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Free vibration analysis of rotating beams with random properties

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Abstract. In this paper, free vibration of rotating beam with random properties is studied. The cross-sectional area, elasticity modulus, moment of inertia, shear modulus and density are modeled as random fields and the rotational speed as a random variable. To study uncertainty, stochastic finite element method based on second order perturbation method is applied. To discretize random fields, the three methods of midpoint, interpolation and local average are applied and compared. The effects of rotational speed, setting angle, random property variances, discretization scheme, number of elements, correlation of random fields, correlation function form and correlation length on "Coefficient of Variation" (C.O.V.) of first mode eigenvalue are investigated completely. To determine the significant random properties on the variation of first mode eigenvalue the sensitivity analysis is performed. The results are studied for both Timoshenko and Bernoulli-Euler rotating beam. It is shown that the C.O.V. of first mode eigenvalue of Timoshenko and Bernoulli-Euler rotating beams are approximately identical. Also, compared to uncorrelated random fields, the correlated case has larger C.O.V. value. Another important result is, where correlation length is small, the convergence rate is lower and more number of elements are necessary for convergence of final response.

Key words: rotating beam; random properties; random fields; stochastic finite element method; second order perturbation.

1. Introduction

The vibration analysis of rotating beam is important in understanding behavior of rotating structures such as propellers, turbine blades, helicopter rotors and satellite booms. To study vibration of rotating beams, Abbas (1986) and Yokoyama (1988) used finite element methods. Lee and Lin (1994) studied the vibration of non-uniform rotating beam by neglecting the Coriolis effect. Yang and Tsao (1997) studied the vibration and stability of a pretwisted blade under nonconstant rotating speed. Pohit *et al.* (1999) investigated free out-of-plane vibration of a rotating beam with a nonlinear elastomeric constraint. Al-Bedoore and Hamdan (2001) developed a mathematical method for a rotating flexible arm undergoing large planar flexural deformations. Bazoune *et al*. (2001) used finite element method to study the dynamic response of spinning tapered Timoshenko beams. Vibration of a rotating damped blade with an elastically restrained root was investigated by Lin *et al.*

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(2004). The comparison between Bernoulli-Euler and Timoshenko beam theories in a composite rotating beam is performed by Jung *et al.* (2001) and Chandiramani *et al.* (2003).

Aforementioned studies are limited to vibration of rotating beams with deterministic properties. However, in reality, properties of structures and mechanical systems are random. The first study in vibration analysis of a system with random properties was carried out by Boice and Goodman (1964) using perturbation method. Singh (1985) studied the turbine blade reliability with random properties. Iwan and Haung (1996) presented a procedure for obtaining the dynamic response of nonlinear system with parameter uncertainty. Chakraborty and Dey (1998) proposed a stochastic finite element method in the frequency domain for analysis of structural dynamic problems involving uncertain parameters. Ishida (2001) studied the eigenvalue problem of uniform and optimum beam with uncertain cross-sectional area. Lin (2000) evaluated the influence of the random parameter changes to the dynamic behavior of rotating Timoshenko beam. Also, Lin (2001) investigated the probabilistic behavior of rotating Timoshenko beam. The stochastic finite element method (SFEM) was developed for analysis of a system with random properties by Combou (1975) and Handa and Anderson (1981). An important procedure in SFEM is to discretize random fields, e.g. midpoint method (Handa and Anderson 1981), local average method (Vanmarcke and Grigoriu 1983), and interpolation method (Liu *et al.* 1986).

In this paper, free vibration of rotating beam with random properties is studied. This work is based on the Lin's paper (2000, 2001). He considered the free vibration of rotating beam with uncertain material, geometry properties and rotating speed. In his papers, these parameters are modeled as random variables, and used analytical methods for investigation of the free vibration of a rotating beam. In our work the cross-sectional area, elasticity modulus, moment of inertia, shear modulus and density are modeled as random fields and the rotational speed as a random variable. To study uncertainty, stochastic finite element method based on second order perturbation method is applied. To discretize random fields, the three methods of midpoint, interpolation and local average are applied and compared.

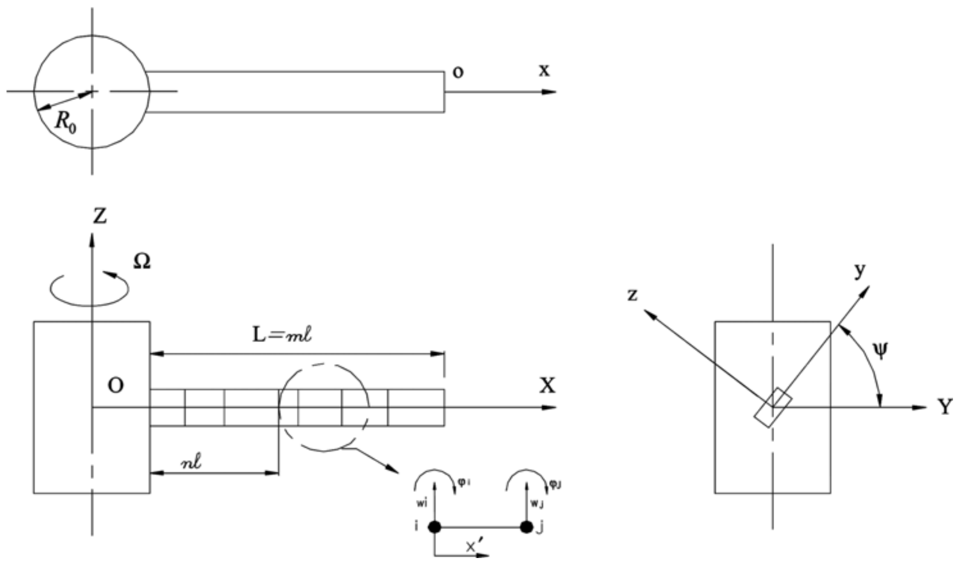

Fig. 1 Timoshenko rotating beam schematic

2. Finite element formulation of Timoshenko rotating beams

Fig. 1 shows a uniform beam with hub radius R_0 and length L, rotating about the Z-axis with constant angular velocity Ω. The origin of X, Y, Z coordinate axes is at the hub center. The origin of x, y, z coordinate axes is at the hub periphery connected to the neutral axis. The x-axis is along the beam length and coincident with the X-axis, the y-axis is along the beam width and the z-axis is along the thickness. The beam midplane makes angle Ψ with the rotational plane called setting angle. One may divide the beam into m elements ($l = L/m$) as shown in Fig. 1. Using the Timoshenko beam theory, strain energy in element level is:

$$U^e = \frac{1}{2}\int_0^l EI\left(\frac{\partial\varphi}{\partial x'}\right)^2 dx' + \frac{1}{2}\int_0^l KGA\left(\frac{\partial w}{\partial x'} - \varphi\right)^2 dx' + \frac{1}{2}\int_0^l N_{x'}\left(\frac{\partial w}{\partial x'}\right)^2 dx' \tag{1}$$

The first, second and third terms are bending, shear and centrifugal strain energies, respectively. The centrifugal force $N_{x'}$ is calculated as follows:

$$N_{x'} = A\int_{x'}^l \rho\Omega^2(R_0 + nl + x')dx' + A\int_{(n+1)l}^{ml} \rho\Omega^2(R_0 + x)dx =$$

$$\rho A\Omega^2 l^2\left[\frac{R_0}{l}(m-n) + \frac{1}{2}(m^2 - n^2) - \left(\frac{R_0}{l} + n\right)\left(\frac{x'}{l}\right) - \frac{1}{2}\left(\frac{x'}{l}\right)^2\right] \tag{2}$$

and the kinetic energy is calculated as:

$$T^e = \frac{1}{2}\int_0^l \rho A\left(\frac{\partial w}{\partial t}\right)^2 dx' + \frac{1}{2}\int_0^l \rho I\left(\frac{\partial\varphi}{\partial t}\right)^2 dx' \tag{3}$$

due to translational and rotational kinetic energy contributions.

Due to centrifugal force $F_{z'}$ that affects in Z direction, the potential energy is obtained as:

$$W^e = \frac{1}{2}\int_0^l F_{z'} w A dx' \tag{4}$$

where $F_{z'} = \rho\Omega^2 w \sin^2\psi$. Using two-node elements with 8 degrees of freedom for beam discretization, the transverse displacement w and bending rotation φ in each element length are approximated by Hermite shape functions with respect to nodal displacements:

$$\begin{aligned} w &= \{f_1,\ 0,\ f_3,\ 0,\ f_5,\ 0,\ f_7,\ 0\}\{q\} = \{N_w\}^T\{q\} \\ \varphi &= \{0,\ f_2,\ 0,\ f_4,\ 0,\ f_6,\ 0,\ f_8\}\{q\} = \{N_\varphi\}^T\{q\} \\ \{q\}^T &= \{w_i,\ \varphi_i,\ w_i',\ \varphi_i',\ w_j,\ \varphi_j,\ w_j',\ \varphi_j'\} \end{aligned} \tag{5}$$

where ($'$) shows derivative with respect to spatial variable and

$$\begin{aligned} f_1 = f_2 &= 1 - 3(\xi)^2 + 2\xi^3 & f_3 = f_4 &= \xi - 2\xi^3 + \xi^3 \\ f_5 = f_6 &= 3\xi^2 - 2\xi^3 & f_7 = f_8 &= -\xi^2 + \xi^3 \end{aligned} \tag{6}$$

where $\zeta = \frac{x'}{l}$. Hence, the stiffness and mass matrices are obtained:

$$
\begin{aligned}
[K_b]^e &= \int_0^l \{B_b\} EI \{B_b\}^T dx' \\
[K_s]^e &= \int_0^l \{B_s\} KGA \{B_s\}^T dx' \\
[K_c]^e &= \int_0^l N_{x'} \{B_w\}\{B_w\}^T dx' \\
[M_t]^e &= \int_0^l \{N_w\} \rho A \{N_w\}^T dx' \\
[M_r]^e &= \int_0^l \{N_\varphi\} \rho I \{N_\varphi\}^T dx'
\end{aligned}
\tag{7}
$$

Where

$$
\{B_b\} = \frac{\partial}{\partial x'}\{N_\varphi\} \quad \text{and} \quad \{B_s\} = \frac{\partial}{\partial x'}\{N_w\} - \{N_\varphi\} = \{B_w\} - \{N_\varphi\}
$$

Using the usual assemblage of matrices, one may obtain stiffness and mass matrices. Finally, using the Hamilton principle, the following equation is obtained for vibration of rotating beam:

$$
([K_0] - \Omega^2 \sin^2 \psi [M_t])\{q\} + [M]\{\ddot{q}\} = \{0\} \tag{8}
$$

Where

$$
[M] = [M_r] + [M_t] \quad \text{and} \quad [K_0] = [K_b] + [K_s] + [K_c]
$$

For a harmonic motion, the following eigenvalue equation is obtained:

$$
([K] - \lambda [M])\{q\}^* = \{0\} \tag{9}
$$

Where $[K] = [K_0] - \Omega^2 \sin^2 \psi [M_t]$, and λ is the eigenvalue of rotating beam and $\{q\}^*$ is the vibration amplitude.

3. Free vibration of rotating beam with random properties

In this paper, we assume that the rotating beam has uncertainty in its properties, i.e., elasticity modulus, shear modulus, moment of inertia and density which are modeled as random fields and the rotational speed as a random variable. If α_i, $i = 1 \dots 6$ is a random field then using $\alpha_i = \bar{\alpha}_i(1 + \varepsilon_i)$ where $\bar{\alpha}_i$ is expectation of α_i, the random field ε_i can be obtained with zero expectation.

To consider for uncertainty in Eq. (9), the second-order perturbation method is applied. In this method, all random elements of an equation are expanded about expectation of random variables using the Taylor series up to second order terms. If this method is applied to Eq. (9), one gets:

$$
\begin{aligned}
[M] &= [M]^0 + [M]^{,p}\varepsilon_p + \frac{1}{2}[M]^{,ps}\varepsilon_p\varepsilon_s \\
[K] &= [K]^0 + [K]^{,p}\varepsilon_p + \frac{1}{2}[K]^{,ps}\varepsilon_p\varepsilon_s \\
[q] &= [q]^0 + [q]^{,p}\varepsilon_p + \frac{1}{2}[q]^{,ps}\varepsilon_p\varepsilon_s \\
[\lambda] &= [\lambda]^0 + [\lambda]^{,p}\varepsilon_p + \frac{1}{2}[\lambda]^{,ps}\varepsilon_p\varepsilon_s
\end{aligned}
\qquad p, s = 1 \dots N
\tag{10}
$$

where N is total number of random variables. Substituting Eq. (10) in Eq. (9):

$$
\begin{aligned}
&([K]^0 - \lambda^0[M]^0)\{q\}^0 = \{0\} \\
&([K]^0 - \lambda^0[M]^0)\{q\}^{,p} = ([K]^{,p} - \{\lambda\}^{,p}[M]^0 - \{\lambda\}^0[M]^{,p})\{q\}^0 \\
&([K]^0 - \lambda^0[M]^0)\{q\}^{,ps} = ([K]^{,ps} - \{\lambda\}^{,ps}[M]^0 - 2\lambda^{,p}[M]^{,s} - \lambda^0[M]^{,ps})\{q\}^0 \\
&\quad + 2([K]^{,p} - \lambda^{,p}[M]^0 - \lambda^0[\lambda]^{,p})\{q\}^{,s}
\end{aligned}
\tag{11}
$$

Note here, the summation law for repeated indices is applied. The comma shows partial derivative, and $[.]^0$ and $\{.\}^0$ show deterministic value of matrix and vector. Eqs. (11) should be solved successively to obtain the final response.

4. Calculation of first and second derivative of eigenvalues and eigenvectors

It is seen from Eqs. (11) that to solve the above equations, the first and second derivatives of eigenvalues and eigenvectors should be calculated. By applying eigenvalue properties, the following equations can be obtained (Kleiber and Hien 1992):

$$
\lambda^{,p} = \frac{\{q\}^{0T}([K]^{,p} - \lambda^0[M]^{,p})\{q\}^0}{\{q\}^{0T}[M]^0\{q\}^0} \tag{12}
$$

$$
\lambda^{,ps} = \frac{\{q\}^{0T}([K]^{,ps} - 2\lambda^{,p}[M]^{,s} - \lambda^0[M]^{,ps})\{q\}^0}{\{q\}^{0T}[M]^0\{q\}^0} + \frac{2\{q\}^{0T}([K]^{,p} - \lambda^0[M]^{,p} - \lambda^{,p}[M]^0)\{q\}^{,s}}{\{q\}^{0T}[M]^0\{q\}^0} \tag{13}
$$

Superscript "T" shows transpose. It is necessary to note that Lin (2000, 2001) used the finite difference method to compute the derivatives of eigenvalues. Two methods are proposed for calculation of derivatives of eigenvectors (Kleiber and Hien 1992). In the first method, the derivative of the eigenvector is obtained from linear combination of all eigenvectors. In the second method, the derivative of the eigenvector is obtained by additional relations found by derivation of orthogonal relations with respect to random variables. If the second method is applied, we have:

$$
\{q\}^{,p} = -([A]^T[A]^{-1}[A]^T[B])\{q\}^0
$$

where

$$
[A] = \begin{bmatrix} [K]^0 - \lambda^0[M]^0 \\ 2\{q\}^{oT}[M]^0 \end{bmatrix}, \qquad [B] = \begin{bmatrix} [K]^{,p} - \lambda^{,p}[M]^0 - \lambda^0[M]^{,p} \\ \{q\}^{oT}[M]^{,p} \end{bmatrix} \tag{14}
$$

Upon application of expectation and variance operators to Eqs. (10) and using only the first two moments of random variables, one gets:

$$
\begin{aligned}
E[\lambda] &= \lambda^0 + \frac{1}{2}\lambda^{,ps}E[\varepsilon_p\varepsilon_s] \\
Var[\lambda] &= \lambda^{,p}\lambda^{,s}E[\varepsilon_p\varepsilon_s]
\end{aligned}
\tag{15}
$$

Using the above equations, one may calculate expectation, variance and coefficient of variation (C.O.V.) of first eigenvalue of rotating beam.

5. Sensitivity analysis of rotating beam

The purpose of sensitivity analysis is to determine that with variation of a parameter, how much variation one gets in response. One of the important advantages of sensitivity analysis is to determine the parameters, which have the most effects on the response characteristics. On the other hand, this analysis determines the random parameters that have the least effects on deviation of response; thus one can assume them as deterministic parameters.

In this paper following the sensitivity analysis of Lin (2000, 2001), the effect of uncertainty of random variables on response deviation is represented by the following equation:

$$\eta_i = \frac{\partial S_\lambda}{\partial S_{\varepsilon_i}} \frac{S_{\varepsilon_i}}{S_\lambda} \tag{16}$$

Here S_{ε_i} and S_λ are standard deviation of response and that of random variables respectively. The parameter η_i is defined as relative sensitivity factor. If Eq. (15) is used, η_i will be equal to:

$$\eta_i = (\lambda'^i)^2 \frac{S^2_{\varepsilon_i}}{S^2_\lambda} \tag{17}$$

6. Discussions

In this paper, shear modulus, $G(x)$, cross-sectional area, $A(x)$, elasticity modulus, $E(x)$, moment of inertia, $I(x)$, and density, $\rho(x)$, are modeled as random fields and rotational speed, Ω, is modeled as a random variable. It is necessary to note that Lin (2000, 2001) used the parameters "T" and "H" (thickness and width of the cross section) instead of "$I(x)$" and "$A(x)$". To discretize random fields, three methods of midpoint, interpolation and local average are applied and results are calculated using these methods. Here, the following property values are used (Lin 2001):

$$E[\rho(x)] = 7830\ (\mathrm{kg/m^2}), \quad E[E(x)] = 165.44\ (\mathrm{Gpa}), \quad E[G(x)] = 63.44\ (\mathrm{Gpa})$$

$$E[A(x)] = 6 \times 10^{-4}\ (\mathrm{m^2}), \quad E[I(x)] = 8 \times 10^{-8}\ (\mathrm{m^4}), \quad L = 0.8(\mathrm{m})$$

$$R_0 = 0.08\ (\mathrm{m}), \quad K = 0.85$$

Where $E[.]$ is expectation operator and the correlation function of random fields is defined as:

$$\rho(r) = \exp\left(-\frac{a|r|}{L}\right) \tag{18}$$

Where r is the length between two arbitrary points of the beam and "a" is a constant. The ratio L/a is called "correlation length". If $a = 0$, two points in random field are fully correlated and if $a = \infty$, the two points will be fully uncorrelated. Variance vector and correlation function matrix of random fields are defined as:

$$\{Var\} = \begin{Bmatrix} \sigma_{\varepsilon_1}^2 \\ \sigma_{\varepsilon_2}^2 \\ \sigma_{\varepsilon_3}^2 \\ \sigma_{\varepsilon_4}^2 \\ \sigma_{\varepsilon_5}^2 \end{Bmatrix}, \qquad [\rho(r)] = \begin{bmatrix} \rho_{11}(r) & \rho_{12}(r) & \rho_{13}(r) & \rho_{14}(r) & \rho_{15}(r) \\ \rho_{21}(r) & \rho_{22}(r) & \rho_{23}(r) & \rho_{24}(r) & \rho_{25}(r) \\ \rho_{31}(r) & \rho_{32}(r) & \rho_{33}(r) & \rho_{34}(r) & \rho_{35}(r) \\ \rho_{41}(r) & \rho_{42}(r) & \rho_{43}(r) & \rho_{44}(r) & \rho_{45}(r) \\ \rho_{51}(r) & \rho_{52}(r) & \rho_{53}(r) & \rho_{54}(r) & \rho_{55}(r) \end{bmatrix} \tag{19}$$

Subscripts 1, 2, 3, 4, 5 show $\rho(x), I(x), E(x), A(x), G(x)$ respectively. Correlation matrix of random fields is obtained as:

$$[R(r)] = \{Var\}[\rho(r)]\{Var\}$$

Rotational speed is defined as random variable α_6.

In Figs. 2-5 coefficient of variation (C.O.V.) of first mode eigenvalue is plotted versus rotational speed. The variance of all random fields equals to 0.01 and random fields are assumed independent and $a = +\infty$, so:

$$\{Var\} = \begin{Bmatrix} 0.01 \\ 0.01 \\ 0.01 \\ 0.01 \\ 0.01 \end{Bmatrix}, \qquad [\rho(r)] = \begin{bmatrix} A & 0 & 0 & 0 & 0 \\ 0 & B & 0 & 0 & 0 \\ 0 & 0 & C & 0 & 0 \\ 0 & 0 & 0 & D & 0 \\ 0 & 0 & 0 & 0 & E \end{bmatrix}, \qquad Var(\Omega) = 0.01F \tag{20}$$

In these figurers, four curves are plotted:

Curve (a): all properties are assumed random $(A = B = C = D = E = F = 1)$

Curve (b): only material properties are assumed random ($A = C = E = 1$ and the rest are zero)

Curve (c): only geometry properties are assumed random ($B = D = 1$ and the rest are zero),

Curve (d): only rotational speed is assumed random ($F = 1$ and the rest are zero).

The accuracy of results is investigated by Monte-Carlo method. Simulation is carried out by 10000 samples and is compared with second-order perturbation method. It is realized that there is a good agreement between second-order perturbation method and Monte-Carlo method, which exhibits accuracy of results for all rotational speed range.

Fig. 2 shows C.O.V. for Timoshenko type, where the setting angle is equal to zero. It is seen that if rotational speed is random (curve (d)), C.O.V. curve is ascending with respect to rotational speed and if material (curve (b)) and geometry (curve (c)) properties are random, C.O.V. curves are descending. If all properties are random (curve (a)), the C.O.V. curves are descending due to having random of material and geometry and then due to rotational speed they are ascending. If rotational speed increases, the effect of randomness of this parameter (Ω) increases, and consequently, the C.O.V. curve of this parameter will be ascending. On the other hand, if rotational speed increases, the mean value of the first mode eigenvalue increases. But the variance of eigenvalue remains constant (the variances of geometrical and material random properties are constant) and consequently the C.O.V (variance over mean value) curves of material and geometry will be

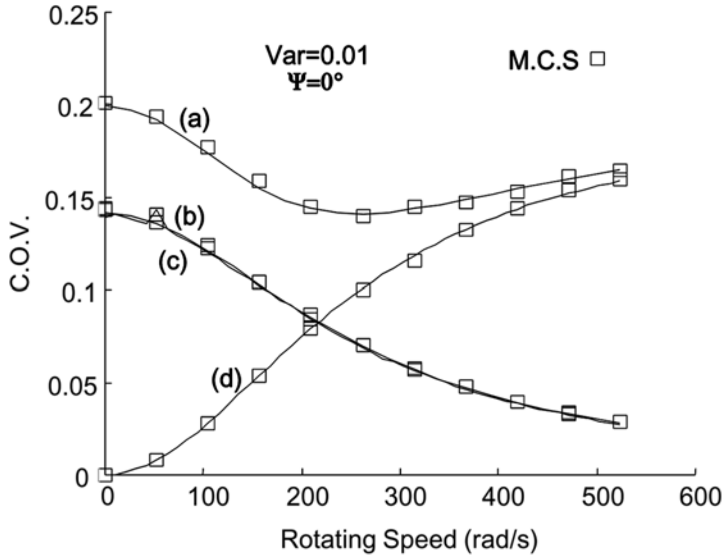

Fig. 2 C.O.V. of first eigenvalue of Timoshenko rotating beam versus rotating speed, setting angle is 0 degree

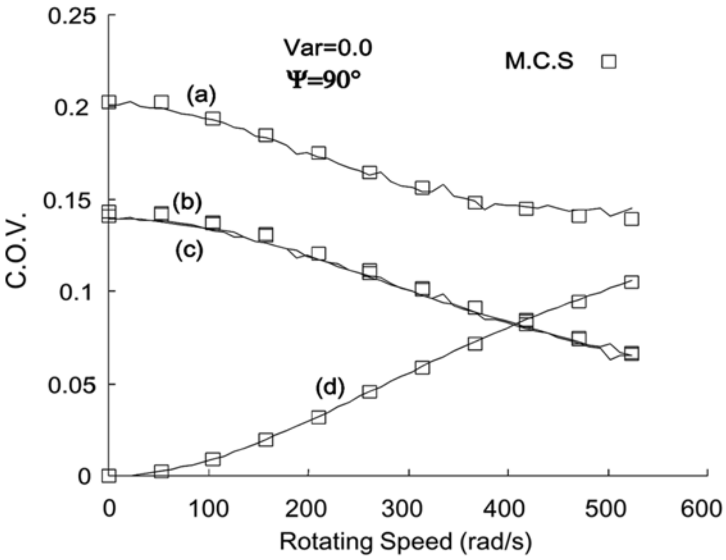

Fig. 3 C.O.V. of first eigenvalue of Timoshenko rotating beam versus rotating speed, setting angle is 90 degrees

decreasing with respect to rotational speed. The next point to consider is the close similarity between the curves of geometry and material. To justify the similarity between curves (b) and (c) in Fig. 2, we simply assume Bernoulli-Euler rotating beam with zero rotational speed for which (Eqs. (8) and (9)):

$$[K_b]\{q\}^* = \lambda[M_t]\{q\}^* \tag{21}$$

By factoring "EI" and "ρA" in Eq. (7) we have

$$[K_b] = EI[K_a], \quad [M_t] = \rho A[M_a] \tag{22}$$

Then one can rewrite Eq. (21) as

$$EI[K_a]\{q\}^* = \lambda \rho A[M_a]\{q\}^* \tag{23}$$

Above equation shows that the "order of magnitude" of geometrical properties (A, I) and material properties (ρ, E) are the same. So, with having identical variation in geometrical and material properties, then one will have identical variation in eigenvalues. This leads to the same C.O.V. curves.

The properties of Figs. 3 and 2 are similar except that the setting angle in Fig. 3 is assumed to be 90 degrees. It is seen from Fig. 3 that with the increase of setting angle, curves become smoother and the randomness effect of rotational speed decreases. To justify this, we rewrite the centrifugal stiffness matrix (Eqs. (2) and (7)) as:

$$[K_c] = \Omega^2[K_\gamma] \tag{24}$$

then Eq. (8) will be

$$([K_b] + [K_s] + \Omega^2([K_\gamma] - \sin^2\psi[M_t]))\{q\}^* = \lambda[M]\{q\}^* \tag{25}$$

Since the translational mass matrix $[M_t]$ is a positive definite matrix and the expression $([K_\gamma] - \sin^2\psi[M_t])$ is multiplied by Ω^2, if $\Psi = 0$, then the variation in rotating speed has the largest effect on variation in the first mode eigenvalue.

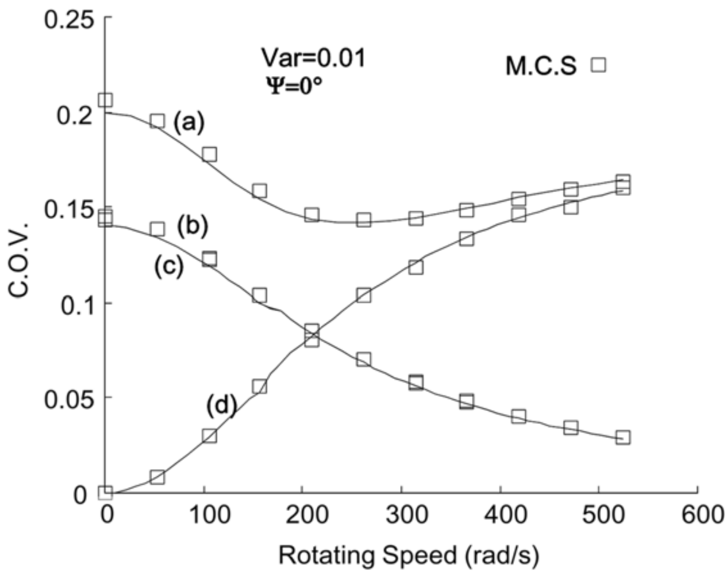

Fig. 4 C.O.V. of first eigenvalue of Bernoulli-Euler rotating beam versus rotating speed, setting angle is 0 degree

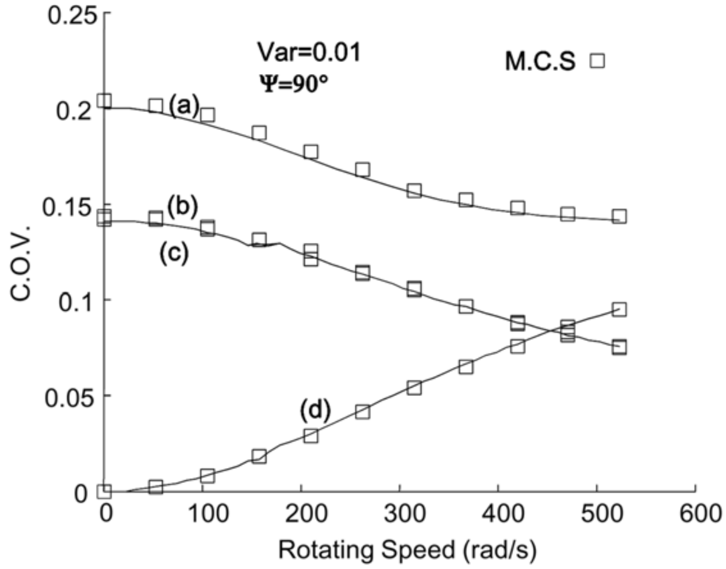

Fig. 5 C.O.V. of first eigenvalue of Bernoulli-Euler rotating beam versus rotating speed, setting angle is 90 degrees

Figs. 4 and 5 show curves of Bernoulli-Euler rotating beam. It can be seen that aforementioned figures are similar to corresponding Figs. 2-3 for the Timoshenko beam. Consequently, C.O.V. of first mode eigenvalue of Timoshenko and Bernoulli-Euler rotating beams are approximately identical. The reason is the first mode eigenvalue of Timoshenko and Bernoulli-Euler rotating beams are nearly identical and consequently the variation of corresponding eigenvalues and finally their C.O.V. curves are nearly same.

Figs. 6 and 7 show the C.O.V. of rotating Timoshenko beam versus rotational speed with different variances. All the curves are plotted by assuming that all properties are random and variances are identical and equal to 0.0025, 0.01, 0.0225, 0.04. Fig. 6 is plotted with setting angle equal to zero. Monte-Carlo simulation with 10000 samples is plotted to be compared with second-order perturbation method. It is seen that, agreement is very good. Also, it is realized that if variance of random properties increases, the C.O.V. increases accordingly. The properties of Fig. 7 are similar to those of Fig. 6 except that the setting angle is assumed to be 90 degrees. Theses curves are smoother than the curves of Fig. 6. To justify this, using Eq. (25), since the translational mass matrix $[M_t]$ is a positive definite matrix, and the expression $([K_\gamma] - \sin^2\psi[M_t])$ is multiplied by Ω^2, when $\Psi = 0$, the variation in rotating speed has the largest effect on variation in eigenvalue which creates sharp changes in the curve pattern, while when $\Psi = 90$, this effect is the smallest, so, the curve pattern will be smoother.

Fig. 8 shows the C.O.V. of first mode eigenvalue as a function of standard deviation of random properties with different rotational speeds. It is realized that with increasing of standard deviation of random properties, the C.O.V. increases. Also, with increasing of rotational speed, the slope of curves decreases. The relation between C.O.V. and standard deviation is linear.

The aforementioned results are obtained with the assumption that each random field of a random property is fully-correlated. If random fields are semi-correlated, the random fields as well as beam should be discretized. Consequently, three methods for discretization of random fields may be applied.

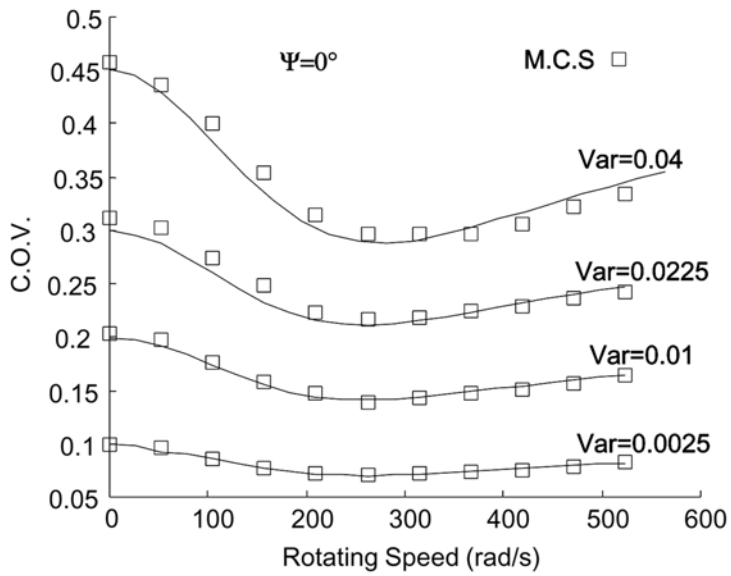

Fig. 6 C.O.V. of first eigenvalue of Timoshenko rotating beam versus rotating speed with different variances

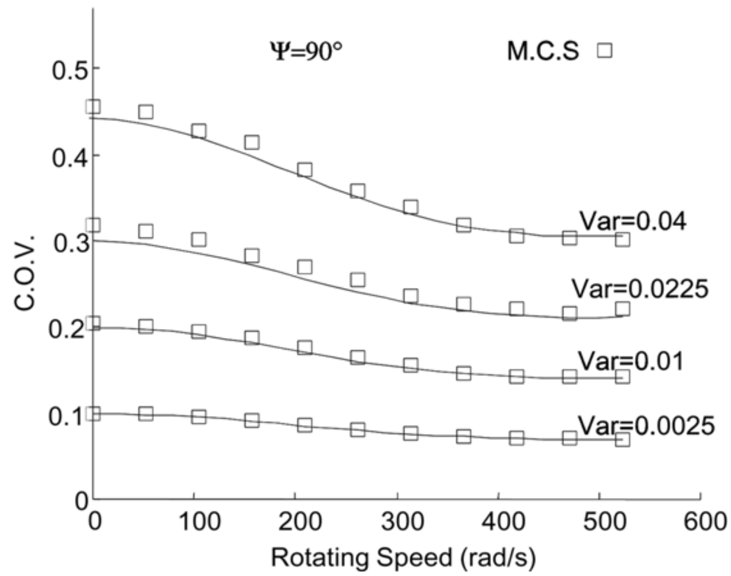

Fig. 7 C.O.V. of first eigenvalue of Timoshenko rotating beam versus rotating speed with different variances

To discretize random fields, methods of midpoint, interpolation and local average are applied. $\rho(x)$ and $E(x)$ are assumed random and their variances are identical and equal to 0.01. As a result, the correlation matrix has 4 nonzero elements:

$$[\rho(r)]_{3,3} = [\rho(r)]_{5,5} = \rho_0(r), \qquad [\rho(r)]_{5,3} = [\rho(r)]_{3,5} = b^*\rho_0(r) \tag{21}$$

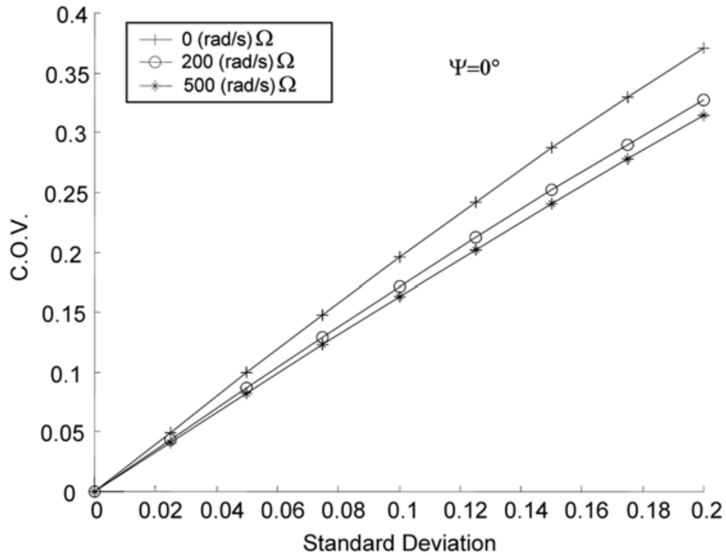

Fig. 8 C.O.V. of first eigenvalue of Timoshenko rotating beam versus standard deviation of random properties with different rotating speed.

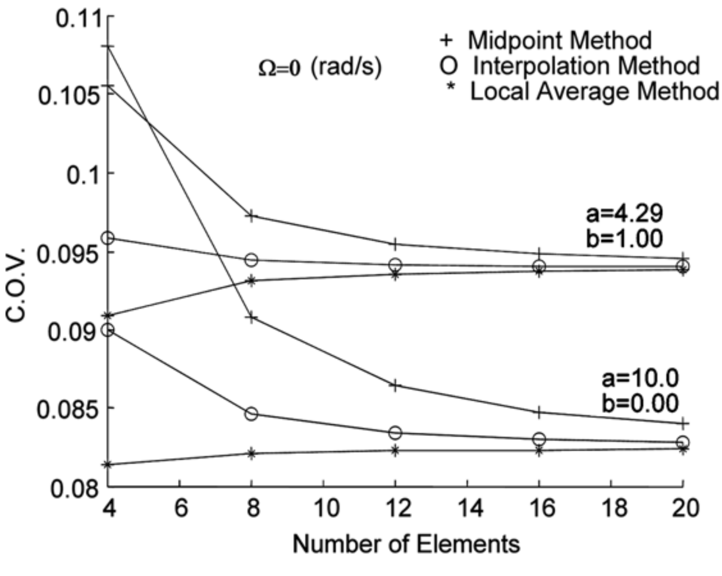

Fig. 9 C.O.V. of first eigenvalue of Timoshenko rotating beam versus number of elements with different discretization methods for correlated and uncorrelated random fields

Where $\rho_0(r) = \exp(-a|r|/L)$, and b is a constant that defines the correlation between random fields $\rho(x)$ and $E(x)$.

Fig. 9 shows C.O.V. versus the number of elements for the three methods of discretization. The figures represent correlated and uncorrelated random fields. The rotational speed is zero. It is seen that compared to uncorrelated case the correlated case has large C.O.V. Also, if the value "a" is large or if correlation length is small, the convergence rate is lower and more number of elements

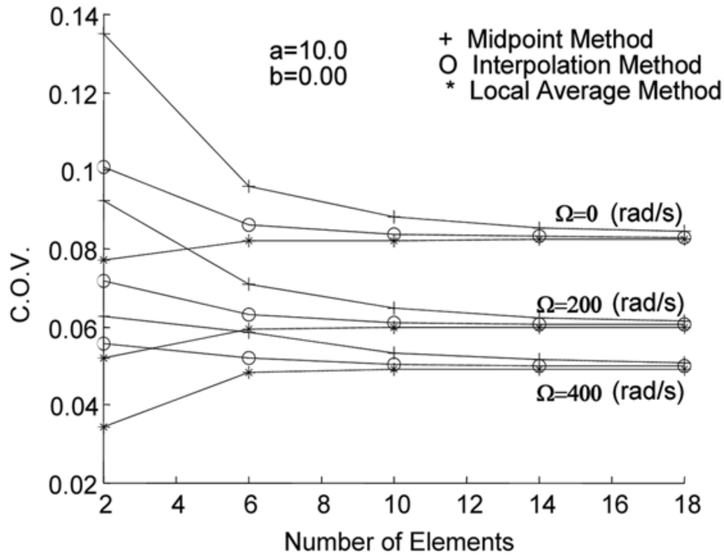

Fig. 10 C.O.V. of first eigenvalue of Timoshenko rotating beam versus number of elements with different discretization methods and rotating speed

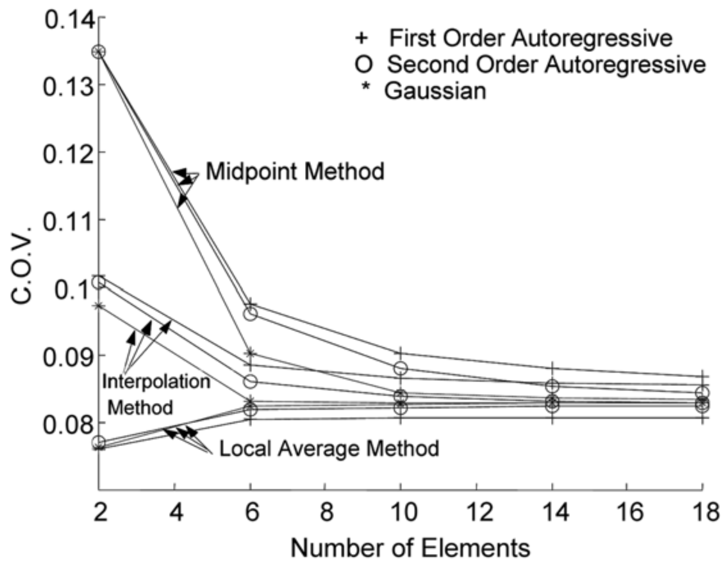

Fig. 11 C.O.V. of first eigenvalue of Timoshenko rotating beam versus number of elements with different discretization methods and three correlation functions

are necessary for convergence of final response.

It is realized that in correlated case ($a = 4.29$, $b = 1$) the convergence rate of interpolation and local average methods is higher than that midpoint method. As it is seen from Fig. 9, the midpoint and interpolation methods converge to the solution from upper limit where the local average method approaches the solution from the lower limit. The curves of uncorrelated case ($a = 10$, $b = 0$) are also investigated. It is seen again, that the convergence rate of interpolation and local average methods is higher than that of midpoint method. Consequently, in general, the convergence rate of interpolation and local average methods is higher than that of midpoint method.

Fig. 10 shows the C.O.V. versus the number of elements for three different rotational speeds. The most important result of the figure is that with the increase of rotational speed, the C.O.V. decreases. It can be concluded that the change of rotational speed has a small effect on convergence rate. In summary, the method of discretization has significant effect on the convergence rate of response. The number of elements necessary for a good convergence rate has direct relationship with correlation length. The smaller correlation length, the larger the number of elements. On the other hand, if the number of elements is large, the stiffness and mass matrices will also be large. This creates problems in the numerical calculation of these matrices.

Fig. 11 shows the effect of correlation function on convergence rate. The curves are plotted versus the number of elements and for three correlation functions of $\rho_0(r) = \exp(-(a|r|/L)^2)$, $\rho_0(r) = \exp(-a|r|/L)$, $\rho_0(r) = (1 + a|r|/L)\exp(-a|r|/L)$ namely "Gaussian", "first order autoregressive" and "second order autoregressive" respectively. It can be inferred that the correlation function has a small effect on convergence rate.

Fig. 12 shows the C.O.V. versus $a = L/(Correlation\ Length)$ for three different rotational speeds ($b = 0$). It is apparent here that for small rotational speed as the parameter "a" increases, the C.O.V decreases. In other words, when the correlation length decreases, the C.O.V. also decreases. Likewise, as the rotational speed increases, the slope of curves decreases. It is necessary to note that in rotational speed $\Omega = 400$ rad/s the curve is not monotonic.

Fig. 13 is plotted assuming the random fields $E(x)$ and $\rho(x)$ are correlated ($b = 1$). It is seen that as the rotational speed increases the C.O.V. decreases. Likewise the C.O.V increases with respect to "a" in range of $0 < a < 3.5$ and if $a > 3.5$ the C.O.V. decreases. This feature is approximately apparent for all rotational speeds.

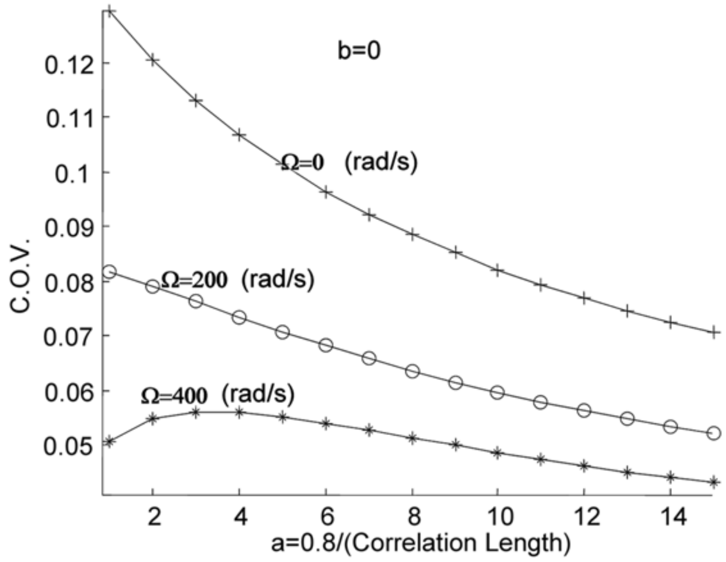

Fig. 12 C.O.V. of first eigenvalue of Timoshenko rotating beam versus parameter "$a = 0.8$/(Correlation Length)", for uncorrelated random fields

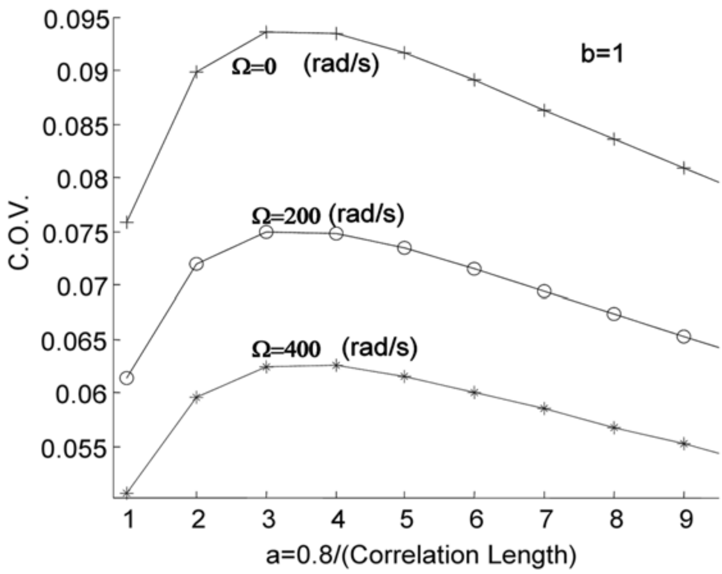

Fig. 13 C.O.V. of first eigenvalue of Timoshenko rotating beam versus parameter "$a = 0.8$/(Correlation Length)", for correlated random fields

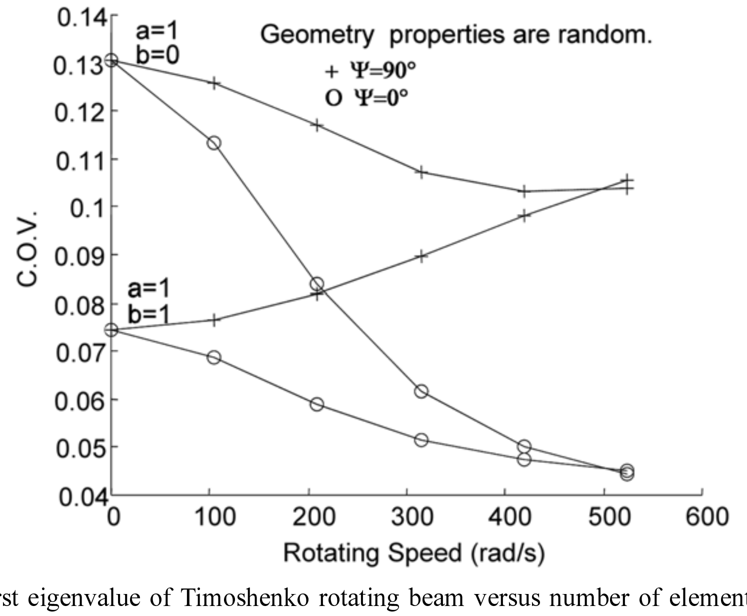

Fig. 14 C.O.V. of first eigenvalue of Timoshenko rotating beam versus number of elements for correlated and uncorrelated random fields

Fig. 14 shows the C.O.V. versus rotational speed using 14 elements. It should be taken into account that here only the material properties are random $(E(x), \rho(x), G(x))$. Curves are plotted for correlated and uncorrelated random fields and setting angle 0 and 90 degrees ($a = 1$). The C.O.V. corresponding to uncorrelated random field is larger than the correlated random field. For correlated and uncorrelated cases, the C.O.V. corresponding to setting angle of 90 degrees is higher

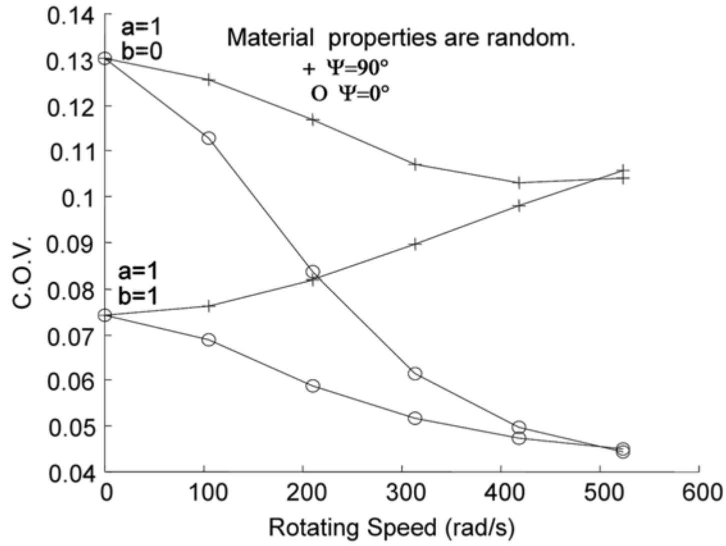

Fig. 15 C.O.V. of first eigenvalue of Timoshenko rotating beam versus number of elements for correlated and uncorrelated random fields

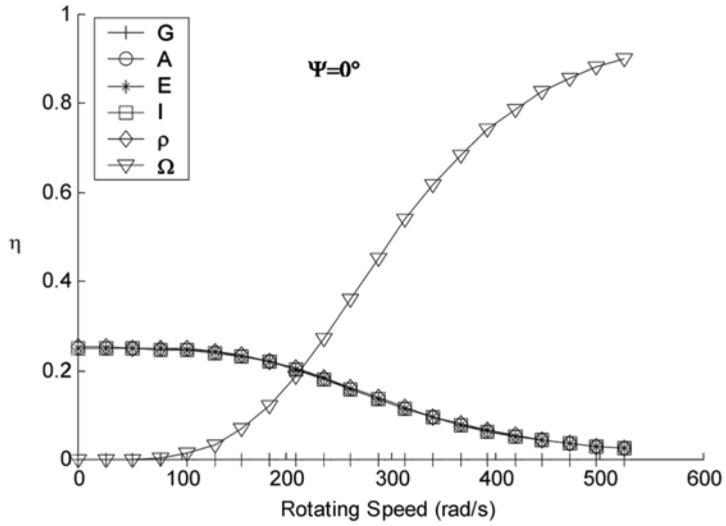

Fig. 16 Relative sensitivity coefficient of first eigenvalue of Timoshenko rotating beam versus rotating speed, setting angle is 0 degree

than setting angle of 0 degree. The reason might be the existence of term $([K_\gamma]-\sin^2\psi[M_t])$ in stiffness matrix (Eq. (25)) and the fact that where the setting angle increases the randomness effects of $\rho(x)$ increase (matrix $[M_t]$ is function of $\rho(x)$).

Fig. 15 is plotted with properties of Fig. 14, with the exception that the geometry properties ($A(x)$, $I(x)$) are assumed to be random. The figure is similar to the previous one. Also, in this figure the C.O.V. corresponding to uncorrelated random field is larger than the correlated random field. Likewise, because matrix $[M_t]$ is function of $A(x)$, where the setting angle increases the randomness effects of $A(x)$ increase.

For sensitivity analysis of random properties on response, parameter "η" (relative sensitivity coefficient) is defined. This parameter determines the effect of randomness of each property on deviation of response. The advantage of the analysis is that it determines the random weight of each random property. As a result, the random properties with small η can be assumed to be deterministic. For this aim, Figs. 16 to 19 are plotted. Here the random fields are assumed uncorrelated and any field is fully-correlated and the variance is 0.01. In Fig. 16 "η", is plotted

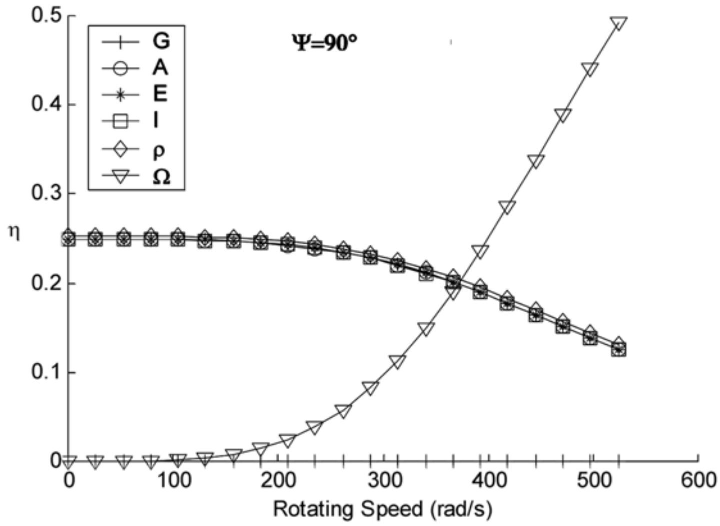

Fig. 17 Relative sensitivity coefficient of first eigenvalue of Timoshenko rotating beam versus rotating speed, setting angle is 90 degrees

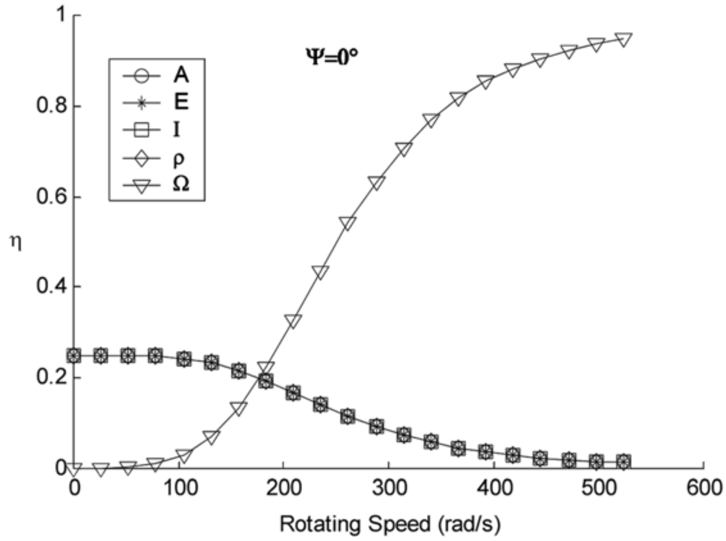

Fig. 18 Relative sensitivity coefficient of first eigenvalue of Bernoulli-Euler rotating beam versus rotating speed, setting angle is 0 degree

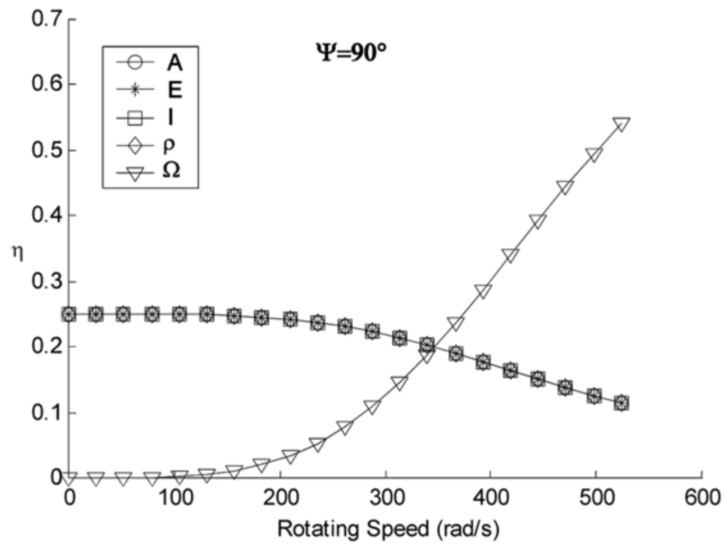

Fig. 19 Relative sensitivity coefficient of first eigenvalue of Bernoulli-Euler rotating beam versus rotating speed setting, angle is 90 degrees

versus rotational speed for Timoshenko beam and the setting angle is 0 degree. It is seen the effect of randomness of parameter "$G(x)$" is approximately zero. It means that without any error we can assume $G(x)$ to be a deterministic property. In the first mode eigenvalue of a rotating beam, the effects of shear deformation is very small and because the shear deformation is directly proportional to shear modulus $G(x)$, then the variation of shear modulus has very small effects on variation of first mode eigenvalue. On the other hand, the value of parameters A, ρ, I, E which have medium η will decrease with an increase of rotational speed. The curves corresponding to rotational speed is ascending i.e., this parameter has high η in high rotational speed. The setting angle in Fig. 17 is 90 degrees. The figure is similar to previous one with the exception that here, the curves are smoother. The justification is the presence of term $-\Omega^2 \mathrm{Sin}^2\Psi[M_t]$ in stiffness matrix, will change the stiffness as ψ changes. Consequently, the randomness of properties will bear different results.

Figs. 18 and 19 are plotted for Bernoulli-Euler beam with setting angle of 0 and 90 degrees. Here, one gets the same results as the ones for Timoshenko beam and there are not much differences between the parameter η of these two types.

7. Conclusions

In this paper, free vibration of rotating beam with random properties is studied. Cross-sectional area, elasticity modulus, moment of inertia, shear modulus and density are modeled as random fields and the rotational speed as a random variable. To study uncertainty, stochastic finite element method based on second order perturbation method is applied. To discretize random fields, the three methods of midpoint, interpolation and local average are applied and compared. The most important results of the paper as follows:

1. If rotational speed is random, C.O.V. curve is ascending with respect to rotational speed and if

material or geometry properties are random, C.O.V. curves are descending. If all properties are random, The C.O.V. curves are first descending due to randomness of material and geometry and then due to rotational speed they are ascending.

2. C.O.V. of first mode eigenvalue of Timoshenko and Bernoulli-Euler rotating beams are approximately identical.
3. If variance of random properties increases, the C.O.V. increases accordingly.
4. Compared to uncorrelated random fields the correlated case has larger C.O.V. value.
5. If correlation length is small, the convergence rate is lower and more number of elements are necessary for convergence of final response.
6. The convergence rate of interpolation and local average methods is higher than that midpoint method.
7. The midpoint and interpolation methods converge to the solution from upper limit where the local average method approaches the solution from the lower limit.
8. With the increase of rotational speed, the C.O.V. decreases.
9. The rotational speed has fairly small effect on convergence rate.
10. The correlation function has fairly small effect on convergence rate.
11. When the correlation length decreases, the C.O.V. also decreases.
12. The C.O.V. corresponding to uncorrelated random field is larger than the correlated random field.
13. The effect of randomness of parameter "$G(x)$" is approximately zero. It means that without any error we can assume G to be a deterministic property. On the other hand, the parameters A, ρ, I, E have medium η that with the increase of rotational speed their value decrease.
14. There are not much differences between the parameter η of Timoshenko and Bernoulli-Euler types.

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