Structural Engineering and Mechanics, Vol. 20, No. 3 (2005) 279-292

DOI: http://dx.doi.org/10.12989/sem.2005.20.3.279

Direct determination of influence lines and surfaces by F.E.M.

Engin Orakdöğen† and Konuralp Girgin‡

Faculty of Civil Engineering, Technical University of İstanbul, 34469 Ayazağa, İstanbul, Turkey

(Received June 14, 2004, Accepted March 16, 2005)

Abstract. In this study, element loading matrices are defined for static application of classical Müller-Breslau principle to finite element method. The loading matrices are derived from existing element matrices using Betti's law and known governing equations of F.E.M. Thus, the ordinates of influence lines and influence surfaces may be easily obtained from structural analysis for the loading matrices derived from governing equations, instead of through introduced unit force or displacement techniques. An algorithm for a computer program and comparative numerical examples are also presented to illustrate the procedure for determination of influence line and surface ordinates.

Key words: influence lines; influence surfaces; Betti's law, Müller-Breslau principle, finite element method.

1. Introduction

The internal force and displacement influence functions of a structure are of prime importance in engineering mechanics, especially when live loads are considered. An influence function at a particular point of a structure represents the variation in any response such as displacements and internal forces due to unit external forces moving on the structure and they are very useful concepts for obtaining maximum or minimum values of responses of moving and live loads. One of the classical techniques for obtaining influence functions is to analyze the structure for different positions of unit external effects. Since this technique is time consuming, a more efficient technique, based on Müller-Breslau Principle, is applied in general, Ghali and Neville (1978). According to this principle, the influence function of any response relating to a structure, whether statically determinate or indeterminate, is proportional to the deflected shape of the structure obtained by inducing a known displacement or discontinuity in the direction of the response. Fu (1973) defined an equivalent load vector for influence surface ordinates by inserting a relative deformation to nodes. However, the method requires a corrective vector due to the average deformation considerations along element edges and it is concluded from numerical examples that the accuracy of this solution depends on the mesh sizes. Cifuentes and Paz (1991) have developed an algorithm based on the Müller-Breslau Principle applicable to frames and shells. Sample input data are also

† Associate Professor, Corresponding author, E-mail: orak@itu.edu.tr
‡ Assistant Professor

given for the MSC/NASTRAN finite element code. However, this method requires revising the input data, since extra nodes and constraints should be defined to be able to give a relative unit displacement according to the Müller-Breslau Principle. Shen (1992) has extended the Müller-Breslau Principle to structures consisting of finite elements by introducing a loading vector for average stress at any point on the structure. Since the average stress is considered, the structure should be finely meshed in the vicinity of the point at which influence function ordinates are to be determined and constant strain fields called Standard Displacement Modes (SDM) need to be defined for each type of finite element considered in the problem. SDMs for n-node isoparametric plane stress elements and three-node triangular plate bending elements are given in the study. Belegundu (1986, 1988) proposed a method called the Adjoint Method for influence lines. In this method, an adjoint variable vector is calculated for any response function using the adjoint equations and then influence line ordinates are obtained by solving the equilibrium equations. Memari and West (1991) suggested a remedy for adjoint variable vectors since a correction to the adjoint variable vector is necessary in the case of the response function in the directions of constrainted degrees of freedom. Kwak and Song (2001), have used Pusher's influence surfaces (1977), to find the most unfavourable internal forces due to the vehicle loads. Akesson *et al*. (1995), have utilized the classical Müller-Breslau influence function technique for determination of stress intensity factors at the crack tip. Hanson *et al*. (2004) and Yamashita *et al*. (2004), have used the displacement influence functions of elastic bodies for crack growth simulations. In these studies, classical approach based on the unit force or stress loading is used to obtain influence coefficients.

In this paper, element loading matrices are defined by using Betti's law and governing equations of finite element method for direct determination of the influence lines or surfaces in frame and shell structures. In contrast to other methods in the literature, since the matrices are derived by using existing finite element matrices, it may be applied to the structures consisting of any type of finite element without revising the input data or defining any SDMs or adjoint variable vectors. This proposed approach corresponds to the direct application of the classical Müller-Breslau Principle to finite element method. This paper also explains the statement *If the coefficients of a stress matrix are used as right-hand sides, the solutions are the stresses resulting from unit loads on each variable in turn* outlined in Irons and Ahmad (1986) and illustrates it's numerical applications to frame and shell structures.

2. Influence function ordinates of nodal displacement components of frame and shell elements

It is well known from structural mechanics that, according to Betti's Law, influence function ordinates of any displacement component in a linear-elastic structure can be obtained as the displacement ordinates due to the unit loading in the direction of considered displacement components. If the structure is discretized by frame or shell finite elements, the resulting displacement vector **d** which is obtained by the solution of linear simultaneous equations

$$\mathbf{S}\,\mathbf{d} = \mathbf{q} \tag{1}$$

gives the influence function ordinates at nodes or so called influence coefficients, where **S** is the system stiffness matrix and **q** is the system loading vector whose element in the direction of the displacement component considered is equal to 1 while the remaining elements are zero.

If the Eq. (1) is rewritten for influence coefficients of all nodal displacement components of a particular element, it yields

$$\mathbf{S}\,\mathbf{D} = \mathbf{Q} \tag{2}$$

where **D** is a matrix of which each column consists of influence coefficients of an independent nodal displacement component and **Q** is the system loading matrix of which each column consisting of **q** global loading vectors due to the unit loadings in the directions of independent nodal displacement components of the considered element. Furthermore, when the matrix **S** and **Q** are rearranged such that the node numbers of the particular element are consecutive, the matrix **Q** includes an identity matrix

$$\mathbf{Q} = \begin{bmatrix} 0 \\ \mathbf{I} \\ 0 \end{bmatrix} \tag{3}$$

In other words, if the element loading matrix for determining the influence coefficients of the nodal displacement components of a particular element is represented by **R**, it yields an identity matrix

$$\mathbf{R} = \mathbf{I} \tag{4}$$

3. Element loading matrix for influence coefficients of stress components

The influence coefficients of stress components at a particular point on an element may be obtained by using the governing equation of finite element method in terms of the influence coefficients of nodal displacement components as follows:

$$\boldsymbol{\sigma} = \mathbf{E}\,\mathbf{B}\,\mathbf{D} \tag{5}$$

Substituting

$$\mathbf{G} = \mathbf{E}\,\mathbf{B} \tag{6}$$

in (5) gives

$$\boldsymbol{\sigma} = \mathbf{G}\,\mathbf{D} \tag{7}$$

or

$$\boldsymbol{\sigma}^{\mathbf{T}} = \mathbf{D}\,\mathbf{G}^{\mathbf{T}} \tag{8}$$

where,

$\boldsymbol{\sigma}$ is a matrix consisting of the influence coefficients of the stress components at a particular point on the element,

E is the elasticity matrix of the element,

B is a matrix consisting of the derivatives of the element's shape functions written in terms of the local point coordinates,
G is the element stress matrix, and
D is a matrix consisting of the influence coefficients of the nodal displacement components of the element.

Although the influence coefficients of stress components may be obtained by matrix multiplication in terms of influence coefficients of nodal displacement components by using (5), they may also be directly obtained as in the following:
If the element loading matrix in (4) is taken as

$$\mathbf{R} = \mathbf{I}\,\mathbf{G}^{\mathrm{T}} = \mathbf{G}^{\mathrm{T}} \tag{9}$$

the resulting displacement matrix gives $\mathbf{D}\,\mathbf{G}^{\mathrm{T}}$ as the product or $\boldsymbol{\sigma}^{\mathrm{T}}$, which, according to (2) and (8), consists of influence coefficients of stress components at particular points on the element. In conclusion, for obtaining influence coefficients of stress components, transposal of the element stress matrix **G** may be taken as the element loading matrix **R**. It should also be noted that, transformation of the element loading matrix **R** from the local axis to the global ones is necessary for constructing the global loading matrix **Q**. If the influence surface coefficients of any stress component are to be determined separately, the column of matrix $\mathbf{G}^{\mathrm{T}}$ corresponding to stress component should be taken as the element loading vector **r**. However, when the influence surface coefficients of average stress components are to be obtained, the averages of the element stress vectors connected at a node must be applied together as an unique loading case.

4. Element loading matrices for influence coefficients of displacement and strain components

Similar to the explanations in the previous section, the influence coefficients of displacement and strain components at a particular point on the element may be directly obtained by using the finite element equations

$$\mathbf{u}^{\mathrm{T}} = \mathbf{D}\,\mathbf{N}^{\mathrm{T}} \tag{10}$$

and

$$\boldsymbol{\varepsilon}^{\mathrm{T}} = \mathbf{D}\,\mathbf{B}^{\mathrm{T}} \tag{11}$$

where,
u is a matrix consisting of the influence coefficients of the displacement components at a particular point on the element,
$\boldsymbol{\varepsilon}$ is a matrix consisting of the influence coefficients of the strain components at a particular point on the element, and
N is the shape function matrix of the element written in terms of the local point coordinates.
Matrix **B** has been already defined in section 3.

5. Element loading matrix for influence coefficients of internal force components in frames

The influence coefficients of internal force components in frame structures may also be obtained in terms of the influence coefficients of nodal displacement components by using the governing equation as

$$\mathbf{P} = \mathbf{K}\,\mathbf{D} \tag{12}$$

or

$$\mathbf{P}^T = \mathbf{D}\,\mathbf{K}^T \tag{13}$$

where,

P is a matrix consisting of the influence coefficients of the internal force components of the frame element,

K is the element stiffness matrix, and

D is a matrix consisting of the influence coefficients of the nodal displacement components of the element.

In a manner similar to that explained in Section 3, for obtaining influence coefficients of internal force components in frame structures, transposal of the element stiffness matrix **K** may be taken as the element loading matrix **R**. If the influence line ordinates of any internal force component are to be determined separately, the column of matrix $\mathbf{K}^T$ corresponding to the internal force component should be taken as element loading vector **r**. It is also concluded that the load vector **r** defined herein is precisely the adjoint load vector given in Belegundu (1986, 1988).

6. Computer implementation

A finite element computer program was written for influence cefficients of frames and plates utilizing the derived loading matrices. The algorithm of the computer program is given as follows:

1- Read the finite element input parameters of the structure by identifying the element and defining the local coordinates of the point for the influence coefficient which is to be evaluated,
2- Construct the global stiffness matrix **S**,
3- Construct the global loading vector **Q** for the stress, displacement or strain components by using the matrices $\mathbf{K}^T$, $\mathbf{G}^T$, $\mathbf{N}^T$ or $\mathbf{B}^T$ of the considered element,
4- Perform the static analysis of the structure and find the displacement matrix **D**,
5- Separate the vector **D** into sub-vectors to obtain the influence coefficients for the external loads in the directions of the nodal displacement components.
6- Correct the influence coefficients in the direction of response, since an initial unit displacement is introduced.
7- Obtain the influence line or surface ordinates within the elements using element shape functions.

7. Numerical examples

In this chapter, four numerical examples are given to illustrate the present formulation and to compare the results with those obtained in previous studies.

7.1 Example 1

Consider the truss shown Fig. 1(a). This example is taken from Belegundu (1988), Cifuentez and Paz (1991).

For the influence line for the axial force in rod 3-9, a loading vector is defined by using the columns P_1 or P_3 of the transposed element stiffness matrix **K**, since the axial force is constant along the element length. Positive sign convention and transposal of element stiffness matrix K in local axes are shown in Fig. 2, the loading vector $\mathbf{r}^{3\text{-}9}$ in global axes is given in Table 1.

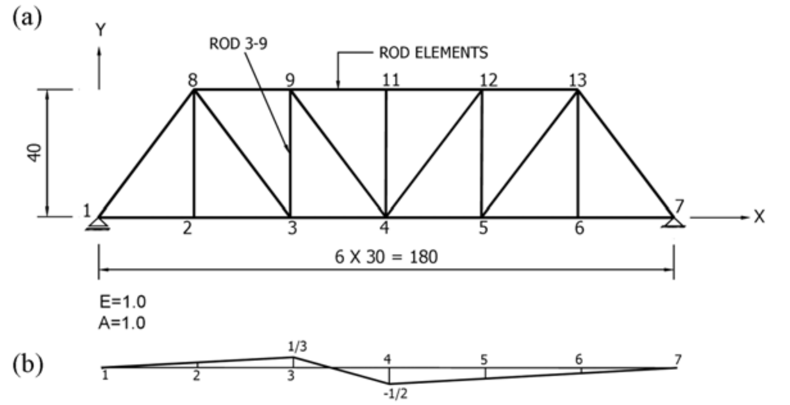
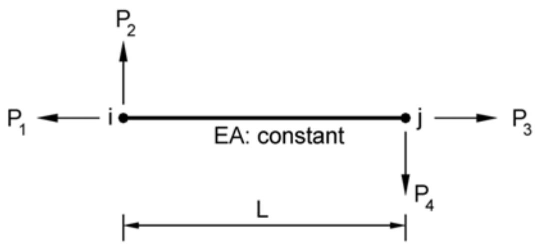


Fig. 1 (a) Plane truss, (b) influence line for the axial force in rod 3-9



$$
K^T = \begin{bmatrix} P_1 & P_2 & P_3 & P_4 \\ \dfrac{EA}{L} & 0 & \dfrac{EA}{L} & 0 \\ 0 & 0 & 0 & 0 \\ \dfrac{EA}{L} & 0 & \dfrac{EA}{L} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
$$

Fig. 2 Positive sign convention and transposal of element stiffness matrix in local axes

Table 1 Element loading vector of rod 3-9 in global axes

Freedom #	r^{3-9}
1	0
2	−0.025
3	0
4	0.025

As is shown in Fig. 1(b), the influence line coefficients are the same as those given by Belegundu (1988) and Cifuentes and Paz (1991).

7.2 Example 2

Consider the three span continious beam shown in Fig. 3(a). This example is taken from Mc Cormac (1984) and Cifuentes and Paz (1991).

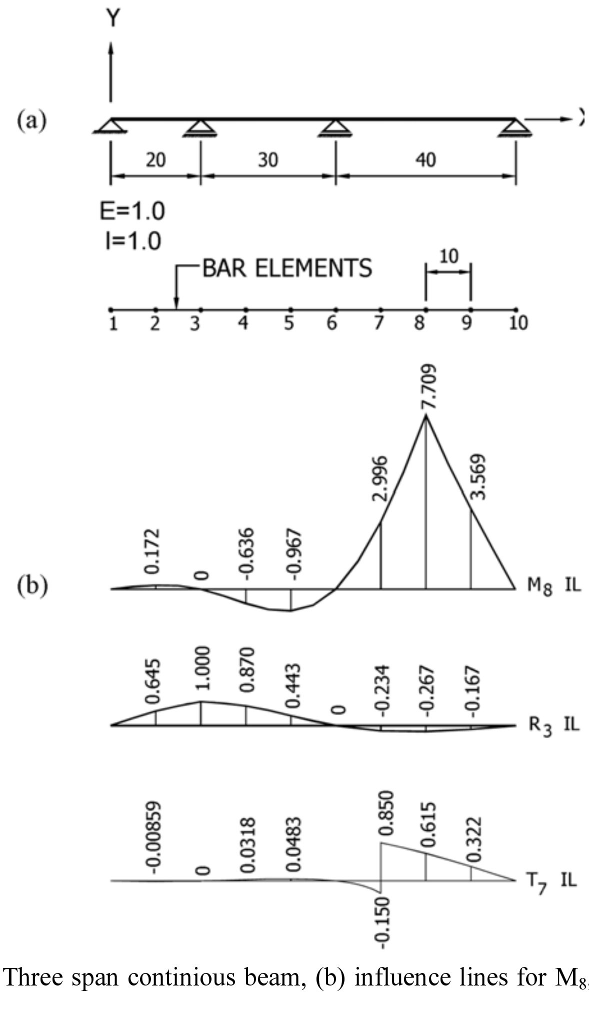
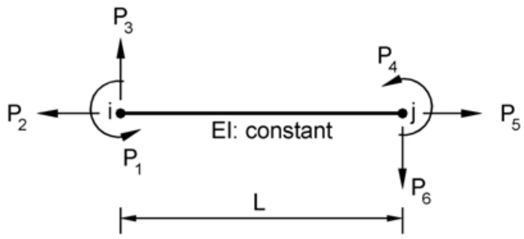


Fig. 3(a) Three span continious beam, (b) influence lines for M_8, R_3 and T_7

For obtaining influence line of bending moment M_8, column P_4 of the member stiffness matrix has been chosen as the loading vector of member 7-8. In a similar manner, column P_6 of stiffness matrix was chosen as the loading vector of member 6-7 for obtaining the influence line of shear force T_7. However, column P_6 for member 2-3 and column P_3 of member 3-4 were chosen together as loading vectors for support reaction R_3 since the vertical equilibrium equation at node 3 is $R_3 = T_3^{2-3} - T_3^{3-4}$. In influence line R_3, the vertical displacement of node 3 is obtained as equal to zero from the analysis as the displacement is restrained. But, since the loading vectors correspond to the unit vertical displacement, the resulting vertical displacements should be superimposed with this initial relative unit displacement. Thus, the vertical displacement of node 7 is obtained as 0.850 and influence line ordinate of shear force T_7 at the 7 end of element 6-7 should be equal to −0.150. The positive sign convention and the member stiffness matrix are shown in Fig. 4, the element loading vectors used for the influence lines are given in Table 2.



$$K^T = \begin{bmatrix} P_1 & P_2 & P_3 & P_4 & P_5 & P_6 \\ \frac{4EI}{L} & 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & \frac{6EI}{L^2} \\ 0 & \frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 \\ \frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & \frac{6EI}{L} & 0 & \frac{12EI}{L^3} \\ \frac{2EI}{L} & 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & \frac{6EI}{L^2} \\ 0 & \frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 \\ \frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} \end{bmatrix}$$

Fig. 4 Positive sign convention and element stiffness matrix

Table 2 Element loading vectors used for influence line ordinates

Freedom #	$\mathbf{r}^{7\text{-}8}$	$\mathbf{r}^{2\text{-}3}$	$\mathbf{r}^{3\text{-}4}$	$\mathbf{r}^{6\text{-}7}$
1	−0.200	0.060	−0.060	−0.060
2	0	0	0	0
3	−0.060	0.012	−0.012	0.012
4	−0.040	0.060	−0.060	0.060
5	0	0	0	0
6	−0.060	0.012	−0.012	0.012

It is concluded from this example, all the influence line ordinates shown in Fig. 3(b) are in close agreement with those of Mc Cormac (1984) and Cifuentes and Paz (1991).

7.3 Example 3

Consider the simply supported square plate shown in Fig. 5 and the corresponding finite element mesh Cifuentes and Paz (1991). The stiffness and stress matrices of fully compatible, 16 DOF plate finite element shown in Fig. 6, are taken from Bogner *et al.* (1965).

For the influence surface coefficients of M_x for node 41, the column of the element stress matrix corresponding to M_x is taken as loading vector. The loading vectors used for the analysis are given in Table 3. Since the average M_x stress is considered, the average loading vectors of the four elements connected at node 41 are loaded together. Thus, $\frac{1}{4}^{th}$ of the element loading vectors are used for the analysis.

After the analysis of the plate for given element loading vectors, influence surface coefficients of

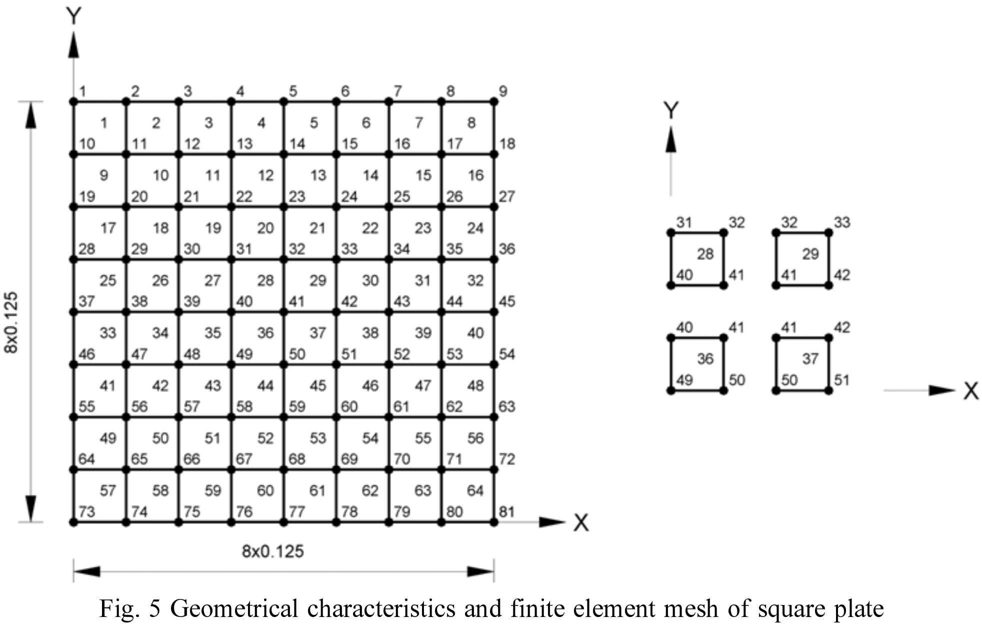


Fig. 5 Geometrical characteristics and finite element mesh of square plate

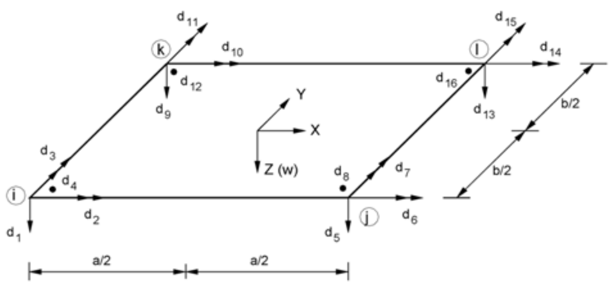


Fig. 6 16 DOF plate finite element

Table 3 Loading vectors for the influence surface coefficients of M_x

Freedom #	$\mathbf{r}^{29}$	$\mathbf{r}^{28}$	$\mathbf{r}^{37}$	$\mathbf{r}^{36}$
1	499.200	−384.000	−115.200	0.000
2	−9.600	0.000	4.800	0.000
3	32.000	−16.000	0.000	0.000
4	0.000	0.000	0.000	0.000
5	−384.000	499.200	0.000	−115.200
6	0.000	−9.600	0.000	4.800
7	16.000	−32.000	0.000	0.000
8	0.000	0.000	0.000	0.000
9	−115.200	0.000	499.200	−384.000
10	−4.800	0.000	9.600	0.000
11	0.000	0.000	32.000	−16.000
12	0.000	0.000	0.000	0.000
13	0.000	−115.200	−384.000	499.200
14	0.000	−4.800	0.000	9.600
15	0.000	0.000	16.000	−32.000
16	0.000	0.000	0.000	0.000

Table 4 Influence surface ordinates of M_x at chosen nodes for square plate

Node #	IS coefficient of M_x
11	0.01077
17	0.01077
21	0.04447
25	0.04447
31	0.11645
33	0.11645
39	0.05777
41	0.34609
49	0.11645
51	0.11645
57	0.04447
61	0.04447
65	0.01077
71	0.01077

M_x are obtained. The coefficients at chosen nodes are given in Table 4.

Influence surface coefficient of M_x at node 39 was obtained as −0.06 by Cifuentes and Paz (1991). The negative sign of the ordinate obtained by Cifuentes and Paz (1991) may be due to positive sign convention. It should be noted that the element stiffness and stress matrices given by Bogner *et al.* (1965) were used in computer code for illustration of the procedure. However, the procedure is open to the stiffness and stress matrices of any kind of finite element. The contour plot of influence surface of M_x for node 41 is also given in Fig. 7.

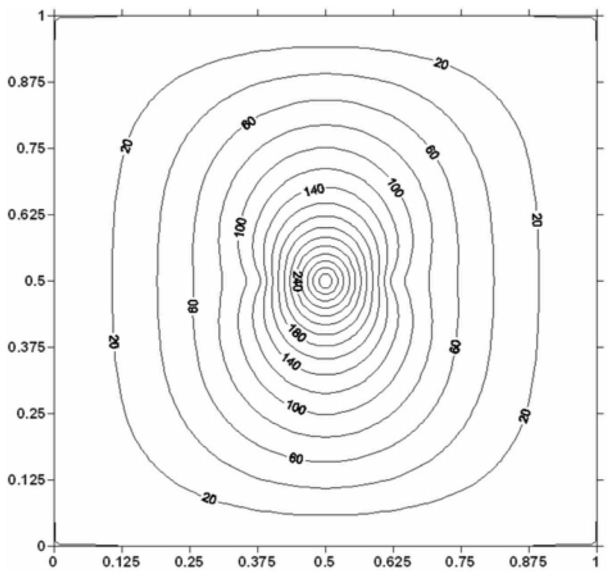


Fig. 7 Contour plot of influence surface of M_x for node 41 – values to be multiplied by 10^{-3}

7.4 Example 4

Consider the simply supported L shaped plate shown in Fig. 8 and the corresponding finite element mesh. It was intended to obtain the influence surface coefficients of M_x and M_{xy} for node 41.

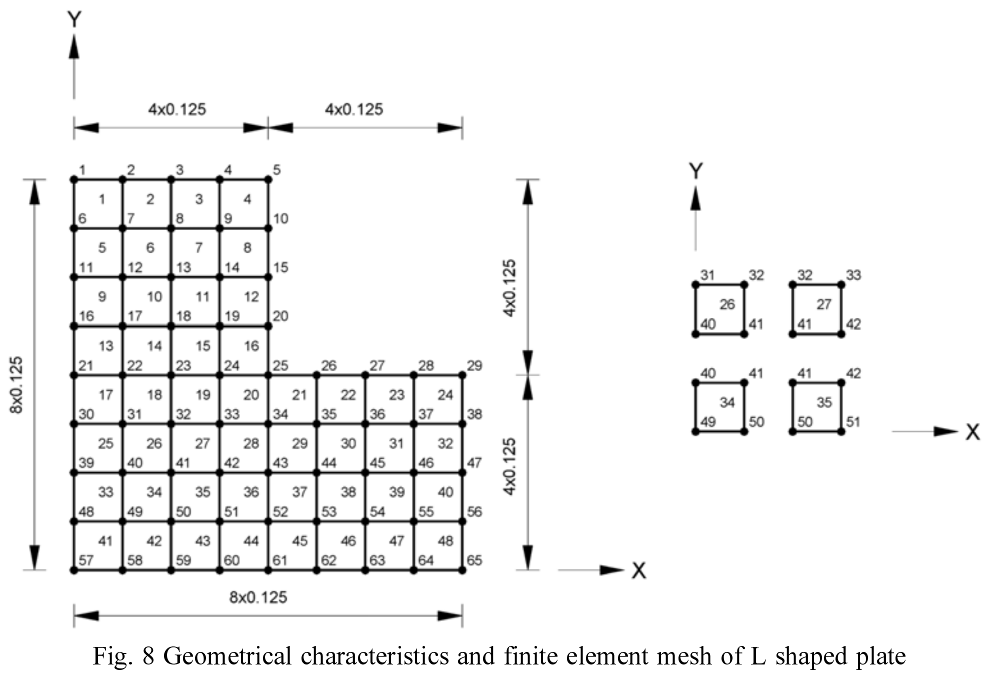


Fig. 8 Geometrical characteristics and finite element mesh of L shaped plate

The loading vectors used for the influence surface coefficients are given in Table 5. For the average influence surface coefficients, the average loading vectors of four elements connected at node 41 are loaded together. Thus, ¼th of element loading vectors are used for the analysis.

After the analysis of plate for given loading vectors, influence surface coefficients of M_x and M_{xy} are obtained. The chosen coefficients are given Tables 6 and 7.

Table 5 Loading vectors for influence surface coefficients of M_x and M_{xy}

Freedom #	r^{27}		r^{26}		r^{35}		r^{34}	
	M_x	**M_{xy}**	**M_x**	**M_{xy}**	**M_x**	**M_{xy}**	**M_x**	**M_{xy}**
1	499.200	0.000	−384.000	0.000	−115.200	0.000	0.000	0.000
2	−9.600	0.000	0.000	0.000	4.800	0.000	0.000	0.000
3	32.000	0.000	−16.000	0.000	0.000	0.000	0.000	0.000
4	0.000	−0.700	0.000	0.000	0.000	0.000	0.000	0.000
5	−384.000	0.000	499.200	0.000	0.000	0.000	−115.200	0.000
6	0.000	0.000	−9.600	0.000	0.000	0.000	4.800	0.000
7	16.000	0.000	−32.000	0.000	0.000	0.000	0.000	0.000
8	0.000	0.000	0.000	−0.700	0.000	0.000	0.000	0.000
9	−115.200	0.000	0.000	0.000	499.200	0.000	−384.000	0.000
10	−4.800	0.000	0.000	0.000	9.600	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	32.000	0.000	−16.000	0.000
12	0.000	0.000	0.000	0.000	0.000	−0.700	0.000	0.000
13	0.000	0.000	−115.200	0.000	−384.000	0.000	499.200	0.000
14	0.000	0.000	−4.800	0.000	0.000	0.000	9.600	0.000
15	0.000	0.000	0.000	0.000	16.000	0.000	−32.000	0.000
16	0.000	0.000	0.000	0.000	0.000	0.000	0.000	−0.700

Table 6 Influence surface coefficients of M_x at the chosen nodes for the L shaped plate

Node #	IS coefficient of M_x
7	0.00460
9	0.00439
17	0.02307
19	0.02048
31	0.06661
33	0.06277
35	−0.00417
37	−0.00236
38	0.00000
39	0.00000
41	0.28257
49	0.04823
51	0.04730
53	−0.00299
55	−0.00223

Table 7 Influence surface coefficients of M_{xy} at chosen nodes for L shaped plate

Node #	IS coefficient of M_{xy}
7	0.00050
9	0.00023
17	0.00344
19	−0.00072
31	0.02106
33	−0.01893
35	−0.00072
37	0.00023
41	0.00219
49	−0.01656
51	0.02106
53	0.00344
55	0.00050

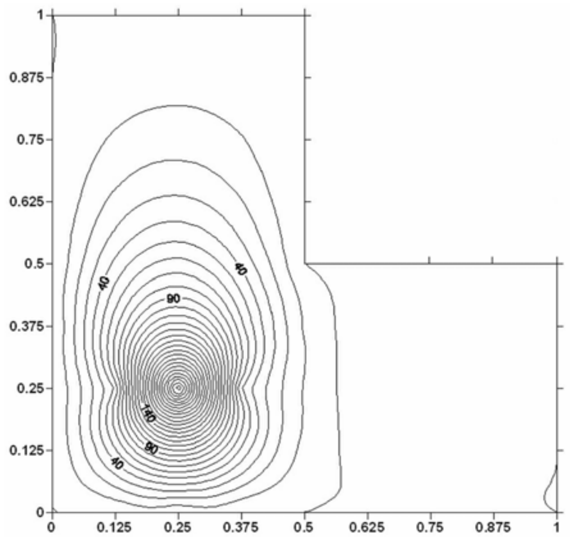


Fig. 9 Contour plot of influence surface of M_x for point 41– values to be multiplied by 10^{-3}

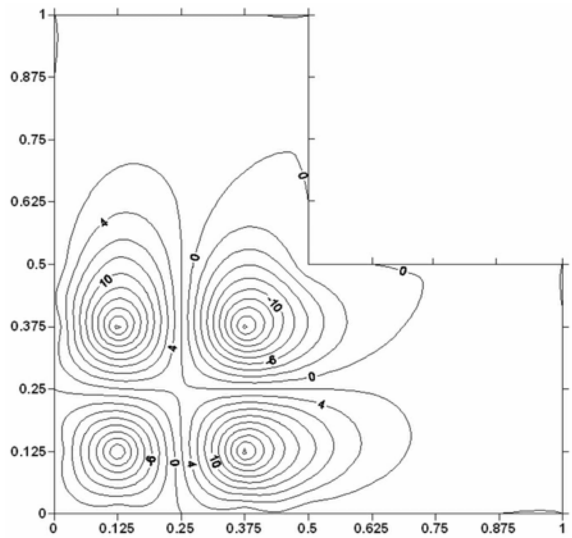


Fig. 10 Contour plot of influence surface of M_{xy} for point 41– values to be multiplied by 10^{-3}

The contour plots of the influence surfaces of M_x and M_{xy} for node 41 are also shown in Fig. 9 and Fig. 10 respectively.

8. Conclusions

General loading matrices are defined for determination of influence line or surface coefficients of internal force, stress, displacement or strain components in linear-elastic structures by using governing equations and basic finite element matrices. Thus, the influence line or surface coefficients of any internal force, stress, displacement or strain components can be directly obtained from the analysis of structure. Once the nodal values of the influence functions are determined, those within the elements may be easily calculated using the element shape functions. The proposed technique is very effective for the finite element codes, since it utilizes the existing finite element matrices to obtain influence line or surface coefficients without any revision of input data or the definition of loading vectors by calculating new displacement fields. Moreover, the technique presented herein may also be utilized for crack growth problems solved by step by step linearization, instead of through introduced unit force or displacement techniques.

References

Akesson, B.A., Bjarnehed, H.L., Andersson, H.O. and Josefson, B.L. (1995), "Routine FE determination of stress intensity factors using Müller-Breslau influence function technique", *Fatique Fract. Engng. Mater. Struct.*, **18**, 1115-1132.

Belegundu, A.D. (1986), "Interpreting adjoint equations in structural optimization", *J. Struct. Eng.,* ASCE, **112**(8), 1971-1976.

Belegundu, A.D. (1988), "The adjoint method for determining influence lines", *Comput. Struct.*, **29**(2), 345-350.

Bogner, F.K., Fox, R.L. and Schmit, L.A. (1965), "The generation of inter-element-compatible stiffness and mass matrices by the use of interpolation formulas", *Proc. of Conf. on Math. Meth. in Struc. Mech*., Wright-Patterson AFB, Ohio.

Cifuentes, A. and Paz, M. (1991), "A note on the determination of influence lines and surfaces using finite elements", *Finite Elements in Analysis and Design*, **7**, 299-305.

Fu, H. (1973), "Indirect structural analysis by finite element method", *Proc. ASCE, J. Struct. Div.*, **99**(ST1), 91-111.

Ghali, A. and Neville, A.M. (1978), *Structural Analysis*, Chapman and Hall, London.

Hanson, J.H., Bittencourt, T.N. and Ingraffea, A.R. (2004), "Three-dimensional influence coefficient method for cohesive crack simulations", *Engineering Fracture Mechanics*, **71**, 2109-2124.

Irons, B. and Ahmad, S. (1986), *Techniques of Finite Elements*, Ellis Horwood Limited, Market Cross House, Cooper Street, Chichester, West Sussex, PO191EB, England.

Kwak, H. and Song, J. (2001), "Live load design moments for parking garage slabs considering support deflection effect", *Comput. Struct.*, **79**, 1735-1751.

Mc Cormac, J.C. (1984), *Structural Analysis*, Harper & Row, New York, 4th edition.

Memari, A.M. and West, H.H. (1991), "Computation of bridge design forces from influence surfaces", *Comput. Struct.*, **38**(5/6), 547-556.

Pucher, A. (1977), *Influence Surface of Elastic Plates*. Berlin: Springer

Shen, W. (1992), "The generalized Müller-Breslau principle for higher-order elements", *Comput. Struct.*, **44**, 207-212.

Yamashita, Y., Shinozaki, M., Ueda, Y. and Sakano, K. (2004), "Fatique crack growth life prediction for surface crack located in stress concentration part based on the three-dimensional finite element method", *Transactions of ASME*, **126**, 160-166.