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Uncertain-parameter sensitivity of earthquake input energy to base-isolated structure

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Abstract. The input energy to a base-isolated (BI) building during an earthquake is considered and formulated in the frequency domain. The frequency-domain approach for input energy computation has some notable advantages over the conventional time-domain approach. Sensitivities of the input energy to the BI building are derived with respect to uncertain parameters in the base-isolation system. It is demonstrated that the input energy can be of a compact form via the frequency integration of the product between the input component (Fourier amplitude spectrum of acceleration) and the structural model component (so-called energy transfer function). With the help of this compact form, it is shown that the formulation of earthquake input energy in the frequency domain is essential for deriving the sensitivities of the input energy to the BI building with respect to uncertain parameters. The sensitivity expressions provide us with information on the most unfavorable combination of the uncertain parameters which leads to the maximum energy input.

Key words: earthquake input energy; base-isolation; frequency-domain analysis; response sensitivity; parameter uncertainty.

1. Introduction

It is commonly recognized (for example, Naeim and Kelly 1999) that base-isolation (BI) systems are very useful in reducing the earthquake response of buildings except for absolute base displacement (for example, Barbat *et al.* 1995, Meirovitch and Stemple 1997, Morales 2003) and are being installed in many buildings and facilities after the Hyogo-ken Nanbu earthquake (1995). It is also true that the mechanical properties of the BI system are fairly uncertain and uncertainty analysis is often implemented in the actual structural design of base-isolated (BI) buildings. For example, the dependence of the mechanical properties of the BI system on temperature, amplitude of deformation, velocity of deformation, axial stress of isolators, etc. and the degree of variability of these factors have to be taken into account appropriately.

In this paper, uncertainty in BI buildings and its effect on earthquake energy input are investigated. That uncertainty is assumed to result from the variability in the modeling of mechanical properties of the BI system. For this purpose, a shear building model supported by a BI

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system is treated. The analysis of BI buildings is well established and some computer programs can be used for the analysis of BI buildings. It is also true that, while the analysis of BI effects has been focused mainly on the investigation in terms of deformation and acceleration, much attention has never been directed to the investigation in terms of the earthquake input energy to the BI building. However, the energy concept may be appropriate especially in the analysis of BI buildings which consist of multiple components with completely different properties. In practice, the energy concept is often used in the preliminary design of BI buildings (AIJ 1989, 2001). This method is referred to as 'an envelope analysis method'.

From this point of view, the earthquake input energies to a superstructure and to an overall system of the BI building are chosen as the response quantities in the evaluation of the effect of the uncertain parameters. The frequency-domain approach is used to evaluate the earthquake input energies in an analytical way. It is shown that the earthquake input energies to a superstructure and to an overall system can be obtained in a compact form by taking advantage of the frequency-domain approach. The transfer function necessary in the evaluation of the input energy in the frequency domain is obtained in closed form by utilizing an explicit expression of the inverse of the tri-diagonal coefficient matrix in the equations of motion. It is also shown that the sensitivity of the earthquake input energy with respect to uncertain parameters can also be obtained in closed form by taking advantage of the frequency-domain approach. The sensitivity of the transfer function needed in the evaluation of the sensitivity of the earthquake input energy is derived in closed form by using the equations of motion in the frequency domain. It will be confirmed through comparison with results by the finite difference method that the proposed method has a reasonable accuracy, and its reliability and efficiency are remarkable.

2. Earthquake input energy in frequency domain

Consider an N-story shear building model, as shown in Fig. 1, supported by a BI system. Let m_i, k_i, c_i denote the mass of the $(i+1)$-th floor, the stiffness of the i-th story and the corresponding damping coefficient, respectively. It is assumed here that the BI system can be modeled by a linear elastic spring and a linear viscous damper justified for BI systems based on linear rubber bearing (LRB) that include systems composed solely of LRB and systems constituted by LRB and hydraulic dampers. The horizontal stiffness and the damping coefficient of the BI system are denoted by k_0 and c_0, respectively. This model is subjected to a horizontal acceleration $\ddot{u}_g(t) = a(t)$ at the ground surface. Let u_i denote the horizontal displacement of the $(i+1)$-th floor relative to the ground.

The equations of motion for this BI building may be expressed as

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = -\mathbf{M}\mathbf{1}\ddot{u}_g \tag{1}$$

where $\mathbf{u} = \{u_0 \ u_1 \ \ldots \ u_N\}^T$. $\mathbf{M}$, $\mathbf{K}$, $\mathbf{C}$ and $\mathbf{1}$ are the following mass, stiffness and damping matrices and influence coefficient vector, respectively.

$$\mathbf{M} = \mathrm{diag}(m_0 \ m_1 \ \ldots \ m_N) \tag{2a}$$

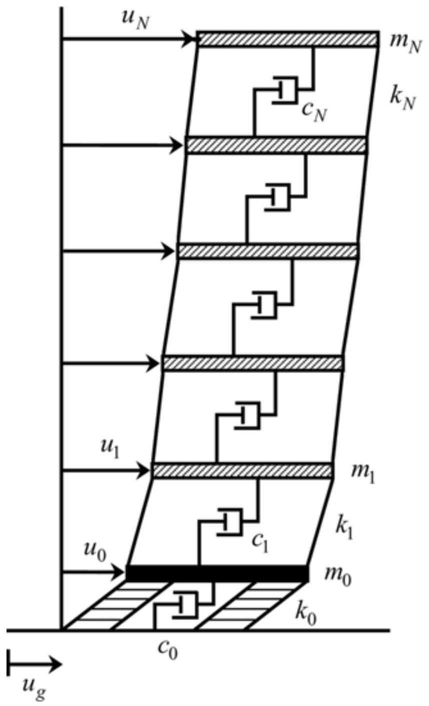


Fig. 1 N-story shear building model supported by a base-isolation system

$$\mathbf{K} = \begin{bmatrix} k_0 + k_1 & -k_1 & & \mathbf{0} \\ -k_1 & \ddots & \ddots & \\ & \ddots & k_{N-1} + k_N & -k_N \\ \mathbf{0} & & -k_N & k_N \end{bmatrix} \tag{2b}$$

$$\mathbf{C} = \begin{bmatrix} c_0 + c_1 & -c_1 & & \mathbf{0} \\ -c_1 & \ddots & \ddots & \\ & \ddots & c_{N-1} + c_N & -c_N \\ \mathbf{0} & & -c_N & c_N \end{bmatrix} \tag{2c}$$

$$\mathbf{1} = \{1 \ \ 1 \ \ \ldots \ \ 1\}^T \tag{2d}$$

Consider the earthquake input energy (Housner 1959, Housner and Jennings 1977, Akiyama 1985, Uang and Bertero 1990, Trifunac *et al.* 2001, Takewaki 2004a, b) to the present model. Premultiplication of $\dot{\mathbf{u}}^T$ on Eq. (1) and integration of the resulting equation from 0 to t_0 lead to

$$[(1/2)\dot{\mathbf{u}}^T\mathbf{M}\dot{\mathbf{u}}]_0^{t_0} + \int_0^{t_0}\dot{\mathbf{u}}^T\mathbf{C}\dot{\mathbf{u}}dt + [(1/2)\mathbf{u}^T\mathbf{K}\mathbf{u}]_0^{t_0} = -\int_0^{t_0}\dot{\mathbf{u}}^T\mathbf{M}\mathbf{1}\ddot{u}_g dt \equiv E_I^A \tag{3}$$

Integration by parts of E_I^A and its rearrangement by use of $\dot{u}_g(0) = \dot{u}(t_0) = 0$ provide

$$\begin{aligned} E_I^A &= -[\dot{\mathbf{u}}^T\mathbf{M1}\dot{u}_g]_0^{t_0} + \int_0^{t_0}\ddot{\mathbf{u}}^T\mathbf{M1}\dot{u}_g dt \\ &= \int_0^{t_0}\{\textstyle\sum_{i=0}^N m_i\ddot{u}_i\}\dot{u}_g dt \\ &= \int_0^{t_0}\{\textstyle\sum_{i=0}^N m_i(\ddot{u}_g+\ddot{u}_i) - \sum_{i=0}^N m_i\ddot{u}_g\}\dot{u}_g dt \\ &= \int_0^{t_0}\{\textstyle\sum_{i=0}^N m_i(\ddot{u}_g+\ddot{u}_i)\}\dot{u}_g dt - [(1/2)(\sum_{i=0}^N m_i)\ \dot{u}_g^{\ 2}]_0^{t_0} \\ &= \int_0^{t_0}\{\textstyle\sum_{i=0}^N m_i(\ddot{u}_g+\ddot{u}_i)\}\dot{u}_g dt \end{aligned} \tag{4}$$

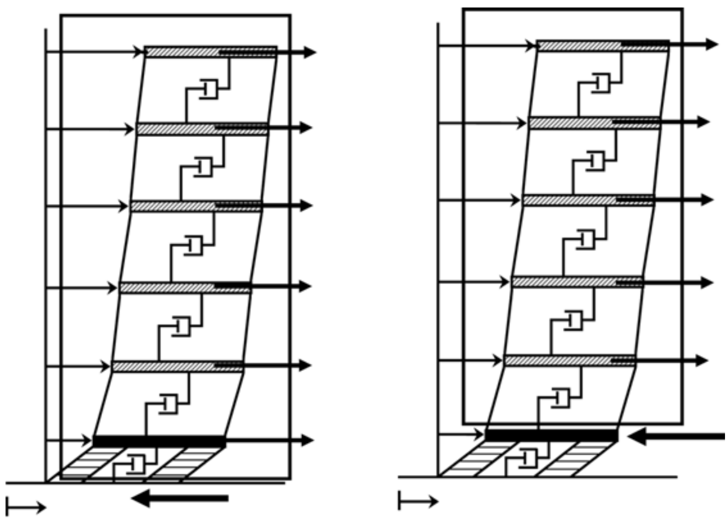
The expression in the braces in the last equation indicates the sum of inertial forces acting on the base-isolated floor mass and building floor masses. Eq. (4) implies that the work by the ground on the total system of the BI building is equal to E_I^A; see Fig. 2(a).

It is known (Lyon 1975, Ohi *et al*. 1985, Kuwamura *et al*. 1994, Ordaz *et al*. 2003, Takewaki 2004a, b) that, in linear elastic structures, the earthquake input energy can also be expressed in the frequency domain. It should be noted that, while the previous formulation is restricted to the total system (Takewaki 2004b), the present formulation includes a new formulation of the evaluation of input energy to a subsystem, i.e., the input energy to a superstructure.

Let $\ddot{U}_i$, $\ddot{U}_g$ denote the Fourier transforms of $\ddot{u}_i$, $\ddot{u}_g$, and $H_{\ddot{U}_i}(\omega)$ denote the transfer functions of $\ddot{u}_i$ to $\ddot{u}_g$.

$$\ddot{U}_i/\ddot{U}_g = H_{\ddot{U}_i}(\omega) \tag{5}$$

These quantities can be derived from the Fourier transformed equations of Eq. (1).



(a) E_I^A (b) E_I^S

Fig. 2 Free-body diagram for computation of E_I^A and E_I^S

The Fourier inverse transformation of Eq. (4) after the extension of lower and upper limits from $(0, t_0)$ to $(-\infty, \infty)$ and use of Eq. (5) lead to

$$
\begin{aligned}
E_I^A &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{\textstyle\sum_{i=0}^{N} m_i(\ddot{U}_g + \ddot{U}_i)\}e^{i\omega t}\dot{u}_g dt d\omega \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty}\{\textstyle\sum_{i=0}^{N} m_i\ddot{U}_g(\omega)(1 + H_{\ddot{U}_i}(\omega))\}(\int_{-\infty}^{\infty} e^{i\omega t}\dot{u}_g dt)d\omega \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty}\{\textstyle\sum_{i=0}^{N} m_i\ddot{U}_g(\omega)(1 + H_{\ddot{U}_i}(\omega))\}\dot{U}_g(-\omega)d\omega \qquad (6) \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty}\{\textstyle\sum_{i=0}^{N} m_i\ddot{U}_g(\omega)(1 + H_{\ddot{U}_i}(\omega))\}(\ddot{U}_g(-\omega)/(-i\omega))d\omega \\
&= -\frac{1}{\pi}\int_{0}^{\infty}\frac{1}{\omega}\mathrm{Im}[\textstyle\sum_{i=0}^{N} m_i H_{\ddot{U}_i}(\omega)]|\ddot{U}_g(\omega)|^2 d\omega
\end{aligned}
$$

where the symbol i denotes the imaginary unit and Im[·] indicates the imaginary part of a complex number. It is known that the earthquake input energy to a linear elastic structure does not depend on the phase characteristics of input motions (Lyon 1975, Ohi *et al.* 1985, Kuwamura *et al.* 1994, Ordaz *et al.* 2003, Takewaki 2004a, b). Eq. (6) clearly supports this fact.

Consider next the work by the base-isolated floor on the superstructure alone; see Fig. 2(b). This quantity indicates the input energy to the superstructure alone and is expressed by

$$E_I^S = \int_0^{\infty}\{\textstyle\sum_{i=1}^{N} m_i(\ddot{u}_g + \ddot{u}_i)\}(\dot{u}_g + \dot{u}_0)dt \qquad (7)$$

The internal story shear force in the first story is in equilibrium with the inertial forces $-\sum_{i=1}^{N} m_i$ $(\ddot{u}_g+\ddot{u}_i)$ and does the work on $-(\dot{u}_g+\dot{u}_0)dt$. The Fourier inverse transformation of the terms in Eq. (7) after the extension of the lower limit from 0 to $-\infty$ and use of the transfer functions defined in Eq. (5) provide

$$
\begin{aligned}
E_I^S &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{\textstyle\sum_{i=1}^{N} m_i(\ddot{U}_g + \ddot{U}_i)\}e^{i\omega t}(\dot{u}_g + \dot{u}_0)dt d\omega \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty}\{\textstyle\sum_{i=1}^{N} m_i\ddot{U}_g(\omega)(1 + H_{\ddot{U}_i}(\omega))\}\{\int_{-\infty}^{\infty} e^{i\omega t}(\dot{u}_g + \dot{u}_0)dt\}d\omega \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty}\{\textstyle\sum_{i=1}^{N} m_i\ddot{U}_g(\omega)(1 + H_{\ddot{U}_i}(\omega))\}\{\dot{U}_g(-\omega) + \dot{U}_0(-\omega)\}d\omega \qquad (8) \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty}\{\textstyle\sum_{i=1}^{N} m_i\ddot{U}_g(\omega)(1 + H_{\ddot{U}_i}(\omega))\}\{\ddot{U}_g(-\omega)(1 + H_{\ddot{U}_0}(-\omega))/(-i\omega)\}d\omega \\
&= -\frac{1}{\pi}\int_{0}^{\infty}\frac{1}{\omega}\mathrm{Im}[\{\textstyle\sum_{i=1}^{N} m_i(1 + H_{\ddot{U}_i}(\omega))\}\{1 + H_{\ddot{U}_0}(-\omega)\}]\cdot|\ddot{U}_g(\omega)|^2 d\omega
\end{aligned}
$$

Let us simplify the expressions of Eqs. (6) and (8) by use of the functions $F_A(\omega)$ and $F_S(\omega)$ to be called energy transfer functions.

$$E_I^A = \int_0^\infty F_A(\omega) \left|\ddot{U}_g(\omega)\right|^2 d\omega \tag{9a}$$

$$E_I^S = \int_0^\infty F_S(\omega) \left|\ddot{U}_g(\omega)\right|^2 d\omega \tag{9b}$$

where

$$F_A(\omega) = -\frac{1}{\pi\omega}\mathrm{Im}\left[\sum_{i=0}^N m_i H_{\ddot{U}_i}(\omega)\right] \tag{10a}$$

$$F_S(\omega) = -\frac{1}{\pi\omega}\mathrm{Im}\left[\left\{\sum_{i=1}^N m_i(1 + H_{\ddot{U}_i}(\omega))\right\}\left\{1 + H_{\ddot{U}_0}^*(\omega)\right\}\right] \tag{10b}$$

The symbol $(\ )^*$ in Eq. (10) denotes complex conjugate. It is interesting to note that, while the input energy cannot be decomposed into the term for the structural parameters and that for the input parameters in the conventional time-domain approach (Eqs. (4), (7)), it is possible in the proposed frequency-domain approach (Eqs. (9a, b)).

3. Uncertain-parameter sensitivities of earthquake input energy to overall system and structure

Let the symbol $(\ )'$ denote the differentiation with respect to one of the uncertain parameters k_0 and c_0. The sensitivity of the earthquake input energy to the structure with respect to one of the uncertain parameters may be expressed by

$$E_I^{S\prime} = \int_0^\infty F_S(\omega)' \left|\ddot{U}_g(\omega)\right|^2 d\omega \tag{11}$$

where

$$F_S(\omega)' = -\frac{1}{\pi\omega}\mathrm{Im}\left[\left\{\sum_{i=1}^N m_i H_{\ddot{U}_i}(\omega)'\right\}\left\{1+H_{\ddot{U}_0}^*(\omega)\right\} + \left\{\sum_{i=1}^N m_i(1+H_{\ddot{U}_i}(\omega))\right\} H_{\ddot{U}_0}^*(\omega)'\right] \tag{12}$$

Similarly, the sensitivity of the earthquake input energy to the overall BI building with respect to one of the uncertain parameters may be expressed by

$$E_I^{A\prime} = \int_0^\infty F_A(\omega)' \left|\ddot{U}_g(\omega)\right|^2 d\omega \tag{13}$$

where

$$F_A(\omega)' = -\frac{1}{\pi\omega}\mathrm{Im}\left[\sum_{i=0}^N m_i H_{\ddot{U}_i}(\omega)'\right] \tag{14}$$

Let us define $\mathbf{H}_U = \{H_{U_0}(\omega)\ \ H_{U_1}(\omega)\ \dots\ H_{U_N}(\omega)\}^T$. It is noted that

$$\mathbf{H}_U = -\frac{1}{\omega^2}\left\{H_{\ddot{U}_0}(\omega)\ \ H_{\ddot{U}_1}(\omega)\ \ \dots\ \ H_{\ddot{U}_N}(\omega)\right\}^T \tag{15}$$

The tri-diagonal coefficient matrix of the Fourier transformed equation of Eq. (1) may be expressed as

$$\mathbf{A} = -\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K} \tag{16}$$

Then the transfer function $\mathbf{H}_U$ can be derived in closed form (Yanai 1980, Takewaki *et al.* 1996, Takewaki 1999) as

$$\mathbf{H}_U = -\mathbf{A}^{-1}\mathbf{M1} \tag{17}$$

It is important to keep in mind that only the first component $H_{U_0}(\omega)$ in $\mathbf{H}_U$ is needed in the following analysis. Therefore the first row of the symmetric matrix $\mathbf{A}^{-1}$is required which is the same as the transpose of the first column. The closed-form expression of the first column of $\mathbf{A}^{-1}$ is given in Appendix I and $H_{U_0}(\omega)$ can be obtained in closed form.

Note that the differentiation of the equations of motion $\mathbf{AH}_U = -\mathbf{M1}$ in the frequency domain with respect to an uncertain parameter provides $\mathbf{AH}_U' + \mathbf{A}'\mathbf{H}_U = \mathbf{0}$. Then the sensitivities of the transfer function with respect to the uncertain parameters may be derived in closed form as

$$\frac{\partial}{\partial k_0}\mathbf{H}_U = -\mathbf{A}^{-1}\left(\frac{\partial \mathbf{A}}{\partial k_0}\right)\mathbf{H}_U \tag{18a}$$

$$\frac{\partial}{\partial c_0}\mathbf{H}_U = -\mathbf{A}^{-1}\left(\frac{\partial \mathbf{A}}{\partial c_0}\right)\mathbf{H}_U \tag{18b}$$

where

$$\frac{\partial \mathbf{A}}{\partial k_0} = \mathrm{diag}(1 \;\; 0 \;\; \dots \;\; 0) \tag{19a}$$

$$\frac{\partial \mathbf{A}}{\partial c_0} = i\omega \times \mathrm{diag}(1 \;\; 0 \;\; \dots \;\; 0) \tag{19b}$$

It should be noted that, because of the expression of Eqs. (19a, b), only the first component $H_{U_0}(\omega)$ in $\mathbf{H}_U$ and only the first column of $\mathbf{A}^{-1}$ are needed in Eqs. (18a, b). $H_{U_0}(\omega)$ has been obtained in closed form in Eq. (17).

Let us define an uncertain parameter vector $\boldsymbol{\alpha} = \{\alpha_1 \;\; \alpha_2\}^T = \{k_0 \;\; c_0\}^T$. Then the earthquake input energy around a nominal parameter vector may be expressed approximately by

$$E_I^S(\boldsymbol{\alpha} + \Delta\boldsymbol{\alpha}) \cong E_I^S(\boldsymbol{\alpha}) + \sum_{i=1}^{2} \frac{\partial E_I^S(\boldsymbol{\alpha})}{\partial \alpha_i}\Delta\alpha_i \tag{20}$$

$$E_I^A(\boldsymbol{\alpha} + \Delta\boldsymbol{\alpha}) \cong E_I^A(\boldsymbol{\alpha}) + \sum_{i=1}^{2} \frac{\partial E_I^A(\boldsymbol{\alpha})}{\partial \alpha_i}\Delta\alpha_i \tag{21}$$

Eqs. (20), (21) can be used as an effective prediction tool of the most unfavorable combination of the uncertain parameters k_0 and c_0. If the sensitivity $\partial E_I^S(\boldsymbol{\alpha})/\partial\alpha_i$ (or $\partial E_I^A(\boldsymbol{\alpha})/\partial\alpha_i$) is positive, the corresponding increment $\Delta\alpha_i$ should be chosen as a positive value. On the contrary, if the sensitivity

$\partial E_I^S(\boldsymbol{\alpha})/\partial\alpha_i$ (or $\partial E_I^A(\boldsymbol{\alpha})/\partial\alpha_i$) is negative, the corresponding increment $\Delta\alpha_i$ should be chosen as a negative value. It should be noted that this treatment is valid only if the range of variation of the uncertain parameters k_0 and c_0 is relatively narrow and the degree of nonlinearity of $E_I^S(\boldsymbol{\alpha}), E_I^A(\boldsymbol{\alpha})$ with respect to $\boldsymbol{\alpha}$ is small. In case that the degree of nonlinearity is not small, higher-order approximation should be introduced. The higher-order coefficients may be derived by differentiating the equation $\mathbf{A}\mathbf{H}_U' + \mathbf{A}'\mathbf{H}_U = \mathbf{0}$. For example, the second-order coefficients may be derived as

$$\mathbf{H}_U'' = -2\mathbf{A}^{-1}\mathbf{A}'\mathbf{H}_U' \tag{22}$$

$$\dot{\mathbf{H}}_U' = -\mathbf{A}^{-1}(\dot{\mathbf{A}}\mathbf{H}_U' + \mathbf{A}'\dot{\mathbf{H}}_U) \tag{23}$$

where prime and super-dot denote differentiation with respect to distinct uncertain parameters, i.e., k_0 or c_0. It is interesting to note that $\mathbf{A}'$ and $\dot{\mathbf{A}}$ include non-zero components only in (1, 1) and the first column of $\mathbf{A}^{-1}$ alone is needed in Eqs. (22) and (23). The closed-form expression of the first column of $\mathbf{A}^{-1}$ is shown in Appendix I.

4. Numerical examples

4.1 Earthquake input energy

Numerical examples for 5-story, 10-story and 15-story shear building models with BI systems are presented. Originally BI systems were applied to rather low-rise buildings. However, BI systems are being installed in mid-rise or high-rise buildings especially in Japan. The floor masses are $m_i = 3.20 \times 10^5$ (kg)(i = 1, ..., N) and $m_0 = 9.60 \times 10^5$ (kg). The story stiffnesses of the buildings are determined so that the 5-story, 10-story and 15-story shear building models with fixed-base have the fundamental natural periods of 0.5(s), 1.0, 1.5, respectively, and their lowest eigenmodes of the models with fixed-base are straight. As a result, the lowest eigenmode (superstructural part) of the BI system is not straight. This treatment is based on the inverse problem approach (Nakamura and Yamane 1986, Takewaki 2000). It is also assumed that the damping matrix of the superstructure with fixed base is proportional to the stiffness matrix of the superstructure and the damping ratio in the lowest mode of the superstructure with fixed base is 0.05. The stiffness k_0 and damping coefficient c_0 of the BI system have been determined so that the natural periods of the base-isolated rigid building models ($k_i \to \infty$ (i = 1, ..., N)) are 3.0(s), 4.0, 5.0, respectively, for the 5-story, 10-story and 15-story shear building models and the damping ratio of the base-isolated rigid building models is 0.15.

Figs. 3(a)-(c) show the plots of the energy transfer functions $F_A(\omega)$, $F_S(\omega)$ for the 5-story, 10-story and 15-story shear building models. The unit of the ordinate is $\mathrm{N}\cdot\mathrm{s}^3/\mathrm{m}$. It can be observed that $F_A(\omega)$ has the peak around the natural circular frequency of the BI building and $F_S(\omega)$ is negligible compared to $F_A(\omega)$. It is also seen in Fig. 3 that $F_S(\omega)$ have multiple peaks.

Table 1 shows the comparison of input energies to overall system E_I^A under El Centro NS (Imperial Valley 1940) by the conventional method in time domain and by the proposed method in frequency domain. The time increment of the ground motion acceleration data and of the time-domain numerical analysis is 0.02(s). The Fourier amplitude of El Centro NS (Imperial Valley 1940) is plotted in Fig. 4. In the conventional time-domain method, the average acceleration method

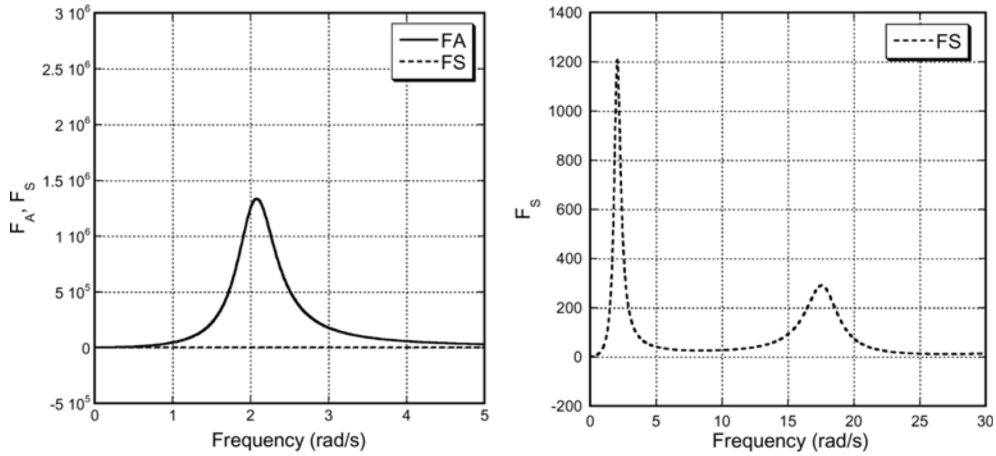


Fig. 3(a) Energy transfer functions $F_A(\omega)$, $F_S(\omega)$ for 5-story model (unit: N · s^3/m)

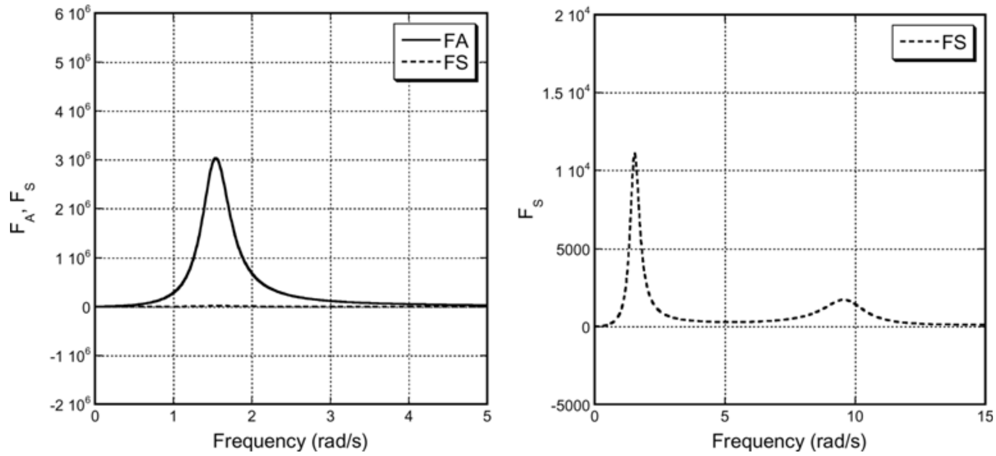


Fig. 3(b) Energy transfer functions $F_A(\omega)$, $F_S(\omega)$ for 10-story model

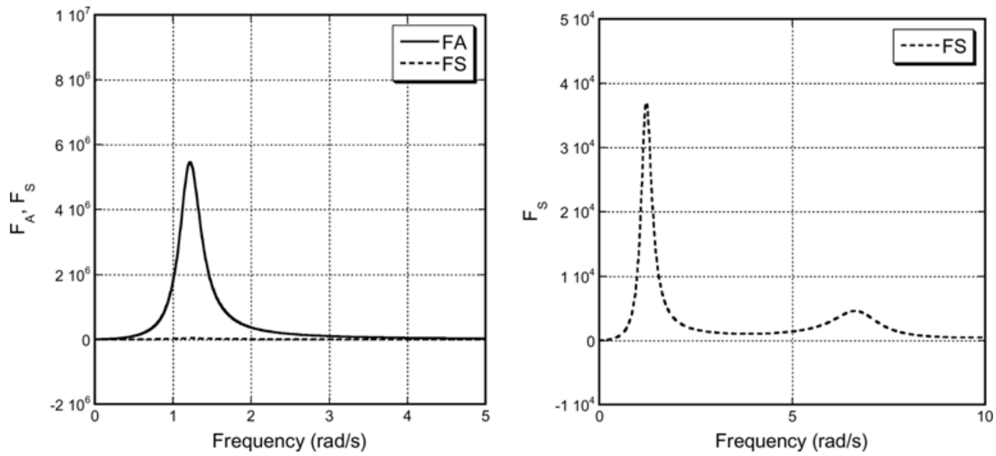
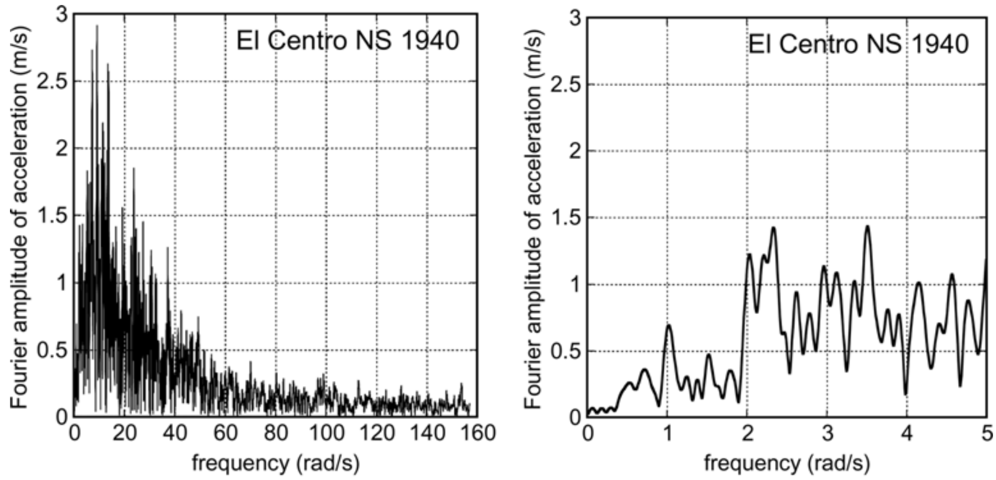


Fig. 3(c) Energy transfer functions $F_A(\omega)$, $F_S(\omega)$ for 15-story model

Table 1 Comparison of input energies to overall system E_I^A under El Centro NS 1940 by the conventional method in time domain (T) and by the proposed method in frequency domain (F)

	Model	5-story	10-story	15-story
E_I^A (J)	T	0.990×10^6	0.671×10^6	0.678×10^6
	F	0.982×10^6	0.667×10^6	0.682×10^6



(a) overview (b) magnified

Fig. 4 Fourier amplitude of acceleration of El Centro NS (Imperial Valley 1940)

has been used as the numerical integration scheme of the equation of motion and the trapezoidal rule has been used for integration of Eq. (4). On the other hand, in the proposed frequency-domain method, the trapezoidal rule has been used for numerical integration of Eq. (6). Furthermore, the trailing zeros have been added to the original ground motion to enhance the accuracy of analysis in the frequency domain. As a result, time duration as eight times the original time duration has been adopted. It can be concluded that the proposed frequency-domain approach corresponds to the time-domain approach within the accuracy of 1% in the computation of the overall input energy E_I^A.

It is interesting to note that the overall input energy to the 10-story and 15-story models is smaller than that to the 5-story model.

4.2 Sensitivity of earthquake input energy with respect to uncertain parameters

The stiffness and damping coefficient of the base-isolation system evaluated in the previous section are referred to as the nominal values. The sensitivities of the earthquake input energies with respect to uncertain parameters are considered at this point.

The solid lines in Fig. 5(a) illustrate the sensitivities $\partial F_A(\omega)/\partial k_0$, $\partial F_S(\omega)/\partial k_0$ for the 5-story model obtained from the closed-form expression, Eqs. (12), (14). The dotted lines in Fig. 5(a) indicate the sensitivities $\partial F_A(\omega)/\partial c_0$, $\partial F_S(\omega)/\partial c_0$ for the 5-story model. It can be seen that, while both positive and negative values can appear in the stiffness sensitivities $\partial F_A(\omega)/\partial k_0$ and $\partial F_S(\omega)/\partial k_0$

around the fundamental natural frequency of the BI building, large absolute values of negative numbers can appear in the damping sensitivities $\partial F_A(\omega)/\partial c_0$ and $\partial F_S(\omega)/\partial c_0$ around such frequency range. This is because, while the frequency, i.e., the fundamental natural frequency of the BI building, giving the peak value of the energy transfer function shifts according to the variation of the stiffness of the BI system, that does not change in the case of the variation of the damping coefficient of the BI system. If the frequency corresponding to the peak value shifts to the positive direction, the sensitivity in the range of frequency lower than the fundamental natural frequency of the BI building becomes negative and that in the range of frequency higher than the fundamental natural frequency of the BI building becomes positive. The corresponding figures for 10-story and 15-story models are shown in Figs. 5(b) and (c).

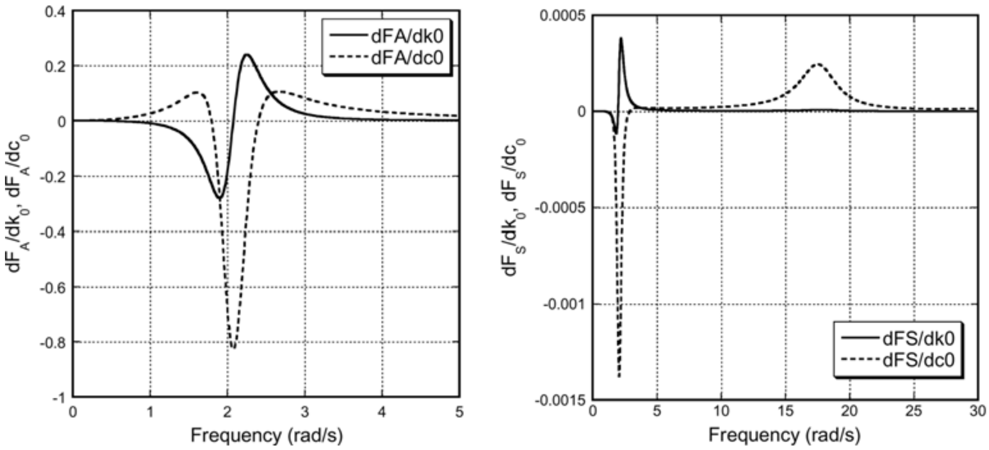


Fig. 5(a) Sensitivities of energy transfer functions $F_A(\omega)$, $F_S(\omega)$ for 5-story model with respect to k_0, c_0 by proposed analytical method (unit for $\partial F_A/\partial k_0$, $\partial F_S/\partial k_0$ is s^3 and that for $\partial F_A/\partial c_0$, $\partial F_S/\partial c_0$ is s^2)

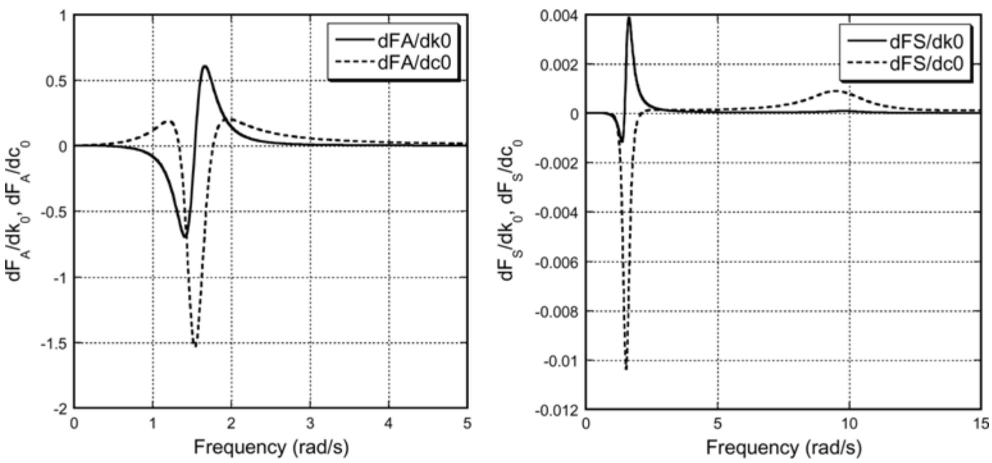


Fig. 5(b) Sensitivities of energy transfer functions $F_A(\omega)$, $F_S(\omega)$ for 10-story model with respect to k_0, c_0 by proposed analytical method

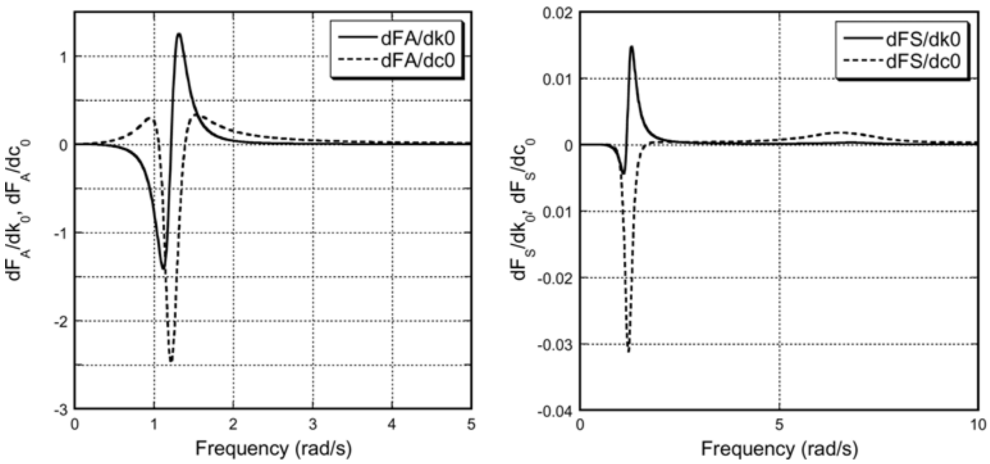


Fig. 5(c) Sensitivities of energy transfer functions $F_A(\omega)$, $F_S(\omega)$ for 15-story model with respect to k_0, c_0 by proposed analytical method

Table 2 Input energies and their sensitivities with respect to stiffness and damping parameters in base-isolated system

Model	5-story	10-story	15-story
E_I^A (J)	0.982×10^6	0.667×10^6	0.682×10^6
E_I^S (J)	0.210×10^4	0.132×10^5	0.256×10^5
$\partial E_I^A/\partial k_0$ (m^2)	0.885×10^{-1}	0.415×10^{-1}	-0.233×10^{-1}
$\partial E_I^A/\partial c_0$ (m^2/s)	-0.785×10^{-1}	0.198×10^0	0.171×10^0
$\partial E_I^S/\partial k_0$ (m^2)	0.263×10^{-3}	0.104×10^{-2}	0.185×10^{-2}
$\partial E_I^S/\partial c_0$ (m^2/s)	0.654×10^{-3}	0.678×10^{-2}	0.115×10^{-1}

Table 2 shows the input energy sensitivities under El Centro NS 1940 with respect to uncertain parameters by the proposed method in frequency domain (Eqs. (11) and (13)). It is noted that, while the present evaluation method includes the closed-form expression in the integrand and is reliable from this point of view, it requires a numerical integration in the frequency domain which does not reduce the reliability. In this sense, it may be appropriate to call the present method a nearly exact method. To check the accuracy of the proposed method, the finite difference method (1% difference scheme) has been adopted. For the nearly exact value of $\partial E_I^A/\partial k_0$ as 0.885×10^{-1} by Eq. (13), the value by the finite difference method through the proposed frequency-domain approach is 0.873×10^{-1} (1.4% error) and that through the conventional time-domain approach is 0.837×10^{-1} (5.4% error). Furthermore, for the nearly exact value of $\partial E_I^A/\partial c_0$ as -0.785×10^{-1} by Eq. (13), the value by the finite difference method through the proposed frequency-domain approach is -0.746×10^{-1} (5.0% error) and that through the conventional time-domain approach is -0.871×10^{-1} (11.0% error). These results imply that the proposed frequency-domain approach is more reliable than the conventional time-domain approach in the computation of input energy sensitivities.

It should be noted that, while the response $\dot{\mathbf{u}}, \ddot{\mathbf{u}}$ and its response sensitivity $\dot{\mathbf{u}}', \ddot{\mathbf{u}}'$ in the integrand of the differentiated expression $E_I^{S\prime}, E_I^{A\prime}$ of Eqs. (4) and (7) with respect to an uncertain parameter have to be evaluated by some numerical integration schemes, the proposed method through Eqs. (11)-(19) includes the closed-form expression in the integrand of Eqs. (11) and (13). This characteristic leads to a stable and reliable result in the input energy sensitivity computation.

Fig. 6 presents the comparison of sensitivities of the energy transfer functions $F_A(\omega)$, $F_S(\omega)$ for 5-story model with respect to k_0, c_0 by the proposed analytical method and by the finite difference method (1% difference scheme). It can be observed that the result by the proposed method corresponds fairly well to the result by the finite difference method and it may be said that the proposed method is reliable.

Fig. 7 shows the variation of the input energy E_I^A with respect to variation of uncertain parameters k_0 and c_0 (80% - 120% of nominal values). It can be seen that, while a linear approximation may be possible for damping variation, a higher-order approximation may be necessary for stiffness variation. With the help of Eq. (20), it can be understood that the most unfavorable combination of uncertain parameters corresponds to the largest value of k_0 and the smallest value of c_0. The structural designer can obtain useful information from this analysis for the degree of robustness of the designed BI building.

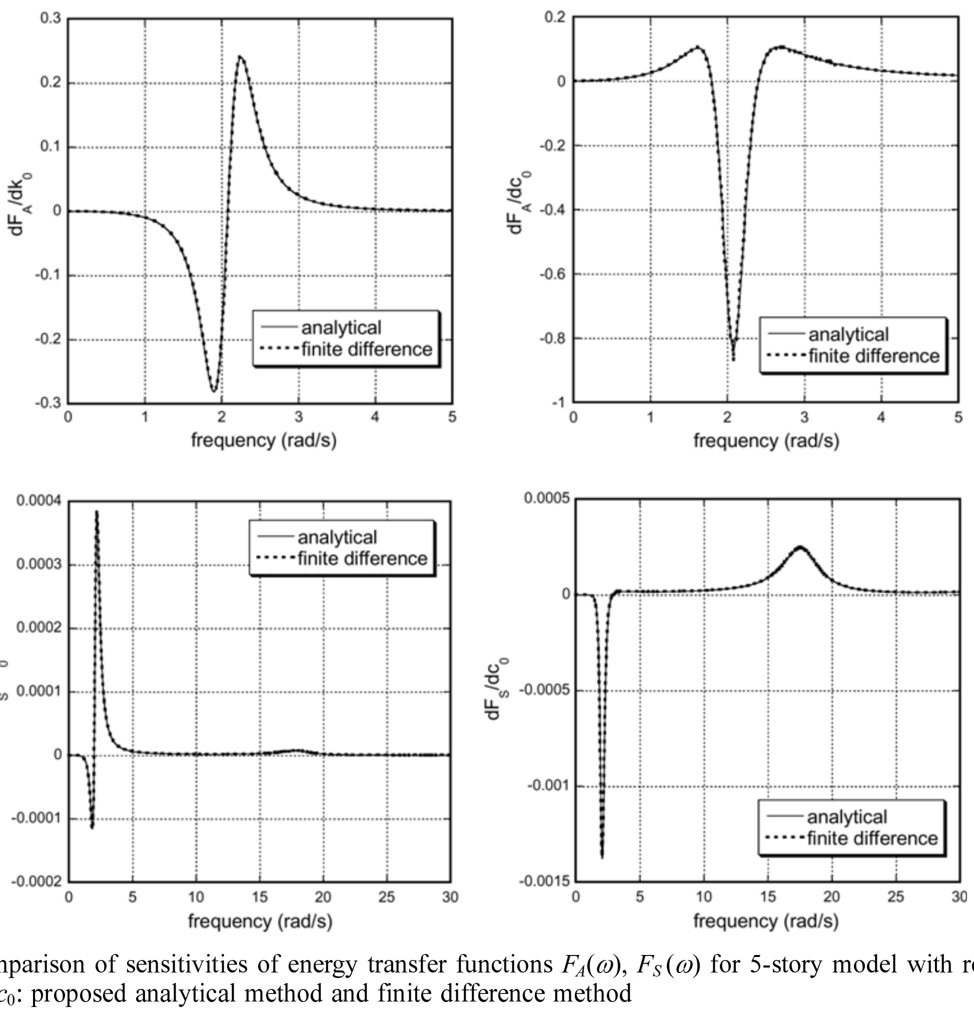


Fig. 6 Comparison of sensitivities of energy transfer functions $F_A(\omega)$, $F_S(\omega)$ for 5-story model with respect to k_0, c_0: proposed analytical method and finite difference method

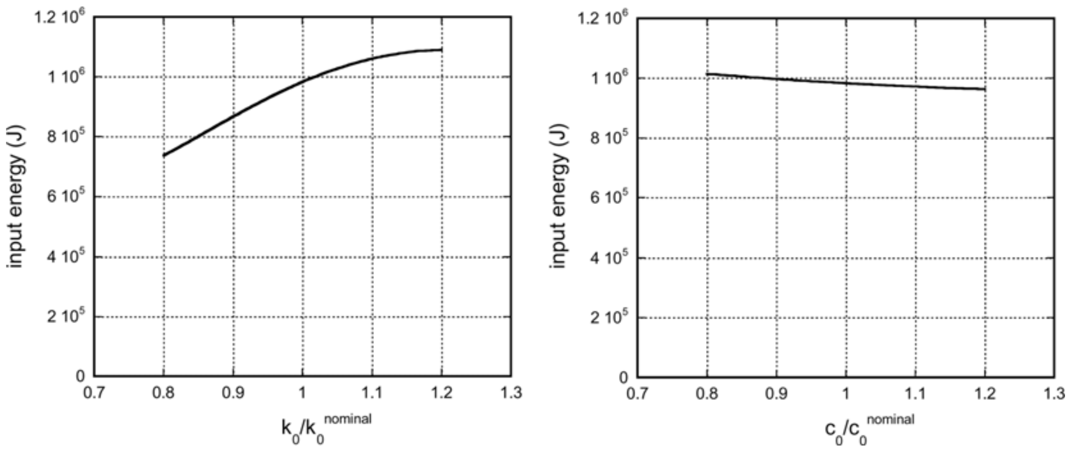


Fig. 7 Variation of earthquake input energy with respect to uncertain parameters

5. Conclusions

The following conclusions may be drawn.

(1) The earthquake input energy is an appropriate performance measure in the uncertainty analysis of base-isolated buildings which consist of multiple components with completely different properties. It can provide a global performance measure for base-isolated buildings compared to deformation indices.
(2) The earthquake input energies to a superstructure and to an overall base-isolated building can be obtained in a compact form by taking advantage of a frequency-domain approach. The transfer function necessary in the evaluation of the input energy in the frequency domain can be obtained in closed form by utilizing an explicit expression of the inverse of the coefficient matrix in the equations of motion in the frequency domain.
(3) The proposed frequency-domain method has a reasonable accuracy in the computation of earthquake input energy in comparison with the conventional time-domain method.
(4) The sensitivities of the earthquake input energies to a structure and to an overall base-isolated building with respect to uncertain base-isolation parameters can also be obtained in closed form by taking advantage of the frequency-domain approach. The sensitivity of the transfer function needed in the evaluation of the sensitivities of the earthquake input energies to a structure and to an overall base-isolated building can be derived in closed form by using the equations of motion in the frequency domain.
(5) It has been confirmed through comparison with results by the finite difference method that the proposed method has a reasonable accuracy and its reliability and efficiency are remarkable.

In this paper, only the linear elastic response has been considered. In the case where base-isolation devices exhibit non-linear responses, deterministic and stochastic equivalent linearization techniques could be used. This topic will be discussed in the future. It is commonly recognized that the degree of uncertainty of structural parameters in fixed-base buildings is smaller than that in BI buildings, especially in base-isolation devices. In this sense, the present uncertainty analysis plays a key role in BI buildings.

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Appendix I: Inverse of matrix A defined in Eq. (16)

Let us re-express the matrix **A** defined in Eq. (16) as

$$\mathbf{A} = \begin{bmatrix} d_0 & -e_1 & & \mathbf{0} \\ -e_1 & \ddots & \ddots & \\ & \ddots & d_{N-1} & -e_N \\ \mathbf{0} & & -e_N & d_N \end{bmatrix} \tag{A1}$$

where d_j, e_j are as follows.

$$d_j = k_j + k_{j+1} + i\omega(c_j + c_{j+1}) - \omega^2 m_j \; (j = 0, ..., N-1) \tag{A2}$$

$$d_N = k_N + i\omega c_N - \omega^2 m_N \tag{A3}$$

$$e_j = k_j + i\omega c_j \; (j = 1, ..., N) \tag{A4}$$

Let us define the following ordered set $\{P_j\}$ of the principal minors of **A**.

$$P_0 = 1, \; P_1 = d_N, \; P_2 = \begin{vmatrix} d_{N-1} & -e_N \\ -e_N & d_N \end{vmatrix}, \; \ldots, \; P_{N+1} = \det\mathbf{A} \tag{A5}$$

It is known that $\{P_j\}$ satisfies the following recurrence relation.

$$P_{j-1} = d_{N+2-j}P_{j-2} - e_{N+3-j}^{\,2}P_{j-3} \quad (j = 3, \ldots, N+1) \tag{A6}$$

It is therefore concluded that $\{P_j\}$ and det **A** can be obtained systematically without difficulty. Then the first column of the inverse of **A** may be expressed as Yanai (1980)

$$\frac{1}{\det\mathbf{A}}\left\{P_N \;\; e_1 P_{N-1} \; \ldots \; \left(\prod_{i=1}^{N+1-j} e_i\right)P_{j-1} \;\; \left(\prod_{i=1}^{N+1-(j-1)} e_i\right)P_{j-2} \; \ldots \; \left(\prod_{i=1}^{N+1-1} e_i\right)P_0\right\}^T \tag{A7}$$