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Frequency domain analysis of concrete arch dams by decoupled modal approach

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Abstract. A modal approach is proposed for dynamic analysis of concrete arch dam-reservoir systems in frequency domain. The technique relies on mode shapes extracted by considering the symmetric parts of total mass and stiffness matrices. Based on this method, a previously developed program is modified, and the response of Morrow Point arch dam is studied for various conditions. The method is proved to be very effective and it is an extremely convenient modal technique for dynamic analysis of concrete arch dams.

Key words: decoupled modal approach; concrete arch dams; dynamic analysis.

1. Introduction

Different techniques can be implemented for modal analysis of general concrete arch dam-reservoir systems in the frequency domain. Some are dependent on sub-structuring approaches and employ the mode shapes of the dam with an empty reservoir (Fok and Chopra 1986, Tan and Chopra 1995). The more direct method relies on the mode shapes of the coupled dam-reservoir system. However, in this latter alternative, standard eigenvalue computation methods are not applicable due to the fact that the coupled system is not symmetric (Zienkiewicz and Taylor 2000). Although, it is possible to achieve the symmetrization, it requires introduction of additional variables and complications in computer programming (Ohayon 1979, Fellipa 1988).

In the present study, a modal analysis method is proposed which is dependent on mode shapes evaluated from the symmetric part of the original eigen-problem of the system. The formulation of this method is presented initially and a special computer program “MAP-76” (Lotfi 2001), is enhanced based on this approach. The program was originally developed based on direct method in frequency domain and in this way the modal analysis option is also included. Utilizing this tool, the frequency response functions are calculated for an idealized model of Morrow Point arch dam as an example. The convergence of the method is verified for normal compressible water with different assumptions imposed at the reservoir boundaries, as well as for a pseudo-incompressible impounded water condition.

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2. Method of analysis

The modal analysis technique utilized in this study is designed for the FE-(FE-HE) concept of discretization (i.e., Finite Element-(Finite Element-Hyperelement)), which is applicable for a general concrete arch dam-reservoir system. In this manner, the dam is discretized by solid finite elements. The reservoir is divided into two parts, a near-field region (usually an irregular shape) in the vicinity of the dam and a far-field part (assuming uniform channel), which extends to infinity. The former region is discretized by fluid finite elements and the latter part is modeled by a three-dimensional fluid hyper-element.

The formulation can be explained much easier, if one concentrates initially on a dam with finite reservoir system (basically the same as a model of dam and reservoir near-field), and subsequently adds the effects of reservoir far-field region for the more complicated case. For this purpose, let us begin with this simpler formulation and then complete it on that basis.

2.1 Dam with finite reservoir system

This is the problem, which can be totally modeled by finite element method. It can be easily shown that in this case, the coupled equations of the system may be written as (Lotfi 2002):

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{B} & \mathbf{G} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{r}} \\ \ddot{\mathbf{p}} \end{bmatrix} + \begin{bmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{L} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{r}} \\ \dot{\mathbf{p}} \end{bmatrix} + \begin{bmatrix} \mathbf{K} & -\mathbf{B}^T \\ \mathbf{0} & \mathbf{H} \end{bmatrix} \begin{bmatrix} \mathbf{r} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} -\mathbf{MJa}_g \\ -\mathbf{BJa}_g \end{bmatrix} \tag{1}$$

$\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ in this relation represent the mass, damping and stiffness matrices of the dam body, while $\mathbf{G}$, $\mathbf{L}$, $\mathbf{H}$ are assembled matrices of fluid domain. The unknown vector is composed of $\mathbf{r}$, which is the vector of nodal relative displacements and the vector $\mathbf{p}$ that includes nodal pressures. Meanwhile, $\mathbf{J}$ is a matrix with each three rows equal to a 3×3 identity matrix (its columns correspond to unit rigid body motion in cross-canyon, stream, and vertical directions) and $\mathbf{a}_g$ denotes the vector of ground accelerations. Furthermore, $\mathbf{B}$ in the above relation is often referred to as interaction or coupling matrix.

The relation (1) can also be written alternatively in a more compact form as:

$$\overline{\mathbf{M}}\ddot{\bar{\mathbf{r}}} + \overline{\mathbf{C}}\dot{\bar{\mathbf{r}}} + \overline{\mathbf{K}}\bar{\mathbf{r}} = -\overline{\mathbf{M}}\overline{\mathbf{J}}\mathbf{a}_g \tag{2}$$

Where $\bar{\mathbf{r}}$ and $\overline{\mathbf{J}}$ are defined as follows:

$$\bar{\mathbf{r}} = \begin{bmatrix} \mathbf{r} \\ \mathbf{p} \end{bmatrix} \tag{3}$$

$$\overline{\mathbf{J}} = \begin{bmatrix} \mathbf{J} \\ \mathbf{0} \end{bmatrix} \tag{4}$$

The exact form of $\overline{\mathbf{M}}$, $\overline{\mathbf{C}}$ and $\overline{\mathbf{K}}$, are well apparent by matching relations (1) and (2) together. Obviously, these matrices can also be written as sum of the symmetric and unsymmetric parts as below:

$$\overline{\mathbf{M}} = \mathbf{M}_S + \mathbf{M}_U \tag{5a}$$

$$\overline{\mathbf{C}} = \mathbf{C}_S \tag{5b}$$

$$\overline{\mathbf{K}} = \mathbf{K}_S + \mathbf{K}_U \tag{5c}$$

It is noted from Eq. (1) that the damping matrix is totally symmetric, and the following relation also holds:

$$\mathbf{K}_U = -\mathbf{M}_U^T \tag{6}$$

For harmonic ground excitations $\mathbf{a}_g(t) = \mathbf{a}_g(\omega)e^{i\omega t}$ with frequency ω, displacements and pressures will all behave harmonic, and the Eq. (2) can be expressed as:

$$[-\omega^2\overline{\mathbf{M}} + i\omega\overline{\mathbf{C}} + \overline{\mathbf{K}}]\overline{\mathbf{r}} = -\overline{\mathbf{M}}\overline{\mathbf{J}}\,\mathbf{a}_g \tag{7}$$

In direct frequency domain method, this relation is solved for unknown vector $\overline{\mathbf{r}}$ at different frequencies. Although, the total dynamic stiffness matrix (resulting left hand side matrix of relation (7)) is not symmetric, it can be easily made symmetric (Lotfi 2005). Therefore, usual symmetric skyline solvers can be employed utilizing complex number arithmetic.

In modal approach, which is the basis of the present study, the method relies on obtaining the natural frequencies and mode shapes of the system. Thereafter, the solution can be estimated as usual based on the combination of these modes.

2.1.1 Decoupled modal technique

The eigenvalue problem corresponding to relation (2) can be written as follows:

$$\overline{\mathbf{K}}\overline{\mathbf{X}}_j = \bar{\lambda}_j\overline{\mathbf{M}}\overline{\mathbf{X}}_j \tag{8}$$

Physically, it is clear that the eigenvalues of this system are real and free vibration modes exist. However, it is noted from the form of matrices $\overline{\mathbf{K}}, \overline{\mathbf{M}}$ (relation (5)) that the system is not symmetric and standard eigenvalue computation methods are not directly applicable. Although there are techniques available to arrive at a symmetric form and reduce the problem to a standard eigenvalue problem, it is computationaly costly and additional variables are required to be introduced. Therefore, this path is not pursued in the present study. As a substitute, it was preferred to work with the eigenvalues and vectors extracted from the following eigen-problem:

$$\mathbf{K}_S\mathbf{X}_j = \lambda_j\mathbf{M}_S\mathbf{X}_j \tag{9}$$

Where $\mathbf{K}_S$, $\mathbf{M}_S$ are the symmetric parts of the $\overline{\mathbf{K}}$, $\overline{\mathbf{M}}$ matrices, as mentioned previously (relation (5)). Of course, the eigenvectors obtained through the above relation, are not the true mode shapes of the coupled system. However, these can be presumed as Ritz' vectors which can be similarly combined to estimate the true solution. The solution of this substitute eigen-problem is easily obtained by standard methods, since the involving matrices are symmetric and positive definite. Having the orthogonality condition and normalizing the modal matrix with respect to mass matrix, one would have:

$$\mathbf{X}^T\mathbf{M}_S\mathbf{X} = \mathbf{I} \tag{10a}$$

$$\mathbf{X}^T\mathbf{K}_S\mathbf{X} = \Lambda \tag{10b}$$

Where $\mathbf{I}$ is the identity matrix and Λ is a diagonal matrix containing the eigenvalues of the symmetric substitute system. The following relations are also derived easily based on relations (5), (6) and (10):

$$\mathbf{X}^T\overline{\mathbf{M}}\mathbf{X} = \mathbf{I} + \mathbf{X}^T\mathbf{M}_U\mathbf{X} \tag{11a}$$

$$\mathbf{X}^T\overline{\mathbf{K}}\mathbf{X} = \Lambda - \mathbf{X}^T\mathbf{M}_U^T\mathbf{X} \tag{11b}$$

As usual in modal techniques, the solution is written as a combination of different modes:

$$\bar{\mathbf{r}} = \mathbf{X}\mathbf{Y} \tag{12}$$

The vector $\mathbf{Y}$ contains the participation factors of the modes. Substituting this relation into (7) and multiplying both sides of that equation by $\mathbf{X}^T$, it yields:

$$[-\omega^2\mathbf{X}^T\overline{\mathbf{M}}\mathbf{X} + i\omega\mathbf{X}^T\overline{\mathbf{C}}\mathbf{X} + \mathbf{X}^T\overline{\mathbf{K}}\mathbf{X}]\mathbf{Y} = -\mathbf{X}^T\overline{\mathbf{M}}\overline{\mathbf{J}}\mathbf{a}_g \tag{13}$$

Or alternatively, the following relation is obtained by employing (11):

$$[-\omega^2(\mathbf{I} + \mathbf{X}^T\mathbf{M}_U\mathbf{X}) + i\omega\mathbf{C}^* + (\Lambda - \mathbf{X}^T\mathbf{M}_U^T\mathbf{X})]\mathbf{Y} = \mathbf{F}^* \tag{14}$$

In this relation, additional matrix definitions are utilized as below:

$$\mathbf{C}^* = \mathbf{X}^T\overline{\mathbf{C}}\mathbf{X} \tag{15a}$$

$$\mathbf{F}^* = -\mathbf{X}^T\overline{\mathbf{M}}\overline{\mathbf{J}}\mathbf{a}_g \tag{15b}$$

The vector of participation factors can be solved through relation (14), which is dependent on the excitation frequency. Thereafter, the unknown vector is obtained by Eq. (12) as usual in modal procedure. It must be mentioned that the left-hand side matrix of relation (14) is in general unsymmetrical. However, it may be easily transformed to a symmetric matrix by multiplying certain rows of that matrix relation by an appropriate factor. This can be shown as follows:

It is noticed through (1) and (5) that the symmetric matrices, utilized in the substitute eigen-problem, correspond to the decoupled dam-reservoir system. Therefore, the natural frequencies and eigenvectors are actually related to this decoupled system. It should be noted that in actual programming, one can modify the usual subspace iteration routines to converge to the desired lowest modes of the dam first, and similarly for the finite reservoir region afterwards by appropriate selection of initial vectors. On the other hand, they can also be obtained as two separate problems.

Let us now assume for simplicity, without loss of generality that the mode shapes for the dam are ordered first and the ones for the finite reservoir considered subsequently in the modal matrix,

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{X}_2 \end{bmatrix} \tag{16}$$

Similar arrangements are presumed for the eigenvalues in the diagonal matrix Λ.

$$\Lambda = \begin{bmatrix} \Lambda_1 & \mathbf{0} \\ \mathbf{0} & \Lambda_2 \end{bmatrix} \tag{17}$$

Then, it is clear that the following relations hold:

$$\mathbf{X}^T\mathbf{M}_U\mathbf{X} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{X}_2^T\mathbf{B}\mathbf{X}_1 & \mathbf{0} \end{bmatrix} \tag{18a}$$

$$\mathbf{X}^T\mathbf{M}_U^T\mathbf{X} = \begin{bmatrix} \mathbf{0} & \mathbf{X}_1^T\mathbf{B}^T\mathbf{X}_2 \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{18b}$$

$$i\omega\mathbf{C}^* = \begin{bmatrix} 2\beta_d i\Lambda_1 & \mathbf{0} \\ \mathbf{0} & i\omega\mathbf{X}_2^T\mathbf{L}\mathbf{X}_2 \end{bmatrix} \tag{18c}$$

Of course, it must be mentioned that in the last relation, it is assumed that the damping matrix of the dam is of hysteretic type. This means:

$$\mathbf{C} = \frac{2\beta_d}{\omega}\mathbf{K} \tag{19}$$

Where β_d is the constant hysteretic factor of the dam body.

Substituting relations (18) into (14), the following equation is concluded:

$$\begin{bmatrix} -\omega^2\mathbf{I} + \Lambda_1(1+2\beta_d i) & -\mathbf{X}_1^T\mathbf{B}^T\mathbf{X}_2 \\ -\omega^2\mathbf{X}_2^T\mathbf{B}\mathbf{X}_1 & -\omega^2\mathbf{I} + \Lambda_2 + i\omega\mathbf{X}_2^T\mathbf{L}\mathbf{X}_2 \end{bmatrix} \begin{bmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{F}_1^* \\ \mathbf{F}_2^* \end{bmatrix} \tag{20}$$

It is noticed that the vector of participation factors, is also assumed to be partitioned into two parts in this relation, and as before the indices 1, 2 correspond to dam and reservoir modes, respectively. Relation (20) is equivalent to (14) and as mentioned previously, this is in general an unsymmetrical system of equations. However, it is now revealing that this matrix relation could become symmetrical by multiplying the lower matrix equation by a factor of ω^{-2}, which yields the following relation:

$$\begin{bmatrix} -\omega^2\mathbf{I} + \Lambda_1(1+2\beta_d i) & -\mathbf{X}_1^T\mathbf{B}^T\mathbf{X}_2 \\ -\mathbf{X}_2^T\mathbf{B}\mathbf{X}_1 & \omega^{-2}(-\omega^2\mathbf{I} + \Lambda_2 + i\omega\mathbf{X}_2^T\mathbf{L}\mathbf{X}_2) \end{bmatrix} \begin{bmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{F}_1^* \\ \omega^{-2}\mathbf{F}_2^* \end{bmatrix} \tag{21}$$

2.2 Dam-reservoir system

The formulation for a dam with finite reservoir was already presented. For the case where the reservoir extends to infinity, a hyper-element must be used along with the fluid finite elements utilized for reservoir near-field. The governing relation for hyper-element and its derivation can be found elsewhere in detail (Lotfi 2004). Therefore, if the matrices of the hyper-element assemble in conjunction with the fluid finite elements, Eq. (7) would now become:

$$\left[-\omega^2\overline{\mathbf{M}} + i\omega\overline{\mathbf{C}} + \overline{\mathbf{K}} + \begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & \overline{\mathbf{H}}_h(\omega)\end{bmatrix}\right]\overline{\mathbf{r}} = -\overline{\mathbf{M}}\overline{\mathbf{J}}\mathbf{a}_g + \begin{bmatrix}\mathbf{0}\\ \overline{\mathbf{R}}_p(\omega)\end{bmatrix}\mathbf{a}_g \tag{22}$$

Where $\overline{\mathbf{H}}_h(\omega)$ and $\overline{\mathbf{R}}_p(\omega)$ are the expanded form of $\mathbf{H}_h(\omega)$ and $\mathbf{R}_p(\omega)$ matrices which cover the entire fluid domain pressure degrees of freedom. Assuming that the pressure degrees of freedom related to hyper-element are ordered first in the unknown pressure vector, then these matrices would have the following forms:

$$\overline{\mathbf{H}}_h(\omega) = \begin{bmatrix}\mathbf{H}_h(\omega) & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix} \tag{23a}$$

$$\overline{\mathbf{R}}_p(\omega) = \begin{bmatrix}\mathbf{R}_p(\omega)\\ \mathbf{0}\end{bmatrix} \tag{23b}$$

The relation (22) is the equation to be used instead of (7), when the reservoir is extended to infinity and one is considering the direct approach in frequency domain. Then, it is easy to see that in modal analysis, the relation (21) must be modified as follows when the hyper-element is also included.

$$\begin{bmatrix}-\omega^2\mathbf{I} + \Lambda_1(1+2\beta_d i) & -\mathbf{X}_1^T\mathbf{B}^T\mathbf{X}_2\\ -\mathbf{X}_2^T\mathbf{B}\mathbf{X}_1 & \omega^{-2}(-\omega^2\mathbf{I} + \Lambda_2 + i\omega\mathbf{X}_2^T\mathbf{L}\mathbf{X}_2 + \mathbf{X}_2^T\overline{\mathbf{H}}_h\mathbf{X}_2)\end{bmatrix}\begin{bmatrix}\mathbf{Y}_1\\ \mathbf{Y}_2\end{bmatrix} = \begin{bmatrix}\mathbf{F}_1^*\\ \omega^{-2}(\mathbf{F}_2^* + \mathbf{X}_2^T\overline{\mathbf{R}}_p\mathbf{a}_g)\end{bmatrix} \tag{24}$$

3. Modeling and basic parameters

A special computer program “MAP-76” (Lotfi 2001) is used as the basis of this study. The program was already capable of analyzing a general dam-reservoir system by direct approach in the frequency domain (Lotfi 2004). In this study, the modal analysis option is also included in the program based on the formulation presented in the previous section.

The program is based on the FE-(FE-HE) concept. This means, the dam is treated by solid finite elements, while the reservoir is divided into two parts, the near-field region in the vicinity of the dam, which is discretized by fluid finite elements, and the far-field part is modeled by a three-dimensional fluid hyper-element.

3.1 Models

An idealized symmetric model of Morrow Point arch dam is considered. The geometry of the dam may be found in (Hall and Chopra 1983).

The dam is discretized by 40 isoparametric 20-node finite elements (Fig. 1a). The water domain is divided into two regions (Fig. 1b). The near-field part is considered as a region, which extends to a length of 0.2 H in upstream direction at dam mid-crest point. H being the dam height or maximum water depth in the reservoir. The far-field region starts from that point and extends to infinity in the upstream direction. Both these regions combined are assumed to form a uniform reservoir shape, similar to the work of Tan and Chopra (1995). The former domain is discretized by 80

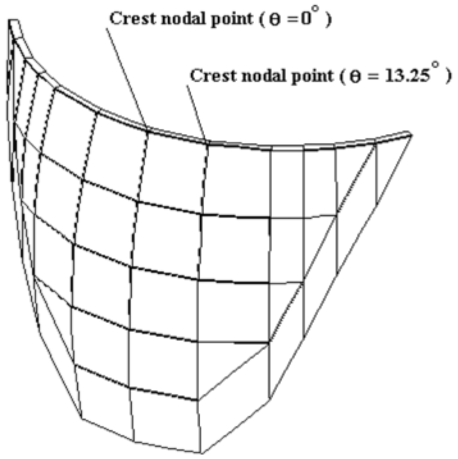

Fig. 1(a) Finite element mesh of the dam body

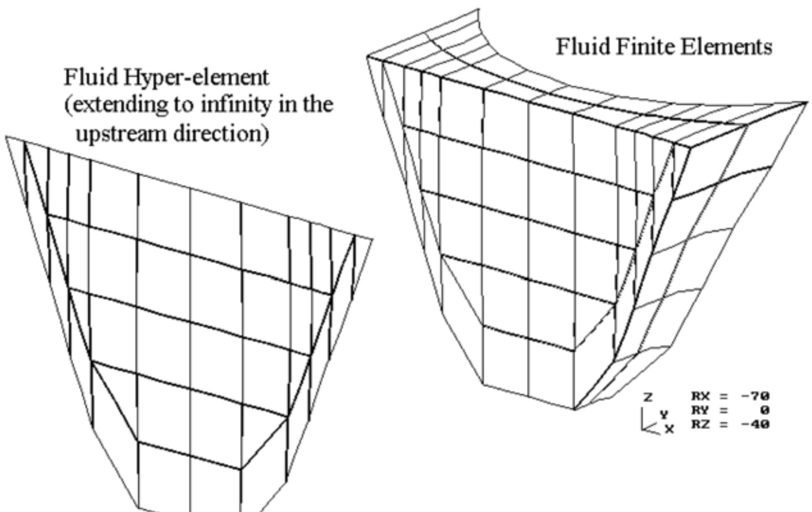

Fig. 1(b) Discretization of water domain (fluid finite elements and the fluid hyper-element) isoparametric 20-node fluid finite elements, while the latter part is modeled by a hyper-element which itself is costructed from 40 isoparametric 8-node sub-elements. Furthermore, the foundation rock is assumed rigid.

3.2 Basic parameters

The dam body is assumed to be homogeneous and isotropic with linearly viscoelastic properties for mass concrete: Elastic modulus (E_d) = 27.5 GPa., Poisson's ratio = 0.2, unit weight = 24.8 kN/m^3, and hysteretic damping factor $\beta_d = 0.05$.

The impounded water is taken as inviscid fluid with unit weight = 9.81 kN/m^3. The water is initially assumed as a compressible fluid with a pressure wave velocity c = 1440. m/sec. However, in a later stage, the incompressible case is also treated. For this purpose, a relatively high value of $c = 1440. \times 10^6$ m/sec is considered as a substitute for the actual infinite value. This is referred to as the pseudo-incompressible case.

4. Results

As mentioned before, the water is initially taken as a compressible fluid with its normal pressure wave velocity of c = 1440. m/sec. As a first step, the eigen-problem is solved based on the symmetric parts of the total mass and stiffness matrices. This is actually a decoupled system, and the natural frequencies obtained correspond to either the dam or the finite reservoir (reservoir near-field) region. Similar to the values for the dam with an empty reservoir, or the natural frequencies of the finite reservoir assuming rigid boundaries except at the water surface. The first five natural frequencies of each domain are listed in Table 1.

The first mode shape of the dam is displayed in Fig. 2(a), which is an anti-symmetric mode. Meanwhile, the first mode shape of the finite reservoir region is also depicted in Fig. 2(b), and that corresponds to a symmetric mode.

In the next step, the modal analyses are carried out based on two different sets of modes employed and the results are presented in Figs. 3-5. The first case corresponds to a set of 30, 60 modes utilized for the dam and the reservoir near-field domains, respectively. In the second case, these numbers are increased to 50, 100 modes. Of course, at this step the hyper-element is also considered to model the effects of reservoir far field.

As mentioned, the foundation is assumed rigid in the present study. However, two different alternatives of reservoir boundary absorption coefficient $\alpha = 1$, and 0.5 were investigated. $\alpha = 1$

Table 1 Natural frequencies of the dam and reservoir near-field region

Mode number (i)	Natural frequencies f_i (Hz)	
	Dam	Reservoir near-field
1	3.75	3.42
2	4.20	6.59
3	6.05	7.87
4	6.71	10.21
5	7.69	11.73

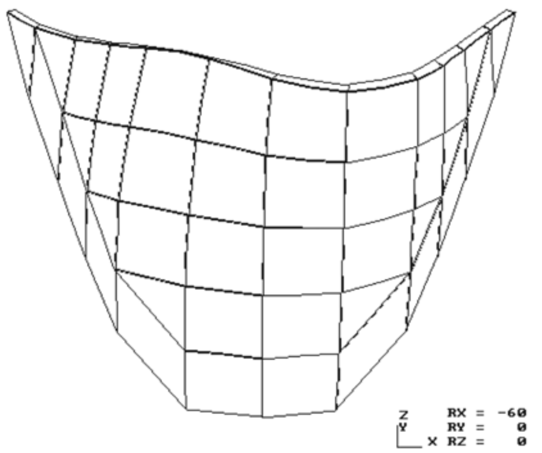

Fig. 2(a) The first decoupled mode shape of the dam on rigid foundation

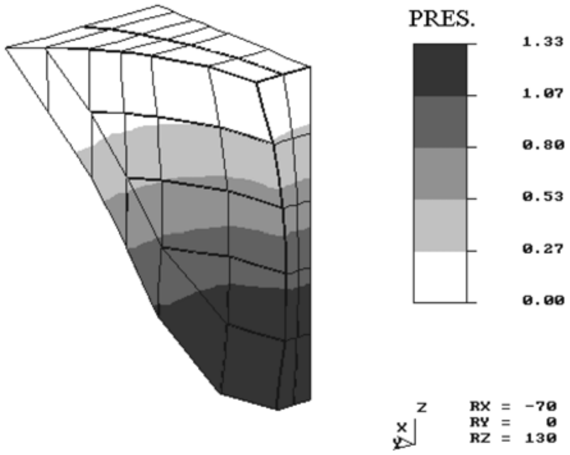

Fig. 2(b) The first mode of the reservoir near-field region

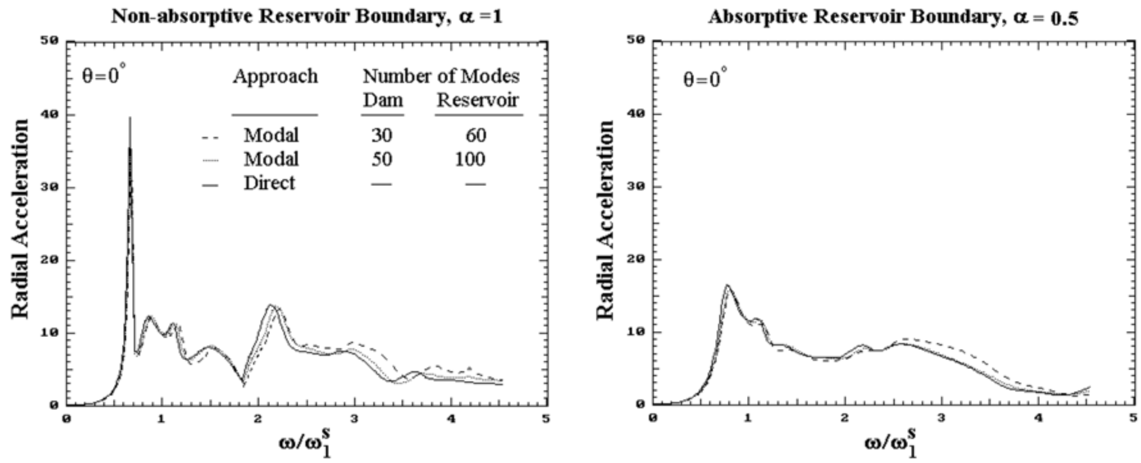

Fig. 3 Response at dam crest due to upstream ground motion

represents full reflection, while $\alpha = 0.5$ allows for partial refraction of waves impinging at reservoir-foundation boundaries (Fenves and Chopra 1984). The result for the direct approach is also shown in each graph as a reference. This can be taken as an exact response, which could be obtained if all of the modes of the dam and the reservoir near-field are considered in the analysis.

Furthermore, it should be mentioned that the response quantities plotted are the amplitudes of the complex valued radial accelerations for two points located at dam crest (Fig. 1a). This is either the mid-crest point ($\theta = 0^{o}$) selected for upstream or vertical excitations or a point located at ($\theta = 13.25^{o}$) which is used for the case of cross-stream excitation. This is due to the fact that radial acceleration is diminished at mid-crest for the cross-stream type of ground motion.

In each case, the amplitude of radial acceleration is plotted versus the dimensionless frequency for a significant range. The dimensionless frequency for upstream and vertical excitation is defined as ω/ω_1^S where ω is the excitation frequency and ω_1^S is the fundamental frequency of the dam on rigid

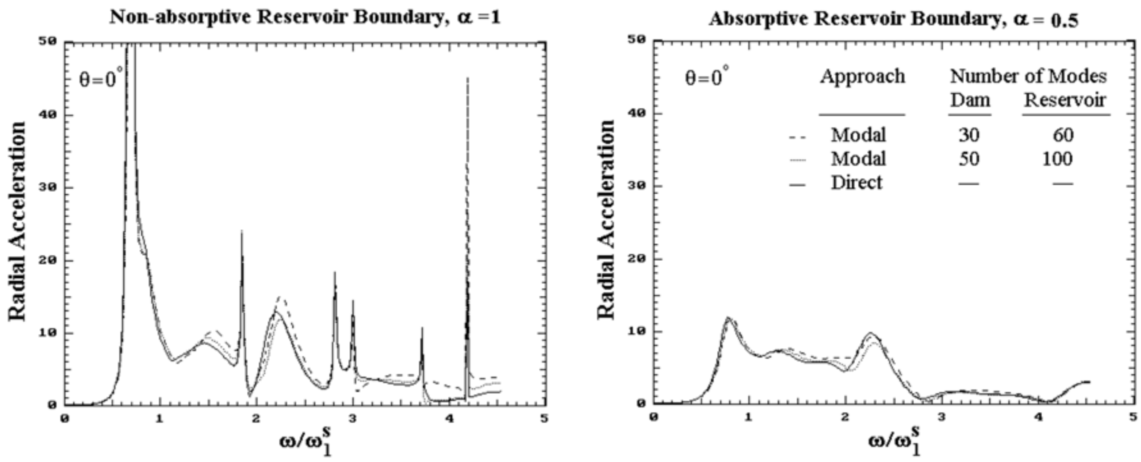

Fig. 4 Response at dam crest due to vertical ground motion

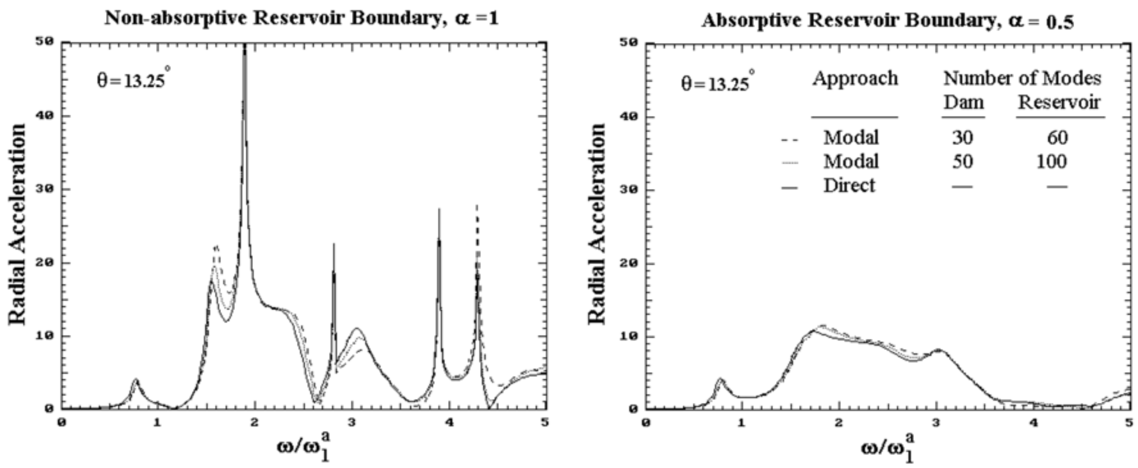

Fig. 5 Response at dam crest due to cross-stream ground motion

foundation with empty reservoir for a symmetric mode. For the cross-stream excitation cases, the dimensionless frequency is defined as ω/ω_1^a, where ω_1^a is the fundamental resonant frequency of the dam on rigid foundation with empty reservoir for an anti-symmetric mode.

Based on these results (Figs. 3-5), it is clearly noticed that the modal solution is converging to the exact solution (direct approach results) as the number of modes increase for all three excitations and both reservoir boundary absorption assumptions considered. Moreover, it is clear that the errors are negligible in the frequency range considered (up to approximately five times ω_1^S) for upstream and vertical excitations, even for the set of 30, 60 modes employed. For cross-stream excitation, the error become actually negligible for the higher mode set utilized (i.e., 50, 100 modes) in the case of non-absorptive reservoir boundary.

Following from this, the pressure wave velocity is increased to a relatively high value of $c = 1440. \times 10^6$ m/sec. As mentioned, this can be envisaged as a pseudo-incompressible case. Again, the responses are compared for different sets of number of modes, against the response obtained

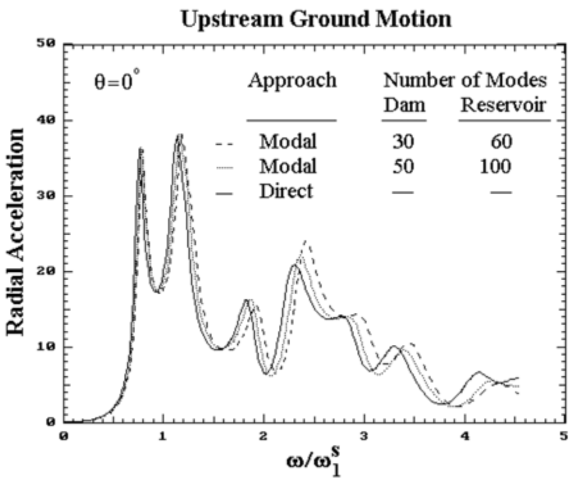

Fig. 6 Response at dam crest due to upstream excitation for the pseudo-incompressible condition

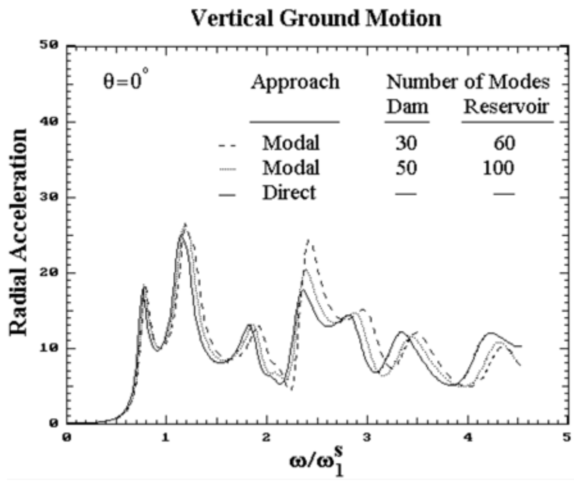

Fig. 7 Response at dam crest due to vertical excitation for the pseudo-incompressible condition

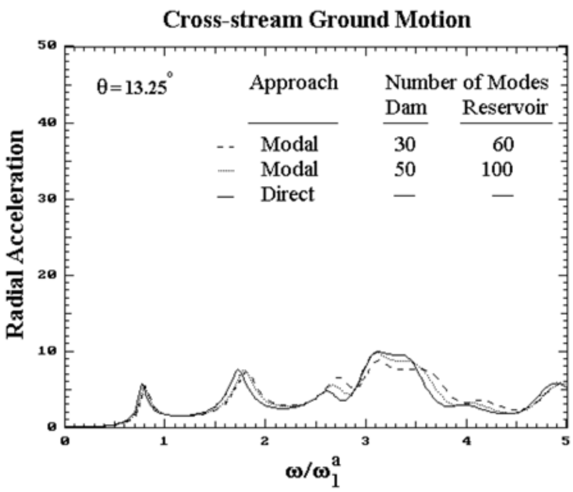

Fig. 8 Response at dam crest due to cross-stream excitation for the pseudo-incompressible condition

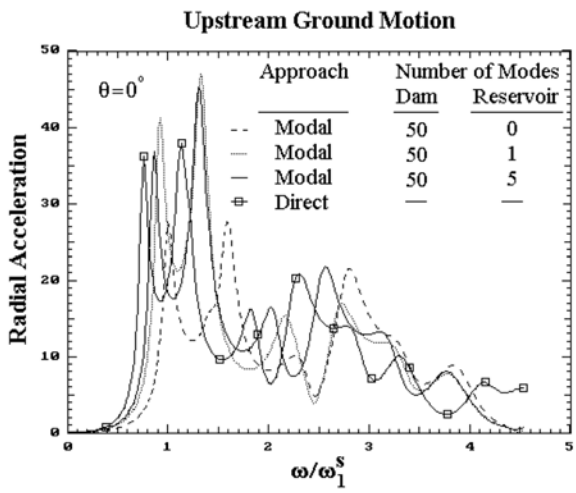

Fig. 9 Response at dam crest due to upstream ground motion by utilizing very low number of reservoir near-field modes for the pseudo-incompressible condition

based on the direct approach (Figs. 6-8). The convergence trend is quite evident similar to previous cases. Therefore, it is proved that the technique can be employed to obtain the incompressible solution by modal approach without any difficulty or special requirements. Finally, it seemed interesting to study the results that could be obtained for this pseudo-incompressible problem if one employs very low number of finite reservoir modes. For this purpose, the number of modes utilized for the dam is fixed (50 modes) and three cases are considered corresponding to 0, 1 and 5 modes for the finite reservoir region.

The results are compared in Fig. 9 along with the direct approach solution in the case of upstream excitation. Of course, it is clear that the response obtained with no mode of the finite reservoir region being considered is actually the same as the response of the dam with an empty reservoir. It is noted that the first peak is occurring at ω/ω_1^S (the fundamental frequency of the dam on rigid foundation with empty reservoir for a symmetric mode) in this case as expected.

Again, it is noticed that the responses are shifting toward the true solution (direct approach results) as the number of modes combined increase. However, even with five modes, the accuracy is not acceptable. This is in spite of the fact that the first natural frequency of the finite reservoir region for this pseudo-incompressible case is several orders of magnitude higher than the first natural frequency of the dam. Therefore, it is noted that the number of modes required for the reservoir near-field region does not necessarily decrease for the pseudo-incompressible case. Although accurate solutions can be obtained with relatively low number of modes for the reservoir near-field region (Fig. 6), the accuracy is not acceptable if very low number of modes of that region are employed (Fig. 9).

5. Conclusions

A formulation is presented for the dynamic analysis of general concrete arch dam-reservoir systems in the frequency domain. The method is based on the modal approach applied for the FE-(FE-HE) technique (i.e., Finite Element-(Finite Element-Hyperelement)). The special computer program MAP-76 is modified based on the theory explained and the dynamic behavior of an idealized symmetric model of Morrow point arch dam is considered as a controlling example. The water is initially assumed compressible with a normal pressure wave velocity of c = 1440. m/sec. Subsequently; this is increased to a relatively high value ($c = 1440. \times 10^6$ m/sec.), which is referred to as the pseudo-incompressible case. Specifically, this investigation leads to the following conclusions:

For the normal compressibility condition, it is verified that the modal solution converges to the exact solution (direct approach results) as the number of mode increases. It is observed that the errors are negligible in the frequency range considered (up to approximately five times ω_1^S) for upstream and vertical excitations, even for the low set of 30, 60 modes selected for the dam and the reservoir near-field region, respectively. The response due to cross-stream excitation converges a little slower in the case of non-absorptive reservoir boundary.

For the pseudo-incompressible case, it is noted that the number of the modes required for the reservoir near field region does not decrease in comparison to the normal compressible water case. Moreover, the accuracy is not acceptable if very low number of modes are utilized for this fluid region. This is in spite of the fact that the first natural frequency of the finite fluid region for this pseudo-incompressible case is several orders of magnitude higher than the first natural frequency of the dam. However, accurate solutions can still be obtained with relatively low set of number of the modes similar to the normal compressible case.

The formulation introduced is proved to be an effective and convenient procedure for dynamic analysis of general concrete arch dam-reservoir systems based on the modal approach in frequency domain. The method is not dependent on non-standard eigenvalue extraction routines, while it converges to the exact solution if all the modes are considered. Furthermore, it is also shown that the technique can be employed to obtain the solution for the incompressible case without any difficulty or special requirements.

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