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Serviceability reliability analysis of cable-stayed bridges

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Abstract. A reliability analysis method is proposed in this paper through a combination of the advantages of the response surface method (RSM), finite element method (FEM), first order reliability method (FORM) and the importance sampling updating method. The accuracy and efficiency of the method is demonstrated through several numerical examples. Then the method is used to estimate the serviceability reliability of cable-stayed bridges. Effects of geometric nonlinearity, randomness in loading, material, and geometry are considered. The example cable-stayed bridge is the Second Nanjing Bridge with a main span length of 628 m built in China. The results show that the cable sag that is part of the geometric nonlinearities of cable-stayed bridges has a major effect on the reliability of cable-stayed bridge. Finally, the most influential random variables on the reliability of cable-stayed bridges are identified by using a sensitivity analysis.

Key words: reliability analysis; failure probability; response surface method (RSM); importance sampling; serviceability limit state; Monte Carlo simulation (MCS); cable-stayed bridges; geometric non-linearity.

1. Introduction

Cable-stayed bridges have been used extensively in the construction of long span bridges in recent years. The increasing popularity of cable-stayed bridges among bridge engineers can be attributed to: (1) the appealing aesthetics; (2) the full and efficient utilization of structural materials; (3) the increased stiffness over suspension bridges; (4) the efficient and fast mode of construction; and (5) the relatively small size of the bridge elements (Ren 1999).

The static behavior of cable-stayed bridges has been studied by many researchers, including Fleming (1979), Nakai *et al.* (1985), Hegab (1986), Aboul-ella (1988), Nazmy and Abdel-Ghaffar (1990), Self and Dilger (1990) and Ren (1999). These studies were based on the assumption of complete determinacy of structural parameters. This is usually referred to as deterministic analysis. In reality, however, there are uncertainties in design variables. These uncertainties include geometric properties (cross-sectional properties and dimensions), material mechanical properties (modulus and strength, etc.), load magnitude and distribution, etc. Thus the deterministic analysis cannot provide complete information regarding static behavior of cable-stayed bridges. Therefore, the static behavior of cable-stayed bridges must be studied under a probabilistic viewpoint.

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Reliability analysis provides the tool of incorporating structural modeling uncertainties in the analysis of the structural response by describing the uncertainties as random variables. Liu and Der Kiureghian (1991) investigated the reliability of a square plate with a hole of random geometry in its center by a finite element reliability method for geometrically nonlinear structures under static loads that employs the first- and second-order reliability methods, namely, FORM and SORM. Imal and Frangopol (2001, 2002) extended the finite element reliability method to investigate the reliability of suspension bridges. However, studies of the reliability of cable-stayed bridges under static loads have rarely been reported. Bruneau (1992) investigated the reliability of cable-stayed bridges under static loads, and assessed the practicability of system-reliability analytical methods to assist in the design of cable-stayed bridges. Unfortunately, there are three disadvantages in his study. First, all geometric nonlinear sources in cable-stayed bridges were not considered. However, the calculation results of Nazmy and Abdel-Ghaffar (1990) had shown that the geometric nonlinearities significantly affect the static behavior of long span cable-stayed bridges. Second, in his study, the structural reliability index was obtained using the first-order second-moment method (FOSM). In FOSM methods, the information on the distribution of random variables is ignored (Haldar 2000). Third, a complete sensitivity analysis was not included in his study. Recently, Chen (2000) studied the serviceability reliability of cable-stayed bridges. The sensitivity index that indicates the influence of each of the random variables on the overall reliability analysis was not computed. Further, the study concentrated on implementation of the response surface method (RSM) in the reliability analysis of cable-stayed bridges.

Several techniques exist to perform structural reliability analyses. These techniques may be divided into three categories as: (1) FORM and SORM, (2) RSM, and (3) Monte Carlo simulation (MCS).

FORM and SORM are the earliest and most widely used methods in structural reliability analysis. Extensive reviews of these methods are found in Zhao and Ono (1999a, 2001), Rackwitz (2001). The main idea of these methods is to estimate the probability of failure using first-order or second-order approximations to the limit state at the design point. These methods require the evaluation of the derivatives of the response functions or limit state functions with respect to the random variables. When these functions are explicit functions of the random variables, it is easy to compute the derivatives of these functions. However, in many cases, particularly for complicated structures, the limit state functions are usually implicit in terms of the random variables. Therefore, derivatives of the limit state functions are not readily available. This restricts the applicability of these methods to the reliability analysis of complicated structures where the limit state surfaces are not known explicitly. Liu and Der Kiureghian (1991) proposed an improved FORM/SORM method in conjunction with a probabilistic finite element analysis to solve this problem. In this procedure, response gradients for geometrically nonlinear structures with parametric uncertainties need to be computed. Unfortunately, the existing deterministic finite element code available to design engineers cannot compute response gradients. Therefore, to use the method, it is necessary to modify the existing deterministic finite element code. Furthermore, errors in using this approach for nonflat or implicit limit state functions are not easy to determine (Guan and Melchers 1997).

RSM could be pursued for the reliability analysis of structures with implicit limit state functions. The basic idea of the method is to approximate the original complex and implicit limit state functions by simple response surface function (RSF). A major advantage of the method is that the implicit limit state functions are represented in an explicit form. Thus full advantage can be taken of existing computational methods developed for reliability analyses. Due to its advantage, the method has widely been applied in reliability analysis of structures (Bucher and Bourgund 1990, Rajashekhar and Ellingwood 1993, Huh and Haldar 2002). However, the method has the following disadvantages: (1) an arbitrary parameter in RSM, h_i, has a considerable effect on the estimated probability of failure; (2) it may not be accurate when the probability of failure is extremely small.

MCS is another method for the reliability analysis of structures with implicit limit state functions. The method uses randomly generated samples of the input variables for each deterministic analysis, records the numbers of times that failure occurs, and estimates probability of failure after numerous repetitions of the deterministic analysis. This method is robust, simple and easy to use. Therefore, the method is often used to validate other analysis techniques. However, the method has one drawback: it needs an enormously large amount of computation time. To reduce the computational cost, different variance reduction techniques such as Importance sampling (Harbitz 1983, Shinozuka 1983) and Adaptive sampling (Karamchandani *et al.* 1989) can be employed. Liu and Moses (1994) proposed an RSM-based Monte Carlo Importance Sampling (RSM-MCIS) to estimate structural reliability. The basic procedure of the method is: (1) use RSM to approximately obtain the limit state surface; (2) based on the obtained limit state surface, apply Monte Carlo Importance Sampling to evaluate structural reliability. However, when the difference between limit state surface obtained by RSM and actual limit state surface is large, RSM-MCIS estimate will produce significant errors. The weakness in RSM-MCIS is discussed in Section 3.

The purposes of this paper are to propose an efficient method to overcome the drawbacks of the previous reliability methods, to investigate the serviceability reliability of cable-stayed bridges using the proposed method, and to conduct a sensitivity analysis to ascertain the effect of parameter uncertainty on the final results.

2. Proposed method

2.1 Principle

The proposed method is a hybrid method, consisting of RSM, FEM, FORM and the importance sampling updating method. The method is based on three key concepts: (1) approximation of the limit state by RSM; (2) deterministic finite element analysis by FEM; and (3) estimation of the failure probability through a combination of FORM and importance sampling updating method.

The limit state functions may be categorized into two types: (1) explicit limit state function; and (2) implicit limit state function. If the limit state function is explicit in terms of the basic random variables, it becomes easy to conduct reliability analysis of structures. This is mainly because many available methods for reliability analysis such as FORM and SORM can be used. However, in practice limit state function is not explicitly known. In other words, the limit state function is usually implicit. To solve this problem, RSM is used. The main idea of the RSM is to approximate the implicit limit state function by simple and explicit polynomial. A second-order polynomial without cross terms is adopted here. The second-order polynomial can be represented as

$$\hat{g}(X) = b_0 + \sum_{i=1}^{k} b_i X_i + \sum_{i=1}^{k} b_{ii} X_i^2 \tag{1}$$

Where $\hat{g}(X)$ = the approximate limit state function; $X_i(i = 1, 2, \ldots, k)$ = ith random variable; and

b_0, b_i, b_{ii} = unknown coefficients to be determined by solving a set of simultaneous equations. The number of unknown coefficients in (1) is $p = 2k + 1$. Consider three values of each random variable, namely, a low value, a medium value, and a high value (e.g., $X_i^C - h_i\sigma_{X_i}$, X_i^C, $X_i^C + h_i\sigma_{X_i}$), where the value of h_i is considered to be 3.0 for the first iteration and 0.99 for the subsequent iterations; X_i^C and σ_{X_i} = coordinates of the center point and standard deviation of a random variable X_i. The initial center point can be the mean value point. For determining the location of the center point, the iterative linear interpolation scheme suggested by Bucher and Bourgund (1990) is used in this study. A detailed description of the RSM can be obtained from Haldar and Mahadevan (2000). It should be pointed out that the approximate limit state function in Eq. (1) is only determined one (first) time in the analytical procedures of the proposed method. In other words, the iterative linear interpolation scheme used in the traditional RSM is unnecessary for the proposed method. This is because the importance sampling updating technique is introduced in the proposed method. The importance sampling updating technique is described later.

The use of the proposed method may involve in deterministic finite element analysis. FEM is considered to be the most reliable analysis method. In this paper, the primary purpose of applying FEM is to extend the proposed method to implement finite element reliability analysis. For more details concerning the FEM used in this paper, the reader is referred to Cheng (2000).

FORM and SORM can be used to estimate the failure probability. As indicated in Cambier *et al.* (2002), these methods have a significant disadvantage: the accuracy of the results is very difficult to be validated. To circumvent the disadvantage of these methods, Hohenbichler and Rackwitz (1988) developed an importance sampling updating method to improve the SORM estimate. In the method proposed in this paper, the importance sampling updating method is extended to improve the FORM estimate. For completeness, the importance sampling updating method is briefly described. A more detailed description may be obtained from Hohenbichler and Rackwitz (1988).

The probability of failure through sampling of a correction factor to the SORM second order estimate can be written as Cambier *et al.* (2002)

$$P_f = \Phi(-\beta)\prod_{i=1}^{n-1}\left(1 + \frac{\phi(-\beta)}{\Phi(-\beta)}\kappa_i\right)^{-1/2} E[Z(v)] \tag{2}$$

$$Z(v) = \frac{\Phi(-f(v))}{\Phi(-\beta)}\exp\left(\frac{1}{2}\frac{\phi(-\beta)}{\Phi(-\beta)}\sum_{i=1}^{n-1}\kappa_i(v_i)^2\right) \tag{3}$$

where Φ = the standard normal cumulative probability; ϕ = the standard normal density; κ_i = the main curvatures of the limit state function at the design point u^*; β = the first-order reliability index; $E[Z(v)]$ = the correction factor that is determined by simulation; v = the independent normal vector with means $E[v_i] = 0$, variances $Var[v_i] = \left(1 + \frac{\phi(-\beta)}{\Phi(-\beta)}\kappa_i\right)^{-1}$ and such that $v^T \cdot u^* = 0$; $f(v)$ = the root of $g(v, f(v)) = 0$; and g = the actual limit state function in the standard space.

Let $\kappa_i = 0$ in Eqs. (2) and (3), the probability of failure through sampling of a correction factor to the FORM first order estimate is given by

$$P_f = \Phi(-\beta)E[Z(v)] \tag{4}$$

$$Z(v) = \frac{\Phi(-f(v))}{\Phi(-\beta)} \tag{5}$$

where v = the independent normal vector with means $E[v_i] = 0$ and variances $Var[v_i] = 1$.

The unique feature of the proposed method is the combination of the advantages of RSM, FEM, FORM and the importance sampling updating method. It should be noted that the proposed method is different from the RSM-based Monte Carlo Importance Sampling method (RSM-MCIS) presented by Liu and Moses (1994). As stated before, Monte Carlo Importance Sampling in RSM-MCIS is based on the approximate limit state surface obtained by using RSM. Therefore, when the noise is added in the approximate limit state function, RSM-MCIS will produce significant errors as will be shown later. This problem can be resolved by the proposed method because the importance sampling updating technique in the proposed method is based on the actual limit state surface. The efficiency and accuracy of the proposed method are verified in the subsequent section.

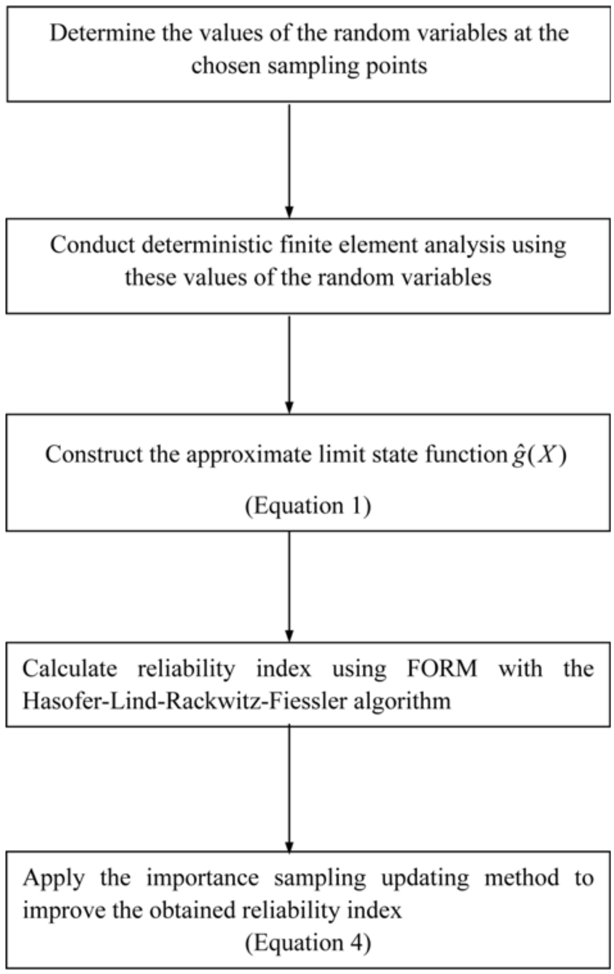


Fig. 1 Flow chart for the procedures the proposed method

2.2 Procedure for the proposed method

The procedures of the proposed method are:

(1) Determine the values of the random variables at the chosen sampling points. For problems involving k variables, $(2k+1)$ samples described above are chosen in this paper.

(2) Conduct a deterministic finite element analysis using these values of the random variables.

(3) Use the response surface method (RSM) to construct the approximate limit state function $\hat{g}(X)$.

(4) After the approximated limit state function $\hat{g}(X)$ is determined, the FORM with the Hasofer-Lind-Rackwitz-Fiessler algorithm is applied to obtain the reliability index.

(5) Apply the importance sampling updating method to improve the obtained reliability index.

A flow chart for the above procedures is given in Fig. 1.

3. Verification examples and investigations

The main objective here is to investigate the computation efficiency and accuracy of the proposed method for reliability analysis. Five examples are presented in this section. The first example considers linear limit state function. The second example considers general parabolic limit state function. This example demonstrates the accuracy and efficiency of the proposed method for a problem with a large number of random variables and large curvatures. The third and the fourth examples are given to demonstrate the application where the limit state function is not available in closed form and finite element analysis is required to compute $\hat{g}(X)$. The last example is considered to demonstrate the applicability of the proposed method in geometrically nonlinear finite element reliability analysis of structures.

For the purpose of these investigations, the proposed method was implemented in a reliability analysis program. The program was developed using the FORTRAN77/90 computer language. For comparison, other reliability analysis methods such as FORM, SORM, RSM, MCS, RSM-MCIS were also implemented in the computer program. The program NASAB (Cheng 2003) was used for the deterministic finite element analysis.

In the following reliability analyses, the computation time is referred to that of a Pentium III, 800 MHz PC. In the following tables, the figures in the parentheses refer to the number of the simulations.

3.1 Example 1: Linear limit state function (explicit limit state function)

The example is taken from Der Kiureghian *et al.* (1987). The limit state function is

$$G(x) = x_1 + 2x_2 + 2x_3 + x_4 - 5x_5 - 5x_6 \tag{6}$$

The statistics of the six random variables in this limit state function are listed in Table 1. The results are shown in Table 2. Fig. 2 illustrates the effectiveness of the different methods (MCS, RSM-MCIS and the proposed method). It shows the coefficient of variation of the probability of failure versus the number of Monte Carlo Simulations. The exact results are obtained by using MCS with 100,000 samples.

Table 1 Statistics of the random variables for Example 1

Variable	Mean	Standard deviation	Distribution
x_1	120	12	Lognormal
x_2	120	12	Lognormal
x_3	120	12	Lognormal
x_4	120	12	Lognormal
x_5	50	15	Lognormal
x_6	40	12	Lognormal

Table 2 Results of Example 1

Method	β	p_f	Number of the simulations
FORM	2.348	9.433e-3	-
SORM	2.271	1.157e-2	-
RSM	2.348	9.433e-3	-
MCS (exact results)	2.251	1.220e-2	100,000
RSM-MCIS	2.249	1.225e-2	57,300
Proposed method	2.252	1.215e-2	30,000

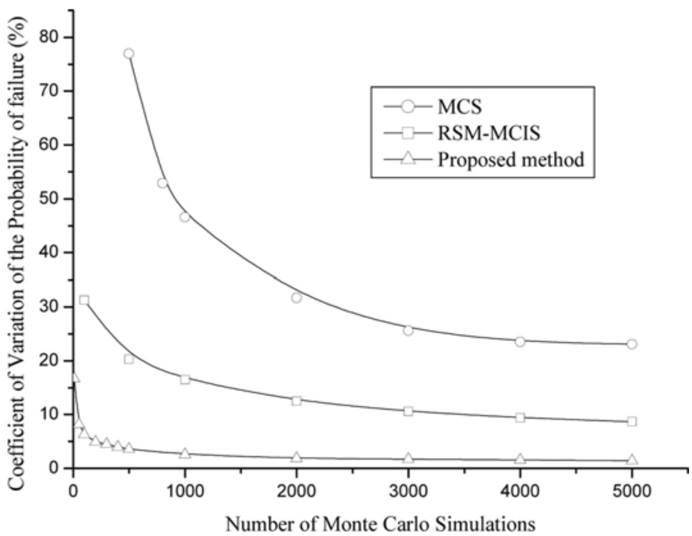


Fig. 2 Coefficient of variation of the probability of failure for Example 1

From Table 2, it can be seen that: (1) FORM and RSM give the same results. This indicates that the actual limit state function is very closely approximated by a second-order polynomial without cross terms; (2) Both the proposed method and RSM-MCIS achieve excellent accuracy, whereas the FORM, SORM and RSM estimates are quite approximate for this example; (3) The proposed method requires fewer samples than both MCS and RSM-MCIS for comparable accuracy. Thus the

CPU time spent by the proposed method is less than the time required by the MCS and RSM-MCIS.

It is seen from Fig. 2 that the proposed method converges very quickly, since with 1,000 samples, the coefficient of variation of the probability of failure is less than 3%.

3.2 Example 2: General parabolic limit state function (explicit limit state function)

Considering the following limit state function in standardized space, which has been used in Der Kiureghian and Lin (1987) and Zhao and Ono (1999b).

$$G(x) = \beta_F - u_n + \frac{1}{2}\sum_{j=1}^{n-1} jau_j^2 \tag{7}$$

Here u is standard normal variables, β_F is taken to be 2.0, and a is taken to be 0.03. A range of n values between 5 and 30 is considered. The results for different values of n are listed in Table 3. The exact values for $n = 5$, 10, 15 and 20 are obtained using MCS with 10,000,000 samples. The exact values for $n = 30$ are obtained from Cambier *et al.* (2002). This is because that MCS is not appropriate for a value of $n = 30$ (Cambier *et al.* 2002). From Table 3 one can see that when the number of random variables is large (e.g. $n = 30$), the errors in these methods (FORM, SORM, RSM and RSM-MCIS) become very large. However, there is very good agreement between the results obtained using the proposed method and the exact results with any number of random variables, n. Furthermore, it is noteworthy that the RSM failed to converge to a solution in the case of $n = 30$. This is because that there are slight differences in the approximate limit state function obtained by RSM and the actual limit function, which are shown in Table 4. Note that Table 4 gives

Table 3 Results of Example 2

Method		Parameter				
		$n = 5$	$n = 10$	$n = 15$	$n = 20$	$n = 30$
FORM	β	2.000	2.000	2.000	2.000	2.000
	p_f	2.275e-2	2.275e-2	2.275e-2	2.275e-2	2.275e-2
SORM	β	2.114	2.446	2.900	3.412	4.49
	p_f	1.726e-2	7.216e-2	1.866e-2	3.224e-4	3.550e-6
RSM	β	2.000	2.000	2.000	2.000	not converged
	p_f	2.275e-2	2.275e-2	2.275e-2	2.275e-2	not converged
RSM-MCIS	β	2.133	2.558	3.135	3.803	5.402
	p_f	1.645e-2 (10,000)	5.265e-3 (10,000)	8.604e-4 (10,000)	7.153e-5 (10,000)	3.301e-8 (100,000)
Proposed method	β	2.136	2.551	3.135	3.820	5.266
	p_f	1.634e-2 (10,000)	5.374e-3 (10,000)	8.593e-4 (10,000)	6.672e-5 (10,000)	6.971e-8 (100,000)
Exact results	β	2.136	2.551	3.139	3.809	5.264
	p_f	1.634e-2	5.366e-3	8.482e-4	6.990e-5	7.060e-8

Table 4 Comparison of coefficients between fitted and exact values

Coefficients (Eq. 1)	Fitted values (Eq. 7)	Exact values (Eq. 7)
b_0	2.000	2.000
b_1~b_{29}	0.000	0.000
b_{30}	−1.000	−1.000
b_{0101}	1.499999966472387e-002	1.500e-2
b_{0202}	2.999999932944775e-002	3.000e-2
b_{0303}	4.499999899417162e-002	4.500e-2
b_{0404}	5.999999865889549e-002	6.000e-2
b_{0505}	7.499999832361937e-002	7.500e-2
b_{0606}	8.999999798834324e-002	9.000e-2
b_{0707}	0.104999997653067	0.105
b_{0808}	0.119999997317791	0.120
b_{0909}	0.134999996982515	0.135
b_{1010}	0.149999996647239	0.150
b_{1111}	0.164999996311963	0.165
b_{1212}	0.179999995976686	0.180
b_{1313}	0.194999995641410	0.195
b_{1414}	0.209999995306134	0.210
b_{1515}	0.224999994970858	0.225
b_{1616}	0.239999994635582	0.240
b_{1717}	0.254999994300306	0.255
b_{1818}	0.269999993965030	0.270
b_{1919}	0.284999993629754	0.285
b_{2020}	0.299999993294477	0.300
b_{2121}	0.314999992959201	0.315
b_{2222}	0.329999992623925	0.330
b_{2323}	0.344999992288649	0.345
b_{2424}	0.359999991953373	0.360
b_{2525}	0.374999991618097	0.375
b_{2626}	0.389999991282821	0.390
b_{2727}	0.404999990947545	0.405
b_{2828}	0.419999990612268	0.420
b_{2929}	0.434999990276992	0.435

a comparison of the coefficients in (1) for the sake of simplicity. From Table 4, one can see that the noise is added in the approximate limit state function. As indicated in Liu and Der Kiureghian (1991), FORM with the Hasofer-Lind-Rackwitz-Fiessler algorithm does not perform well for the noise problem. Thus, RSM based on the FORM became invalid. The proposed method can be employed to solve this problem. In the proposed method, to obtain the first-order reliability index, the tolerance criteria used in the Hasofer-Lind-Rackwitz-Fiessler algorithm needs to be modified temporarily. However, this does not affect the finial results since the inaccuracy first-order reliability index can be improved using importance sampling updating algorithm in the proposed method. It should be pointed out that the tolerance criteria mentioned above refers to the tolerance between

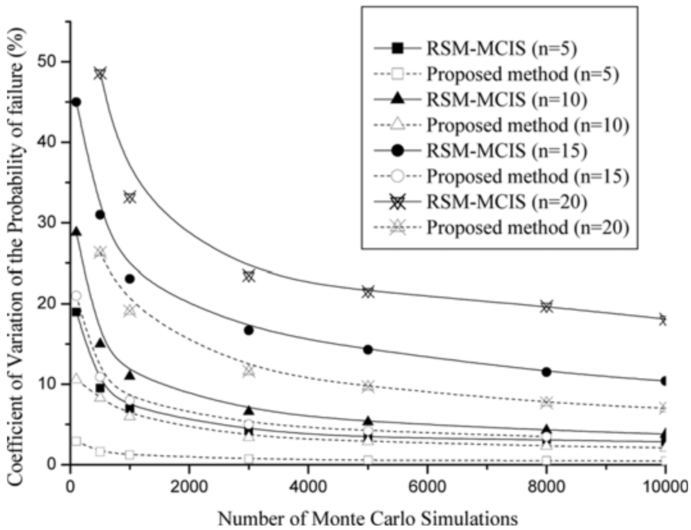


Fig. 3 Coefficient of variation of the probability of failure with different values of $n = 5, 10, 15, 20$ for Example 2

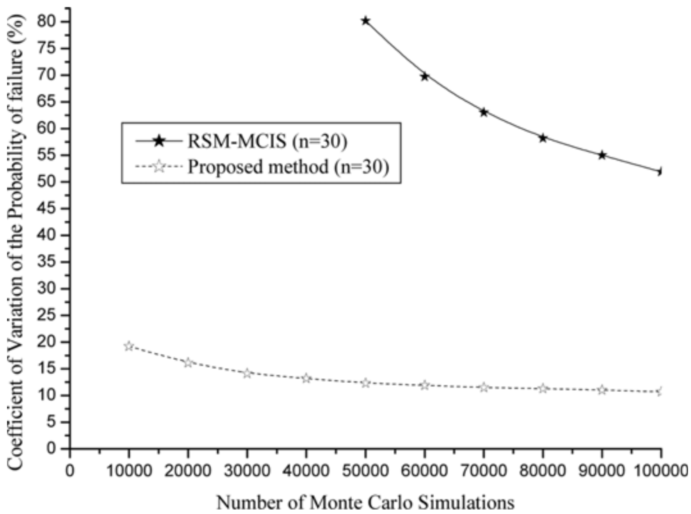


Fig. 4 Coefficient of variation of the probability of failure with the value of $n = 30$ for Example 2

design point values as the iteration progresses.

Figs. 3 and 4 show the coefficient of variation of the probability of failure for different methods with different values of n. From these figures, it can be seen that the proposed method is the most efficient method for all values of n.

3.3 Example 3: Linear frame structure with one story and one bay (implicit limit state function)

The third example is a linear frame structure of one story and one bay as shown in Fig. 5.

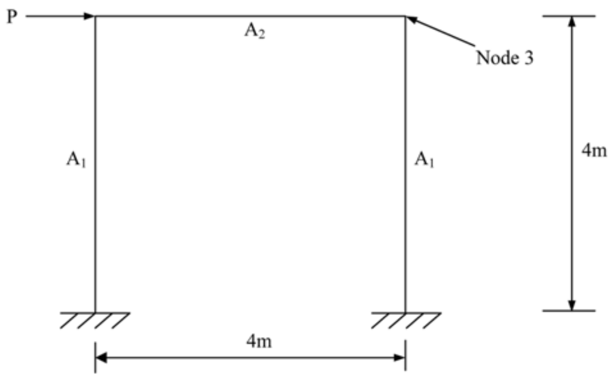


Fig. 5 Linear portal frame of Example 3

Table 5 Statistics of the random variables for Example 3

Variable	Mean	Standard deviation	Dimension	Distribution
A_1	0.36	0.036	m^2	Lognormal
A_2	0.18	0.018	m^2	Lognormal
P	20	5.0	KN	Type I largest

Different cross sectional areas A_i and horizontal load P are treated as independent random variables; their statistics are listed in Table 5. The sectional moments of inertia are expressed as $I_i = \alpha_i A_i^2$ $(\alpha_1 = 0.08333, \alpha_2 = 0.16670)$. The Young's modulus, E, is treated as deterministic. $E = 2.0 \times 10^6$ KN/m^2. Of interest is the probability that the horizontal displacement at node 3 exceeds 0.01 m. Thus, the limit state function is expressed as

$$G(A_1, A_2, P) = 0.01 - u_3(A_1, A_2, P) \tag{8}$$

The results are listed in Table 6. The exact values in Table 6 are obtained from Zhao (1996). It can be observed from Table 6 that the results from the proposed method are closer to the exact results than those of both RSM and RSM-MCIS. Also, the proposed method requires few samples and numbers to form response surface function. Thus, the proposed method promise to save computational effort, particularly when a deterministic finite element analysis requires large amount

Table 6 Results of Example 3

Method	β	p_f	Numbers of forming RSF	CPU time (s)
RSM	2.791	2.625e-3	16	29.46
RSM-MCIS	2.710	3.361e-3	16	29.64(10000)
Proposed method	2.808	2.490e-3	2	100.45(100)
Exact results (Zhao 1996)	2.831	2.322e-3	-	-

Note: RSF is the so-called approximation function, $\hat{g}(X)$ mentioned in this paper

Table 7 Effect of initial value of h_i on the estimates

Method	Initial value of h_i									
	h_i = 1.0		h_i = 1.5		h_i = 2.0		h_i = 2.5		h_i = 3.0	
	β	p_f	β	p_f	β	p_f	β	p_f	β	p_f
RSM	2.612	4.496e-3	2.698	3.486e-3	2.731	3.160e-3	2.772	2.786e-3	2.791	2.625e-3
Proposed method	2.820	2.398e-3	2.819	2.409e-3	2.817	2.427e-3	2.813	2.453e-3	2.808	2.49e-3

of computation time or the number of FEM calculations is large. Note that the CPU time required for the proposed method is large compared to both RSM and RSM-MCIS. This is mainly because the importance sampling updating technique in the proposed method is based on the actual limit state surface. However, it should be pointed out that the CPU time required for the proposed method is acceptable. Although more CPU time is required when compared with the RSM and RSM-MCIS, the results obtained with the proposed method are much more accurate. Another observation from this table is that the RSM-MCIS gives worse results than the RSM. This is because (1) Monte Carlo Importance Sampling in the RSM-MCIS is based on the approximate limit state function obtained by RSM; and (2) the difference between the approximate limit state function and actual limit state function is large.

Guan and Melchers (2001) investigated the effects of an arbitrary parameter in RSM, h_i on the estimated failure probability. They concluded that the value of h_i could have a considerable effect. On the other hand, the different values of h_i also affect the accuracy of the results obtained from RSM. However, these problems can be overcome using the proposed method. To validate this point, we set the parameter h_i to be different values and solve the previous example (Example 3) using the proposed method and RSM. The results are listed in Table 7. From this table, it can be seen that the proposed method does offer a significant improvement over the RSM results.

3.4 Example 4: Linear frame structure with twelve stories and three bays (implicit limit state function)

The fourth example is a linear frame structure with twelve stories and three bays as shown in Fig. 6. Different cross sectional areas A_i and horizontal load P are treated as independent random variables; their statistics are listed in Table 8. The sectional moments of inertia are expressed as $I_i = \alpha_i A_i^2$ $(\alpha_1 = \alpha_2 = \alpha_3 = 0.08333, \alpha_4 = 0.26670, \alpha_5 = 0.2000)$. The Young's modulus, E, is treated as deterministic. $E = 2.0 \times 10^7$ KN/m^2. Element types are indicated in Fig. 6. Of interest is the probability that the horizontal displacement at node A exceeds 0.096 m. Thus, the limit state function is expressed as

$$G(A_1, A_2, A_3, A_4, A_5, P) = 0.096 - u_A(A_1, A_2, A_3, A_4, A_5, P) \tag{9}$$

The results are shown in Table 9. From the table, one can see that the proposed method with few simulations achieves excellent accuracy. The exact values in Table 10 are obtained from Zhao (1996).

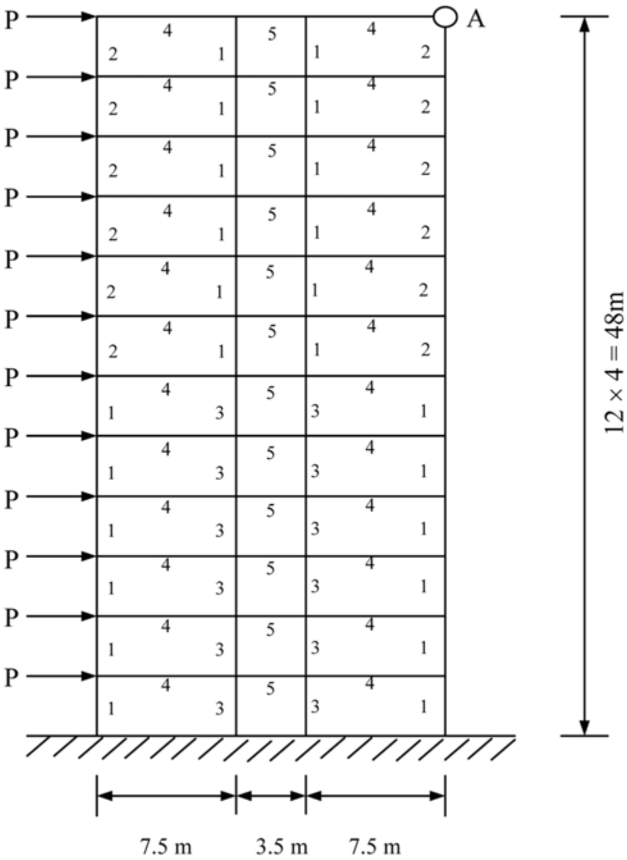


Fig. 6 Linear portal frame of Example 4

Table 8 Statistics of the random variables for Example 4

Variable	Mean	Standard deviation	Dimension	Distribution
A_1	0.25	0.025	m^2	Lognormal
A_2	0.16	0.016	m^2	Lognormal
A_3	0.36	0.036	m^2	Lognormal
A_4	0.20	0.020	m^2	Lognormal
A_5	0.15	0.015	m^2	Lognormal
P	30.0	7.5	kN	Type I largest

Table 9 Results of Example 4

Method	β	p_f
RSM	1.4469	7.396e-2
Proposed method	1.4264(300)	7.688e-2
Exact results (Zhao 1996)	1.4391(2000)	7.5058e-2

Table 10 Statistics of the random variables for Example 5

Variable	Mean	Standard deviation	Dimension	Distribution
P	4.5 9.0 13.5 18.0	2.25 4.5 6.75 9.00	KN	Normal
E_1	200	10.0	GPa	Normal

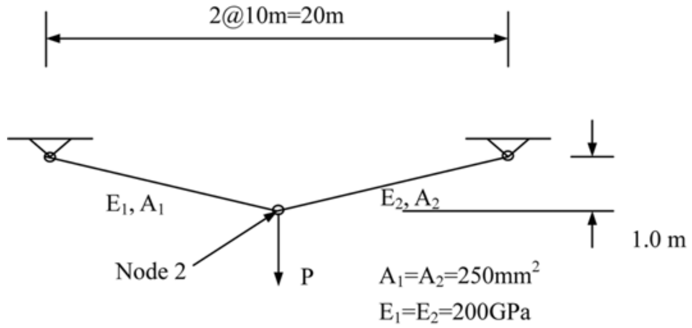


Fig. 7 Geometrically nonlinear truss of Example 5

3.5 Example 5: Geometrically nonlinear truss (implicit limit state function)

A geometrically nonlinear truss, studied by Frangopol and Imai (2000), is considered in this example. The geometry of the truss as shown in Fig. 7 is assumed to be deterministic. The cross-sections of the members ($A_1 = A_2 = 250\ \text{mm}^2$) are assumed to be identical and deterministic. The statistics of the random variables considered are listed in Table 10. Taken from Frangopol and Imai (2000), the limit state function is defined as

$$G(P, E) = 0.25 - v_2(P, E) \tag{10}$$

where $v_2(P, E)$ = the vertical displacement at the node 2.

The reliability of the truss was analyzed using the proposed method and RSM. Two types of reliability analysis are considered: (1) linear reliability analysis; and (2) geometrically nonlinear reliability analysis. The computational results are presented in Table 11. Note that the exact values in this table are obtained using the proposed method with a coefficient of variation less than 6%. From this table, one can find that when the exact reliability index is small, both the proposed method with few simulations and the RSM give very good approximations for the linear and geometrically nonlinear reliability analyses. When the exact reliability index is extremely large (e.g. $\beta = 7.85$), the RSM produces large errors, even not converged. The proposed method with few simulations provides good results.

Table 11 Results of Example 5

Load (KN)	Linear reliability analysis			Nonlinear reliability analysis		
	RSM	Proposed method	Exact results	RSM	Proposed method	Exact results
P = 4.5	β = 7.574 p_f = 1.810e-14	β = 7.822 p_f = 2.600e-15 (300)	β = 7.850 p_f = 2.600e-15 (30611)	Not converged	β = 10.560 p_f = 2.283e-26 (300)	β = 10.581 p_f = 1.825e-26 (30902)
P = 9.0	β = 3.341 p_f = 4.174e-4	β = 3.345 p_f = 4.114e-4 (300)	β = 3.350 p_f = 4.041e-4 (6324)	β = 5.289 p_f = 6.149e-8	β = 5.284 p_f = 6.320e-8 (300)	β = 5.289 p_f = 6.149e-8 (3029)
P = 13.5	β = 1.622 p_f = 5.240e-2	β = 1.621 p_f = 5.251e-2 (300)	β = 1.622 p_f = 5.240e-2 (1000)	β = 3.013 p_f = 1.293e-3	β = 3.012 p_f = 1.298e-3 (300)	β = 3.013 p_f = 1.293e-3 (1586)
P = 18.0	β = 0.730 p_f = 0.233	β = 0.727 p_f = 0.234 (300)	β = 0.729 p_f = 0.233 (1000)	β = 1.800 p_f = 3.593e-2	β = 1.800 p_f = 3.593e-2 (300)	β = 1.800 p_f = 3.593e-2 (1000)

4. Application to an example long span cable-stayed bridge

4.1 Description of the example bridge

The example cable-stayed bridge studied here is the Second Nanjing Bridge, with a 628 m central span length, which is now the longest cable-stayed bridge in China. The bridge span arrangements are (58.5 + 246.5 + 628 + 246.5 + 58.5) m. There are six traffic lanes. The elevation view of the bridge is shown in Fig. 8.

The bridge deck is a 37.2 m wide and 3.144 m deep steel box. The bridge towers are the diamond-shaped concrete towers of 196 m high. For more details of the bridge, the reader is referred to Tang (2001), Xiang (1997).

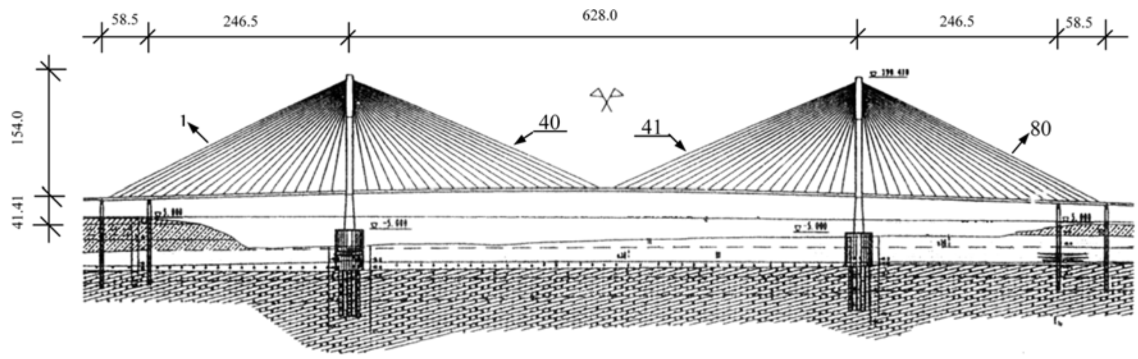


Fig. 8 Elevation of the 2nd Nanjing Bridge (Unit: m)

4.2 Nonlinear considerations and finite element modeling

A long-span cable-stayed bridge exhibits geometric nonlinear characteristics under loadings. These geometric nonlinearities come from the cable sag effect, axial force-bending interaction effect, and large displacement effect.

The major structural components of a cable-stayed bridge are the cables, the towers and the stiffening girders. The finite element modeling of these components can be accomplished with the aid of two basic elements: truss element (cable element) and beam element. Plane beam elements were used to model the girder and towers. The cables were modeled by plane truss (cable) elements. Three finite element models are considered in the following reliability analysis. In Model 1, geometric nonlinearities are not considered. In Model 2, only axial force-bending interaction effect and large displacement effect of the geometric nonlinearities mentioned above are considered. Each cable is treated as a plane truss element. In Model 3, all geometric nonlinearities mentioned above are accurately considered. A single two-node catenary element proposed by Karoumi (1999) is used in modeling each cable. The main advantage of the element is its ability to accurately account for cable sag effects. In this paper, an incremental-iterative method based on the Newton-Raphson method is employed for the solution of geometric nonlinear problems. The solution method has been implemented in a deterministic analysis program called NASAB (Cheng 2003).

There is no restraint between the girder and towers. In this case the bridge girder swings freely at towers (called a floating system). All other supports of the girder are assumed to be simply supported (moveable hinge restraints).

4.3 Reliability analysis

Sectional properties (A_i, I_i), elastic modulus (E_i), weight per unit volume of material (γ_i), and applied live loads (q) are chosen as the random variables of interest for this study. The statistical descriptions of these random variables are shown in Table 12. They are similar to the ones given by Chen (2000). For simplicity, the applied live loads are assumed to be uniformly distributed on the bridge deck. Two uniform live load cases are considered in the reliability analysis. Live load case I is the live load uniformly distributed in all spans. Live load case II is the live load uniformly distributed only in the central span. It should be pointed out that the live load model and mean value of load is simply taken as the design value found from the Chinese design code (Highway Cable-stayed Bridge Design Specification in China (JTJ027-96) 1996).

After the long span cable-stayed bridge is completed, and before the live load is applied, the bridge has sustained large dead load deformations and stresses in each member. To consider the effect of dead load, the reliability analysis should involve two steps: (1) the initial shape of the cable-stayed bridge under dead loads is determined. The finite element computation procedure proposed by Wang *et al.* (1993) is adopted for determining the initial shape of cable-stayed bridges under dead loads. The computation procedure has been implemented in a deterministic analysis program called NASAB; and (2) based on the determined initial shape, the reliability analysis is performed. In the analysis, serviceability failure is defined to occur when a deflection exceeds an allowable deflection limit. The allowable displacement at midpoint of center span, δ_{allow}, is considered not to exceed $L/400$, where L is the central span length. The value 400 is obtained from (Highway Cable-stayed Bridge Design Specification in China (JTJ027-96) 1996). Of course, any other value can be used for this purpose. For this example, δ_{allow} becomes 1.57 m and the

Table 12 Statistics of the random variables for example bridge

Random variables		Substructures	Distribution types	Mean value	Standard deviation
Elastic modulus	E_1	Girder	Normal	2.1×10^8 kN/m^2	2.1×10^7 kN/m^2
	E_2	Towers	Normal	3.5×10^7 kN/m^2	3.5×10^6 kN/m^2
	E_3	Cables	Normal	1.85×10^8 kN/m^2	1.85×10^7 kN/m^2
Cross sectional areas	A_1	Girder	Lognormal	1.697 m^2	0.08485 m^2
	A_2	Towers, 152.11-195.41 m	Lognormal	18.19 m^2	0.9095 m^2
	A_3	Towers, 41.41-152.11 m	Lognormal	18.44 m^2	0.922 m^2
	A_4	Towers, 0-41.41 m	Lognormal	33.07 m^2	1.6535 m^2
	A_5	Cables (no.15-19,22-26,55-59,62-66)	Lognormal	1.0698×10^{-2} m^2	5.349×10^{-4} m^2
	A_6	Cables (no.11-14,20-21,27-30,51-54, 60-61,67-70)	Lognormal	1.2546×10^{-2} m^2	6.273×10^{-4} m^2
	A_7	Cables (no. 8-10,31-35,46-50,71-73)	Lognormal	1.5316×10^{-2} m^2	7.658×10^{-4} m^2
	A_8	Cables (no.3-7,36-38,43-45,74-78)	Lognormal	1.8548×10^{-2} m^2	9.274×10^{-4} m^2
	A_9	Cables (no.1-2,39-42,79-80)	Lognormal	2.0396×10^{-2} m^2	1.020×10^{-3} m^2
Sectional moments of inertia	I_1	Girder	Lognormal	3.404 m^4	0.1702 m^4
	I_2	Towers, 152.11-195.41 m	Lognormal	41.397 m^4	2.06985 m^4
	I_3	Towers, 41.41-152.11 m	Lognormal	43.40 m^4	2.17 m^4
	I_4	Towers, 0-41.41 m	Lognormal	108.12 m^4	5.406 m^4
Weight per unit volume of material	γ_1	Girder	Normal	139.75 kN/m^3	6.9875 kN/m^3
	γ_2	Towers	Normal	24.5 kN/m^3	1.225 kN/m^3
	γ_3	Cables	Normal	76.85 kN/m^3	3.8425 kN/m^3
Live load	q	Girder	Normal	47.431 kN/m 59.288 kN/m 71.146 kN/m 83.004 kN/m	6.166 kN/m 7.707 kN/m 9.249 kN/m 10.79 kN/m

serviceability limit state can be represented as:

$$G(x) = 1.57 - y_{mid}(x) \tag{11}$$

where $y_{mid}(x)$ = the vertical displacement at midpoint of center span. It should be pointed out that the following computation results are obtained using the proposed method with a coefficient of variation less than 5%.

Fig. 9 shows the effect of geometric nonlinearities on the serviceability reliability of a long span cable-stayed bridge under live load case I. From the figure, it can be seen that (1) the reliability results obtained by the model 1 are higher than those obtained by the models 2 and 3. This implies that reliability analyses neglecting all geometric nonlinear effects overestimate the serviceability reliability of the cable-stayed bridge under live load case I; (2) the difference in the reliability index between the two models 1 and 2 is small. This implies that the effects of axial force-bending

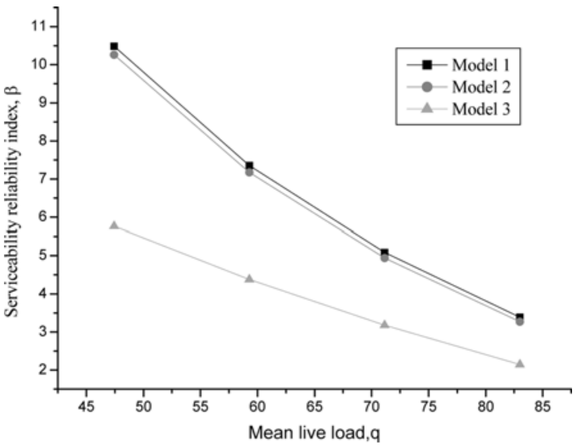


Fig. 9 Serviceability reliability index, β for several mean live loads, q for models 1,2 and 3 (live load case I)

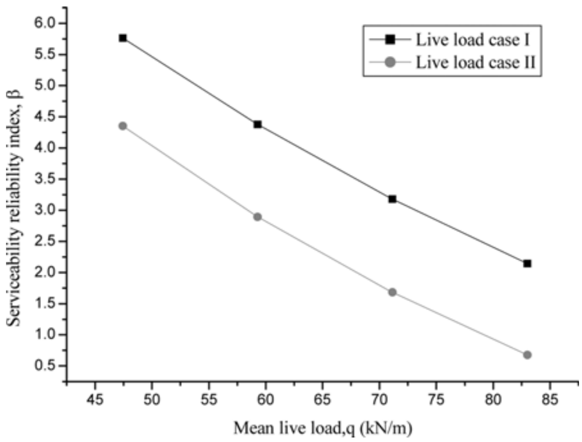


Fig. 10 Serviceability reliability index, β for several mean live loads, q for live load cases I and II (Model 3)

interaction and large displacement in the geometric nonlinearities are negligible. However, the difference in the reliability index between the two models 2 and 3 is large. This implies that the cable sag in the geometric nonlinearities significantly affects the reliability of the cable-stayed bridge. Hence, in the serviceability reliability analysis of long span cable-stayed bridges, the cable sag effect of geometric nonlinearities cannot be ignored. In the following analyses, only model 3 mentioned above is used.

Fig. 10 shows the effect of live loads on the serviceability reliability of a long span cable-stayed bridge. Two live load cases as mentioned above are considered. From Fig. 10, it can be seen that the reliability of the cable-stayed bridge under live load case II (uniformly distributed only in the central span) is lower than that under live load case I (uniformly distributed only in all spans).

To investigate the effects of the support conditions of the bridge girder on the serviceability reliability of a long span cable-stayed bridge under live load case I, three cases of the girder support conditions used in Ren (1999) are considered. Case I: There is no restraint between the girder and towers. In this case the bridge girder swings freely at towers (called a floating system). All other supports of the girder are assumed to be simply supported (moveable hinge restraints). Case II: the joint between the main girder and left tower is a fixed hinge, whereas another joint between the main girder and right tower is a movable hinge, and all side piers are moveable hinge (roller) supports. Case III: it is assumed that there are some restraints between the girder and towers where the left joint is a fixed hinge restraint and the right joint is a moveable hinge restraint. Nevertheless, both ends of the girder are assumed to have fixed hinge supports. The middle side piers still have moveable hinge supports. It should be pointed out that the floating system denoted as Case I in the paper is the actual connection of the bridge modeled as for design purposes. The other two cases of the girder support conditions are chosen in this paper for comparative purposes. The reliability indices in these three cases are summarized in Table 13. From this table, it can be seen that the serviceability reliability of long span cable-stayed bridges is dependent on the manner of the support conditions of the bridge girder. Small restraints result in lower reliability, whereas stronger restraints lead to higher reliability. Therefore, it is very important to select the appropriate support conditions of the bridge girder to enhance overall reliability of long-span cable-stayed bridges.

Table 13 Comparison of reliability indices for various cases

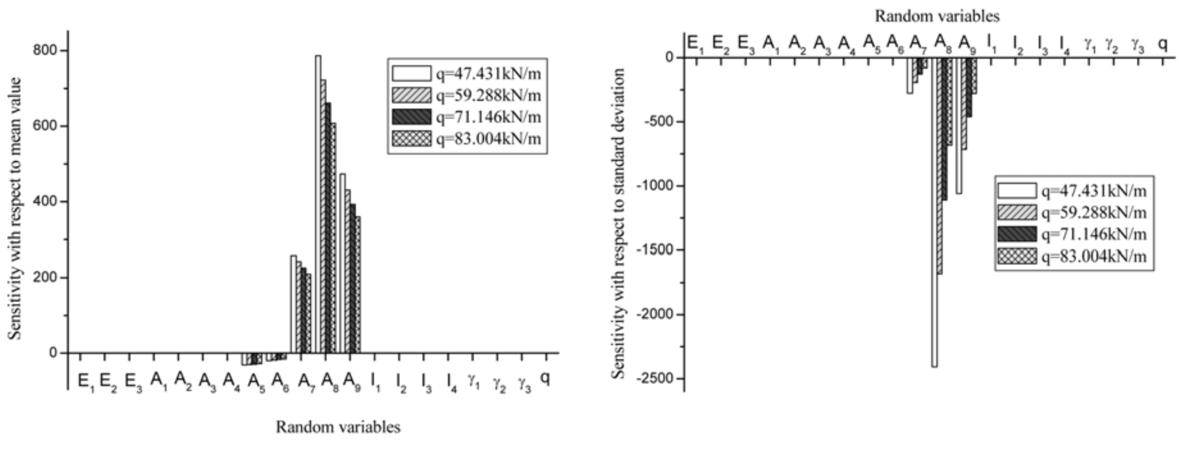
Case	Case I	Case II	Case III
β	5.764	5.854	6.454

4.4 Sensitivity analysis

An important step in the structural reliability analyses is the sensitivity analysis of reliability indices. This helps identify the important parameters. On the other hand, sensitivity analysis is also useful in reducing the size of problems with a large numbers of random variables. This is because that, in general, only a few variables have a significant effect on the structural reliability (Haldar 2000). Some literature is available on sensitivity analysis of reliability indices. Liu and Der Kiureghian (1991) presented extensive analyses of reliability sensitivities with respect to parameters defining the random fields. The results indicate relative importance of the basic variables and fields for the static behavior of the plate. Imai and Frangopol (2001, 2002) carried out sensitivity analyses of reliability indices with respect to the mean values and standard deviations of the variables of interest. The results indicate that the wind load and resistance are the most influential variables on the reliability of suspension bridges. To the writers' knowledge, there has been no investigation on the influences of the random variables on the reliability of cable-stayed bridges. In this section, sensitivity analysis is used to identify the influences of the random variables on the reliability of cable-stayed bridges. Of particular interest in this study is sensitivity measures with respect to the mean and standard deviation of each random variable. The sensitivity measures are formulated as indicated by Bjerager and Krenk (1989)

$$\frac{d\beta}{d\theta} = \alpha^T \frac{du^*}{d\theta} \tag{12}$$

$$u^* = \beta \cdot \alpha$$



(a) (b)

Fig. 11 Sensitivity of reliability indexes with respect to (a) mean value, (b) standard deviation (live load case I and Model 3) where $\beta = |u^*|$ and $\alpha =$ the outward unit normal vector to the limit state surface in u^*. Here θ is the mean or standard deviation of each random variable.

The results of the sensitivity analysis are presented in Fig. 11. The results show that the cable area has the most influential effect on the reliability of cable-stayed bridges. This illustrates the reasons why the cable sag effect of geometric nonlinearities cannot be ignored for evaluating the reliability of cable-stayed bridges. It can also be seen from Fig. 11 that as the loads increase, the effect of the cable area on the bridge reliability reduces. The reason is the increase in stiffness with increasing loads.

5. Conclusions

A reliability analysis method has been proposed in this paper through a combination of the advantages of RSM, FEM, FORM and the importance sampling updating method. Using the RSM method combined with FORM, the proposed method can calculate the reliability of complex structures of which the limit state functions are not known explicitly. Also, it is possible to use the existing deterministic finite element code without modifying it. By introducing the FEM, the proposed method can be used to perform finite element reliability analysis. The use of importance sampling updating technique in the proposed method has the following advantages: (1) it makes the proposed method obtain very good results with few simulations. Thus, significant reduction in computing time is achieved compared to the direct Monte Carlo simulation (MCS); (2) it makes the proposed method be insensitive to noise; (3) it makes the proposed method provide better results than using the RSM and RSM-MCIS methods. This is because the importance sampling updating technique is based on the actual limit state surface. Another advantage of the proposed method over the RSM is that it is insensitive to the values of an arbitrary parameter in RSM, h_i. The proposed method is particularly useful for extremely small failure probability problems. The accuracy and efficiency of the proposed method is compared through five examples.

The proposed method has been applied to estimate the serviceability reliability of a long span cable-stayed bridge. It was found that: (1) the cable sag in the geometric nonlinearities of cable-stayed bridges has a major effect on the reliability of cable-stayed bridge; (2) with regard to live loads, in the most cases, live load case II (uniformly distributed only in the central span) is more risky to the long span cable-stayed bridge; (3) separating the girder from the towers reduces the serviceability reliability of the long span cable-stayed bridge; (4) the effect of the cable area on the reliability of cable-stayed bridges is significant.

It should be pointed out that the application of the proposed method is not limited to the serviceability reliability analysis of cable-stayed bridges. Wider application of the proposed method is being explored.

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