

Neural network based modeling of infilled steel frames

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Abstract. A neural network based model is developed for the structural analysis of masonry infilled steel frames, which can account for the non-linearities in the material properties and structural behaviour. Using the data available from the analytical methods, an ANN model with input parameters consisting of dimension of frame, size of infill, properties of steel and infill was developed. It was found to be acceptable in predicting the failure modes of infilled frames and corresponding failure load subject to limitations in the training data and the predicted results are tested using the available experimental results. The study shows the importance of validating the ANN models in simulating structural behaviour especially when the data are limited. The ANN model was also compared with the available experimental results and was found to perform well.

Key words: neural network-model; infilled steel frames; failure modes; collapse load.

1. Introduction

Artificial intelligence is an area of computer science concerned with designing intelligent computer systems, that is, systems that exhibit the characteristics associated with intelligence in human behaviour. In order to improve the analysis, design and control of the behaviour of the various structural systems, structural engineers have turned to the area of modelling of the physical processes. The behaviour of such systems is highly complex, which is governed by a large number of variables making the process a tedious one. The traditional approach used in most research in modelling is to start with an empirical equation, which is followed by a regression analysis using experimental data to determine the unknown coefficients such that these equations will fit the data.

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With this approach, based on the assumption and details of the experiment, different equations were derived-ranging from simple to complex. But these models have been found to be suitable only to interpret their own experimental results or data used.

The present study examines the feasibility of using artificial neural networks to predict the failure modes and corresponding failure load of infilled frames using available data from experiments. Since the behaviour of infilled frames is influenced by various factors with a complex and sometimes unknown interrelationship and the experimental data available are noisy and limited, ANN may be appropriately used as an alternative tool to the problem of predicting the failure mode and the corresponding failure load, initially focussing on steel frames infilled with masonry.

The effect of infill panels on the stiffness, ultimate load capacity and failure modes of framed structures has been one of the most interesting research topics in the last five decades. Liauw and Kwan (1985) proposed a unified plastic analysis, which accounts for the stress distribution due to the development of cracks together with the crushing of the infill towards collapse and the shear strength at the infill-frame interface depending upon the interface conditions. This theory is applicable to single storey and multi-storey single bay infilled frames.

Kwan *et al.* (1990) analysed 6 large scale models of multibay, multi-storey infilled steel frames with concrete infill, as an extension of Liauw and Kwan's plastic theory approach and the results show fairly good agreement with experimental results. This study was an important step towards the development of a general plastic theory for multibay infilled frames.

Jenkins (1995) studied the application of back propagation networks to approximate structural analysis. A simple six-storey structural steel frame with rigid joints was selected for the analysis. He concluded that ANNs could approximate the analysis of such structures provided sufficient training data is supplied and the number of units in the hidden layer is sufficient to represent the internal features and relationship connecting input and output values.

Muralikrishna and Gangadharam (1999) had proposed a new method based on ANN for the analysis of infilled frames. They used the data available from previously conducted experiments and applied it for the analysis of single bay single storey portal frame subjected to inplane nodal loads. They developed the neural net using backpropagation simulator and validated the analysis of single bay single storey portal frame subjected to inplane nodal forces. The application was run with 600 training patterns and 10 test patterns generated using STAAD, FEM package. The input variables for the training/testing patterns are the span of the frame, height of the frame, beam to column moment of inertia ratios and the nodal loads. The support reactions were taken as the output variables.

2. Neural network modelling

The system developer must go through a period of trial and error in the design decisions before coming up with a satisfactory design. The design issues in neural network are complex and are the major concerns of system developers.

Modelling a neural network consists of:

- arranging neurons in various layers.
- deciding the type of connections among neurons for different layers.
- deciding the number of neurons in the hidden layer.
- determining the strength of connection within the network by allowing the networks learn the

appropriate values of connection weights by using a training data set.

- presenting the data set in a proper way so as to avoid paralysis of network (Topping and Bahreininejad 1997).

2.1 Selecting input layer

The input layer has to be configured taking into account the possible parameters that may influence the output. These parameters vary greatly from problem to problem. Although the network is supposed to map the unknown functional relationship between input and output parameters, the performance for unseen problem depends upon the input parameters. Since the behaviour of an infilled frame is a highly non-linear one, this factor is very important. In the present work, the dimensions of frame and infill, its properties and the plastic moment capacity of the frame are taken into account.

2.2 Selecting output vector

Selecting the output vector is the simplest task in the net development. Normally the number of desired output parameters decides the output vector automatically. But, sometimes, it may be required to provide a node for a parameter which is not desired, to facilitate easy mapping of the functional relationship between input and output parameters. In the present case, the different modes of failure and failure loads are taken as the output vectors.

2.3 Selection of threshold function

This factor depends mainly upon the intended use of the network. Since back propagation is used in the network for the prediction problem, the sigmoidal non-linear function is used. The sigmoidal function is a continuous one with low output for low input and high output for sufficiently high input.

2.4 Configuring hidden layers

The selection of the number of hidden layers and the nodes is the most challenging part in the total network development process. Moreover, there are no fixed guidelines available for this purpose and hence trial and error method is adopted. The number of hidden layers as well as the nodes were altered to achieve the accuracy and to improve the generalization capacity.

2.5 Normalising input and output parameters

Neural networks are very sensitive to absolute magnitudes. The output of the sigmoid transfer function used in the back propagation algorithm lies between 0 and 1. Hence, the output values of the training patterns should be scaled to the same limit. It should also be noted that the system cannot actually reach its extreme values of 0 or 1 without infinitely large weights. Input patterns may also have high numerical values, which, when combined with the weights, produce high outputs. When these are employed in the transfer function, they force the output of the sigmoid function to travel near 0 or 1. Initialising the weights to small random values would help to avoid

this situation. However it is found to be better to normalise the input patterns as well as output patterns to lie between 0.2 and 0.8.

2.6 Presenting training data

The training data is presented to the network till it learns the functional relationship between input and output parameters. Normally it takes a long time for the successful training of feed forward networks using back propagation algorithm. In addition, to start with, the weights are randomly chosen. This has a large impact on the training time. For a particular set of weights, the net may get paralysed. As a result of which the user has to restart the learning process with a new set of weights. But, once the network is trained, it can take any value within the range of the given data.

2.7 Evaluating net performance

The performance of the network is best evaluated by watching its behaviour during testing towards unseen or new examples. Sometimes, there is a possibility of a net having mapped a different functional relationship between the given input and output vectors during the testing phase. This may happen when the relationships obtained during the testing phase is different from that already formed during training. Hence the user has to evaluate the performance of the net by testing it for a number of unseen examples before the network is used for prediction.

The extent of generalization achieved by the network has been studied in the present work. Small tolerance limit has been specified so as to increase the accuracy of the network. It has been observed that the generalised functional data, as generated by the network, is valid within an approximately defined domain only. As far as the application domain of network is concerned, it depends on the comprehensiveness of the training.

2.8 Organising training set

The training set for the development of a network comprises of examples from which the network is supposed to learn the internal representation of the system. The training set has to be selected carefully and it should reflect all concepts that the neural network is supposed to learn. At the same time, care should be taken to present the training set in a proper order so as to avoid its adverse effects on the learning such as network paralysis etc. (Flood and Kartam 1994).

3. Infilled frame structures

Structural frames are often filled with masonry walls for the purpose of partition or as architectural elements. Although these infills are usually not considered in the structural design, their influence on the behaviour of the frame is considerable. The frame, while directly carrying some of the load, primarily serves to transfer and distribute the major part of the load to the infill. The stiffness response of the infill is influenced, to a considerable extent by the way in which the frame distributes the loads to it and simultaneously, the frame's contribution to the over-all stiffness is affected by the change in its mode of distortion, as a result of reaction of the infill.

The strength of infill frame is influenced by the interaction between the frame and infill (Ghosh

and Amde 2002). The nature of interaction controls the stress distribution in the infill and, therefore, affects its strength and modes of failure. Thus the mutual interaction between the frame and the infill plays an important role in controlling the stiffness and strength of the infilled frame and so ignoring the effect of infill in stiffening and strengthening the surrounding frame is not always a conservative approach. Such an assumption usually leads to a substantial inaccuracy in predicting the lateral stiffness and strength of the frame. Different methods are available for the analysis of infilled frames.

3.1 Analytical models

There are several analytical methods to predict the behaviour, strength and stiffness of infilled frames. Some of these methods are empirical or semi-empirical, and some are more rational and use sophisticated mathematical models for geometry and materials. These analytical methods can be grouped into two categories (a) Macroscopic approach, which try to predict the overall behaviour and (b) Microscopic approach, modelling mechanical properties of the materials to predict the behaviour.

Macroscopic models usually idealize the panel by an equivalent beam or strut. Although these methods require less computational effort, changes in the topology of the panel due to crack opening and closing, and change in material properties in macro structure cannot be taken into account in simplified model implementations.

Microscopic methods employ principles of mechanics of solids to model the frame and the infill behaviour. But the drawbacks are (1) the boundary conditions and connections cannot be modelled properly using this method (2) friction between the frame and the infill cannot be modelled with reasonable accuracy. (3) elements are assumed to be isotropic, whereas they can be non-isotropic. These methods will result in highly complicated and complex problem formulations when the non-linearity has to be incorporated.

3.2 Experimental methods

Considerable experimental works have been done related to the behaviour of infilled frames. However, the following comments can be made on the tests conducted earlier.

- Test specimens were carefully designed and manufactured in laboratories. Therefore they do not represent the real structures with inherent weaknesses related to design and construction.
- In general, very small-scaled models were used in most of these tests.
- In most of the tests, loading was monotonic. In majority of the tests, single-storey specimens were employed. Such specimens do not represent the boundary conditions realistically.

3.3 Application of ANN to infilled frames

ANN has emerged as a computationally powerful tool in Artificial Intelligence with the potential of mapping an unknown nonlinear relationship between the given set of inputs and outputs. The actual mapping process of input into output patterns requires simple computation effort, and is therefore ideally suited for non-linear structural analysis. ANN has been proved successful in solving many civil and structural engineering problems such as in preliminary design, structural analysis and optimization (Andres and Kawashima 2003).

Table 1 Possible modes of failure (Liauw and Kwan 1985)

Type of failure	Collapse load
Mode1 Corner crushing with failure in columns and infill-beam connections.	$H_u = \sqrt{(2(M_{pj} + M_{pc})/\sigma_c t h^2)} \sigma_c t h + s(l - h/2)$
Mode2 Corner crushing with failure in beams and infill-column connections.	$H_u = \frac{1}{\tan \theta} \sqrt{(2(M_{pj} + M_{pc})/\sigma_c t h^2)} \sigma_c t h + s h/2$
Mode3 Diagonal crushing with failure in infill-beam connection.	$H_u = \frac{4M_{pj}}{h} + \sigma_c t h \left(\frac{2}{3} \times \beta - \frac{1}{2} \times \beta^2 \right) + s \left(l - \frac{h}{2} \right)$
Mode4 Diagonal crushing with failure in infill-column connection	$H_u = \frac{4M_{pj}}{h} + \frac{\sigma_c t h}{6 \tan^2 \theta} + \frac{s h}{2 \tan \theta}$

The present paper illustrates the application of ANN in the analysis of infilled frames so as to get the failure modes of infilled steel frames. The method based on ANN can accommodate most of the uncertainties like the non-linear behaviour of infill/frame materials, lack of fit at the frame/infill, non-homogeneity of materials etc. The unified plastic analysis developed by Liauw and Kwan (1985) for infilled frames accounts for the stress redistribution due to the development of cracks together with the crushing of infill towards collapse and shear strength at the infill-frame interface depending upon the interface condition, thus compensating most of the limitations of microscopic and macroscopic methods. So this method is used for framing the training data and the results are verified by using the available experimental results.

In developing a unified plastic analysis for infilled frames with different interface conditions, the most important aspect is to recognize the characteristics of interaction between the infill and the frame. The theory for fully integral infilled frames is first developed taking into account the shear strength of the connectors. Then non-integral theory is directly derived from the fully integral models by setting the shear strength of the connector to zero.

Since the experimental data available (Liauw and Kwan 1985, Kwan *et al.* 1990) are limited, only two test data are presented for the given network. The actual collapse load is the minimum of the values derived from all the kinematically possible failure modes. Collapse load for non-integral infilled frames is obtained by setting the shear strength to zero.

4. Case studies

4.1 Integral infilled frames and non-integral infilled frames

Training data was formed for single bay single storey integral infilled frame and two bay two storey models using Liauw and Kwan's plastic theory, which accounts for the interface conditions. Later this was extended to single bay single storey non-integral models as well. Data was trained accommodating different possible parameters influencing output and was tested for the available experimental results (Liauw and Kwan 1985, Kwan *et al.* 1990).

4.1.1 Integral infilled frames

Tables 2, 3 present the training data and test data for the single bay single storey infilled frames incorporating all possible failure modes. Fig. 1 shows the comparison of the results for predicted values using ANN with those based on plastic theory as well as the experimental values (Liauw and Kwan 1985).

Learning rate parameter	- 0.2
Neural network architecture	- 7-5-4 (10000 cycles)
Average error per cycle	- 0.027795
Error tolerance	- 0.006
Error last cycle per pattern	- 0.005958

It can be seen from Figs. 1(a), 1(b) and 1(c) that the agreement between the collapse loads computed using ANN and estimated using plastic theory is very good. Since the data for training as well as testing were generated using plastic theory, the implicit assumptions remain same for both and hence the good agreement is as expected.

In the case of experimentally determined collapse loads, the idealisations (inherent in a theoretical approach) are not valid and there is bound to be variations. In the test pattern two in Fig. 1(d), the ANN computed value is more than the experimental value. It is possible that the actual strength etc.

Table 2 Training data for single bay single storey integral infilled frames (Based on plastic theory)

l (m)	h (m)	t (mm)	m	s (N/mm ²)	σ_c (N/mm ²)	f_y (N/mm ²)	Mode1	Mode2	Mode3
1.9	1.58	51	0.08	2.25	27	372	176.682	211.6	379.052
1.9	1.58	51	0.068	2.25	27	372	152.514	182.537	375.451
1.9	1.46	51	0.086	2.25	27	372	176.817	228.709	352.794
1.9	1.46	51	0.074	2.25	27	372	152.649	197.365	348.896
1.9	1.35	51	0.093	2.25	27	372	176.914	247.286	328.902
1.9	1.26	51	0.100	2.25	27	372	177.042	264.552	309.053
1.9	1.26	51	0.086	2.25	27	372	152.874	228.18	304.998
1.9	1.18	51	0.092	2.25	27	372	152.964	243.621	287.608
1.9	1.10	51	0.113	2.25	27	372	177.211	300.134	277.62
1.9	1.10	51	0.098	2.25	27	372	153.043	258.923	272.495
1.9	1.05	51	0.103	2.25	27	372	153.11	273.597	259.63
1.9	1.58	22	0.042	1.84	33	252	51.787	61.506	197.686

Note: Collapse load (kN) is the minimum of mode1, mode2 and mode3 values.

Table 3 Test data (Based on plastic theory and experiments)

l (m)	h (m)	t (mm)	m	s (N/mm ²)	σ_c (N/mm ²)	f_y (N/mm ²)	Mode1	Mode2	Mode3	Hu (kN)
1.9	1.26	22.22	0.053	1.84	33.1	252.56	52.08	76.72	159.3	53.5
1.9	1.18	51	0.107	2.25	27	372	177.1	282.6	292.4	79

Note: Mode1, Mode2 and Mode3 are based on plastic theory and Hu (Collapse load) is based on experiment.

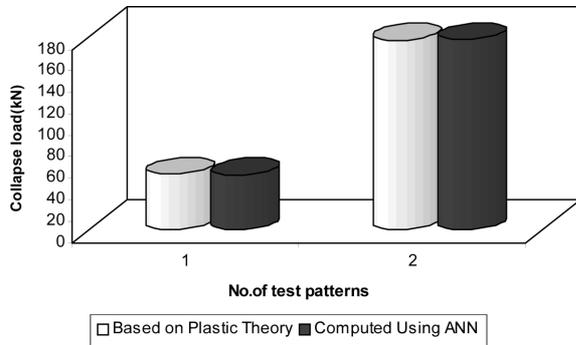


Fig. 1(a) Comparison of results (Mode 1)

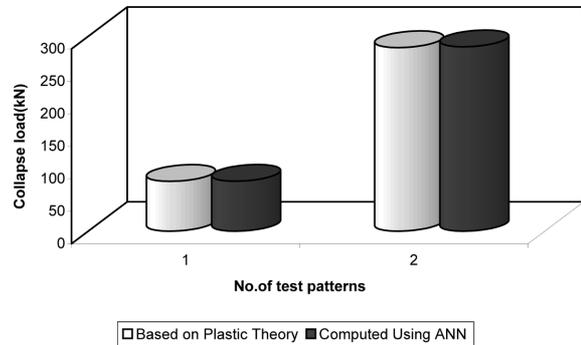


Fig. 1(b) Comparison of results (Mode 2)

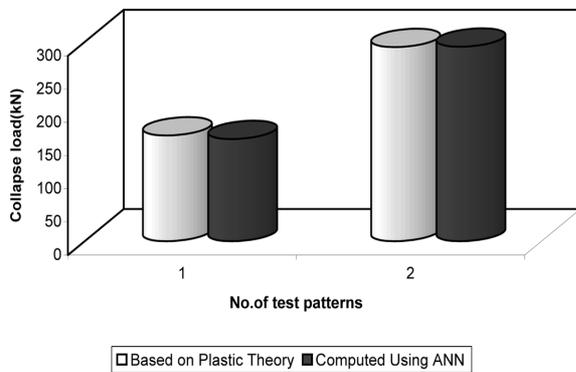


Fig. 1(c) Comparison of results (Mode 3)

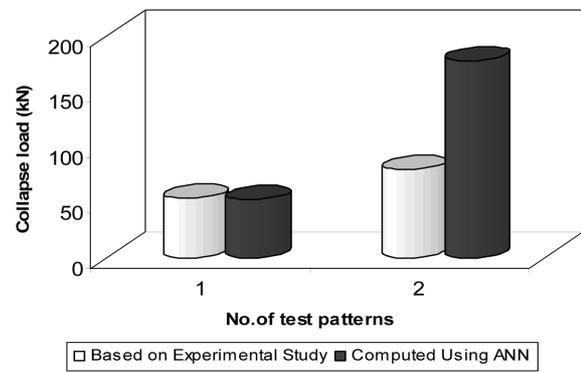


Fig. 1(d) Comparison of results (Collapse load Hu)

attained in the model was less than the stated value and errors could have crept in during the testing phase also.

These shortcomings can be overcome by enlarging the data base (both for training and testing) to include carefully planned and executed experimental results.

4.1.2 Two bay two storey integral infilled frame

Liau and Kwan's plastic theory was extended to multibay infilled frames. Here only three kinematically possible modes in which all the plastic hinges are formed on the columns have been considered. The experimental data (Kwan *et al.* 1990) are used for testing the network. The main philosophy behind Liau and Kwan's plastic theory is the concept that the effect of the infilled panels are equivalent to a set of interaction stresses acting on the frame members at the infill/frame interface, and, the plastic analysis can be simplified to one of the bare frames subjected to lateral loads and interaction stresses. This analysis yielded good results when tested using neural networks.

Tables 4, 5 present the training data and test data. Fig. 2 shows the comparison of neural net and target outputs.

Figs. 2(a), 2(b) and 2(c) show good agreement between the collapse loads predicted using ANN and estimated using plastic theory. Fig. 2(d) shows a reasonably good agreement between the

experimentally determined collapse loads and those estimated using ANN(unlike those in Fig. 1(d)). It can be inferred that the ANN prediction will be good if the experimental investigation is carefully done.

Table 4 Training data for two bay two storey integral infilled frames (Based on plastic theory)

l (m)	h (m)	l/h	m_{pc}	σ_c (N/mm ²)	f_y (N/mm ²)	Mode1	Mode2	Mode3
1.9	1.58	1.2	0.0582	27	372	492.94	489.112	991.019
1.9	1.58	1.2	0.0503	27	372	460.729	457.97	988.241
1.9	1.46	1.3	0.063	27	372	506.629	502.639	950.624
1.9	1.46	1.3	0.0544	27	372	474.499	471.513	647.611
1.9	1.35	1.4	0.0681	27	372	519.338	515.02	913.741
1.9	1.35	1.4	0.0589	27	372	487.122	483.893	910.474
1.9	1.26	1.5	0.073	27	372	529.665	525.039	883.683
1.9	1.26	1.5	0.0631	27	372	497.449	493.99	880.183
1.9	1.18	1.6	0.0779	27	372	538.845	533.905	857.084
1.9	1.18	1.6	0.0674	27	372	506.629	502.935	853.347
1.9	1.11	1.7	0.0828	27	372	546.878	541.626	833.922
1.9	1.11	1.7	0.0716	27	372	514.662	510.735	829.948

Note: Collapse load (kN) is the minimum of mode1, mode2 and mode3 values.

Table 5 Test data (Based on plastic theory and experiment)

l (m)	h (m)	l/h	m_{pc}	σ_c (N/mm ²)	f_y (N/mm ²)	Mode1	Mode2	Mode3	Hu (kN)
1.9	1.15	1.65	0.080	27	372	542.28	537.21	847.14	480
1.9	1.15	1.65	0.069	27	372	510.07	506.28	843.30	620

Note: Mode1, mode2 and mode3 are based on plastic theory and the collapse load, Hu is based on experiment.

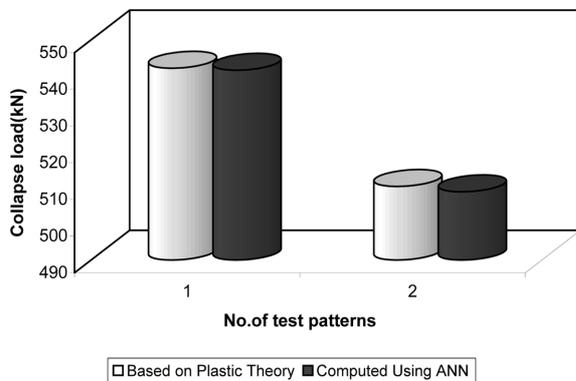


Fig. 2(a) Comparison of results (Mode 1)

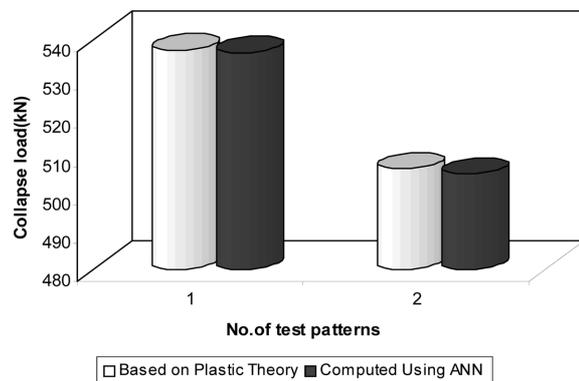


Fig. 2(b) Comparison of results (Mode 2)

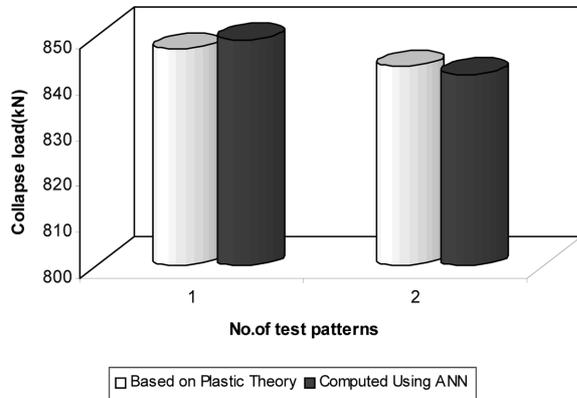


Fig. 2(c) Comparison of results (Mode 3)

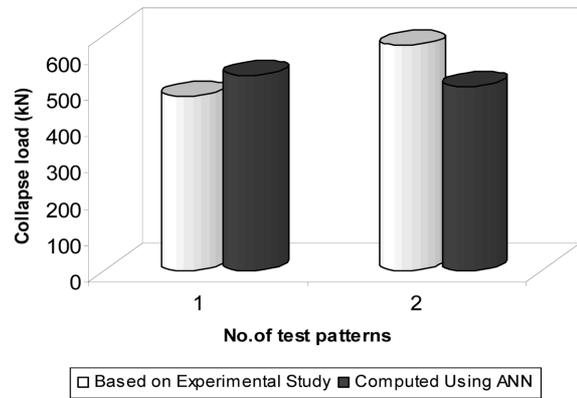


Fig. 2(d) Comparison of results (Collapse load Hu)

4.2 Single bay single storey non-integral infilled frames

Liauw and Kwan's plastic theory was adopted to form the data for single bay single storey non-integral infilled frames. No connectors are provided at the structural interface. By setting the shear strength parameter as zero, the data were formed for steel frames with micro concrete infill using

Table 6 Training data for non-integral infilled frames (Based on plastic theory)

l (m)	h (m)	t (mm)	m	σ_c (N/mm ²)	f_y (N/mm ²)	Mode1	Mode2	Mode3
1.9	1.58	51	0.08	27	372	174.052	209.454	376.55
1.9	1.46	51	0.074	27	372	149.977	195.227	346.263
1.9	1.35	51	0.093	27	372	174.183	245.249	326.145
1.9	1.26	51	0.100	27	372	174.022	262.414	306.306
1.9	1.18	51	0.107	27	372	174.184	280.466	289.482
1.9	1.18	51	0.092	27	372	149.974	241.551	284.66
1.9	1.10	51	0.113	27	372	174.092	297.996	274.594
1.9	1.05	51	0.120	27	372	174.08	315.002	261.959
1.9	1.58	22.22	0.042	33.1	252.5	49.694	59.75	195.643
1.9	1.26	22.22	0.053	33.1	252.5	49.722	74.977	156.983
0.915	0.61	22.22	0.110	33.1	252.5	49.712	74.568	80.231

Note: Collapse load (kN) is the minimum of mode1, mode2 and mode3 values.

Table 7 Test data (Based on plastic theory)

l (m)	h (m)	t (mm)	m	σ_c (N/mm ²)	f_y (N/mm ²)	Mode1	Mode2	Mode3
1.9	0.95	22.22	0.071	33.072	252.56	49.70	99.411	119.89
1.9	1.46	51	0.086	27	372	174.1	226.57	350.16

Note: Collapse load (kN) is the minimum of mode1, mode2 and mode3 values.

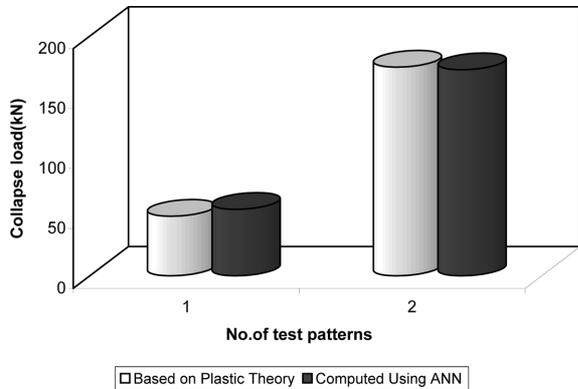


Fig. 3(a) Comparison of results (Mode 1)

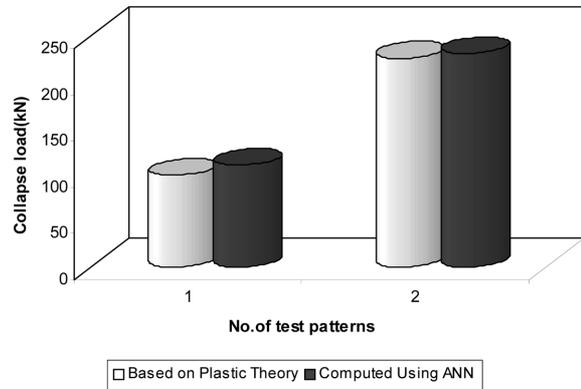


Fig. 3(b) Comparison of results (Mode 2)

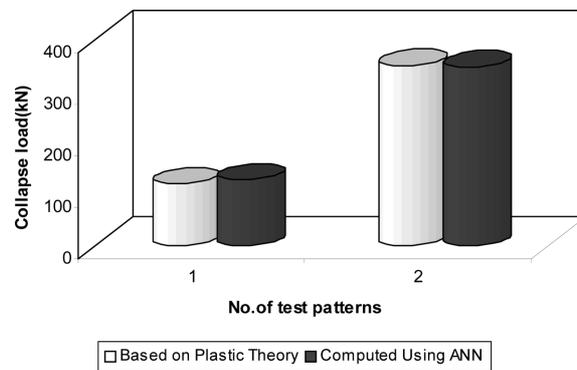


Fig. 3(c) Comparison of results (Mode 3)

the expressions given before considering all possible modes of failure. Tables 6, 7 show the training and testing data. Fig. 3 shows the comparison of collapse load predicted using ANN and computed using plastic theory. The agreement is seen to be very good.

- Maximum number of cycles specified - 1000
- Learning rate parameter - 0.3
- Error tolerance - 0.01
- Error last cycle per pattern - 0.009687

5. Conclusions

The study shows the feasibility of using ANN models for predicting the failure mode and corresponding failure load of infilled steel frames. Because of the scarce and limited data, it is difficult to derive an ANN model which can be applied to frames with a wide range of values in the input parameters. However, the derived ANN model was found to be capable of predicting the failure of infilled frame with the limited available data. If an ANN model with a wide application is desired, there should be an increase in the number and distribution of the training database which

should include results of carefully planned and executed experiments. Only with a sufficient number of data we can develop ANNs, which can completely model the complex interactions among the multiple variables.

The study demonstrated the capability and advantage of using ANNs in modelling physical processes. Unlike in regression analysis, no functional relationship among the variables is assumed before developing an ANN model. ANNs automatically construct the relationships and adapt based on the training data presented to them. The study also shows the importance of validating the performance of ANN models in simulating the physical processes especially when data are insufficient.

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Notation

- l : span of the infilled frame (m)
 h : storey height (m)
 t : thickness of the infilled panel (mm)
 m : relative strength parameter, $\sqrt{4M_p/\sigma_c t h^2}$
 σ_c : crushing strength of the infilled material (N/mm²)
 f_y : yield stress of steel (N/mm²)
 H_u : horizontal collapse load (kN)
 M_p : plastic moment of frame (kN-m)
 M_{pb} : plastic moment of beam (kN-m)
 M_{pc} : plastic moment of column (kN-m)
 M_{pj} : plastic moment of joint, i.e., the smaller value of M_{pb} and M_{pc} (kN-m)
 θ : angle between diagonal of infilled panel and horizontal
 s : shear strength of interface connection (N/mm²)
 βh : length of contact between column and panel in diagonal crushing mode (m)