

Response of a rectangular plate-column system on a tensionless Winkler foundation subjected to static and dynamic loads

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Abstract. The response of a plate-column system having five-degree-of-freedom supported by an elastic foundation and subjected to static lateral load, harmonic ground motion and earthquake motion is studied. Two Winkler foundation models are assumed: a conventional model which supports compression and tension and a tensionless model which supports compression only. The governing equations of the problem are obtained, solved numerically and the results are presented in figures to demonstrate the behavior of the system for various values of the system parameters comparatively for the conventional and the tensionless Winkler foundation models.

Key words: rectangular plate; plate-column system; tensionless foundation; Winkler foundation; static and dynamic loads.

1. Introduction

Plate-column systems, i.e., plates which support a column having a tip mass on elastic foundation, have been studied widely to investigate the soil-structure interaction under static and dynamic loadings. The earthquake response of these systems is usually analyzed under the assumption that the base plate is firmly bounded to the soil, i.e., compressive and tensile stresses can be transmitted between the plate and the foundation. However, when the overturning moment of the inertia forces exceeds the available overturning resistance due to gravity load, a part of the plate loses the contact with the foundation. It is reported that such uplift has been observed in several earthquakes in tower and oil tanks.

The assumption that the foundation reacts in compression as well as in tension, in the analysis of these types of problems simplifies the problem considerable. However, it is rational to expect that the contact between the plate and the foundation is established through the foundation stresses only within the region when the plate penetrates into the foundation. On the other hand, no interaction is expected outside of this region, when the plate lifts off the foundation. In such cases, the problem displays a non-linear character and the solution becomes difficult, since the contact region is not

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known in advance due to the tensionless character of the foundation. The extent of the contact region depends on the geometry of the problem and on the configuration of the loading. Generally, the evaluation of the contact region can be accomplished by using iterative methods. Studies involving plates on tensionless foundation have been carried out by various authors dealing with the rectangular (Dempsey *et al.* 1984, Celep 1988b, Papanikolaou and Doudoumis 2001, Silva *et al.* 2001) and circular plates (Weisman 1970, Villaggio 1983, Celep 1988a, Celep and Turhan 1990, Güler and Celep 1995, Celep and Gençoğlu 2003, Celep and Güler 2004). Solutions are given for a rectangular plate under static load and for circular plates for static and dynamic cases. Recently, buckling and post-buckling behavior of rectangular plates laterally constrained by a tensionless foundation and subjected to in-plane compressive forces are investigated (de Holanda and Gonçalves 2003, Shen and Li 2004). The solutions of static problems are accomplished mostly by applying approximate numerical techniques including nonlinear finite-element formulation and perturbation technique, to the non-linear governing equations of the problem. On the other hand, for the dynamic problems, i.e., for oscillations of a plate on a tensionless foundation, the contact region of the plate depends on time and usually the analysis is carried out numerically by adopting step-wise solution in time domain by updating the contact region continuously.

Housner (1963) investigated the dynamic behaviour of the rigid block including the foundation uplift. Meek (1975) and Psycharis (1983) extended this study by considering earthquake response of flexible structures. Yim and Chopra (1984, 1985) investigated a beam-column system and a multi-story system. The circular plate-column system is studied by including free and forced vibrations as well as the earthquake excitation (Celep and Güler 1991, Celep 1992). The present study deals with a system consisting of a rigid rectangular plate and an elastic column having a tip mass. The static behaviour of the plate-column system on the elastic tensionless Winkler foundation is investigated under a horizontal tip load. The study also includes the investigation of the system under the assumption that it is subjected to a ground harmonic motion and an earthquake motion. Numerical results are reported to demonstrate the effects of the parameters of the system on its static and dynamic behaviour, i.e., the tip mass, the lateral load and its application angle, the plate load and the dimensions of the plate.

2. Statement of the problem

The system considered is given in Fig. 1. It is a rectangular rigid plate of dimension $2A$ and $2B$, of mass M_p subjected to a uniformly distributed load $Q_o(t)$ in addition to its weight. The plate is on a tensionless Winkler foundation of stiffness K_f and supports an axially inextensible and massless column of height L , lateral stiffnesses K_x and K_y and damping C_x and C_y in the directions X and Y , respectively. The column supports a tip mass M_c and the mass is subjected to a horizontal load $P_o(t)$ with an angle α . It is assumed that the system is subjected to a horizontal ground excitation in two directions, $U_{gx}(t)$ and $U_{gy}(t)$. The displaced configuration of the system at any instant of time under the external loads and the horizontal ground excitation can be defined by the horizontal structural displacements $U_x(t)$ and $U_y(t)$, the vertical displacement $W_o(t)$ and the rotation of the rigid plate $\theta_x(t)$ and $\theta_y(t)$ around the axes Y and X , respectively, as shown in Fig. 1. The structural displacements are the displacements which come into being as a result of bending deformations of the column. The rigid plate is penetrated into the foundation and partly lifted off the foundation. The vertical displacement of the rigid plate $W(X, Y, t)$; the vertical load $P_f(t)$ and the moments $M_{fx}(t)$ and $M_{fy}(t)$

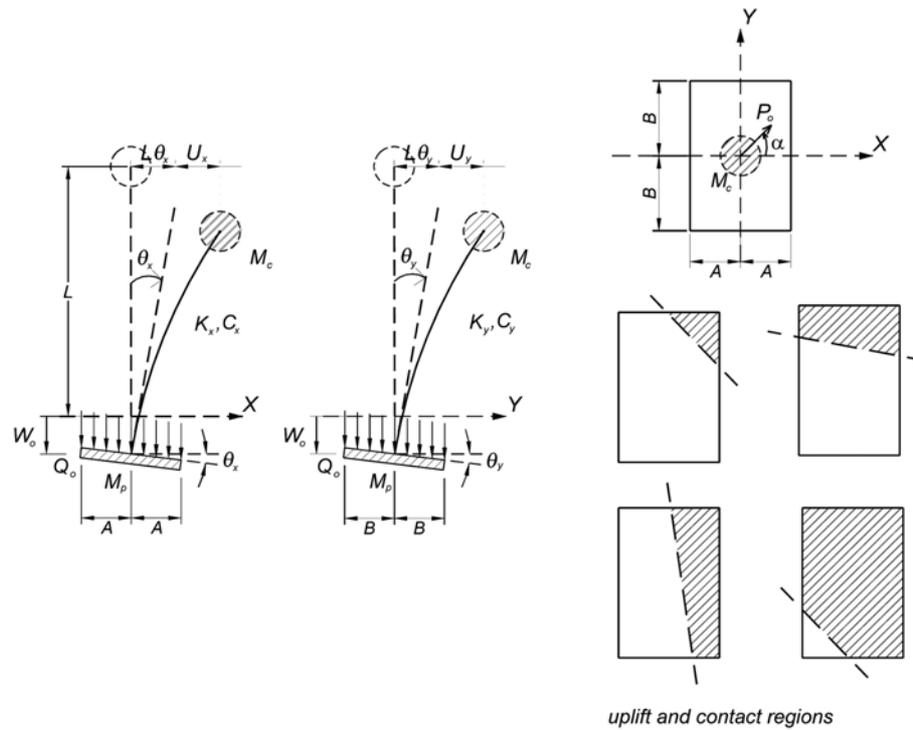


Fig. 1 Rectangular rigid plate-column system supported on a tensionless Winkler foundation

in the directions X and Y applied to the plate at the mid support from the foundation can be expressed as

$$\begin{aligned}
 W(X, Y, t) &= W_o(t) + \theta_x(t)X + \theta_y(t)Y \quad \text{for} \quad -A \leq X \leq A \quad -B \leq Y \leq B \\
 P_f(t) &= \int_{-A}^A \int_{-B}^B W(X, Y, t) K_f H(X, Y, t) dXdY \\
 M_{fx}(t) &= \int_{-A}^A \int_{-B}^B W(X, Y, t) K_f H(X, Y, t) Y dXdY \\
 M_{fy}(t) &= - \int_{-A}^A \int_{-B}^B W(X, Y, t) K_f H(X, Y, t) X dXdY
 \end{aligned} \tag{1}$$

Obviously, the relations $P_f - W_o$, $M_{fx} - \theta_x$ and $M_{fy} - \theta_y$ are linear up to the uplift of the base plate from the foundation. As usual the tensionless character of the Winkler foundation under the rigid plate is taken into consideration in Eq. (1), by introducing the contact function $H(X, Y, t)$ defined as

$$\begin{aligned}
 H(X, Y, t) &= 1 \quad \text{for} \quad W(X, Y, t) > 0 \\
 H(X, Y, t) &= 0 \quad \text{for} \quad W(X, Y, t) \leq 0
 \end{aligned} \tag{2}$$

The contact curve which separates the contact and the lift-off regions is a straight line and can be described as $W_o(t) + \theta_x(t)X + \theta_y(t)Y = 0$ as shown in Fig. 1. Depending on the vertical

displacement and the rotations of the plate, the contact region of the rigid plate has various shapes, such as rectangular, trapezoid and triangle. Two equations governing the small amplitude motion of the system can be derived by considering the lateral equilibrium of forces acting on the tip mass M_c :

$$\begin{aligned} M_c(L\ddot{\theta}_x + \ddot{U}_x + \ddot{U}_{gx}) + C_x\dot{U}_x + K_x U_x &= P_o \cos \alpha \\ M_c(L\ddot{\theta}_y + \ddot{U}_y + \ddot{U}_{gy}) + C_y\dot{U}_y + K_y U_y &= P_o \sin \alpha \end{aligned} \quad (3)$$

where the dots denote the differentiation with respect to the time t . Equations of motion of the rigid plate can be written similarly, by considering the vertical motion and the rotations:

$$\begin{aligned} (M_c + M_p)(\ddot{W}_o - g) &= 4Q_o AB - P_f \\ M_c L(L\ddot{\theta}_x + \ddot{U}_x + \ddot{U}_{gx}) + M_c(\ddot{W} - g)(L\theta_x + U_x) + \frac{1}{3}A^2 M_p \ddot{\theta}_x &= P_o L \cos \alpha + M_{fy} \\ M_c L(L\ddot{\theta}_y + \ddot{U}_y + \ddot{U}_{gy}) + M_c(\ddot{W} - g)(L\theta_y + U_y) + \frac{1}{3}B^2 M_p \ddot{\theta}_y &= P_o L \sin \alpha - M_{fx} \end{aligned} \quad (4)$$

Although the system is assumed to undergo only small displacements and rotations, the second terms in the left side of the last two equations represent the P - Δ effects due to the slenderness of the system. Substitution of the displacement function and the foundation reactions given in (1) into equations of motion (4) yields the following five coupled differential equations

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{k}\mathbf{u} = \mathbf{p} \quad (5)$$

where the dots denote the differentiation with respect to the non-dimensional time τ and

$$\mathbf{u}^T = [u_x \ u_y \ \theta_x \ \theta_y \ w_o] \quad \mathbf{m} = [m_{ij}(\tau)] \quad \mathbf{c} = [c_{ij}] \quad \mathbf{k} = [k_{ij}(\tau)] \quad \mathbf{p} = [p_i(\tau)] \quad (6)$$

and the non-zero elements of the matrices are

$$\begin{aligned} m_{11} = m_{22} = m_{13} = m_{31} = m_{24} = m_{42} = 1 \quad m_{33} = 1 + m_p a^2/3 \quad m_{44} = 1 + m_p b^2/3 \\ m_{55} = 1 + m_p \quad m_{35} = \theta_x + u_x \quad m_{45} = \theta_y + u_y \\ k_{11} = 4\pi^2 \quad k_{22} = 4\pi^2 \frac{k_y}{k_x} \quad k_{31} = k_{42} = -4\pi^2 \frac{m_c}{k_x} \quad k_{33} = 4\pi^2 \frac{a^2}{k_x} H_{xx} + k_{31} \\ k_{34} = k_{43} = 4\pi^2 \frac{ab}{k_x} H_{xy} \quad k_{35} = k_{53} = 4\pi^2 \frac{a}{k_x} H_x \quad k_{44} = 4\pi^2 \frac{b^2}{k_x} H_{yy} + k_{31} \\ k_{45} = k_{54} = 4\pi^2 \frac{b}{k_x} H_y \quad k_{55} = 4\pi^2 \frac{H_o}{k_x} \\ p_1 = p_3 = 4\pi^2 p_o \cos \alpha - \ddot{u}_{gx} \quad p_2 = p_4 = 4\pi^2 p_o \sin \alpha - \ddot{u}_{gy} \quad p_5 = 16\pi^2 \frac{Q_o}{k_x} + 4\pi^2 (1 + m_p) \frac{m_c}{k_x} \end{aligned} \quad (7)$$

The non-dimensional parameters related to the geometry, the load and the foundation introduced are defined as

$$\begin{aligned}
 u_x &= \frac{U_x}{L} & u_y &= \frac{U_y}{L} & u_{gx} &= \frac{U_{gx}}{L} & u_{gy} &= \frac{U_{gy}}{L} & p_o &= \frac{P_o}{K_x L} \\
 k_x &= \frac{K_x}{K_f AB} & k_y &= \frac{K_y}{K_f AB} & c_x &= \frac{C_x}{2\sqrt{M_c K_x}} & c_y &= \frac{C_y}{2\sqrt{M_c K_y}} & q_o &= \frac{Q_o}{K_f L} \\
 m_c &= \frac{M_c g}{K_f LAB} & m_p &= \frac{M_p}{M_c} & T_c &= 2\pi\sqrt{\frac{M_c}{K_x}} & \tau &= \frac{t}{T_c} & a &= \frac{A}{L} & b &= \frac{B}{L}
 \end{aligned}$$

$$\begin{aligned}
 H_o(t) &= \int_{-1}^1 \int_{-1}^1 H(x, y, t) dx dy & H_x(t) &= \int_{-1}^1 \int_{-1}^1 H(x, y, t) x dx dy & (8) \\
 H_y(t) &= \int_{-1}^1 \int_{-1}^1 H(x, y, t) y dx dy & H_{xx}(t) &= \int_{-1}^1 \int_{-1}^1 H(x, y, t) x^2 dx dy \\
 H_{yy}(t) &= \int_{-1}^1 \int_{-1}^1 H(x, y, t) y^2 dx dy & H_{xy}(t) &= \int_{-1}^1 \int_{-1}^1 H(x, y, t) xy dx dy
 \end{aligned}$$

where T_c is the natural period of the rigidly supported structure and c_x and c_y are the corresponding damping ratios. Furthermore, a and b represent the slenderness-ratio parameters and m_p is the ratio of the foundation mass to the superstructure mass. The system of five-degree-of-freedom has five free vibration mode shapes. Two of them are modal lateral vibrations in the two directions and they may be called as column vibrations and they are related to the linear elastic system of the mass M_c and the lateral stiffnesses K_x and K_y . The other two are also lateral mode shapes and they may be called as rocking vibrations and they are related to the system supported to the elastic foundation of stiffness K_f in two directions. The last one is related primarily to the vertical vibrations on the elastic foundation. However, due to the interaction between them, they all related to each other and this interaction depends on the numerical values of the system parameter and it becomes more pronounced, when uplift takes place. In fact the off-diagonal terms in the mass and stiffness matrices show the interaction between these five fundamental mode shapes.

Static and dynamic behaviour of the system is represented by the governing equation of the problem (5), which represents the small amplitude motion of the system including the $P-\Delta$ effect of the tip mass. Due to this effect the mass matrix and due to the tensionless nature of the foundation, the stiffness matrix has time dependent coefficients and the governing Eq. (5) is highly non-linear. The evaluation of the system of non-linear equations requires an iterative solution by the fact that after lift-off these equations depend continuously on the varying degree of the contact between the rigid plate and the foundation. On the other hand, they are relatively simple for a conventional Winkler foundation for which the contact functions can be evaluated easily as

$$H_o = 4 \quad H_x = H_y = H_{xy} = 0 \quad H_{xx} = H_{yy} = 4/3 \quad (9)$$

and consequently the governing Eq. (5) will be a linear one.

The static configuration of the system subjected to the horizontal force P_o at the tip mass and the uniformly distributed load Q_o on the rigid plate can be studied easily by using the static version of the Eq. (5)

$$\mathbf{ku} = \mathbf{p} \quad (10)$$

for both the conventional and the tensionless foundation models.

3. Numerical results and discussion

The static and dynamic response of the system to a specified static load and ground motion can be evaluated by applying a numerical solution procedure to the governing equation of the problem (5). The equation is non-linear mainly by the dependence of the stiffness coefficients on whether the plate is in full contact with the foundation or it is partially uplifted. In the later case, the coefficients depend continuously on the vertical displacements of the plate on the contact area. When a partial contact develops, the solution of the static case requires an iterative solution. The iterative solution is accomplished by assuming an initial contact configuration for the specific loading case and by checking whether the Eq. (10) is satisfied. By updating the contact configuration it is possible to find a solution in a few iteration steps. On the other hand, the dynamic behavior of the system is obtained by assuming an initial condition for the problem and by employing a step-wise numerical solution procedure for the governing differential Eq. (5) along the time axis. At each time step the contact functions H_o , H_x , H_y , H_{xy} , H_{xx} and H_{yy} are evaluated numerically and updated according to the vertical displacement configuration of the plate at the previous time step and the elements of the matrix \mathbf{k} are obtained accordingly. Before producing the results to present, the numerical procedure is verified by considering a number of special cases. Thereafter, for identification of the response of the system numerous results are produced for static and dynamic cases and presented in figures.

Assuming that the system is under static loads, Figs. 2(a), (b) and (c) show the variations of the lateral load p_o a function of the rotation of the plate θ_x as for various values of the direction angle of the load α , the plate length b and the vertical plate load q_o . As it is seen, linear variations are obtained for small values of the lateral load, as long as the full contact is maintained. However, asymptotic variations are displayed for larger values of the load, when a partial contact starts to develop. As the load increases beyond at incipient uplift, the plate separates over increasing lift-off region from the foundation. The linear variation becomes stiffer and the asymptotic values of the load increases, as the angle α , the length b and the vertical load q_o become larger, which indicates that an increase of these parameters delays the onset of the uplift. The linear variations would continue, when a conventional Winkler foundation were taken into consideration. Numerical results corresponding to large values of the displacement should be treated with caution as they may be exceeding the limitations of the small deformation assumption for the system.

Consider that the system is subjected to a horizontal load p_o and it will start to oscillate after removing it. When no uplift takes place, the oscillations come into being as a combination of the free vibration mode shapes. At the onset of the uplift, oscillations of the system come into being by resembling to a harmonic variation and a free vibration period can be evaluated approximately. Figs. 3(a) and (b) show the variations of the non-dimensional period T_o as a function of the lateral load p_o for various values of the direction angle of the load α and the vertical plate load q_o . As it is

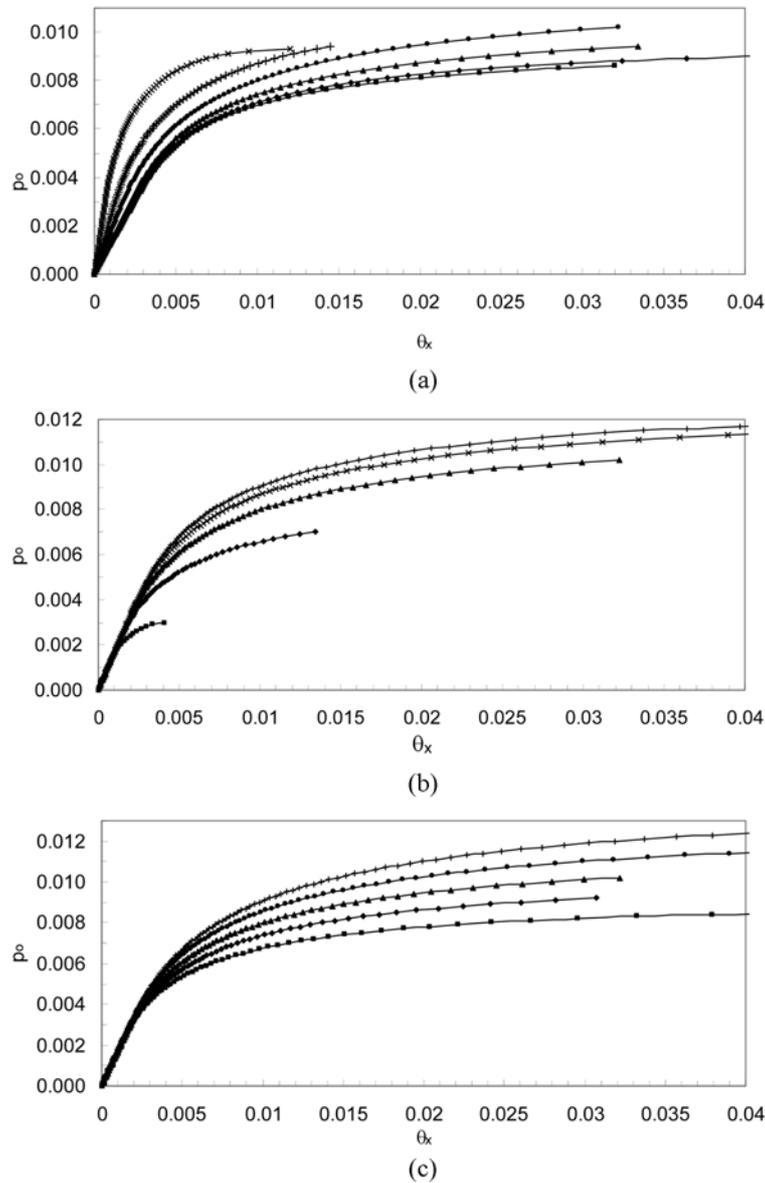


Fig. 2 Variation of the lateral load p_o with the rotation of the plate θ_x for various values of (a) the application angle $\alpha = 0^\circ$, \blacksquare ; 15° , \blacklozenge ; 30° , \blacktriangle ; 45° , \bullet ; 60° , $+$; 75° , \times ; (b) the length of the plate $b = 0.1$, \blacksquare ; 0.2 , \blacklozenge ; 0.3 , \blacktriangle ; 0.4 , \bullet ; 0.5 , $+$; (c) the plate load $q_o = 0$, \blacksquare ; 0.0001 , \blacklozenge ; 0.0002 , \blacktriangle ; 0.0003 , \bullet ; 0.0004 , $+$; for $c_x = c_y = 0.1$, $k_x = k_y = 0.1$, $\alpha = 0$, $m_c = 0.001$, $m_p = 2$, $a = b = 0.3$, $q_o = 0.0002$

seen, oscillations which are forced vibration of the linear system have a period independent of the initial displacement as well as the load p_o . However, the period lengthens as the load, consequently the initial displacement increases and an uplift of the plate takes place. On the other hand, an increase in the plate load q_o delays the onset of the uplift and the increase in the period. As Fig. 3 reveals, the period displays an asymptotic variation. It means that for larger values of the load p_o , a

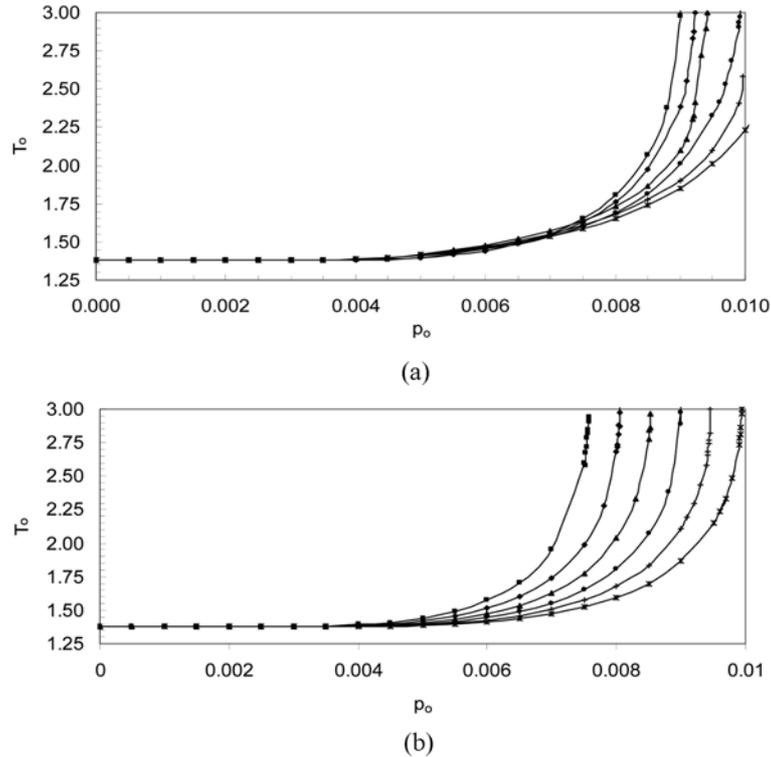


Fig. 3 Variation of the non-dimensional period T_o with the lateral load p_o for various values of (a) the application angle $\alpha = 0$, ■; 15° , ◆; 30° , ▲; 45° , ●; 60° , +; 75° , ×; (b) the plate load $q_o = 0.0005$, ■; 0.00010 , ◆; 0.00015 , ▲; 0.00020 , ●, 0.00025 , +; 0.00030 , ×; for $c_x = c_y = 0.1$, $k_x = k_y = 0.1$, $\alpha = 0$, $m_c = 0.001$, $m_p = 2$, $a = b = 0.3$, $q_o = 0.00020$

static equilibrium configuration can not be established, i.e., the system does not have any displaced configuration which can be assumed as an initial configuration for an oscillatory motion.

Assuming that the system is subjected to a harmonic ground motion such as $\ddot{u}_{gx}(\tau) = \ddot{u}_{go}\sin(\omega_g\tau)$, the governing equation of the problem is evaluated and the time variation of the lateral displacement of the tip mass $v_x(\tau) = u_x(\tau) + \theta_x(\tau)$ is displayed for the conventional and the tensionless Winkler foundation models in Figs. 4(a, b) and Figs. 4(c, d), respectively. In order to display the interaction between the system and the foundation, the ground motion frequency is assumed to be equal to the first (rocking vibration for which the foundation stiffness is more effective) and the second (column vibration for which the column stiffness is more effective) free vibration frequencies of the system in the lateral direction, namely, $\omega_g = 4.59$ and $\omega_g = 38.3$. As the figures show, in the first case mainly the corresponding rocking vibration is excited, whereas the contribution of the column vibration to the oscillation can not be noticed. However, in the second case it can be seen easily that the two vibrations are excited together. The comparison of the figures yields that the rocking motion with larger amplitudes appears to be much more effective than the column vibration. Consequently, the amplitudes of the displacement increase, when the ground motion excites the rocking motion. However, when the ground motion excites the column vibration, the column damping becomes effective and the amplitudes of the displacement decrease, whereas

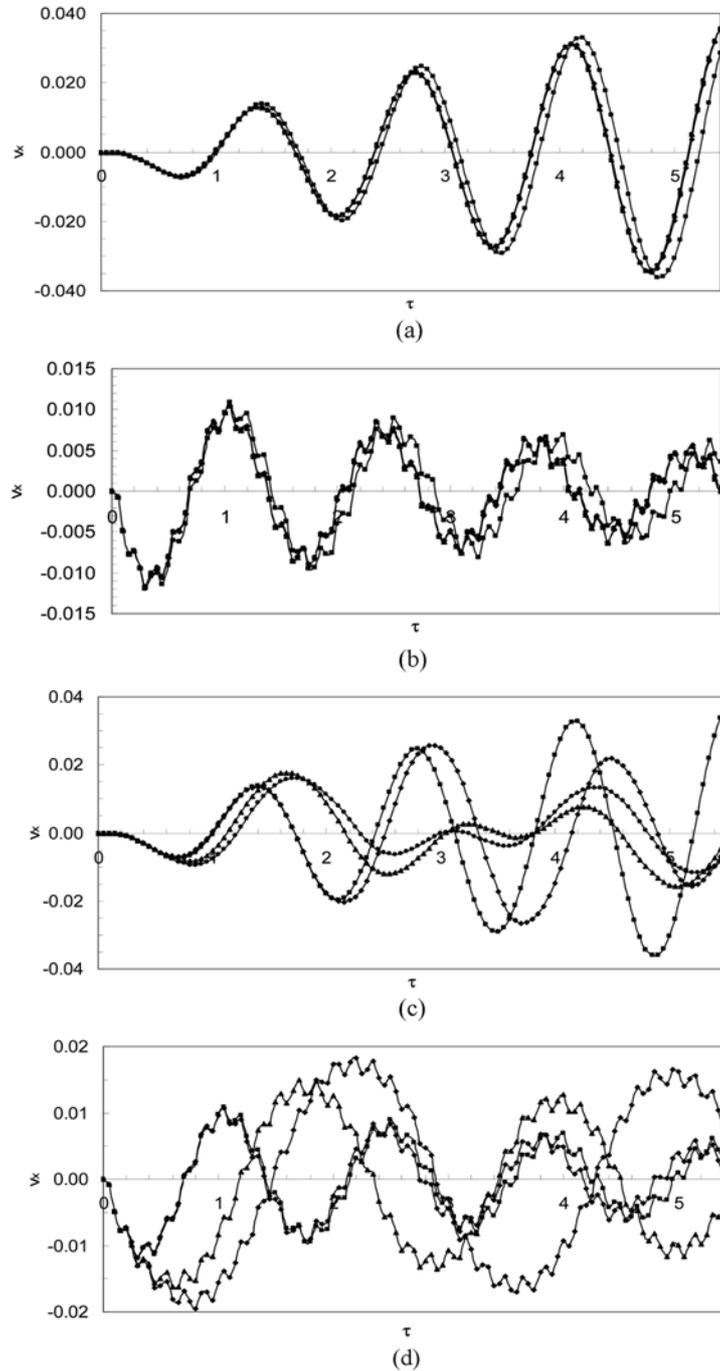


Fig. 4 Variation of the displacement of the tip mass $v_x(\tau) = u_x(\tau) + \theta_x(\tau)$ for various values of the tip mass $m_c = 0.01$, \blacksquare ; 0.001 , \blacklozenge ; 0.0001 , \blacktriangle ; 0.000001 , \bullet ; under the ground motion (a) $\ddot{u}_{gx} = 0.1\sin 4.59\tau$ for the conventional Winkler foundation; (b) $\ddot{u}_{gx} = 2.0\sin 38.3\tau$ for the conventional Winkler foundation; (c) $\ddot{u}_{gx} = 0.1\sin 4.59\tau$ and the tensionless Winkler foundation (d) $\ddot{u}_{gx} = 2.0\sin 38.3\tau$ and for tensionless Winkler foundation for $c_x = c_y = 0.1$, $k_x = k_y = 0.1$, $m_p = 2$, $a = b = 0.3$, $q_o = 0.2$

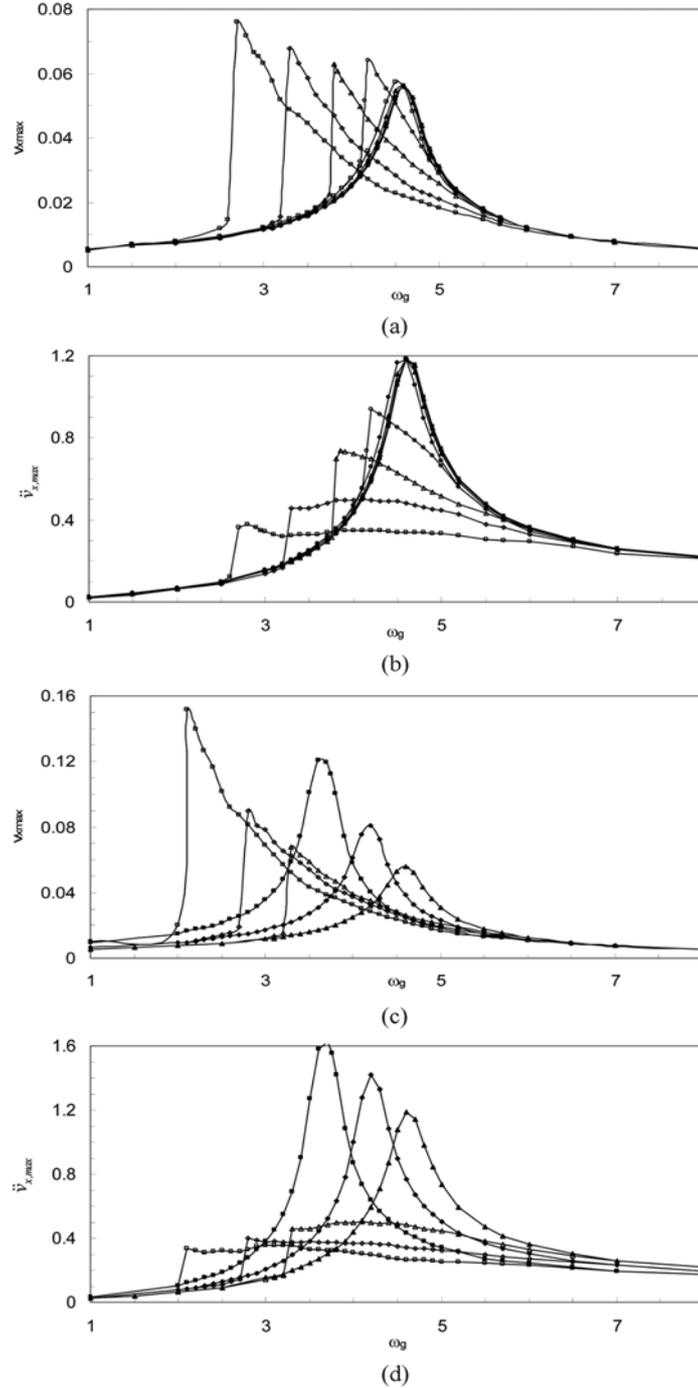


Fig. 5 Spectra of (a, c) the displacement $v_{x,max}$ and (b, d) the acceleration $\ddot{v}_{x,max}$ of the tip mass under the ground motion $\ddot{u}_{gx} = 0.1 \sin \omega_g \tau$ for various values of the tip mass (a, b) $m_c = 0.0005$, $\blacksquare(\square)$; 0.001 $\blacklozenge(\lozenge)$; 0.002, $\blacktriangle(\triangle)$; 0.004, $\bullet(\circ)$; and the length of the plate (c, d) $a = b = 0.1$, $\blacksquare(\square)$; 0.2, $\blacklozenge(\lozenge)$; 0.3, $\blacktriangle(\triangle)$; 0.4, $\bullet(\circ)$; for the conventional (tensionless) foundation for $c_x = c_y = 0.1$, $k_x = k_y = 0.1$, $m_c = 0.001$, $m_p = 2$, $a = b = 0.3$, $q_o = 0.0002$

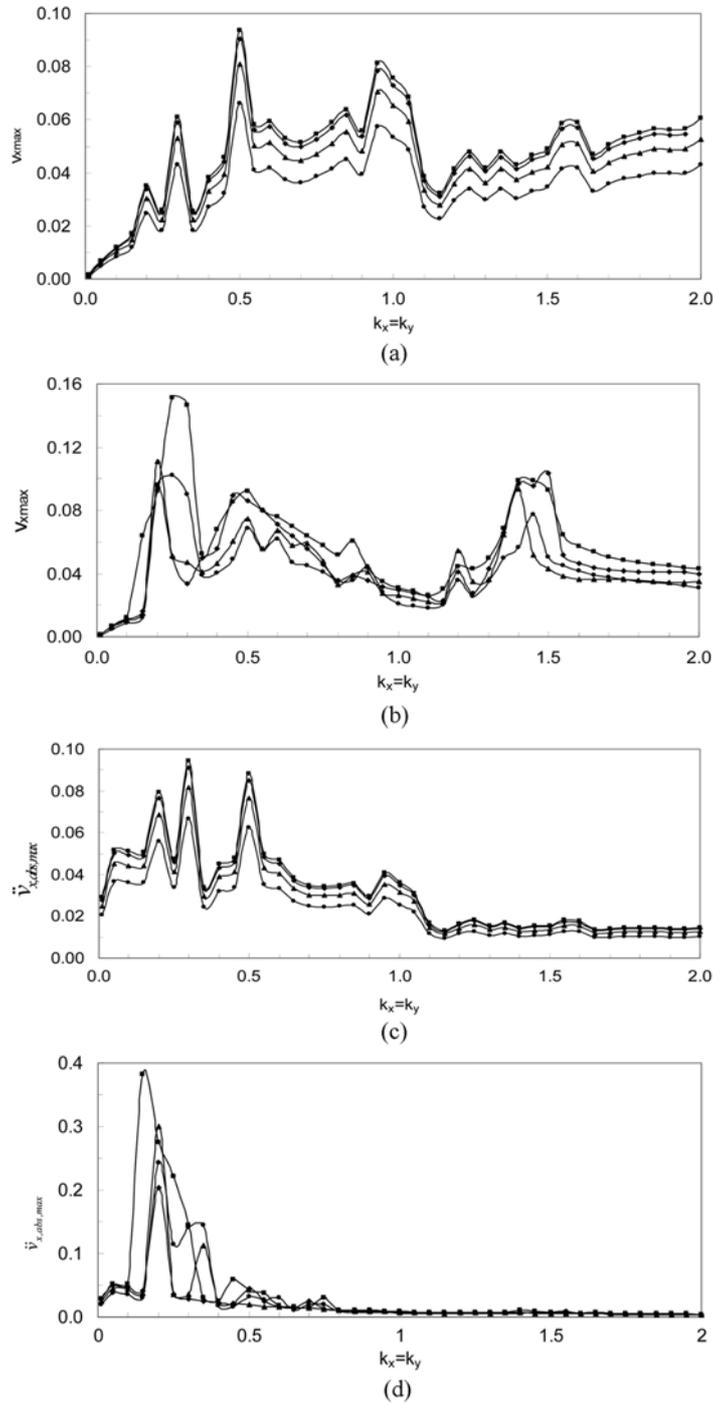


Fig. 6 Spectra of (a, b) the displacement $v_{x,max}$ and (c, d) the absolute acceleration $\ddot{v}_{x,abs,max}$ of the tip mass under the El Centro ground motion as a function of the lateral stiffness $k_x = k_y$ for various values of the angle of excitation direction $a = 0^\circ$, \blacksquare ; 15° \blacklozenge ; 30° , \blacktriangle ; 45° , \bullet ; for (a, c) the conventional and for (b, d) for the tensionless foundation model for $c_x = c_y = 0.1$, $m_c = 0.001$, $m_p = 2$, $a = b = 0.3$, $q_o = 0.0002$

the participation of the column vibration to the displacement becomes more pronounced. In case of the tensionless foundation, the oscillations lengthen and the vibration frequency decreases as the uplift starts, consequently the variations become complex. The figures show that the higher mode (column vibration) contributes little to the lateral displacement and to the overturning moment at the base of the structure. Consequently the uplift of the base plate has little influence on the higher mode response of the system.

In order to study the effects of the foundation uplift on the maximum response of the structure, response spectra of the system are evaluated. Fig. 5 shows spectra of the displacement $v_{x, \max} = [v_x(\tau)]_{\max} = [u_x(\tau) + \theta_x(\tau)]_{\max}$ and the acceleration $\ddot{v}_{x, \max} = [\ddot{v}_x(\tau)]_{\max} = [\ddot{u}_x(\tau) + \ddot{\theta}_x(\tau)]_{\max}$ of the tip mass under the ground motion $\ddot{u}_{gx} = 0.1 \sin \omega_g \tau$ for various values of the tip mass m_c and the length of the plate $a = b$ as a function the ground motion frequency ω_g for the conventional and the tensionless foundation model. As it is expected for the conventional foundation case, bell shape variations are obtained and the maximum spectral values come into being at the free vibration frequency, i.e., at about $\omega_g = 4.59$ for the numerical values of the parameters assumed. For the tensionless case, the response behaviour is strikingly different. The maximum spectral values of the displacement appear earlier and larger depending on onset of the uplift, whereas that of the acceleration become smaller. Furthermore, the bell shape variations alter due to the uplift of the plate. However, when the frequency of the ground motion increases gradually, at the approach to the critical frequency the displacement and the acceleration display a sudden increase, almost like a jump, whereas a smooth decrease appears after passing the critical frequency. The critical frequency depends in varying degree on the parameters of the system including on the slenderness and on the amplitude of the excitation in a complicated manner. As Figs. 5(c) and (d) show, when the length of the base plate decreases, i.e., when the system becomes slender, the spectral displacements and acceleration increases for the conventional foundation model. However, in the tensionless model case also an increase in the spectral displacements is noticed, whereas the numerical values of the spectral accelerations do not show any particular change, except for the critical frequency. Similar variations are given in Figs. 5(a) and (b) for various values of the plate mass m_c .

The above numerical results are presented in a non-dimensional form which applies to all systems having the same non-dimensional parameters, irrespective of their actual dimensional values. However, because actual earthquake motion records are given dimensionally, the earthquake response of the system can be evaluated within the framework of the present formulation only by assuming a numerical value at least for one parameter having acceleration dimension. In the present case the numerical value $T_c^2/L = 0.0045 \text{ s}^2/\text{m}$ is assumed. Figs. 6(a, c) and Figs. 6(b, d) illustrate the response of the system to the north-south component of the El Centro ground motion for the conventional and tensionless foundations, respectively. In figures the spectra of the displacement $v_{x, \max} = [v_x(\tau)]_{\max} = [u_x(\tau) + \theta_x(\tau)]_{\max}$ and the absolute acceleration $\ddot{v}_{x, \text{abs}, \max} = [\ddot{v}_x(\tau) + \ddot{u}_{gx}(\tau)]_{\max} = [\ddot{u}_x(\tau) + \ddot{\theta}_x(\tau) + \ddot{u}_{gx}(\tau)]_{\max}$ of the tip mass for various values of the arrival angle of the ground motion α is presented for two types of the foundation model. The spectral displacement and acceleration display a smooth variation with respect to the arrival angle α , i.e., the spectral values in the direction x decreases as the arrival angle increases. When the lateral stiffness of the column (k_x and k_y) is large with respect to the foundation stiffness, then the rocking motion becomes pronounced compared to the column vibration. On the other hand, the column vibration appears to be significant in comparison of the rocking motion, when k_x and k_y decrease. The corresponding curves for the tensionless case display larger values and complex variations.

4. Conclusions

The static and dynamic behaviour of a rectangular plate-column system under a lateral tip load, a harmonic ground excitation and the El Centro earthquake motion has been investigated for various values of the system parameters by including the effects of the uplift of the base plate. Although the study is based on relatively simple structural idealizations, it incorporates the features of the foundation uplift. Special attention is paid to the non-dimensionalization of the formulation. The numerical results are presented for the conventional and the tensionless Winkler foundation model comparatively. A study of the presented numerical results may lead to the following conclusions:

- (a) Inclusion of the tensionless response of the foundation softens the static and dynamic behavior of the system due to the decrease in the support flexibility. The base plate of a slender system has greater tendency to uplift resulting in greater variations in the behaviour of the system.
- (b) The period of the system increases, when the tensionless model of the foundation is taken into consideration. However, the dynamic behaviour of the displacement and the acceleration becomes more complex.
- (c) When the uplift develops, the displacements of the system under the harmonic ground motion increase, whereas the accelerations and the structural displacements are reduced and the critical frequency shifts by undergoing a gradual decrease. However, in case of the El Centro motion, the same tendency in the variation can be seen particularly for the small lateral column rigidity.
- (d) The uplift of the base plate is influenced mainly by the fundamental mode (rocking oscillations) and the higher modes (column vibrations) have less effect on the process.
- (e) The $P-\Delta$ effect has a negligible effect on the response of the system.

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