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A response spectrum method for seismic response analysis of structures under multi-support excitations

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Abstract. Based on the random vibration theory, a response spectrum method is developed for seismic response analysis of linear, multi-degree-of-freedom structures under multi-support excitations is developed. Various response quantities, including the mean and variance of the peak response, the response mean frequency, are obtained from proposed combination rules in terms of the mean response spectrum. This method makes it possible to apply the response spectrum to the seismic reliability analysis of structures subjected to multi-support excitations. Considering that the tedious numerical integration is required to compute the spectral parameters and correlation coefficients in above combination rules, this paper further offers simplified procedures for their computation, which enhance dramatically the computational efficiency of the suggested method. The proposed procedure is demonstrated for tow numerical examples: (1) two-span continuous beam; (2) two-tower cabled-stayed bridge by using Monte Carlo simulation (MC). For this purpose, this paper also presents an approach to simulation of ground motions, which can take into account both mean and variation properties of response spectrum. Computed results based on the response spectrum method are in good agreement with Monte Carlo simulation results. And compared with the MSRS method, a well-developed multi-support response spectrum method, the proposed method has an incomparable computational efficiency.

Key words: multi-support excitation; response spectrum method; random vibration; seismic reliability analysis.

1. Introduction

Except the temporal variability, the spatial variability is the other important aspect of ground motions. This spatial variability is primarily the consequence of three effects: the wave passage effect, the ground motion ‘incoherence’ effect and the local-soil condition effect (Der Kiureghian and Neuenhofer 1992). Because of its obvious effects on large, multi-support structures, the spatial variability of earthquake ground motion has drawn much attention of researchers.

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Presently, dynamic analysis with spatially varying input excitations is performed mainly by time history approach, spectral analysis approach, and response spectrum method. Using the time history approach, many realistic models of bridges have usually been analyzed for the defined support excitations (Dusseau and Wen 1989, Dumanoglu and Severn 1990, Hyun *et al.* 1992, Nazmy and Abdel-Ghaffar 1992, Nazmy and Konidaris 1994, Price and Eberhard 1998, Zanardo *et al.* 2002). These studies generally concluded that the differential support motions induce response different from those of uniform motions and should be incorporated in the dynamic analysis of these bridges. Appealing for its statistical nature, spectral analysis approach has been performed for studying the effect of spatially varying ground motions in the last two decades (Abdel-Ghaffar and Rubin 1982, Zerva 1990, Hao 1994, Harichandran *et al.* 1996, Lin *et al.* 1997, Allam and Datta 1999, Alkhaleefi and Ali 2002). These studies also underline the significance of the spatial variability of ground motions. Besides, some researchers (Soyluk and Dumanogu 2000, Soyluk *et al.* 2004) attempted to compare the responses of a cable-stayed bridge computed by these tow approaches.

For the engineering community, however, response spectrum method has a great advantage over the above two approaches. The main reason is that most seismic design codes and specifications specify the earthquake motion in terms of the response spectrum. Several attempts have been made to develop a response spectrum method for multiply supported structures. For example, Berrah and Kausel (1992, 1993) suggested a modified response spectrum method, on account of the spatially variability effect by adjusting the spectrum at each support and the existing modal cross-correlation coefficients through two correction factors. Der Kiureghian and Neuenhofer (1992) proposed a multi-support response spectrum (MSRS) method using the fundamental principles of random vibration theory. The method can properly account for the effects of correlation between the support motions as well as between the modes of vibration of the structure. Another methodology, discussed by Heredia-Zavoni and Vanmarcke (1994), is characteristic in the analysis of the dynamic component of the response. Instead of cross-modal terms, the method just needs to compute spectral moments, which are independent of the dynamic properties of a given multi-support structural system. This seems more advantageous in computation and application.

Nevertheless, the existing methodologies are common in at least two disadvantages as below: (a) complex integrals are needed in the process of calculating the correlation coefficients or the spectral parameters, which is quite unappealing for engineering purposes; (b) They can only compute the mean of the peak response, but are unable to compute its standard deviation, so that the seismic reliability analysis cannot be performed reasonably. In this view, based on the random vibration theory, this paper presents a response spectrum method for seismic response analysis of linear systems under multi-support excitations. Various response quantities, including the mean-square of the response, the mean and variance of the peak response, and the response mean frequency are obtained in terms of the ordinates of the mean response spectrum. Meanwhile, procedures are presented to simplify the analysis of the frequency integrals of spectral parameters and cross-correlation coefficient, so that the tedious numerical integrations are not required.

The proposed procedure is demonstrated for tow numerical examples: (1) two-span continuous beam; (2) two-tower cabled-stayed bridge by using Monte Carlo simulation (MC). Since the reasonable input excitations is a key problem for the MC simulation, this paper also presents an approach to simulation of ground motions, which can take into account both mean and variation properties of response spectrum. It is indicated that computed results based on the response spectrum method proposed are in close agreement with results obtained from MC simulation. And compared with the MSRS method, the proposed method reduces the computational time dramatically.

2. Response spectrum analysis of structures under multi-support excitations

2.1 Equation of motion

The equation of motion for a discretized, n-degree-of-freedom linear system subjected to m support motions can be written in the matrix form as (Clough and Penzien 1975):

$$\begin{bmatrix} \boldsymbol{M} & \boldsymbol{M}_c \\ \boldsymbol{M}_c^T & \boldsymbol{M}_g \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{X}} \\ \ddot{\boldsymbol{U}} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{C} & \boldsymbol{C}_c \\ \boldsymbol{C}_c^T & \boldsymbol{C}_g \end{bmatrix} \begin{Bmatrix} \dot{\boldsymbol{X}} \\ \dot{\boldsymbol{U}} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{K} & \boldsymbol{K}_c \\ \boldsymbol{K}_c^T & \boldsymbol{K}_g \end{bmatrix} \begin{Bmatrix} \boldsymbol{X} \\ \boldsymbol{U} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \boldsymbol{F} \end{Bmatrix} \tag{1}$$

where $\boldsymbol{X} = [x_1, \ldots, x_n]^T$ is the n-vector of displacement of the unconstrained degrees of freedom; $\boldsymbol{U} = [u_1, \ldots, u_m]^T$ is the m-vector of prescribed support displacements; $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are the $n \times n$ mass, damping and stiffness matrices associated with the unconstrained degrees of freedom, respectively; $\boldsymbol{M}_g$, $\boldsymbol{C}_g$ and $\boldsymbol{K}_g$ are the $m \times m$ matrices associated with the support degrees of freedom; $\boldsymbol{M}_c$, $\boldsymbol{C}_c$ and $\boldsymbol{K}_c$ are the $n \times m$ coupling matrices associated with both sets of degrees of freedom; and $\boldsymbol{F}$ is the m-vector of the reacting forces at the support degrees of freedom.

2.2 Modal spectral analysis

From the solution to Eq. (1) in Der Kiureghian and Neuenhofer (1992), the power spectral density of $z(t)$ can be written in the form:

$$\begin{aligned} S_{zz}(\omega) = {} & \sum_{k=1}^{m}\sum_{l=1}^{m} a_k a_l S_{u_k u_l}(i\omega) + 2\sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n} a_k b_{li} H_i(-i\omega) S_{u_k \ddot{u}_l}(i\omega) \\ & + \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ki} b_{lj} H_i(i\omega) H_j(-i\omega) S_{\ddot{u}_k \ddot{u}_l}(i\omega) \end{aligned} \tag{2}$$

where $S_{xy}(i\omega)$ denotes the cross-PSD of process x and y, $H_i(i\omega) = [\omega_i^2 - \omega^2 + 2i\zeta_i\omega_i\omega]^{-1}$ represents the frequency response function of mode i, and a_k and b_{ki} denote the effective influence factors and effective modal participation factors. Both of these factors are a function of the structural properties.

Let $S_{zz}^d(i\omega)$ denote the cross-spectral density between the dynamic components, namely the third term on the right-hand side of Eq. (2), then it can be further expressed as (Heredia-Zavoni and Vanmarcke 1994)

$$\begin{aligned} S_{zz}^d(i\omega) = {} & \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n} [\alpha_{kli} + \beta_{kli}(1 - \omega_i^2/\omega^2)] |H_i(i\omega)|^2 S_{\ddot{u}_k \ddot{u}_l}(i\omega) \\ & + i\sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n} [\theta_{kli}(\omega/\omega_i) + \phi_{kli}(\omega/\omega_i)^3] |H_i(i\omega)|^2 S_{\ddot{u}_k \ddot{u}_l}(i\omega) \end{aligned} \tag{3}$$

in which the coefficients α_{kli}, β_{kli}, θ_{kli} and ϕ_{kli} depend on the mass, stiffness and damping properties of the structure only. According to $\sigma_x^2 = \int_{-\infty}^{+\infty} S_{xx}(\omega)d\omega$ and $\sigma_{\dot{x}}^2 = \int_{-\infty}^{+\infty} \omega^2 S_{xx}(\omega)d\omega$, we can have $\sigma_z^{d^2}$ and $\sigma_{\dot{z}}^{d^2}$ from Eq. (3), then:

$$\sigma_z^{d^2} = \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}\left\{\left[\alpha_{kli} + \beta_{kli}\left(1 - \frac{\omega_i^2\gamma_{-2,kli}}{\gamma_{0,kli}}\right)\right]\Gamma_{0,kli} - \theta_{kli}\left(\frac{\Lambda_{1,kli}}{\omega_i}\right) - \phi_{kli}\left(\frac{\Lambda_{3,kli}}{\omega_i^3}\right)\right\}\sigma_{s_{ki}}\sigma_{s_{li}} \quad (4)$$

$$\sigma_{\dot{z}}^{d^2} = \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}\omega_i^2\left\{\left[\alpha_{kli} + \beta_{kli}\left(1 - \frac{\omega_i^2\gamma_{0,kli}}{\gamma_{2,kli}}\right)\right]\left(\frac{\Gamma_{2,kli}}{\omega_i^2}\right) - \theta_{kli}\left(\frac{\Lambda_{3,kli}}{\omega_i^3}\right) - \phi_{kli}\left(\frac{\Lambda_{5,kli}}{\omega_i^5}\right)\right\}\sigma_{s_{ki}}\sigma_{s_{li}} \quad (5)$$

in which $\gamma_{N,kli} = \int_{-\infty}^{+\infty}\omega^N|H_i(\omega)|^2 R_{\ddot{u}_k\ddot{u}_l}(\omega)d\omega\ (N = -2, 0, 2)$; $\sigma_{s_{ki}}$ is the root-mean-square of the normalized modal response $s_{ki}(t)$; the spectral parameters $\Gamma_{0,kli}$, $\Gamma_{2,kli}$, $\Lambda_{1,kli}$, $\Lambda_{3,kli}$ and $\Lambda_{5,kli}$ are defined as:

$$\Gamma_{N,kli} = \frac{1}{\sigma_{s_{ki}}\sigma_{s_{li}}}\int_{-\infty}^{+\infty}\omega^N|H_i(\omega)|^2 R_{\ddot{u}_k\ddot{u}_l}(\omega)d\omega \qquad N = 0, 2 \quad (6)$$

$$\Lambda_{N,kli} = \frac{1}{\sigma_{s_{ki}}\sigma_{s_{li}}}\int_{-\infty}^{+\infty}\omega^N|H_i(\omega)|^2 Q_{\ddot{u}_k\ddot{u}_l}(\omega)d\omega \qquad N = 1, 3, 5 \quad (7)$$

in which $R_{\ddot{u}_k\ddot{u}_l}(\omega)$ and $Q_{\ddot{u}_k\ddot{u}_l}(\omega)$ are the real and imaginary parts of $S_{\ddot{u}_k\ddot{u}_l}(i\omega)$. Vanmarcke (1972) showed that for single degree of freedom systems subjected to ideal white-noise excitations as expressed by

$$\left(1 - \frac{\omega_i^2\gamma_{-2,kli}}{\gamma_{0,kli}}\right) \approx 4\zeta_i^2 \quad \text{and} \quad \left(1 - \frac{\omega_i^2\gamma_{0,kli}}{\gamma_{2,kli}}\right) = 0 \quad (8)$$

and therefore the terms associated with β_{kli} in Eq. (4) are expected to be very small when the system has a small damping ratio ($\zeta_i < 0.1$). As a result, neglecting these terms will lose little accuracy, then Eq. (4), Eq. (5) become:

$$\sigma_z^{d^2} = \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}\left[\alpha_{kli}\Gamma_{0,kli} - \theta_{kli}\left(\frac{\Lambda_{1,kli}}{\omega_i}\right) - \phi_{kli}\left(\frac{\Lambda_{3,kli}}{\omega_i^3}\right)\right]\sigma_{s_{ki}}\sigma_{s_{li}} \quad (9)$$

$$\sigma_{\dot{z}}^{d^2} = \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}\omega_i^2\left[\alpha_{kli}\left(\frac{\Gamma_{2,kli}}{\omega_i^2}\right) - \theta_{kli}\left(\frac{\Lambda_{3,kli}}{\omega_i^3}\right) - \phi_{kli}\left(\frac{\Lambda_{5,kli}}{\omega_i^5}\right)\right]\sigma_{s_{ki}}\sigma_{s_{li}} \quad (10)$$

The total response covariance, σ_z^2, is now obtained from Eqs. (3) and (9):

$$\sigma_{zz}^2 = \sum_{k=1}^{m}\sum_{l=1}^{m}a_k a_l\rho_{u_k u_l}\sigma_{u_k}\sigma_{u_l} + 2\sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}a_k b_{li}\rho_{u_k s_{li}}\sigma_{u_k}\sigma_{s_{li}}$$

$$+ \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}\left[\alpha_{kli}\Gamma_{0,kli} - \theta_{kli}\left(\frac{\Lambda_{1,kli}}{\omega_i}\right) - \phi_{kli}\left(\frac{\Lambda_{3,kli}}{\omega_i^3}\right)\right]\sigma_{s_{ki}}\sigma_{s_{li}} \quad (11)$$

Structures such as long-span bridges, which are affected significantly by multi-support excitations, are often of flexible systems. For these flexible structures, the contribution of the cross-correlation component between pseudo-static and dynamic responses, namely, the second term on the right-hand side of Eq. (11), to the total response is very small (Hao 1993, Nakamura *et al.* 1993, Loh and

Ku 1995, and Harichandran *et al.* 1996). Thus, it is reasonably considered that this cross-correlation component can be neglected without inducing any significant error. And then, Eq. (11) can be reduced further to:

$$\sigma_{zz}^2 = \sum_{k=1}^{m}\sum_{l=1}^{m} a_k a_l \rho_{u_k u_l}\sigma_{u_k}\sigma_{u_l} + \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}\left[\alpha_{kli}\Gamma_{0,kli} - \theta_{kli}\left(\frac{\Lambda_{1,kli}}{\omega_i}\right) - \phi_{kli}\left(\frac{\Lambda_{3,kli}}{\omega_i^3}\right)\right]\sigma_{s_{ki}}\sigma_{s_{li}} \quad (12)$$

where $\rho_{u_k u_l}$ denotes the cross-correlation coefficient between the two support displacements, and

$$\rho_{u_k u_l} = \frac{1}{\sigma_{u_k}\sigma_{u_l}}\int_{-\infty}^{+\infty} S_{u_k u_l}(i\omega)d\omega \quad (13)$$

2.3 Development of the response spectrum method

Based on the random vibration theory, the mean and standard deviation of the peak response $z_{\max}$ have the relations with σ_{zz} as follows:

$$\mu_{z_{\max}} = p_z\sigma_{zz} \quad \text{and} \quad \sigma_{z_{\max}} = q_z\sigma_{zz} \quad (14)$$

in which p_z, q_z are the mean and variance peak factors respectively, and can be obtained by (Davenport 1964):

$$p_z = \sqrt{2\ln\nu T_d} + \frac{0.5772}{\sqrt{2\ln\nu T_d}} \quad (15)$$

$$q_z = \frac{\pi}{\sqrt{6}}\frac{1}{\sqrt{2\ln\nu T_d}} \quad (16)$$

in which ν is the mean zero-crossing rate of the response process; T_d is defined as the period between the first and last crossings of 50% of the maximum absolute acceleration. Similarly, from Eq. (14) $u_{k,\max} = p_{u_k}\sigma_{u_k}$ and $D_k(\omega_i, \zeta_i) = p_{s_{ki}}\sigma_{s_{ki}}$, where $u_{k,\max}$ denotes the mean value of the peak displacement at the k-th support; $D_k(\omega_i, \zeta_i)$ denotes the mean response spectrum for $\omega = \omega_i$ and $\zeta = \zeta_i$ (written as D_{ki} hereafter for convenience); $p_{u_k}, p_{s_{ki}}$ are the mean peak factors of the corresponding process. Using these relations, substituting Eq. (14) into Eq. (12) and considering that p_z/p_{u_k}, $p_z/p_{s_{ki}}$ are near a unity (Der Kiureghian 1980), we obtain:

$$\mu_{z_{\max}} = \left\{\sum_{k=1}^{m}\sum_{l=1}^{m} a_k a_l \rho_{u_k u_l} u_{k,\max} u_{l,\max} + \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}\left[\alpha_{kli}\Gamma_{0,kli} - \theta_{kli}\left(\frac{\Lambda_{1,kli}}{\omega_i}\right) - \phi_{kli}\left(\frac{\Lambda_{3,kli}}{\omega_i^3}\right)\right]D_{ki}D_{li}\right\}^{1/2} \quad (17)$$

$$\sigma_{z_{\max}} = \frac{q_z}{p_z}\left\{\sum_{k=1}^{m}\sum_{l=1}^{m} a_k a_l \rho_{u_k u_l} u_{k,\max} u_{l,\max} + \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}\left[\alpha_{kli}\Gamma_{0,kli} - \theta_{kli}\left(\frac{\Lambda_{1,kli}}{\omega_i}\right) - \phi_{kli}\left(\frac{\Lambda_{3,kli}}{\omega_i^3}\right)\right]D_{ki}D_{li}\right\}^{1/2} \quad (18)$$

Eq. (17) and Eq. (18) are the response spectrum combination rules for seismic response analysis of the multiply supported structure in this paper. In order to compute p_z, q_z through Eq. (15) and

Eq. (16), it needs to give out the response mean frequency $\overline{\omega}$ related to $z(t)$. Since p_z, q_z are not too sensitive to $\overline{\omega}$ (Harichandran 1999) and for the total response $z(t)$, the effect of the dynamic component is dominant, then $\overline{\omega}$ can be approximated as:

$$\overline{\omega} = \pi\nu \approx \frac{\sigma_{\dot{z}}^d}{\sigma_z^d} = \left\{ \frac{\sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n} \omega_i^2 \left[\alpha_{kli}\left(\frac{\Gamma_{2,kli}}{\omega_i^2}\right) - \theta_{kli}\left(\frac{\Lambda_{3,kli}}{\omega_i^3}\right) - \phi_{kli}\left(\frac{\Lambda_{5,kli}}{\omega_i^5}\right)\right] D_{ki} D_{li}}{\sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n} \left[\alpha_{kli}\Gamma_{0,kli} - \theta_{kli}\left(\frac{\Lambda_{1,kli}}{\omega_i}\right) - \phi_{kli}\left(\frac{\Lambda_{3,kli}}{\omega_i^3}\right)\right] D_{ki} D_{li}} \right\}^{1/2} \tag{19}$$

3. Reduction of the computational effort

As they stand, Eq. (6), Eq. (7) and Eq. (13) are not appealing for engineering purposes since the numerical integrations are required. Good approximate methods, however, can be developed to overcome these difficulties as follows.

3.1 Description of ground motions

In general, the spatial variability of earthquake ground motions is characterized by the cross power spectral density (PSD) of ground accelerations. The cross-PSD between the ground accelerations at support k and l can be expressed as

$$S_{\ddot{u}_k\ddot{u}_l}(i\omega) = \gamma_{kl}(i\omega)[S_{\ddot{u}_k\ddot{u}_k}(\omega)S_{\ddot{u}_l\ddot{u}_l}(\omega)]^{1/2} \tag{20}$$

in which $\gamma_{kl}(i\omega)$ denotes the coherence function, and $S_{\ddot{u}_k\ddot{u}_k}(\omega)$ is the auto-PSD at support k.

Various theoretical and empirical models of the coherence function have been developed in recent years (Harichandran and Vanmarcke 1986, Loh and Yeh 1988, Hao *et al.* 1989). In this study, the following model is used (Loh and Yeh 1988):

$$\gamma_{kl}(i\omega) = \exp[-\alpha\omega|\tau|]\exp(i\omega\tau) \tag{21}$$

where α is the incoherence factor, and $\alpha = 1/16\pi$; $\tau = d/v_s$, d denotes the separation distance between two stations k and l, and v_s is the shear wave velocity.

The auto-PSD can be defined by the popular models, such as Kanai-Tajimi spectrum or filtered Kanai-Tajimi spectrum (Clough and Penzien 1975). For the purpose of response spectrum analysis, however, it had better transform the PSD from the given response spectrum. The transformation relationship as proposed by Der Kiureghian (1992) is used:

$$S_{\ddot{u}_k\ddot{u}_k}(\omega) = \frac{\omega^{\theta+2}}{\omega^\theta + \omega_f^\theta}\left(\frac{2\zeta\omega}{\pi} + \frac{4}{\pi T_d}\right)\left[\frac{D(\omega,\zeta)}{p_s(\omega)}\right]^2 \tag{22}$$

where ω_f and θ are two constants which can be obtained by the iteration procedure, and $p_s(\omega)$ is the peak factor for the oscillator response. In this analysis, response spectra of four types of site

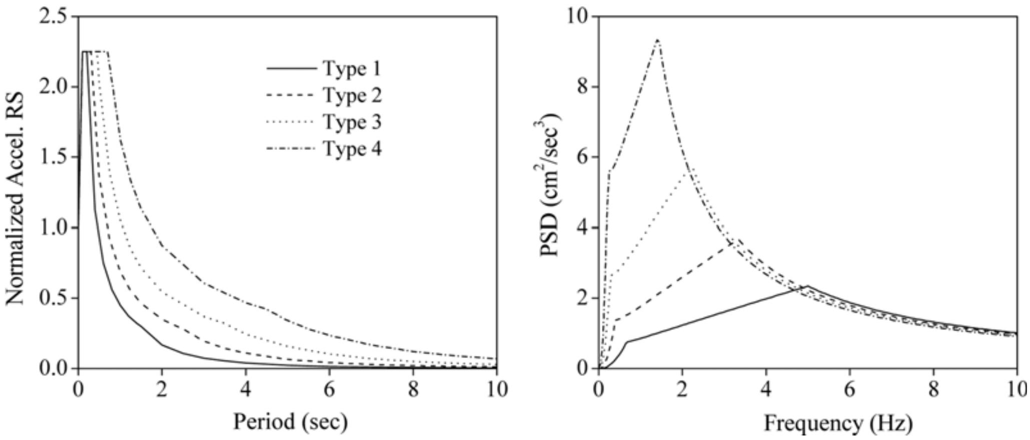

Fig. 1 Normalized response spectra and their corresponding PSD

conditions specified in the Chinese design code for transportation engineering are taken, and the modification of their expressions has been made to fit them to analyze structures whose fundamental period is over 5s (Wang and Fan 1998). Fig. 1 shows these modified spectra, which are normalized, and their corresponding PSD functions when the peak ground acceleration $\ddot{u}_{k,\max} = 0.2g$, $\theta = 2$ and $\omega_f = 1.608, 0.998, 0.698, 0.509$ rad/s for four site conditions, respectively.

3.2 Simplified procedure for spectral parameters

The simplified analysis of integrals of spectral parameters in Eq. (6) and Eq. (7) presupposes that $S_{\ddot{u}_k\ddot{u}_k}(\omega)$ can be approximated by a white noise process, that is $S_{\ddot{u}_k\ddot{u}_k}(\omega) = S_0$. This assumption is feasible mainly due to two points: first, auto-PSD functions transformed from given response spectra (see Fig. 1) are broad-banded; second, the spectral parameters in Eq. (6) and Eq. (7) are expressed as rations of integrals over the spectral shapes, and hence they are only mildly dependent on the specific forms of these functions. As a consequence, substituting Eq. (21) and $S_{\ddot{u}_k\ddot{u}_k}(\omega) = S_0$ into Eq. (6) and Eq. (7), then we can obtain:

$$\frac{\Gamma_{N,kli}}{\omega_i^N} = \frac{\int_{-\infty}^{+\infty}\omega^N|H_i(\omega)|^2\exp(-a|\omega\tau|)\cos(\omega\tau)d\omega}{\omega_i^N\int_{-\infty}^{+\infty}|H_i(\omega)|^2 d\omega} \qquad N = 0, 2 \tag{23}$$

$$\frac{\Lambda_{N,kli}}{\omega_i^N} = \frac{\int_{-\infty}^{+\infty}\omega^N|H_i(\omega)|^2\exp(-a|\omega\tau|)\sin(\omega\tau)d\omega}{\omega_i^N\int_{-\infty}^{+\infty}|H_i(\omega)|^2 d\omega} \qquad N = 1, 3, 5 \tag{24}$$

The following is the simplified computation of $\Gamma_{0,kli}$ discussed in detail as an example. From Eq. (23), it is proved that $\Gamma_{0,kli}$ is the function of only $\omega_i\tau$. This is shown in Fig. 2, where $\Gamma_{0,kli}$ remains constant when ω_i varies, but $\omega_i\tau/2\pi$ equals to 0.4, 1.0, 1.6, 2.2, respectively.

Berrah and Kausel (1992) approximated $|H_i(\omega)|^2$ as follows:

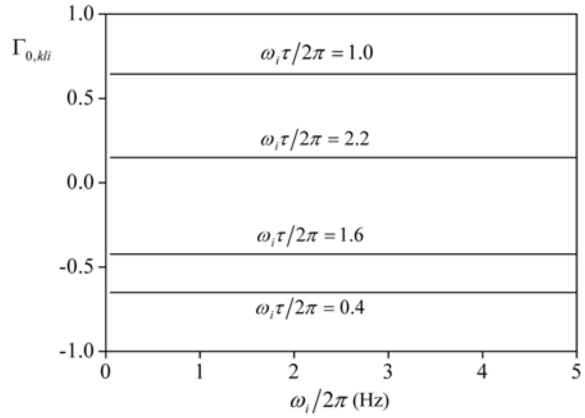

Fig. 2 $\Gamma_{0,kli}$ values when $\omega_i\tau/2\pi = 0.4$, 1.0, 1.6 and 2.2

$$|H_i(\omega)|^2 = \begin{cases} 1/\omega_i^4 & \text{for} \quad \omega \le \omega_i \\ \dfrac{1}{\omega_i^3}\left(\dfrac{\pi}{4\zeta_i} - 1\right)\delta(\omega - \omega_i) \end{cases} \tag{25}$$

with $\delta(\omega - \omega_i)$ being Dirac's delta function. With this simplification, $\Gamma_{0,kli}$ can be evaluated analytically after some algebra, then

$$\Gamma_{0,kli} = \frac{4\zeta_i}{\pi}\left\{\exp(-a\omega_i|\tau|)\left[\frac{\sin(\omega_i\tau)}{\omega_i\tau} - \frac{a\cos(\omega_i\tau)}{\omega_i|\tau|} + \left(\frac{\pi}{4\zeta_i} - 1\right)\cos(\omega_i\tau)\right] + \frac{a}{\omega_i|\tau|}\right\} \tag{26}$$

Since ζ_i is small, generally $\zeta_i < 0.1$, the above equation can be further reduced as shown below with little accuracy lost,

$$\Gamma_{0,kli} = \exp(-a\omega_i|\tau|)\cos(\omega_i\tau) \tag{27}$$

Fig. 3 shows the comparison of $\Gamma_{0,kli}$ values between Eq. (23) using numerical integration and those from Eq. (27) by the above simplification. It is clear from Fig. 3 that there are relatively significant discrepancies between $\Gamma_{0,kli}$ values by using numerical integration and Eq. (27), especially for the greater $\omega_i\tau/2\pi$. This indicates that the assumption in Eq. (25) will bring an obvious error. Nevertheless, $\Gamma_{0,kli}$ values from these two analyses have consistent trends, both varying in an attenuating cosine form. When $\omega_i\tau = n\pi$, they both reach the amplitude, though the numerical results are less than the corresponding approximated ones; and when $\omega_i\tau = (n + 1/2)\pi$, the approximated results equal zero, but the numerical results are close to zero, and lag slightly behind the approximated ones. In view of these features, we can assume that $\Gamma_{0,kli}$ values from the numerical integration have the following form:

$$\Gamma_{0,kli} = \exp(-\alpha_{\Gamma_0}\omega_i|\tau|)\cos(\omega_i\tau + \varphi_{\Gamma_0}) \tag{28}$$

Then by modifying the values of α_{Γ_0} and φ_{Γ_0}, $\Gamma_{0,kli}$ obtained from the above equation can be in

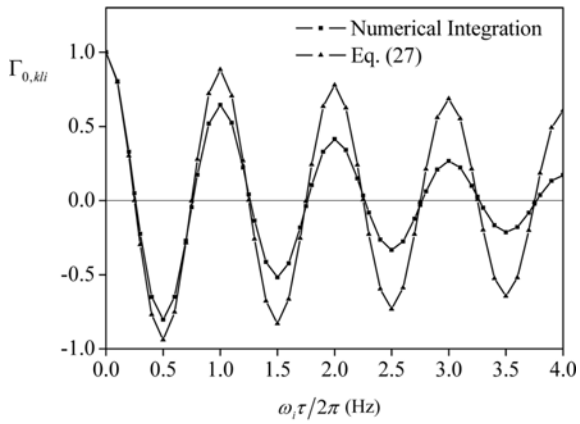

Fig. 3 Comparison of $\Gamma_{0,kli}$ values by using numerical integration and Eq. (27)

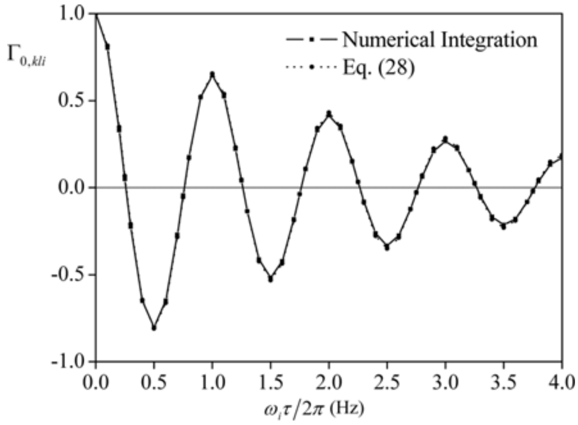

Fig. 4 Comparison of $\Gamma_{0,kli}$ values by using numerical integration and Eq. (28)

close agreement with results from the direct integration. Fig. 4 shows these two sets of results when $\alpha_{\Gamma_0} = 0.419$, $\varphi_{\Gamma_0} = -0.074$.

This simplification procedure is the same with spectral parameters $\Gamma_{2,kli}/\omega_i^2$, $\Lambda_{N,kli}/\omega_i^N$ $(N = 1, 3, 5)$. For explicitness, the approximate equations of these five spectral parameters are listed below, namely

$$\frac{\Gamma_{N,kli}}{\omega_i^N} = \exp(-\alpha_{\Gamma_N}\omega_i|\tau|)\cos(\omega_i\tau + \varphi_{\Gamma_N}) \quad N = 0, 2 \tag{29}$$

$$\frac{\Lambda_{N,kli}}{\omega_i^N} = \exp(-\alpha_{\Lambda_N}\omega_i|\tau|)\sin(\omega_i\tau + \varphi_{\Lambda_N}) \quad N = 1, 3, 5 \tag{30}$$

in which α_{Γ_N}, φ_{Γ_N}, α_{Λ_N} and φ_{Λ_N} are defined as the modification coefficients, whose values are given in Table 1. In the above analysis, the damping ratio ζ_i is 0.05, but for flexible structures, such as a long-span bridge, ζ_i is generally taken as 0.02 or 0.03, so the modification coefficients for the damping ratios 0.02 and 0.03 are also listed in Table 1. From the table, it is found that for the same ζ_i these five spectral parameters have the same modification coefficients α. In Fig. 5, $\Gamma_{2,kli}/\omega_i^2$ and $\Lambda_{N,kli}/\omega_i^N$ $(N = 1, 3, 5)$ values from the direct integration are compared with those from Eq. (29) or Eq. (30), respectively. It is clear from this figure that they agree very well with each other.

Table 1 Values of the modification coefficients

ζ_i	$\Gamma_{0,kli}$		$\frac{\Gamma_{2,kli}}{\omega_i^2}$		$\frac{\Lambda_{1,kli}}{\omega_i}$		$\frac{\Lambda_{3,kli}}{\omega_i^3}$		$\frac{\Lambda_{5,kli}}{\omega_i^5}$	
	α_{Γ_0}	φ_{Γ_0}	α_{Γ_2}	φ_{Γ_2}	α_{Λ_1}	φ_{Λ_1}	α_{Λ_3}	φ_{Λ_3}	α_{Λ_5}	φ_{Λ_5}
2%	0.260	0.000	0.260	0.000	0.260	0.000	0.260	0.060	0.260	0.100
3%	0.325	-0.045	0.325	0.000	0.325	0.000	0.325	0.045	0.325	0.100
5%	0.419	-0.074	0.419	0.060	0.419	0.000	0.419	0.074	0.419	0.165

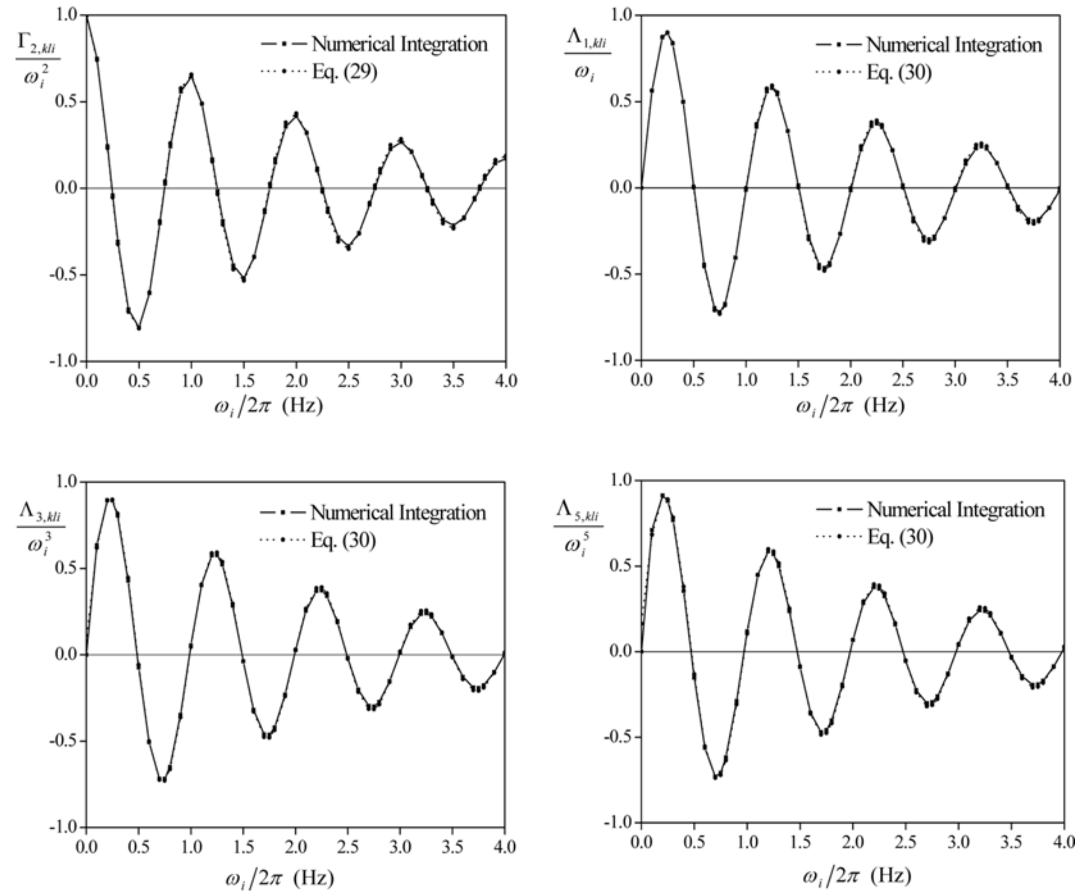

Fig. 5 Comparison of $\frac{\Gamma_{2,kli}}{\omega_i^2}$, $\frac{\Lambda_{N,kli}}{\omega_i^N}$ (N = 1, 3, 5) values by using numerical integration and Eq. (29) or Eq. (30)

3.3 Simplified procedure for $\rho_{u_k u_l}$

Considering that the site conditions underneath the supports of the structure are the same and substituting Eq. (21) into Eq. (13), then we can obtain:

$$\rho_{u_k u_l} = \frac{\int_{-\infty}^{+\infty} \exp(-\alpha\omega|\tau|)\cos(\omega\tau) S_{u_k u_k}(\omega) d\omega}{\int_{-\infty}^{+\infty} S_{u_k u_k}(\omega) d\omega} \tag{31}$$

At this time, the white noise approximation makes the above integral have infinite value at $\omega = 0$ (because $S_{u_k u_k}(\omega) = S_{\ddot{u}_k \ddot{u}_k}(\omega)/\omega^4$). Now $S_{u_k u_k}(\omega)$ takes values from the Eq. (22) and $S_{u_k u_k}(\omega) = S_{\ddot{u}_k \ddot{u}_k}(\omega)/\omega^4$. Fig. 6 shows plots of the cross-correlation coefficient for four types of soil conditions specified in the Chinese design code for transportation engineering. It is concluded that $\rho_{u_k u_l}$ is a monotone decreasing function of τ, decreasing sharply when τ is small ($\tau < 2.0$ sec) and then gently. According to this feature, $\rho_{u_k u_l}$ can be approximated easily by regression analysis in the following form:

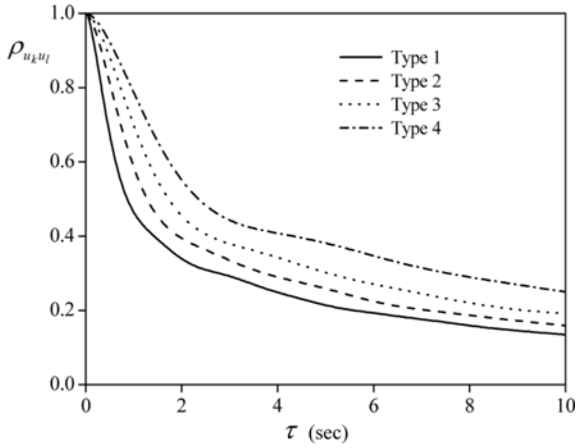

Fig. 6 $\rho_{u_k u_l}$ for sites with four types of soil conditions

Table 2 Values of k_1, k_2 and τ_1

Soil condition	τ_1	k_1	k_2
1	0.60	0.66	0.65
2	1.20	0.40	0.60
3	1.90	0.28	0.55
4	2.30	0.22	0.45

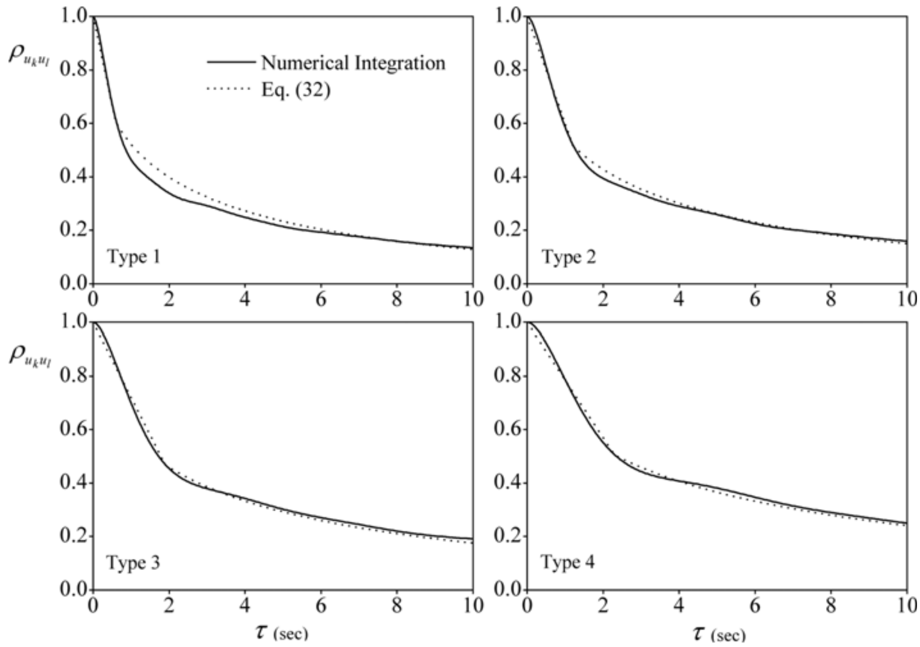

Fig. 7 Comparison of $\rho_{u_k u_l}$ values by using numerical integration and Eq. (32)

$$\rho_{u_k u_l} = \begin{cases} 1 - k_1|\tau| & \tau \le \tau_1 \\ \exp(-k_2\sqrt{|\tau|}) & \tau > \tau_1 \end{cases} \tag{32}$$

where k_1, k_2 and τ_1 have different values for different types of soil conditions (Table 2). Fig. 7 shows the comparison between $\rho_{u_k u_l}$ values for four types of soil conditions through numerical integration and those from Eq. (32). It can be seen from this figure that two sets of results match very well.

4. Multi-support ground motion simulation considering variability of response spectra

In order to use Monte Carlo simulation to demonstrate the proposed procedure reasonably, a prerequisite is that response spectra associated with the input excitations should be the random ones. This means that the stochastic process characteristic should be reflected. Their specified mean is the mean response spectrum in code for design. On the other hand, the coefficient of variation of the response spectrum could be derived from the random vibration theory. Let $y(t)$ denote the response of the single DOF system, and y_m denote the peak responses, then from the Eq. (14), Eq. (15) and Eq. (16) (it needs to change the subscript z into y in this case), the c.o.v. δ_{y_m} can be expressed as:

$$\delta_{y_m} = \frac{\sigma_{y_m}}{\mu_{y_m}} = \frac{\pi}{\sqrt{6}} \frac{1}{0.5772 + 2\ln \nu T_d} \tag{33}$$

To satisfy the above prerequisite, an approach is proposed to simulation of ground motions, which can take into accounts both mean and variation properties of response spectrum. It has the following basic idea: first, we randomly generate an ensemble of response spectra and make their mean and coefficient of variation close to the specified results; then taking these random spectra as target spectrum in turn, we use the method for multi-support ground motion simulation (e.g. Tseng *et al.* 1993) to synthesize the corresponding ground motions, which are compatible with these random spectra respectively. From this point, the key to this approach is how to generate these random spectra. For this purpose, the probabilistic distribution of the response spectrum should be first given out. On disregarding the dependence between the crossings of the response process, the distribution of the peak absolute response has the following form:

$$F_y(y_m) = \exp\left[-\nu T_d \exp\left(-\frac{y_m^2}{2\sigma_y^2}\right)\right] \tag{34}$$

Correspondingly, the mean and standard deviation have the relations like Eq. (14), Eq. (15) and Eq. (16), but it needs to change the subscript z into y in this case.

By solving the inverse function of Eq. (34), we obtain

$$y_m = \sigma_y \cdot \{2\ln \nu T_d - 2\ln[\ln F_y^{-1}(y_m)]\}^{\frac{1}{2}} \tag{35}$$

And substituting the relation $\sigma_y = \mu_{y_m}/p$ [from Eq. (14)] into the above equation,

$$y_m = \frac{\mu_{y_m}}{p} \cdot \{2\ln \nu T_d - 2\ln[\ln F_y^{-1}(y_m)]\}^{\frac{1}{2}} \tag{36}$$

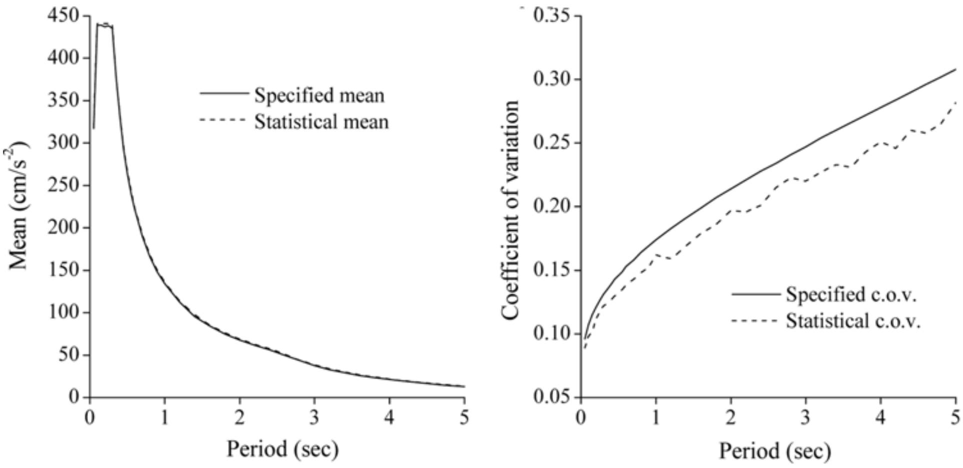

Fig. 8 Comparison between the statistical mean and c.o.v. of 2000 random spectra and the specified ones

According to Eq. (36), random response spectra can be determined as below: first prescribe some natural periods, and generate the random number ς_i corresponding to these periods respectively, and make these random numbers belong to different streams (namely, make them independent), and let ς_i equal to $F_y(y_m)$ one by one, and then obtain the corresponding y_m from Eq. (36), finally connect these sampled y_m in succession so as to generate a random spectrum. Theoretically, an ensemble of random spectra, given by the above procedure, should have the mean and coefficient of variation equal to the specified μ_{y_m} and δ_{y_m}.

Fig. 8 shows the comparison between the statistical mean and coefficient of variation of 2000 random spectra and the specified ones, in which μ_{y_m} is the standard response spectrum (mean response spectrum) relative to $\ddot{u}_{k,\max} = 0.2g$ and the soil site of type 2; δ_{y_m} is given by Eq. (33), T_d is 25s and $\nu = \omega_i / \pi$. It is clear from this figure that the statistical mean of 2000 random spectra is in a very close agreement with the specified one, but the statistical coefficient of variation is clearly lower than the given results. This indicates that the variance of random variable with the distribution shown as Eq. (34) has great error with that computed approximately by Eq. (33), that is, the variance peak factor given by Davenport (1964) could bring a significant error. In this sense, the Eq. (16) should be modified so as to reduce the error between the statistical c.o.v. and the specified one. Introducing a modification coefficient β into Eq. (16), then:

$$q = \frac{\pi}{\sqrt{6}} \frac{\beta}{\sqrt{2 \ln \nu T_d}} \tag{37}$$

Correspondingly, Eq. (33) becomes:

$$\delta_{y_m} = \frac{\sigma_{y_m}}{\mu_{y_m}} = \frac{\pi}{\sqrt{6}} \frac{\beta}{0.5772 + 2 \ln \nu T_d} \tag{38}$$

With the regression analyses for the three cases of T_d = 15s, 20 and 25s, it is found that the statistical coefficients of variation match the computed ones very well when β equals 0.893, 0.900 and 0.908, respectively. For convenience, we assume that β = 0.900. Fig. 9 shows the comparison between the statistical c.o.v of 2000 generated random spectra and the specified ones after modification. It is clear that after modification they become close to a great extent.

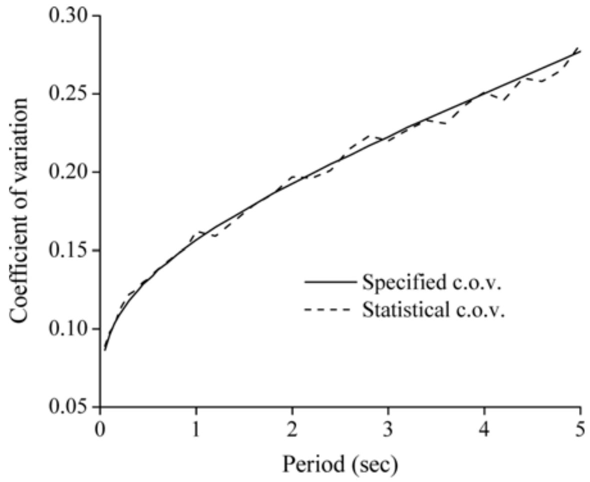

Fig. 9 Comparison between the statistical c.o.v. of 2000 random spectra and the specified ones after modification

5. Numerical example

The proposed procedure is demonstrated for two numerical examples: (1) two-span continuous beam; (2) two-tower cabled-stayed bridge by using Monte Carlo simulation (MC). In the Monte Carlo simulations, the multi-support ground motions used here are obtained by using the simulation method mentioned above. Meanwhile, it is also compared with the multiple-support response spectrum (MSRS) by Der Kiureghian and Neuenhofer (1992). The mean response spectrum in the analysis is the standard one, relative to $\ddot{u}_{k,\max} = 0.2g$ and the soil site of type 2, prescribed in Chinese design code for transportation engineering.

5.1 Two-span continuous beam

Consider a two-span continuous beam, which has been discussed in Der Kiureghian and Neuenhofer (1992), with uniform mass and stiffness properties and simple supports as shown in Fig. 10. In the study, $EI/m = 2.53 \times 10^6$ m^4/s^2, where EI denotes the flexural rigidity and m denotes the mass per unit length of the beam, in order that its fundamental frequency is 1 Hz. 200 sets of vertical multi-support ground motions are used, with a duration of 20s and v_s = 500 m/s.

Comparisons of the maximum displacement, moment along the two-span beam analyzed by the proposed method, MSRS and MC simulation are shown in Fig. 11 and Fig. 12, respectively. From both figures, it can be observed that the mean and standard deviation of the peak response from

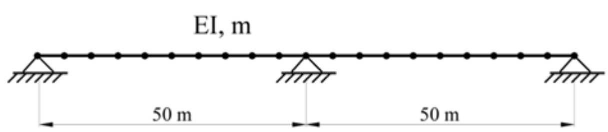

Fig. 10 Two-span continuous beam

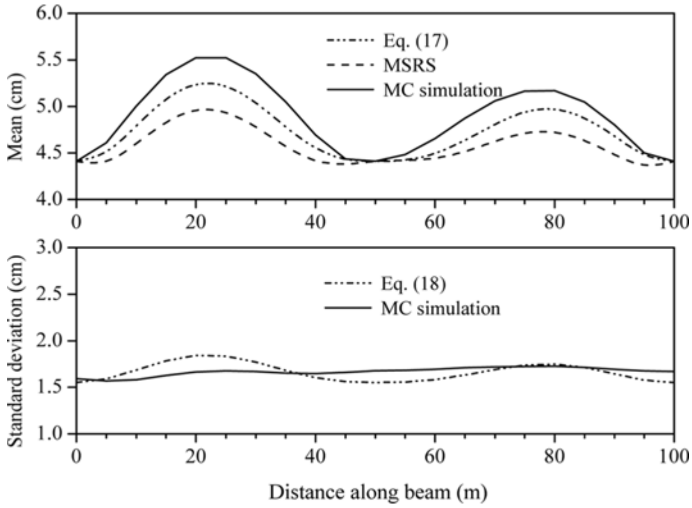

Fig. 11 Comparison of responses of maximum displacement

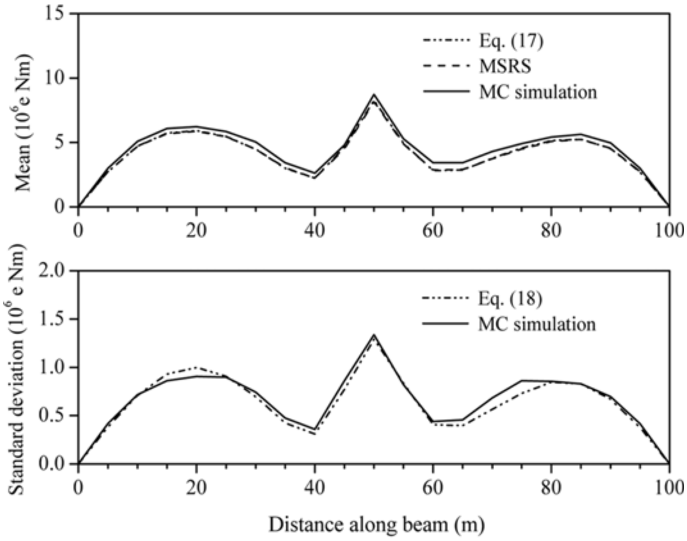

Fig. 12 Comparison of responses of maximum moment

Eq. (17) and Eq. (18) are in close agreement with simulation results. The results from the proposed method and MSRS also match well. However, the proposed method makes it possible a dramatic reduction of the computational time. For example, for every displacement response, moment response, MSRS averagely takes 37.3s, 103.7s (not including time consumed for modal analysis), respectively, but the proposed method just takes less than 1s.

5.2 Two-tower cable-stayed bridge

As shown in Fig. 13, a two-tower cable-stayed bridge has 6 support locations, two towers as high

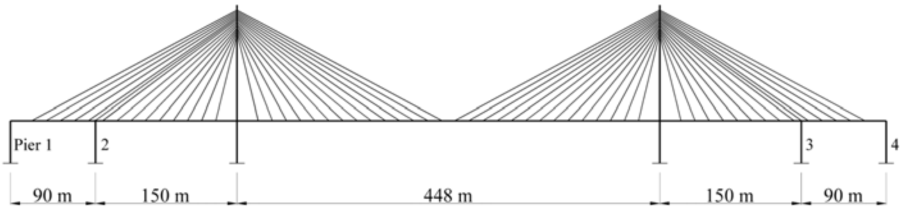

Fig. 13 Two-tower cable-stayed bridge

as 172.10 m and a span of 928 m. Since only its longitudinal response is evaluated, the 2D finite-element model is developed for the analysis. The model has 316 elements connecting 201 nodes (including 6 nodes at the supports) with a total 585 active degrees of freedom. Allowing for the unavailability of response spectra for damping values other than 5%, and extra errors arising if these spectra are modified from the response spectrum for damping value 5%, a damping ratio of 0.05 is used for all modes. The first five modes are 0.215, 0.352, 0.474, 0.675 and 0.804 Hz, respectively. The seismic inputs are longitudinal from the left to the right, with a duration of 25s and v_s = 500 m/s. They have the total number of 1200, divided into 200 sets (a set including 6 inputs at 6 supports).

Table 3 Summary of results for cable-stayed bridge

Response	$\mu_{z_{max}}$					$\sigma_{z_{max}}$		
	MC	MSRS	Eq. (17)	MSRS/MC	Eq. (17)/MC	MC	Eq. (18)	Eq. (18)/MC
Longitudinal displacement at mid-span of bridge deck (cm)	6.689	5.893	7.196	0.88	1.08	1.596	1.798	1.13
Vertical displacement at mid-span of bridge deck (cm)	5.000	4.614	4.331	0.92	0.87	0.793	0.797	1.01
Moment at mid-span of bridge deck (10^6×Nm)	9.730	9.462	9.321	0.97	0.96	1.575	1.581	1.00
Longitudinal displacement at the top of left tower (cm)	8.096	6.711	8.054	0.83	0.99	1.632	1.563	0.96
Longitudinal displacement at the top of right tower (cm)	7.731	6.574	7.925	0.85	1.03	1.608	1.535	0.95
Moment at the bottom of right tower (10^8×Nm)	10.390	9.469	9.761	0.91	0.94	1.305	1.255	0.96
Moment at the bottom of pier 3 (10^8×Nm)	3.632	3.447	3.421	0.95	0.94	0.496	0.455	0.92
Moment at the bottom of pier 4 (10^8×Nm)	4.894	4.602	4.527	0.94	0.93	0.617	0.604	0.98
Shear at the bottom of right tower (10^7×N)	57.310	52.240	53.850	0.91	0.94	7.200	6.927	0.96
Shear at the bottom of pier 3 (10^7×N)	16.960	16.100	15.980	0.95	0.94	2.320	2.128	0.92
Shear at the bottom of pier 4 (10^7×N)	22.860	21.500	21.150	0.94	0.93	2.885	2.822	0.98

Table 3 shows the comparison between the means and standard deviations of peak responses obtained using the proposed method, MSRS and MC simulation. From this table, we can reach the similar conclusion as in the first example, that is, the proposed method can provide enough accuracy. Furthermore, for analyzing the large-sized structure like this cable-stayed bridge, the proposed method has shown more obvious computational efficiency. For example, for every displacement response, moment response and shear response, MSRS averagely takes 460.3s, 872.5s, 726.7s (not including time consumed for modal analysis), respectively, but the proposed method just takes less than 1s.

6. Conclusions

From the results in this study, the following main conclusions can be derived:

(1) A response spectrum method is developed for seismic response analysis of linear, multi-degree-of-freedom systems subjected to multi-support excitations. Modal combination rules are derived for the mean and standard deviation of the peak response and the mean frequency of the response. With these response quantities, the reliability analysis of multi-support structures based on response spectrum theory becomes possible.

(2) Simplified procedures for computation of spectral parameters $\Gamma_{N,kli}/\omega_i^N$ ($N = 0, 2$), $\Lambda_{N,kli}/\omega_i^N$ ($N = 1, 3, 5$) and the cross-correlation coefficient $\rho_{u_k u_l}$ are proposed to avoid the tedious numerical integration. After comparisons, it is found that the results based on the simplified procedures closely agree with those from numerical integration.

(3) In order to validate the proposed method, the Monte Carlo simulation technique is adopted. For this purpose, this paper also presents an approach to simulation of ground motions, which can takes into account both mean and variation properties of response spectra.

(4) Two example structures, one two-span continuous beam and one two-tower cable-stayed bridge, are considered to demonstrate the proposed response spectrum method. As shown, the results based on the proposed method are in close agreement with Monte Carlo simulation results. And compared to the MSRS, the proposed method can reduce the computational time dramatically. These indicate that the multi-support response spectrum developed in this paper can provide enough accuracy and efficiency for the engineering applications.

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References

Abdel-Ghaffar, A.M. and Rubin, L.I. (1982), "Suspension bridge response to multiple-support excitations", *J. Eng. Mech.*, **108**, 419-435.

Alkhaleefi, A.M. and Ali, A. (2002), "An efficient multi-point support-motion random vibration analysis

technique", *Comput. Struct.*, **80**, 1689-1697.
Allam, S.M. and Datta, T.K. (2000), "Analysis of cable-stayed bridges under multi-component random ground motion by response spectrum method", *Eng. Struct.*, **22**, 1367-1377.
Berrah, M. and Kausel, E. (1992), "Response spectrum analysis of structures subjected to spatially varying motions", *EESD*, **21**, 461-470.
Berrah, M. and Kausel, E. (1993), "A modal combination rule for spatially varying seismic motions", *EESD*, **22**, 791-800.
Clough, R.W. and Penzien, J. (1975), *Dynamics of Structures*, McGraw-Hill Book Co., New York.
Davenport, A.G. (1964), "Note on the distribution of the largest value of a random function with application to gust loading", *Proc. Institution of Civil Engineers*, **28**, 187-196, London.
Der Kiureghian, A. (1980), "Structural response to stationary excitation", *J. Eng. Mech.*, **106**, 1195-1213.
Der Kiureghian, A. and Neuenhofer, A. (1992), "Response spectrum method for multi-support seismic excitations", *EESD*, **21**, 713-740.
Dumanoglu, A.A. and Severn, R.T. (1990), "Stochastic response of suspension bridges to earthquake forces", *EESD*, **19**, 133-152.
Dusseau, R.A. and Wen, Y.K. (1989), "Seismic response of deck-type arch bridges", *EESD*, **18**, 701-715.
Hao, H., Olivera, C.S. and Penzien, J. (1989), "Multiple-station ground motion processing and simulation based on SMART-1 array data", *Nucl. Eng. Design*, **111**, 293-310.
Hao, H. (1993), "Arch response to correlated multiple excitations", *EESD*, **22**, 389-404.
Hao, H. (1994), "Ground-motion spatial variation effects on circular arch responses", *J. Eng. Mech.*, **120**(11), 2326-2341.
Harichandran, R.S. and Vanmarcke, E.H. (1986), "Stochastic variation of earthquake motion in space and time", *J. Eng. Mech.*, **112**(2), 154-174.
Harichandran, R.S., Hawwari, A. and Sweidan, B.N. (1996), "Response of long-span bridges to spatially varying ground motion", *J. Struct. Eng.*, **122**(5), 476-484.
Harichandran, R.S. (1999), *Spatial Variation of Earthquake Ground Motion*, Michigan State University, East Lansing, MI.
Heredia-Zavoni, E. and Vanmarcke, E.H. (1994), "Seismic random-vibration analysis of multisupport-structural systems", *J. Eng. Mech.*, **120**(5), 1107-1128.
Hyun, C-H., Yun, C.-B. and Lee, D.-G. (1992), "Nonstationary response analysis of suspension bridges for multiple support excitations", *Probabilistic Engrg. Mech.*, **7**, 27-35.
Lin, J.H., Li, J.J., Zhang, W.S. and Williams, F.W. (1997), "Non-stationary random seismic responsed of multi-support structures in evolutionary inhomogeneous random fields", *EESD*, **26**, 135-145.
Loh, C.H. and Yeh, Y.T. (1988), "Spatial variation of stochastic modeling of seismic different ground motion", *EESD*, **16**, 583-596.
Loh, C.H. and Ku, B.D. (1995), "An efficient analysis of structural response for multiple-support seismic excitations", *Eng. Struct.*, **17**(1), 15-26.
Nakmura, Y., Der Kiureghian, A. and Liu, D. (1993), "Multiple-support response spectrum analysis of the Golden Gate Bridge", Report No. UCB/EERC-93/05, EERC, University of California, Berkeley, CA, 1993.
Nazmy, A.S. and Abdel-Ghaffar, A.M. (1992), "Effects of ground motion spatial variability on the response of cable-stayed bridges", *EESD*, **21**, 1-20.
Nazmy, A.S. and Konidaris, E.G. (1994), "Nonlinear seismic behavior of steel deck-type arch bridges", *Proc. of Fifth U.S. National Conf. on Earthquake Engineering*, Vol. **1**, EERI, Oakland, CA, 367-376.
Price, T.E. and Eberhard, M.O. (1998), "Effects of spatially varying ground motions on short bridges", *J. Struct. Eng.*, **124**(8), 948-955.
Soyluk, K. and Dumanoglu, A.A. (2000), "Comparison of asynchronous and stochastic dynamic response of a cable-stayed bridge", *Eng. Struct.*, **22**, 435-445.
Soyluk, K., Dumanoglu, A.A. and Tuna, M.E. (2004), "Random vibration and deterministic analysis of cable-stayed bridge to asynchronous ground motion", *Struct. Eng. Mech.*, **18**(2), 231-246.
Tseng, W.S., Lilhanand, K. and Yang, M.S. (1993), "Generation of multiple-station response-spectrum-and-coherency-compatible earthquake ground motions for engineering applications", SmiRT-12, K01/3, 13-18.
Vanmarcke, E.H. (1972), "Properties of spectral moments with applications to random vibration", *J. Eng. Mech.*,

98, 425-446.

Wang, J.J. and Fan, L.C. (1998), "Modification of code response spectrum in long-period phase", *Chinese J. of Civil Eng.*, **31**(6), 49-55.

Zanardo, G., Hao, H. and Modena, C. (2002), "Seismic response of multi-span simply supported bridges to a spatially varying earthquake ground motion", *EESD*, **31**, 1325-1345.

Zerva, A. (1990), "Response of multi-span beams to spatially incoherent seismic ground motions", *EESD*, **19**, 819-832.