)^2}\right)^{\frac{1}{2}}, \quad \text{for} \quad b \le x \le 1 \tag{23}$$

$$w(x, 0) = 0, \ \text{for} \ x > 1, 0 < x < b \tag{24}$$

where a_n are unknown coefficients, $P_n^{(1/2,\, 1/2)}(x)$ is a Jacobi polynomial (Gradshteyn and Ryzhik 1980). The Fourier transform of Eqs. (23)-(24) are Erdelyi (1954)

$$A(s) = \bar{w}(s, 0) = \sum_{n=0}^{\infty} a_n F_n G_n(s) \frac{1}{s} J_{n+1}\left(s\frac{1-b}{2}\right) \tag{25}$$

where $F_n = 2\sqrt{\pi}\dfrac{\Gamma\left(n + 1 + \frac{1}{2}\right)}{n!}$, $G_n(s) = \begin{cases} (-1)^{\frac{n}{2}}\cos\left(s\dfrac{1+b}{2}\right), & n = 0, 2, 4, 6, \ldots \\ (-1)^{\frac{n+1}{2}}\sin\left(s\dfrac{1+b}{2}\right), & n = 1, 3, 5, 7, \ldots \end{cases}$

$\Gamma(x)$ and $J_n(x)$ are the Gamma and Bessel functions, respectively.

Substituting Eq. (25) into Eqs. (21)-(22), it can be shown that Eq. (22) are automatically satisfied. Eq. (21) reduces to

$$\frac{4\mu_0}{\pi}\sum_{n=0}^{\infty} a_n F_n \int_0^{\infty} \frac{\gamma}{s} e^{-\frac{s^2}{4p}} G_n(s) J_{n+1}\left(s\frac{1-b}{2}\right)\cos(sx)ds = \tau_0, \quad b \le x \le 1 \tag{26}$$

For a large s, the integrands of Eq. (26) almost all decrease exponentially. So the semi-infinite integral in Eq. (26) can be evaluated numerically by Filon's method (Amemiya and Taguchi 1969). Thus Eq. (26) can be solved for coefficients a_n by the Schmidt method (Morse and Feshbach 1958, Yan 1967). Here, it was omitted. It can be seen in Itou (1978), Zhou *et al.* (1999), Itou (2001).

6. Numerical calculations and discussion

The coefficients a_n are known, so that the entire stress field can be obtained. However, in fracture mechanics, it is important to determine the stress τ_{yz} in the vicinity of the crack tips. In the case of the present study, τ_{yz} along the crack line can be expressed as:

$$\tau_{yz}(x, 0) = -\frac{4\mu_0}{\pi}\sum_{n=0}^{\infty} a_n F_n \int_0^{\infty} \frac{\gamma}{s} e^{-\frac{s^2}{4p}} G_n(s) J_{n+1}\left(s\frac{1-b}{2}\right)\cos(sx)ds \tag{27}$$

When the lattice parameter $a \neq 0$, the semi-infinite integration and the series in Eq. (27) are convergent for any variable x, it gives a finite stress all along $y = 0$, so there is no stress singularity at crack tips. At $b < x < 1$, $\tau_{yz}/(-\tau_0)$ is very close to unity, and for $x > 1$, τ_{yz}/τ_0 possesses finite values diminishing from a finite value at $x = 1$ to zero at $x = \infty$. Since $a/\beta > 1/100$ represents a crack length of less than 100 atomic distances (Eringen *et al.* 1977), and for such submicroscopic sizes, other serious questions arise regarding the interatomic arrangements and force laws, we do not pursue solutions valid at such small crack sizes. The semi-infinite numerical integrals, which occur, are evaluated easily by Filon and Simpson's (Amemiya and Taguchi 1969) methods because the rapid diminution of the integrands. From Itou (1978), Zhou *et al.* (1999), Itou (2001), it can be seen that the Schmidt method is performed satisfactorily if the first ten terms of infinite series to Eq. (27) are retained.

The results are plotted in Figs. 2-8. The following observations are very significant:

(i) The traditional concepts of the non-local theory are extended to solve the dynamic fracture problem of functionally graded materials. The nonlocal elastic solutions yield a finite hoop stress at the crack tips, thus allowing us to use the maximum stress as a fracture criterion. The maximum stress value does not occur at the crack tips, but slightly away from it as shown in Figs. 2-4. This phenomenon has been thoroughly substantiated by Eringen (Itou 1978). The maximum stress value is finite. The distance between the crack tip and the maximum stress point is very small, and it depends on the crack length, the material properties and the lattice parameter. Contrary to the classical elasticity solution, it is found that the present results converge to the classical ones when far away from the crack tips as shown in Figs. 2-4. Simultaneously, for the nonlocal solution, the smaller the lattice parameter is, the more closer to the classical solution as shown in Figs. 2-4.

(ii) The stress of τ_{yz} does not depend on the shear modulus as shown in Eqs. (26)-(27). However, the stress of τ_{yz} depends on the crack length, on the frequency of the incident waves, the parameter

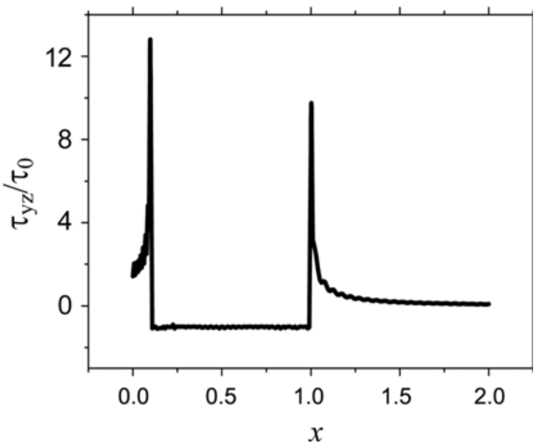

Fig. 2 The stress along the crack line versus x for b = 0.1, ω/c_1 = 0.2, λ = 0.2 and a/β = 0.001

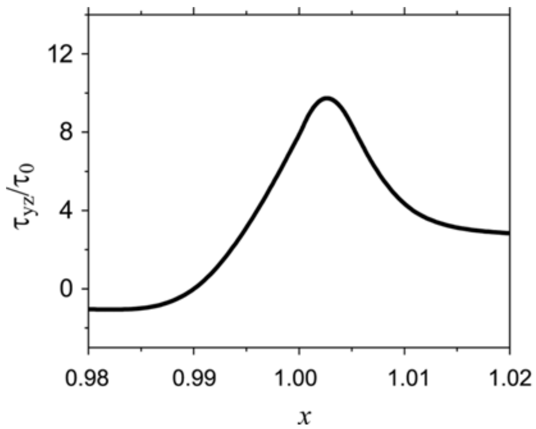

Fig. 3 The local enlarge graph of Fig. 2 near the crack right tip

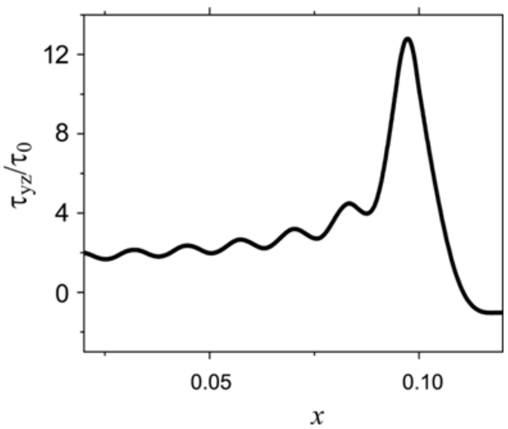

Fig. 4 The local enlarge graph of Fig. 2 near the crack left tip

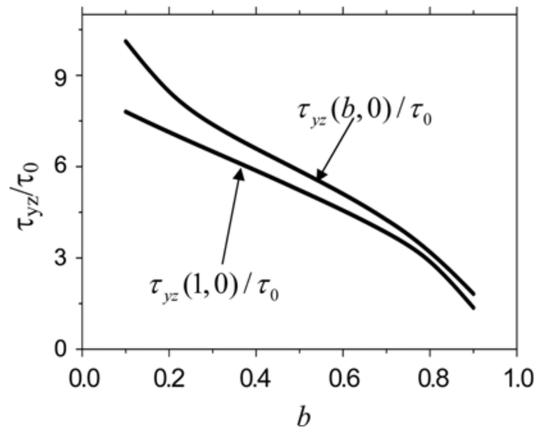

Fig. 5 The stress at the crack tips versus b for a/β = 0.001, λ = 0.2 and ω/c_1 = 0.2

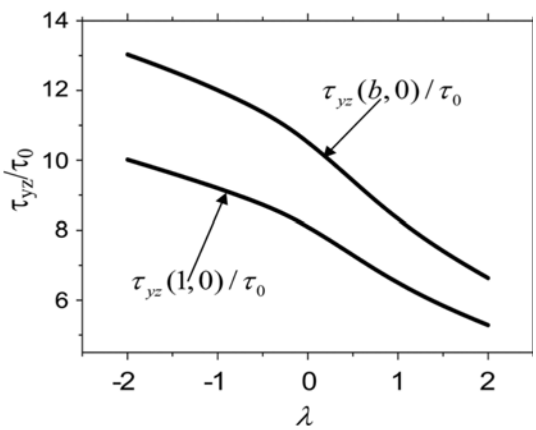

Fig. 6 The stress at the crack tips versus λ for a/β = 0.001, ω/c_1 = 0.2, and b = 0.1

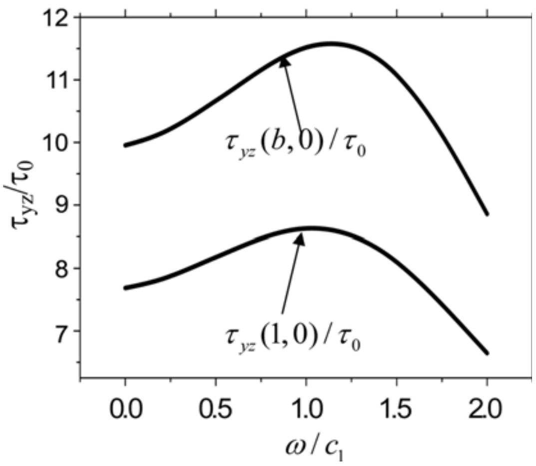

Fig. 7 The stress at the crack tips versus ω/c_1 for a/β = 0.001, λ = 0.2 and b = 0.1

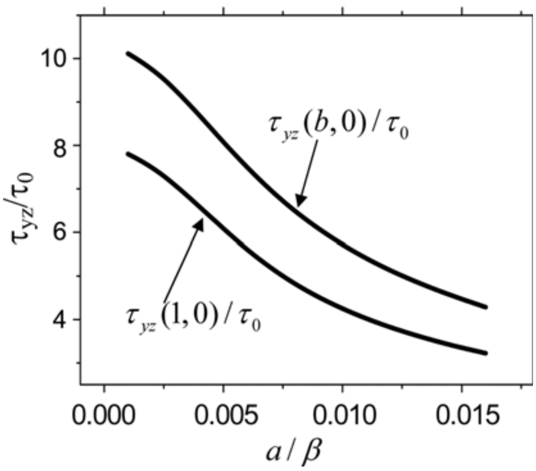

Fig. 8 The stress at the crack tips versus a/β for b = 0.1, λ = 0.2 and ω/c_1 = 0.2

describing the functionally graded materials and the lattice parameter of the materials. This is the same as the anti-plane shear fracture problem in the isotropic homogeneous materials.

(iii) The stress at the crack tips becomes infinite as the lattice parameter distance $a \to 0$. This is the classical continuum limit of square root singularity.

(iv) The value of the stress τ_{yz} at the crack tips increases with increase of the crack length as shown in Fig. 5, i.e., the interaction of two collinear cracks increases with decrease of distance between two cracks. It can be also obtained that the left tip's stress fields are greater than the right tip's ones for the right crack as shown in Fig. 2 and Fig. 5.

(v) The effect of the parameter describing the functionally graded materials on the stress field near the crack tips decreases with increase of the parameter describing the functionally graded materials as shown in Fig. 6. This means that, by decreasing the gradient parameter of FGMs, the dynamic stress fields near the crack tips can be reduced.

(vi) The dynamic stresses of τ_{yz} tend to increase with the frequency reaching a peak and then to decrease in magnitude as shown in Fig. 7. From the results, it can be concluded that the stress fields near the crack tips can be deduced by adjusting the frequency of incident waves in engineering practices.

(vii) The effect of the lattice parameter of the functionally graded materials on the stress field near the crack tips decreases with increase of the lattice parameter as shown in Fig. 8.

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