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Mechanical analysis of non-uniform beams resting on nonlinear elastic foundation by the differential quadrature method

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Abstract. A new approach using the differential quadrature method (DQM) is derived for analysis of non-uniform beams resting on nonlinear media in this study. The influence of velocity dependent viscous damping and strain rate dependent viscous damping is investigated. The results solved using the DQM have excellent agreement with the results solved using the FEM. Numerical results indicated that the DQM is valid and efficient for non-uniform beams resting on non-linear media.

Key words: DQM; non-uniform beam; nonlinear media; mechanical behavior.

1. Introduction

Beams on nonlinear elastic foundations are found in many engineering application. The static and dynamic behavior of non-uniform beam resting on nonlinear elastic foundation is an important topic in structural engineering and several authors have studied it in the past. Tsai and Westmann (1967), Lin and Adams (1987), Weitsman (1970, 1972) studied the static behavior of beams resting on a tensionless elastic foundation. Static analysis of thick, circular and rectangular plates resting on a tensionless elastic foundation have been performed by Akbarov and Kocaturk (1997), Celep (1998a,b), Shen and Yu (2004), Li and Dempsey (1988), Mishra and Chakrabarti (1996), Silva *et al.* (2001), Xiao (2001), and Hong *et al.* (1999). Ma (2004) considered positive solutions for a fourth-order differential equation with nonlinear boundary conditions modeling beams on elastic foundations. Sharma and DasGupta (1975) studied the bending problem of axially constrained beams on nonlinear Winkler-type elastic foundations using Green's functions. Beaufait and Hoadley (1980) solved the problem of elastic beams on linear foundation using the midpoint difference method. Kuo and Lee (1994) investigated the deflection of non-uniform beams resting on a nonlinear elastic foundation using the method of perturbation. Chen (1998) presented the numerical results of the solutions of beams on elastic foundation using the DQEM. In this paper, a new approach using the differential quadrature method (DQM) with its easy to use and the more efficient used to simulate non-uniform beam resting on nonlinear media. The DQM is a numerical technology for solving differential equations. In this study, the DQM was employed to formulate the

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problems in matrix form. The Chebyshev-Gauss-Lobatto point distribution on each beam is employed. The integrity and computational efficiency of the DQM in this problem will be demonstrated through a series of case studies. The effect of the sample point number on the numerical results is studied. To the author's knowledge, very few published papers in the literature have presented the mechanical behavior of non-uniform beams resting on nonlinear media using the DQM.

2. The differential quadrature method

Most science and engineering problems are governed by a set of differential equations. The static and dynamic solutions for many complicated structures have now become achievable using the finite difference method, the finite element method, the finite volume method and the boundary element method. However, to look for an alternative efficient technique is still of prime interest. The concept of the DQM was introduced by Bellman and Casti (1971), Bellman *et al.* (1972). The DQM has been used extensively to solve a variety of problems in different fields of science and engineering. The DQM has been shown to be a powerful contender in solving initial and boundary value problems and thus has become an alternative to the existing methods. Chen and Zhong (1997) pointed out that the DQM is more efficient for nonlinear problems than the traditional finite element and finite difference methods. Quan and Chang (1989a,b) applied Lagrange interpolated polynomials as test function obtained explicit formulations to calculate the DQ weighting coefficients. The key procedure in the DQM lies in the determination of the DQ weighting coefficients. The DQM has been used extensively to solve a variety of engineering problems. One of the fields among which one can find extensive applications of DQM is structural mechanics. Bert *et al.* (1988), Bert and Malik (1996a,b), Bert *et al.* (1993, 1994a,b), Malik and Bert (1996, 1995), Striz *et al.* (1994), Du *et al.* (1996), Sherbourne and Pandey (1991), Jang *et al.* (1989), Feng and Bert (1992), Chen *et al.* (2000), Tomasiello (1998) analyzed static and free vibration of beams and rectangular plates using the DQM. Bert and Malik (1996b) reviewed the recent development of DQM in computational mechanics.

The DQM is employed in the present study. The basic concept of the DQM is that the derivative of a function at a given point can be approximated as a weighted linear sum of the functional values at all of the sample points in the domain of that variable. The differential equation is then reduced into a set of algebraic equations using this approximation. The number of equations is dependent upon the selected number of the sample points. The accuracy of the solution using in this method may be improved by increasing the number of sample points as for any polynomial approach.

For a function $f(x)$, DQM approximation for the mth order derivative at the ith sampling point is given by (Bert and Malik 1996, Bert *et al.* 1993, 1994a)

$$\frac{d^m}{dx^m}\begin{Bmatrix} f(x_1) \\ f(x_2) \\ \vdots \\ f(x_N) \end{Bmatrix} \cong [D_{ij}^{(m)}]\begin{Bmatrix} f(x_1) \\ f(x_2) \\ \vdots \\ f(x_N) \end{Bmatrix} \quad \text{for } i,j = 1, 2, \ldots, N \tag{1}$$

in which N is the number of sample points, x_i is the location of ith sampling point in the domain,

$f(x_i)$ is the functional value at point x_i, and $D_{ij}^{(m)}$ are the DQ weighting coefficients of mth order differentiation attached to these functional values.

In order to overcome the numerical ill-conditions in determining the DQ weighting coefficients $D_{ij}^{(m)}$, a Lagrangian interpolation polynomial was (Bert and Malik 1996, Bert *et al.* 1993, 1994a)

$$f(x) = \frac{M(x)}{(x-x_i)M_1(x_i)} \quad \text{for } i = 1, 2, \ldots, N \tag{2}$$

where

$$\left.\begin{aligned} M(x) &= \prod_{j=1}^{N}(x-x_j) \\ M_1(x_i) &= \prod_{j=1, j\neq i}^{N}(x_i-x_j) \end{aligned}\right\} \quad \text{for } i = 1, 2, \ldots, N$$

Substituting Eq. (2) into Eq. (1) leads to

$$D_{ij}^{(1)} = \frac{M_1(x_i)}{(x_i-x_j)M_1(x_j)} \quad \text{for } i,j = 1, 2, \ldots, N \text{ and } i \neq j \tag{3}$$

and

$$D_{ii}^{(1)} = -\sum_{j=1, j\neq i}^{N} D_{ij}^{(1)} \quad \text{for } i = 1, 2, \ldots, N \tag{4}$$

Once the sample points, i.e., x_i for $i = 1, 2, \ldots, N$, are selected, the coefficients of the DQ weighting matrix can be obtained from Eqs. (3) and (4). It should be emphasized that the number of the test functions must be greater than the highest order of derivative in the governing equations, i.e., $N > m$. The DQ weighting coefficients of second-, third-, and fourth-order derivatives in the DQM, $D_{ij}^{(2)}, D_{ij}^{(3)}, D_{ij}^{(4)}$ may be computed by (Bert and Malik 1996, Bert *et al.* 1993, 1994a)

$$D_{ij}^{(2)} = \sum_{k=1}^{N} D_{ik}^{(1)} D_{kj}^{(1)} \quad \text{for } i,j = 1, 2, \ldots, N \tag{5}$$

$$D_{ij}^{(3)} = \sum_{k=1}^{N} D_{ik}^{(1)} D_{kj}^{(2)} \quad \text{for } i,j = 1, 2, \ldots, N \tag{6}$$

$$D_{ij}^{(4)} = \sum_{k=1}^{N} D_{ik}^{(1)} D_{kj}^{(3)} \quad \text{for } i,j = 1, 2, \ldots, N \tag{7}$$

The computational domain of a non-uniform beam is $0 \leq x \leq L$. A mesh generation is needed to perform for numerical computation. The selection of mesh points always played an important role in the solution accuracy of the DQM. The mesh generation in the x direction for the non-uniform beam can be given by

$$x_i = \frac{L}{2}\left[1 - \cos\frac{(i-1)\pi}{N-1}\right] \quad \text{for } i = 3, 4, \ldots, N-2 \tag{8}$$

The boundary points of the non-uniform beams are defined as

$$x_1 = 0 \tag{9}$$

$$x_N = L \tag{10}$$

The above mesh point distribution is Chebyshev-Gauss-Lobatto point distribution.

3. Bending behavior of the non-uniform beam resting on nonlinear media

Fig. 1 shows the fixed-free beam resting on nonlinear media. It is assumed that the behavior of the beam follows the Euler-Bernoulli hypothesis and that the beam rests on a nonlinear elastic foundation. The strain energy of the non-uniform beam resting on nonlinear media can be simplified as

$$U^e = \frac{1}{2}\int_0^L EI\left(\frac{d^2u}{dx^2}\right)^2 dx + \int_0^L \frac{k_0}{\mu^2}(1 + \mu u - \ln(1 + \mu u))dx \tag{11}$$

where k_0 and μ are the parameters of the foundation. With considering the load, the virtual work δW_1^e done by the non-uniform beam can be derived as

$$\delta W_1^e = \int_0^L w(x)\delta u dx \tag{12}$$

where E is Young's modulus of the non-uniform beam. $I(x)$ is the moment of inertia of cross-sectional area of the non-uniform beam. L is the length of the beam. $w(x)$ is the load. Load $w(x)$ acts on x = 0~L in the beam. Substituting Eqs. (11) and (12) into the principle of the total potential energy

$$\delta W_1^e - \delta U^e = 0 \tag{13}$$

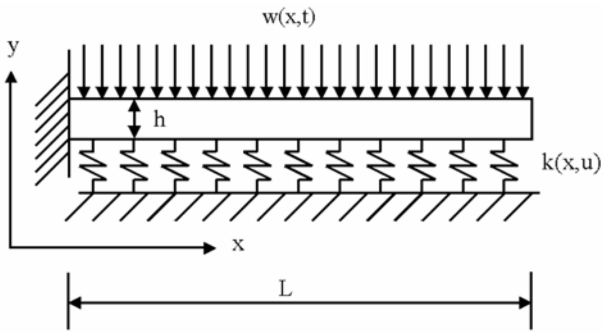

Fig. 1 The geometry of the cantilever beam resting on nonlinear foundation

Solving above equation, the equation of the non-uniform beam resting on nonlinear media is

$$\frac{d^2}{dx^2}\left(EI(x)\frac{d^2u}{dx^2}\right) + \frac{k_0 u}{1+\mu u} = w(x) \tag{14}$$

The corresponding boundary conditions of the clamped-free non-uniform beam are

$$u(0) = 0 \tag{15}$$

$$\frac{du(0)}{dx} = 0 \tag{16}$$

$$EI\frac{du^2(L)}{dx^2} = 0 \tag{17}$$

$$\frac{d}{dx}\left(EI\frac{du^2(L)}{dx^2}\right) = 0 \tag{18}$$

By employing the differential quadrature method, Eq. (1) is substituted into Eq. (14) leads to

$$\left[\left.\frac{d^2EI(x)}{dx^2}\right|_{x=x_i} D_{i1}^{(2)} \ \left.\frac{d^2EI(x)}{dx^2}\right|_{x=x_i} D_{i2}^{(2)} \ \dots \ \left.\frac{d^2EI(x)}{dx^2}\right|_{x=x_i} D_{iN}^{(2)}\right]\{u(x_j)\}$$
$$+\left[2\left.\frac{dEI(x)}{dx}\right|_{x=x_i} D_{i1}^{(3)} \ 2\left.\frac{dEI(x)}{dx}\right|_{x=x_i} D_{i2}^{(3)} \ \dots \ 2\left.\frac{dEI(x)}{dx}\right|_{x=x_i} D_{iN}^{(3)}\right]\{u(x_j)\}$$
$$+[EI(x_i)D_{i1}^{(4)} \ EI(x_i)D_{i2}^{(4)} \ \dots \ EI_{yy}(x_i)D_{iN}^{(4)}]\{u(x_j)\} + \left[\frac{k_0}{1+\mu u(x_i)}\right]\{u(x_i)\} = \{w(x_i)\}$$

for $i = 3, 4, \dots, N-2$ and $j = 1, 2, \dots, N$ (19)

The boundary condition Eq. (15) can be rewritten as

$$[1 \ 0 \ 0 \ \dots \ 0 \ 0]\{u(x_j)\} = \{0\} \quad \text{for } j = 1, 2, \dots, N \tag{20}$$

According to the differential quadrature method, boundary condition Eq. (16) takes the following discrete forms:

$$[D_{11}^{(1)} \ D_{12}^{(1)} \ D_{13}^{(1)} \ \dots \ D_{1,N-1}^{(1)} \ D_{1,N}^{(1)}]\{u(x_j)\} = \{0\} \quad \text{for } j = 1, 2, \dots, N \tag{21}$$

The boundary conditions at the free end can be rearranged into the following discrete forms:

$$[EI(L)D_{N,1}^{(2)} \ EI(L)D_{N,2}^{(2)} \ EI(L)D_{N,3}^{(2)} \ \dots \ EI(L)D_{N,N-1}^{(2)} \ EI(L)D_{N,N}^{(2)} \ ]\{u(x_j)\} = \{0\} \tag{22}$$

for $j = 1, 2, \dots, N$

$$\left[\frac{dEI(x)}{dx}\bigg|_{x=L} D^{(2)}_{N,1}\ \frac{dEI(x)}{dx}\bigg|_{x=L} D^{(2)}_{N,2}\ \frac{dEI(x)}{dx}\bigg|_{x=L} D^{(2)}_{N,3} \cdots\right.$$

$$\left.\frac{dEI(x)}{dx}\bigg|_{x=L} D^{(2)}_{N,N-1}\frac{dEI(x)}{dx}\bigg|_{x=L} D^{(2)}_{N,N}\right]\{u(x_j)\}$$

$$+\,[EI(x)D^{(3)}_{N,1}\big|_{x=L}\ EI(x)D^{(3)}_{N,2}\big|_{x=L}\ EI(x)D^{(3)}_{N,3}\big|_{x=L}\ \cdots\ EI(x)D^{(3)}_{N,N-1}\big|_{x=L}\ EI(x)D^{(3)}_{N,N}\big|_{x=L}]$$

$$\{u(x_j)\} = \{0\} \qquad \text{for } j = 1, 2, \ldots, N \tag{23}$$

4. Dynamic behavior of the non-uniform beam resting on nonlinear media

The kinetic energy of the non-uniform beam resting on nonlinear elastic foundation can be derived as

$$T^e = \frac{1}{2}\int_0^L \rho A\left(\frac{\partial u}{\partial t}\right)^2 dx \tag{24}$$

where ρ is the density of the non-uniform beam and A is the cross-sectional area of the non-uniform beam. For generality, the velocity dependent viscous damping and strain rate dependent viscous damping effects have been considered in the formulation of equation of motion. The velocity dependent viscous damping is a viscous resistance to transverse displacement of the non-uniform beam; and the strain rate dependent viscous damping is a viscous resistance to straining of the non-uniform beam material. The damping force $C_o\frac{\partial u}{\partial t}$ is assumed for the resistance to transverse velocity of the non-uniform beam. The damping force $C_1\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^3 u}{\partial t\,\partial x^2}\right)$ is assumed for the resistance to strain velocity of the non-uniform beam. With considering of the velocity dependent viscous damping and strain rate dependent viscous damping effects in the beam resting on nonlinear elastic foundation, the virtual work $\delta W_2{}^e$ done by the actuator for a virtual displacement δu can be derived as

$$\delta W_2^e = -\int_0^L C_o\frac{\partial u}{\partial t}\delta u\,dx - \int_0^L C_1\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^3 u}{\partial t\,\partial x^2}\right)\delta u\,dx - \int_0^L w\,\delta u\,dx \tag{25}$$

where C_o and C_1 are defined as the velocity dependent viscous damping and the strain rate dependent viscous damping coefficients, respectively. Substituting Eqs. (11), (24), and (25) into Hamilton equation

$$\int_{t_1}^{t_2}(\delta T^e - \delta U^e + \delta W_2{}^e)dt = 0 \tag{26}$$

The equation of motion of the non-uniform beams resting on non-linear media can be derived as

$$\rho A\frac{\partial^2 u}{\partial t^2} + C_0\frac{\partial u}{\partial t} + C_1\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^3 u}{\partial x^2\,\partial t}\right) + \frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 u}{\partial x^2}\right) + \frac{k_0 u}{1+\mu u} = w \tag{27}$$

The dynamic response $u(x, t)$ of a non-uniform beam is governed by above fourth order partial differential equation. The boundary conditions of the clamped-free beam are

$$u(0, t) = 0 \tag{28}$$

$$\frac{\partial u(0, t)}{\partial x} = 0 \tag{29}$$

$$EI\frac{\partial u^2(L, t)}{\partial x^2} = 0 \tag{30}$$

$$\frac{\partial}{\partial x}\left(EI\frac{\partial u^2(L, t)}{\partial x^2}\right) = 0 \tag{31}$$

By employing the DQM, Eq. (1) is substituted into Eq. (27) leads to

$$\begin{aligned}
&[\rho A(x_i)]\left\{\frac{\partial^2 u(x_i, t)}{\partial t^2}\right\} + [C_0]\left\{\frac{\partial u(x_i, t)}{\partial t}\right\} \\
&+\left[C_1\frac{\partial^2 EI(x)}{\partial x^2}\bigg|_{x=x_i} D_{i1}^{(2)}\ \ C_1\frac{\partial^2 EI(x)}{\partial x^2}\bigg|_{x=x_i} D_{i2}^{(2)}\ \ \ldots\ \ C_1\frac{\partial^2 EI(x)}{\partial x^2}\bigg|_{x=x_i} D_{iN}^{(2)}\right]\{u(x_j, t)\} \\
&+\left[2\frac{\partial EI(x)}{\partial x}\bigg|_{x=x_i} D_{i1}^{(3)}\ \ 2\frac{\partial EI(x)}{\partial x}\bigg|_{x=x_i} D_{i2}^{(3)}\ \ \ldots\ \ 2\frac{\partial EI(x)}{\partial x}\bigg|_{x=x_i} D_{iN}^{(3)}\right]\{u(x_j, t)\} \\
&+[EI(x_i)D_{i1}^{(4)}\ \ EI(x_i)D_{i2}^{(4)}\ \ \ldots\ \ EI_{yy}(z_i)D_{iN}^{(4)}]\{u(x_j, t)\} + \left[\frac{k_0}{1 + \mu u(x_i, t)}\right]\{u(x_i, t)\} \\
&+\left[\frac{\partial^2 EI(x)}{\partial x^2}\bigg|_{x=x_i} D_{i1}^{(2)}\ \ \frac{\partial^2 EI(x)}{\partial x^2}\bigg|_{x=x_i} D_{i2}^{(2)}\ \ \ldots\ \ \frac{\partial^2 EI(x)}{\partial x^2}\bigg|_{x=x_i} D_{iN}^{(2)}\right]\{u(x_j, t)\} \\
&+\left[2\frac{\partial EI(x)}{\partial x}\bigg|_{x=x_i} D_{i1}^{(3)}\ \ 2\frac{\partial EI(x)}{\partial x}\bigg|_{x=x_i} D_{i2}^{(3)}\ \ \ldots\ \ 2\frac{\partial EI(x)}{\partial x}\bigg|_{x=x_i} D_{iN}^{(3)}\right]\{u(x_j, t)\} \\
&+[EI(x_i)D_{i1}^{(4)}\ \ EI(x_i)D_{i2}^{(4)}\ \ \ldots\ \ EI_{yy}(z_i)D_{iN}^{(4)}]\{u(x_j, t)\} + \left[\frac{k_0}{1 + \mu u(x_i, t)}\right]\{u(x_i, t)\} \\
&= \{w(x_i, t)\} \qquad \text{for } i = 3, 4, \ldots, N-2 \text{ and } j = 1, 2, \ldots, N
\end{aligned} \tag{32}$$

The boundary condition Eq. (28) can be rewritten as

$$[1\ \ 0\ \ 0\ \ \ldots\ \ 0\ \ 0]\{u(x_j, t)\} = \{0\} \quad \text{for } j = 1, 2, \ldots, N \tag{33}$$

According to the differential quadrature method, boundary condition Eq. (29) takes the following discrete forms:

$$[D_{11}^{(1)}\ D_{12}^{(1)}\ D_{13}^{(1)}\ \ldots\ D_{1,N-1}^{(1)}\ D_{1,N}^{(1)}]\{u(x_j, t)\} = \{0\} \quad \text{for } j = 1, 2, \ldots, N \tag{34}$$

The boundary conditions at the free end can be rearranged into the following discrete forms.

$$[EI(L)D^{(2)}_{N,1}\ EI(L)D^{(2)}_{N,2}\ EI(L)D^{(2)}_{N,3}\ \dots\ EI(L)D^{(2)}_{N,N-1}\ EI(L)D^{(2)}_{N,N}\ ]\{u(x_j,t)\} = \{0\}$$

for $j = 1, 2, \dots, N$ (35)

$$\left[\left.\frac{\partial EI(x)}{\partial x}\right|_{x=L} D^{(2)}_{N,1}\ \left.\frac{\partial EI(x)}{\partial x}\right|_{x=L} D^{(2)}_{N,2}\ \left.\frac{\partial EI(x)}{\partial x}\right|_{x=L} D^{(2)}_{N,3}\ \dots\right.$$
$$\left.\left.\frac{\partial EI(x)}{\partial x}\right|_{x=L} D^{(2)}_{N,N-1}\ \left.\frac{\partial EI(x)}{\partial x}\right|_{x=L} D^{(2)}_{N,N}\right]\{u(x_j,t)\}$$
$$+[EI(x)D^{(3)}_{N,1}\ EI(x)D^{(3)}_{N,2}\ EI(x)D^{(3)}_{N,3}\ \dots\ EI(x)D^{(3)}_{N,N-1}\ EI(x)D^{(3)}_{N,N}]\{u(x_j,t)\} = \{0\}$$

for $j = 1, 2, \dots, N$ (36)

5. Results and discussion

Fig. 2 displays the deflection of fixed-free beam under the load $w = 10$ N/m solved using the DQM and the FEM. The material properties and the geometric dimensions of the beam are $E = 2.4525 \times 10^7$ Pa, $L = 9.0$ m, $k_0 = 600.0$ N/m and $I = 4.5 \times 10^{-4}$ m^4. The feasibility of DQM for the non-uniform beams resting on nonlinear media is studied first. We use the static analysis of a uniform beam resting on nonlinear elastic foundation as an example to demonstrate the high accuracy of the DQM results and the effect of mesh point number. Different number of sample points for the non-uniform beam, i.e., 5, 6, 7, 8, 9, 11 and 13, are selected for accuracy analysis. The results display that the DQM gives good accuracy even if 7 sample points are used. The numerical results calculated from the proposed DQM formulation have the satisfactory accuracy and the solution accuracy is so sensitive to the number of sample points. It may be observed from this figure the curve solved using the DQM closely follows the curve solved using the FEM. The computational time for using the DQM with 5, 6, 7, 8, 9, 11, and 13 sample points are 0.491, 0.641, 0.851, 1.051, 1.272, 1.813 and 2.433 seconds, respectively. However, a computational time over

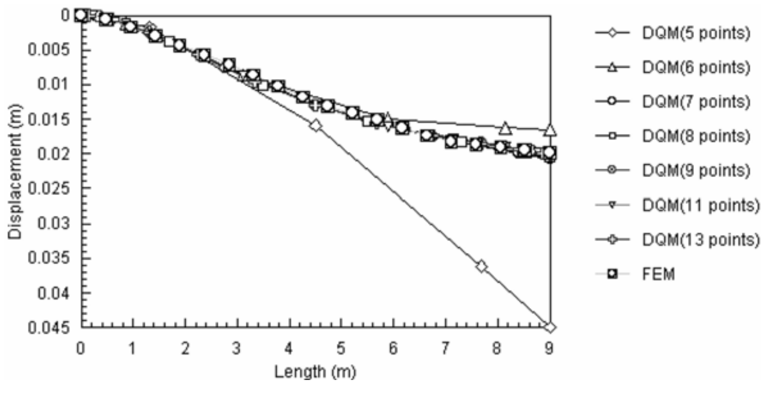

Fig. 2 The deflections of clamped-free beam solved using various sample points

2.934 seconds is required for using the FEM in the similar problem. Results indicate that the DQM is valid for solving such an engineering problem without using a large number of degrees of freedom. Figs. 3 and 4 show the deflections of clamped-free beam and simply supported beam for various loads, respectively. The material properties and the geometric dimensions of the beam are

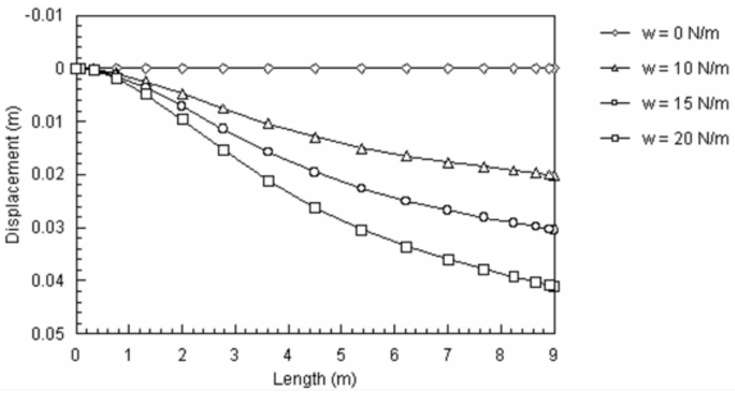

Fig. 3 The deflections of the clamped-free beam for various loads

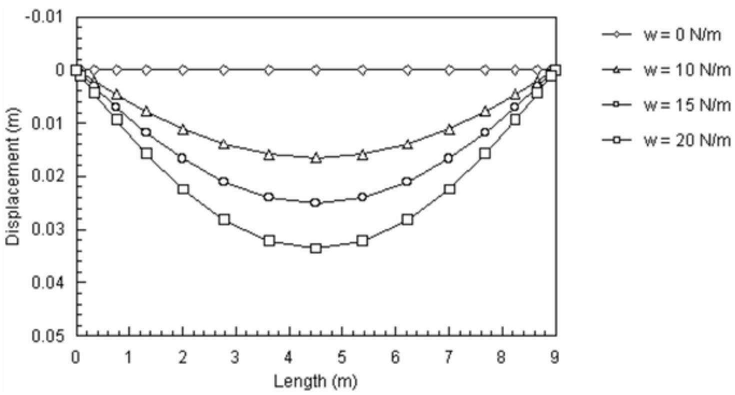

Fig. 4 The deflections of the simply supported beam for various loads

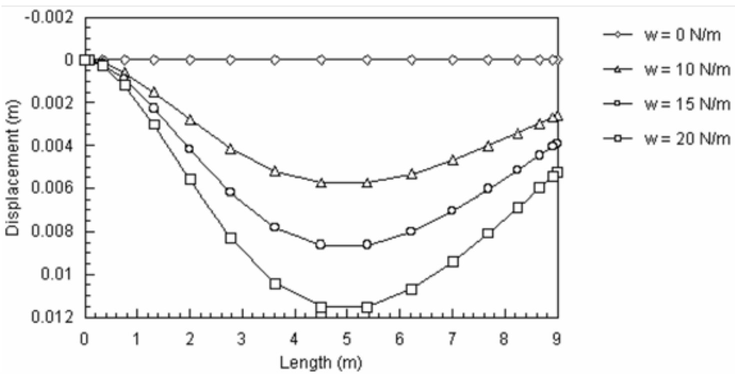

Fig. 5 The deflections of the clamped-free beam for various loads

$E = 2.4525 \times 10^7$ Pa, $L = 9.0$ m, $k_0 = 600.0$ N/m, and $I = 4.5 \times 10^{-4}$ m^4. It is obvious that deflection changes significantly for loads and boundary conditions. Fig. 5 illustrates the clamped-free non-uniform beam deflection for various loads. The material properties and the geometric dimensions of the beam are, $E = 2.4525 \times 10^7$ Pa, $L = 9.0$ m, $k_0 = 600.0$ N/m, $I = I_0(1 + x/L + (x/L)^2)$ and $I_0 = 4.5 \times 10^{-4}$ m^4. It is obvious that the deflection of the beam changes significantly with the inertia shape. Fig. 6 reveals the clamped-free non-uniform beam displacement for various w_0. The material properties and the geometric dimensions of the beam are $E = 2.4525 \times 10^7$ Pa, $L = 9.0$ m, $k_0 = 600.0$ N/m, $I = I_0(1 + x/L + (x/L)^2)$, $I_0 = 4.5 \times 10^{-4}$ m^4 and $w = w_0(1 - x/L)$. It is obvious that the deflection of the beam changes significantly for various loads. The results show that it is efficient to solve the problem of the beam with various bean section shapes and loads using the DQM.

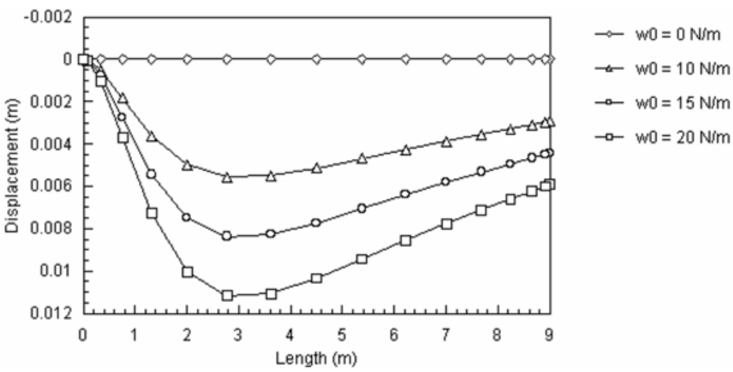

Fig. 6 The deflections of the clamped-free beam for various w_0

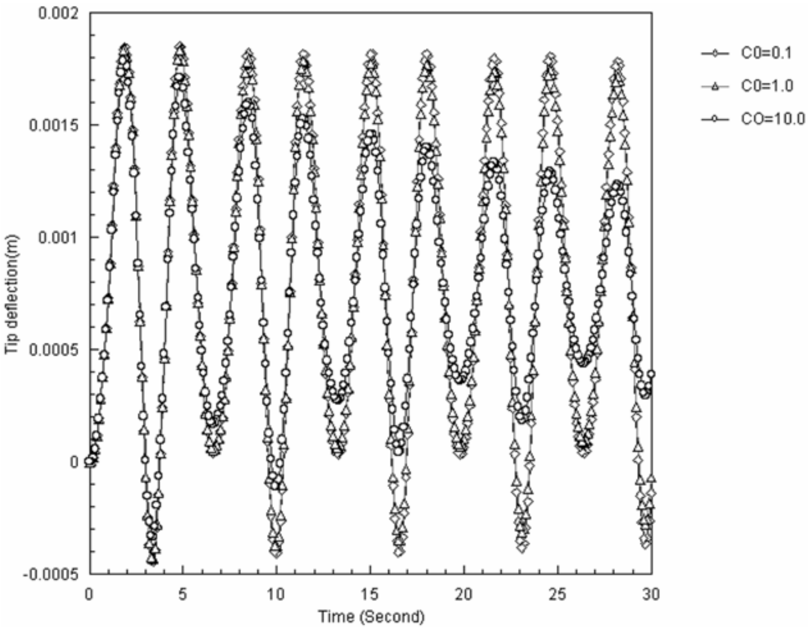

Fig. 7 The tip deflections of the clamped-free beam for load $w = u(t - 0.1)$ N and various velocity dependent viscous damping

Fig. 7 denotes the tip deflection of the clamped-free beam for load $w = u(t - 0.1)$ N/m and various velocity dependent viscous damping. $u(t - 0.1)$ is unit step function. $u(t - 0.1)$ is defines as following:

$$u(t-0.1) = \begin{cases} 1 & t \geq 0.1 \\ 0 & t < 0.1 \end{cases} \tag{36}$$

The material properties and the geometric dimensions of the beam are $E = 2.4525 \times 10^7$ Pa, $\rho = 2300$ kg/m^3, $L = 9.0$ m, $k_0 = 600.0$ N/m, and $I = 4.5 \times 10^{-4}$ m^4. The Newton-Raphson algorithm is used together. It can be clearly seen that there is a significant reduction in the tip deflection by increasing the velocity dependent viscous damping. Through simulation they show that damping can provide reduction in tip deflection response. Fig. 8 displays the stresses near the fixed end of the clamped-free beam for load $w = u(t - 0.1)$ N/m and various velocity dependent viscous damping. It can be clearly seen that there is a significant reduction in the stress near the fixed end by increasing the velocity dependent viscous damping. Simulation results show that damping can provide reduction in beam response. Fig. 9 introduces the tip deflections of the clamped-free beam for load $w = u(t - 0.1)$ N/m and various strain rate dependent viscous damping. It can be clearly seen that there is a significant reduction in the tip deflection by increasing strain rate dependent viscous damping. Fig. 10 shows the stresses near the fixed end of the clamped-free beam for loading $w = u(t - 0.1)$ N/m and various strain rate dependent viscous damping. Simulation results show that damping can provide reduction in the stress response near the fixed end.

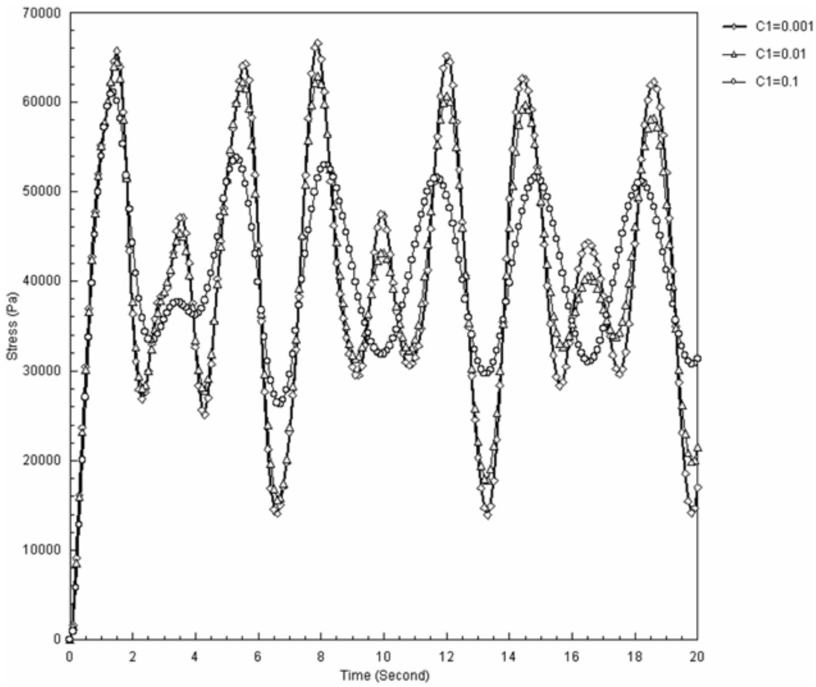

Fig. 8 The stresses near the fixed end of the clamped-free beam for load $w = u(t - 0.1)$ N and various velocity dependent viscous damping

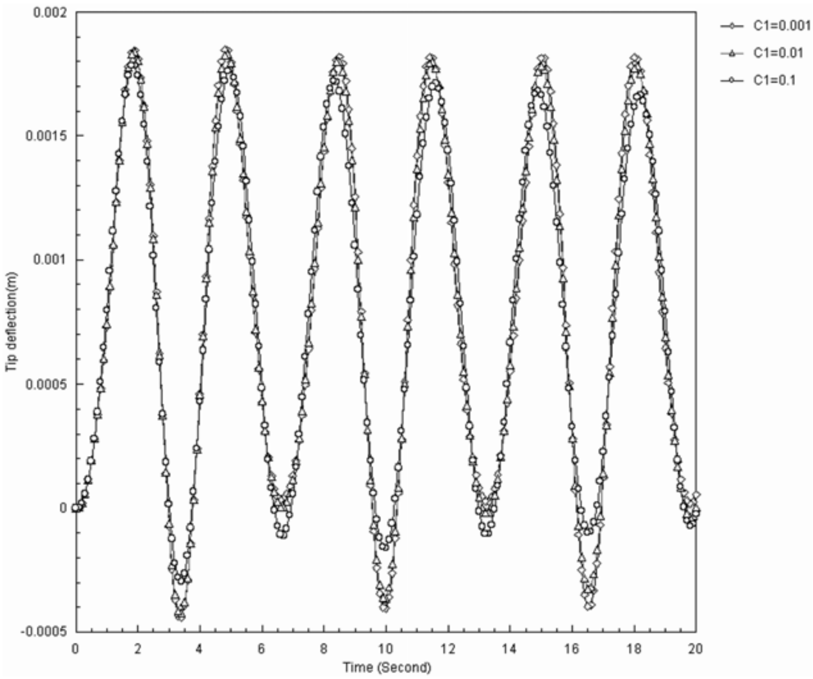

Fig. 9 The tip deflections of the clamped-free beam for load $w = u(t - 0.1)$ N and various strain rate dependent viscous damping

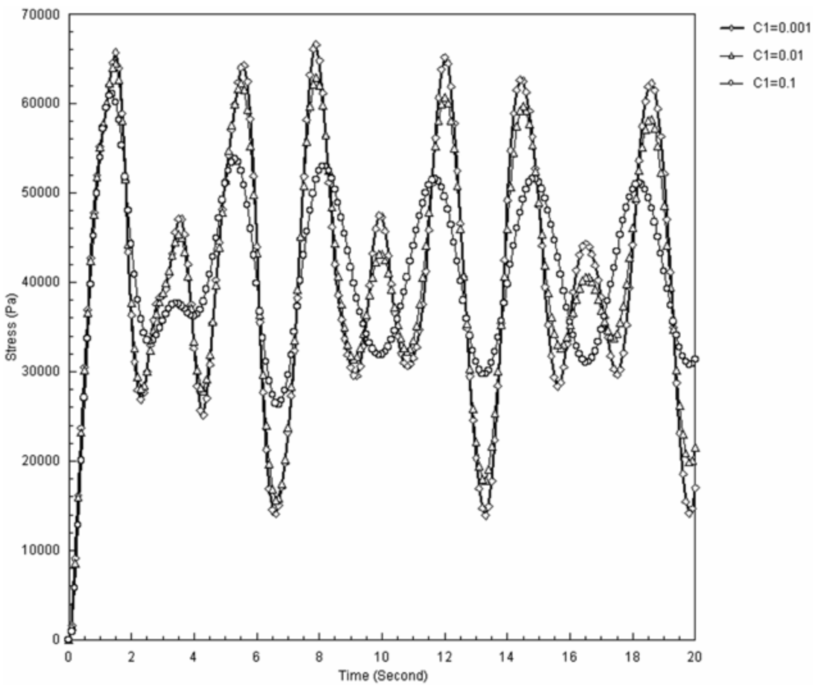

Fig. 10 The stresses near the fixed end of the clamped-free beam for load $w = u(t - 0.1)$ N and various strain rate dependent viscous damping

6. Conclusions

The DQM is proposed to analyze non-uniform beams resting on nonlinear elastic foundation problems in this work. The applicability of the proposed method to the mechanical behavior analysis of non-uniform beams resting on nonlinear elastic foundation is demonstrated. Accurate results are obtained for the problems sensitive to grid point number using the DQM. This approach is convenient for solving problems governed by the fourth or higher order differential equations. In this approach, only seven sample points are needed to achieve convergence. Excellent agreement has been obtained between the calculated results solved using the DQM and the FEM. Numerical results in this work show that the velocity dependent viscous damping and strain rate dependent viscous damping coefficients have a significant influence on the system's dynamic. Results with high accuracy are obtained and fast convergent trend is observed in all study cases. This shows the applicability and efficiency of the DQM. It is expected that the DQM will find a wide range of applications.

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