Structural Engineering and Mechanics, Vol. 20, No. 3 (2005) 265-277
DOI: http://dx.doi.org/10.12989/sem.2005.20.3.265

Application of fractals to study the corroded reinforced concrete beam

Y. F. Fan†, J. Zhou‡ and Z. Q. Hu‡†

State Key Laboratory of Coastal and Offshore Engineering, Dalian University of Technology, Dalian 116024, China

(Received March 2, 2004, Accepted March 21, 2005)

Abstract. This paper is focused on fractal analysis of the surface cracking, a new tool for safety evaluation of corroded reinforced concrete (RC) beams. Comprehensive experimental investigations, including flexural tests, coupon tests on strength evaluation of corroded concrete and rusty rebar, and pullout tests to determine bond strength between concrete and rebar were carried out on nine Corroded Reinforced Concrete Beams (CRCB) exposed to an aggressive environment for more than 10 years. In combination with test results from a previous study on CRCBs fabricated in the laboratory from accelerated methods, it is found that, for both types of beams, the surface cracking distributions are fractal in character at loading and failure stages. Fractal dimension is calculated for all specimens at different corrosion states based on fractal analysis method. Relationships between the fractal dimension and mechanical properties of corroded concrete, rebar corrosion ratio, and ductility of CRCBs are discussed in detail. It is concluded that the fractal dimension can act as a damage index and can be efficiently used to describe the corrosion state of CRCBs.

Key words: condition assessment; corrosion; reinforced concrete beam; fractal; fractal dimension.

1. Introduction

In the 1930s, the current boom in concrete construction began, and concrete has become the material of choice for construction of bridges, dams, highways, and urban transit facilities since then. Recently, cases of premature and serious deterioration of concrete long before their designed service life were reported in several countries (Metha 1997). According to a report by National Materials Advisory Board of the United States, approximately 253,000 concrete bridge decks, some of them less than 20 years old, were in varying states of deterioration and about 35,000 were being added to this list every year (Report of National Materials Advisory Board 1987). The inspection of 42 marine RC structures in Arabian Gulf also showed that 74% were in a severe state of deterioration. Huge financial costs were spent on the repair of deteriorated structures each year (Matta 1993). Therefore, with the increase of cases of premature deterioration of concrete, the effects of the severe environment on the behavior of RC structures, i.e., durability of concrete structures, has become a critical issue and attracted a lot of researchers in the last two decades.

† Associate Professor and Post Doctoral Fellow, Corresponding author, E-mail: fanyf@dlut.edu.cn
‡ Professor, E-mail: zhouj@dlut.edu.cn
‡† Lecturer, E-mail: huzhq@dlut.edu.cn

In order to retrofit the deteriorated structure more effectively, it is necessary to study the causes and processes of the deterioration and structural performance in the damaged state. In contrast to the extensive research that has been done in the area of deterioration process, there are limited studies available on the assessment of CRC members, mostly done in the last decade. For instance, some tests were carried out to study the mechanical properties of CRC members (including beams, slabs and columns), most of which were corroded by accelerated methods in the laboratory (Huang *et al.* 1998, Mangat and Elgarf 1999, Tachibana and Maeda 1990, Yuan and Yu 1997, Lin and Wood 1999, Eyre and Nokhasteh 1992). Based on these test results, simplified models (Huang *et al.* 1998, Mangat and Elgarf 1999), models based on the assumptions of plane section behavior (Lin and Wood 1999, Eyre and Nokhasteh 1992), and Finite Element (FE) model (Dagher and Kulendran 1992) were applied to strength evaluation of CRCBs. Some condition assessment methods based on quantitative descriptions of rebar corrosion ratio were also proposed. In particular, based on tests on RC beams with reinforcement corroded by the electrochemical test method, relationship between the percentage of the reduction of loading capacity or stiffness of CRCB and the thickness loss of rebar were defined, and the corrosion thickness determined from electrochemical parameters were proposed by Huang and Yang as a qualitative parameter for evaluating the loading capacity of beams with corroded reinforcement (Huang and Yang 1997). Considering the corrosion initiation time as a random variable, Enright and Frangopol (1998) developed a probabilistic method for the analysis of resistance degradation of CRCBs. Using reliability theory, Sarveswaran and Roberts (2000) presented a method to assess the reliability of the deteriorated RC beams and conducted sensitivity analysis for identifying important variables qualitatively. However, all aforementioned models are only applicable to some specific cases since not all factors were considered. In practice, the mechanical properties of CRC structures are affected by many factors (such as environmental conditions, material resistance to the action of aggressive substances, rebar corrosion ratio, section geometry, concrete strength, etc.), most of which are stochastic and cannot be described deterministically. Therefore, the state assessment of CRC members based on the evaluation of load carrying capacity is always difficult and lack sufficient accuracy. In addition, the assessment obtained from the accelerated laboratory tests cannot accurately describe the actual corrosion state in field. Bearing all these in mind, it is the objective of this paper to propose a convenient and practical method for the condition assessment of CRCBs.

Since Fractal was introduced in the mid 1970s, the new geometry has been widely adopted to explain cracking phenomena, which can efficiently describe the great variety of natural structures that are irregular, rough and fragmented, having irregularities of various sizes that bear a special 'scaling' relationship to one another (Mandelbrot 1982). Fractal objects are characterized by a fractal dimension estimate d_f. Its physical meaning and practical applications have been investigated by previous studies on the relationship between d_f and the properties of fractured materials (such as fracture toughness, impact energy and dynamic tear energy), showing that irregular geometries of both crack surfaces and crack profiles in a variety of materials could be described by fractal (Mandelbrot *et al.* 1984, Winslow 1985, Bouchaud *et al.* 1993, Borodich 1997, Xie 1990, Aviles and Scholz 1987). Using X-ray scattering technique, it was further demonstrated that the surface of hydrated cement paste was fractal in character, and the large fractal dimension depended somewhat on the water-cement ratio (Winslow 1985). Recent attempts on the study on concrete surfaces also showed that they may be considered fractal in nature (Addison *et al.* 1999, Saouma and Barton 1994, Chiaia *et al.* 1998, Ji *et al.* 1997, Zhou *et al.* 1996). Therefore, the method of fractal analysis is also adopted in this study.

2. Research significance

The objective of this paper is to define a corrosion index to evaluate the damage state of CRCBs through a combined experimental and analytical method. In this vein, experimental investigations on in-situ corroded beams were carried out firstly. Fractal analysis was then used to analyze the test results both from in-situ corroded beams tested in this study and accelerated corroded beams from a previous study. Finally, the relationship between the fractal dimension and mechanical properties of materials and members are discussed in detail. It is concluded that the theory of fractal geometry can be applied to cracking distribution analysis, and the fractal dimension of the surface crack can act as a condition index to evaluate the deterioration of CRC beams. Several examples are provided to illustrate the use of this proposed corrosion index (fractal dimension).

3. Experimental investigation

According to the corrosion process, there are two types of CRCB specimens: (1) laboratory-corroded beams, which is corroded by accelerated corrosion method (such as current, spraying aggressive agent, etc.) in the laboratory, and (2) those delivered directly from the field, which has been in service in the actual aggressive environment for a certain time and has been severely corroded. It is noted that most of the previous research was focused on the first type. As a major contribution, nine beams in the second type were tested in this study.

In this paper, testing results of both kinds of CRCBs were analyzed. Experimental data for the first kind of beams are from Tachibana and Maeda (1990) and Yuan and Yu (1997), and data for the second type are from the tests in this study.

3.1 Preparation of CRC beam

The testing beams, different from those of accelerated corroded in the laboratory, were delivered from a real structure after working in the aggressive environment for more than ten years. According to the aggressive agents, the specimens can be further categorized into two groups. The first group, including CRCB-S1 through CRCB-S6, were from a reinforced concrete frame located

Table 1 Details of CRCBs

No.	b/mm	h/mm	f_y'/MPa	A_s/mm^2	A_s'/mm^2	Stirrups	s/mm	l/mm
CRCB-S1	170	225	210	603	101	Φ8@200	520	1340
CRCB-S2	160	210	210	603	101	Φ8@200	425	1100
CRCB-S3	200	305	310	763	402	Φ8@200	550	1100
CRCB-S4	200	310	310	603	308	Φ8@200	602.5	1450
CRCB-S5	210	230	310	402	402	Φ8@200	530	1360
CRCB-S6	220	220	310	402	308	Φ8@200	580	1460
CRCB-C1	240	310	310	402	402	Φ8@250	750	1800
CRCB-C2	300	410	310	763	308	Φ8@250	830	1660
CRCB-C3	300	370	310	763	308	Φ8@250	920	1840

Note: b, h, l, s are width, depth, length and shear span of the beam, respectively; f_y' is the yield strength of compressive rebar; A_s and A_s' are the area of tensile and compressive rebar.

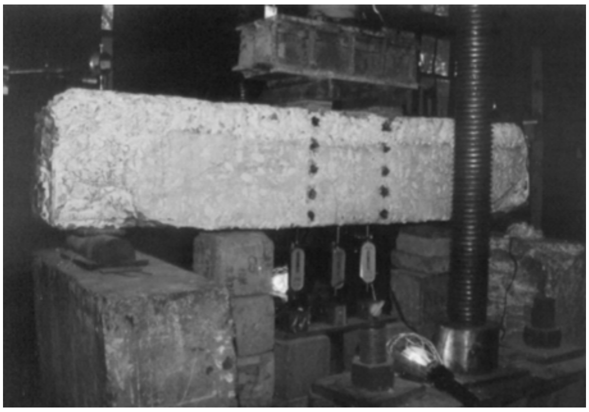


Fig. 1 Flexural test setups

in an ammonium-sulphate plant, where aggressive agents were mainly sulphate, sulfur dioxide and ammonia; and the other group, labeled as CRCB-C1 through CRCB-C3, were from an electronic workshop in a chemical plant and the aggressive agent was mainly Chloride ion. Details of these beams are listed in Table 1.

3.2 Experimental procedure

Coupon tests on mechanical properties of concrete, rusty rebar, bond strength between concrete and rusty rebar, and flexural tests on full scale beams were carried out respectively, with a typical four-point bending test set up shown in Fig. 1.

4. Results and discussions

During tests, specimens in different group behaved differently, as shown in Table 2. Cracking propagation diagrams are given in Fig. 2.

Table 2 Experimental phenomena

Mechanical item	Agent	
	Sulphate	Chloride
Concrete	Apparent internal stress mainly caused by expansion of concrete. Strength of corroded concrete decreases with the deterioration of the concrete.	Apparent internal stress mainly caused by the expansion of rusty rebar. Strength of the corroded concrete decreases with the deterioration of the concrete.
Rebar	Slight rebar corrosion. Corrosion ratio: less than 16%	Severe rebar corrosion. Corrosion ratio: 25-31%.
Bonding strength	Change slightly.	Degraded significantly
Flexural strength	Both cracking and ultimate load increase to some extent, failure mode changes from flexure to shear failure.	Both the loading capacity and ductility dropped significantly, failure mode changes from flexural and shear to bond failure.

(a) CRCB-S2, S3, S1 (From top to bottom) (b) CRCB-C1, C2, C3 (From top to bottom) (c) CRCB- S4, S6, S5 (From top to bottom)

Fig. 2 Cracking pattern of failed CRCBs

5. Fractal description of two types of CRCBs

In fractal analysis, commonly used algorithms for estimating the fractal dimension of surface roughness include ruler method (Aviles and Scholz 1987), box method (Saouma and Barton 1994), and slit island technique (Zhou *et al.* 1996). The box method is employed in this study to calculate the fractal dimension of surface cracks. The method can be briefly described as follows. A sequence of grids (e.g., an initial elementary cell is a square of $L_0 \times L_0$; a secondary elementary cell is a square of $L_0/2 \times L_0/2$, and so on) is firstly placed over maps of the surface cracks. And then the number of cells intersected by surface cracks is counted. Next, the number N is plotted on a *log-log* scale with respect to the inverse of the grid cell size r. If there is a linear relationship, it can be concluded that crack distribution on the surface of CRCB is self-similar, and the fractal dimension can be defined as the slope of the curve, which is

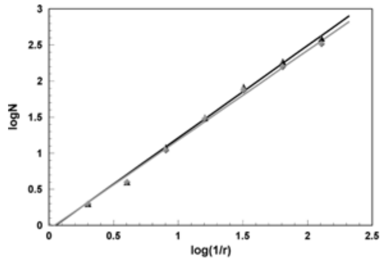
$$d_f = \log N/\log\left(\frac{1}{r}\right) \tag{1}$$

5.1 Fractal analysis of in-situ corroded beams

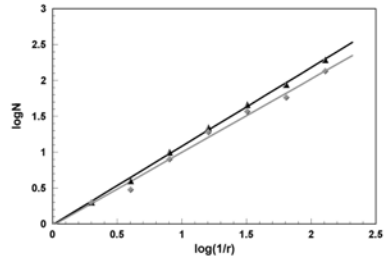
Using the box method described above, $\log N$ vs. $\log(1/r)$ curves of the CRCBs tested in this study are plotted in Fig. 3, where (*a*) through (*f*) are specimens corroded by Sulphate, and those in (*g*) though (*k*) are corroded by Chloride. From Fig. 3, we can observe that there is a perfect liner relationship between $\log N$ and $\log(1/r)$, regardless of the beam type, which indicates that the surface cracks distribution is statistical fractal. As stated above, the slope of all these curves is the fractal dimension d_f. It is noted that fractal dimensions achieved from both sides of the beams are pretty close to each other, illustrating the reliability of the test results.

5.2 Fractal analysis of accelerated corroded beams

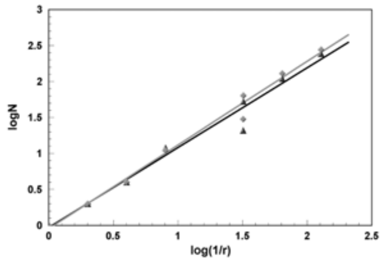
Extensive flexural tests were carried out on accelerated corroded beams by Tachibana and Maeda (1990), and the results are adopted herein for fractal analysis. In their tests, beams were corroded by an accelerated galvanostatic corrosion method in the laboratory for 3 days, 6 days, 10 days and 15 days, respectively. Surface cracking distributions were recorded for each loading step. Box method



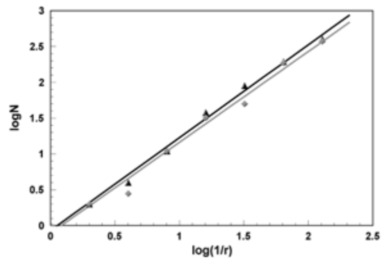
(a) CRCB -S1



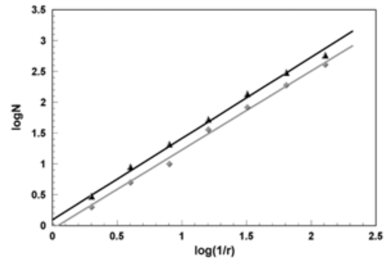
(b) CRCB -S2



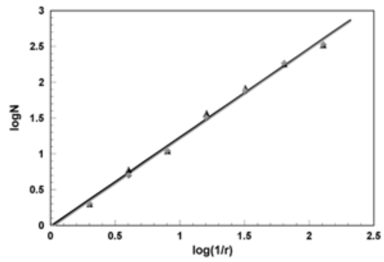
(c) CRCB -S3



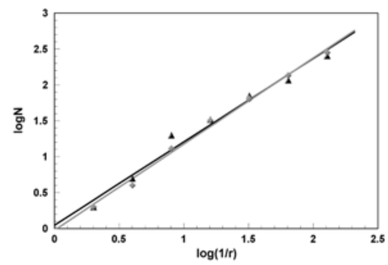
(d) CRCB -S4



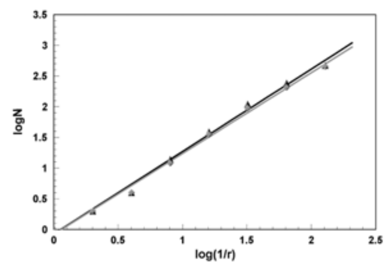
(e) CRCB -S5



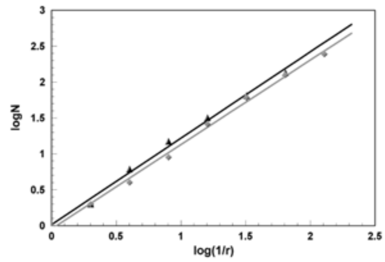
(f) CRCB -S6



(g) CRCB -C1

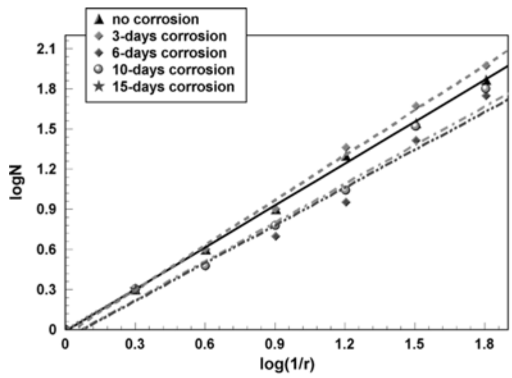


(h) CRCB -C2

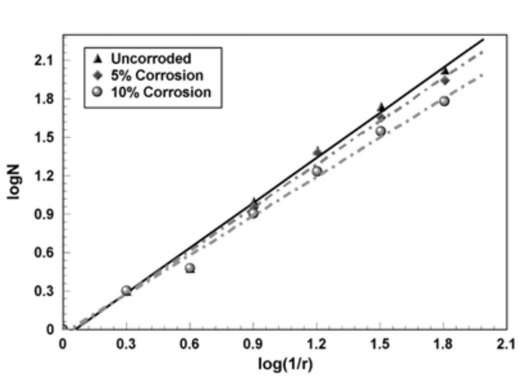


(i) CRCB -C3

Fig. 3 Relation between log N and log(1/r) of the surface cracks on both sides of CRCB

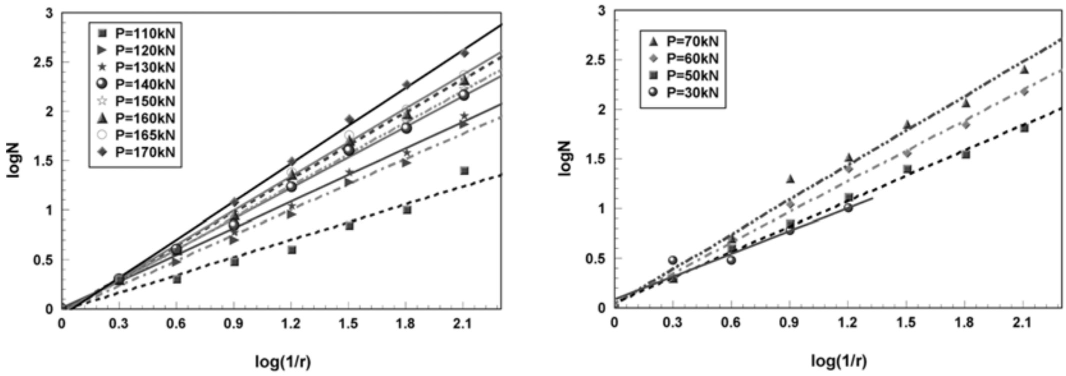


(a) Relation between logN and log (1/r)



(b) Relation between logN and log (1/r)

Fig. 4 Relation between log N and log(1/r) of the surface cracks under every load step



(a) CRCB-S1 (b) CRCB-C1

Fig. 5 Relation between $\log N$ and $\log(1/r)$ of the surface cracks under every load step

is once again used to analyze the crack distribution, with the analysis results shown in Fig. 4(a).

Tachibana and Maeda (1990) also conducted some flexural tests on uncorroded and corroded beams with reinforcements corroded with an accelerated galvanostatic corrosion method for 5% and 10% in the laboratory. Using box methods, the fractal analysis results are shown in Fig. 4(b).

From Fig. 4, a good linear relationship can also be observed between $\log N$ and $\log(1/r)$ curves during the whole loading stage, indicating that the cracking distribution on the accelerated corroded beams also follow a statistical fractal character. Therefore, it can be concluded that the surface cracking distribution is statistical fractal for beams both corroded in field and using accelerated corroded methods in the laboratory.

5.3 Cracking behavior of CRCBs at loading stage

According to the cracking diagrams on the surface of CRCB under each loading step, $\log N$ can be plotted against $\log(1/r)$ for CRCB-S1 and CRCB-C1 in Fig. 5, which follows a linear path, indicating that surface cracks are statistical self-similarity.

The slope of the curves in Fig. 5 denote the fractal dimension d_f for each loading step, as summarized in Figs. 6(a) and (b) for CRCB-S1 and CRCB-C1, respectively. The curves shown in Fig. 6 can be fitted using the following expression to describe the relationship between fractal dimensions and mechanical properties of the beams

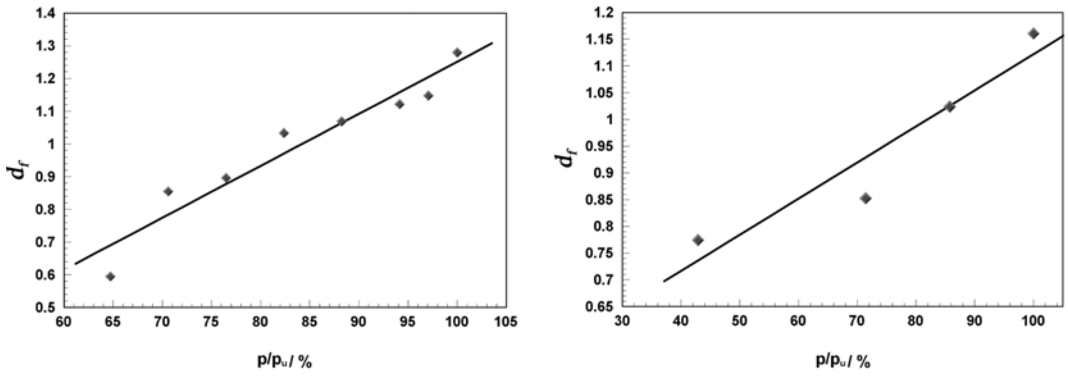
for CRCB-S1,

$$d_f = -0.33936 + 1.59058 p/p_u \tag{2}$$

for CRCB-C1,

$$d_f = 0.44647 + 0.67552 p/p_u \tag{3}$$

Therefore, it is reasonable to assume that there exists a linear relationship between fractal dimension and service load of CRCB as



(a) CRCB-S1 (b) CRCB–C1

Fig. 6 Relation between load and d_f

$$d_f = A + B \cdot p/p_u \tag{4}$$

where A and B are unknown constants to be determined from testing results, which may vary due to different corrosive media; p is the load at stage considered and p_u is the ultimate load.

As demonstrated in Eq. (4), fractal dimension can be used to determine the present loading state and evaluate the safety of the members. From Eq. (2) and Eq. (3), it is noted that the value of constant A can be either negative or positive, corresponding to a ductile and brittle failure, respectively. Therefore, the constant A can be used as an influential index that indicates the effect of different corrosive media CRCBs exposed to.

6. Condition assessment of CRCB

Fractal dimension can be further used to evaluate the mechanical properties (such as strength of corroded concrete, rebar corrosion ratio, ductility degradation and failure mode of the member) of CRCBs, as will be discussed next.

6.1 Strength of corroded concrete

Fig. 7 plots the fractal dimension d_f vs. compressive strength of corroded concrete obtained from the material test, from which we can observe that the fractal dimension of the CRCB will increase as the concrete compressive strength increases, i.e., cracks distribution will decrease with the deterioration of concrete.

It is noted that there is a wide distribution of the data, possibly due to the limited test results and some stochastic influential factors acted on the RC members in the field that cannot be accounted. More tests will be conducted on CRCBs in a following study to further investigate the relationship between d_f and f_c. It is anticipated that more reasonable results between d_f and f_c can be obtained with the accumulation of more experimental data.

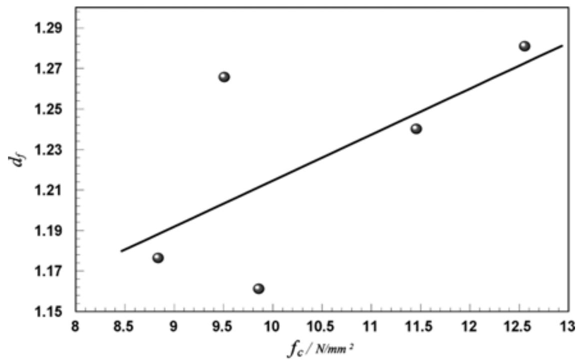
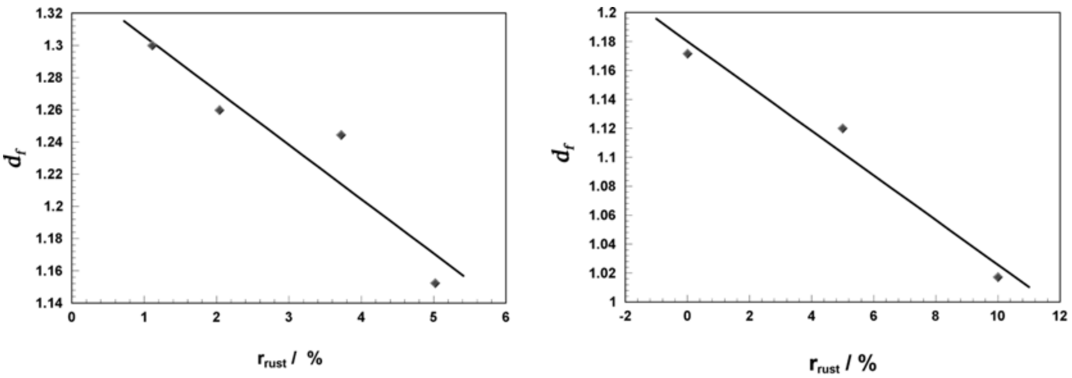


Fig. 7 Relation between d_f and f_c of beams in this paper



(a) Relation between r_{rust} and d_f of beams in this paper (b) Relation between r_{rust} and d_f of beams in Yuan and Yu (1997)

Fig. 8 Relation between fractal dimensions of cracks and rebar corrosion ratio

6.2 Corrosion ratio of rusty rebar

Rebar diameter, corrosion degree of rebar and the concrete cover are major factors that can significantly affect the cracking of the members. Since the rebar diameter and cover are the same for all CRCBs tested in this study, only the effect of the corrosion ratio of the rebar is investigated, and the other two factors will be considered in following studies.

To obtain the mechanical property of rusty rebars in CRCBs, they were pulled out for tensile tests. Corrosion ratio r_{rust} is defined as the ratio of mass loss to the original mass. Fig. 8 shows $d_f - r_{rust}$ curve, which indicates that the fractal dimension d_f will drop significantly as the rebar corrosion ratio increases.

6.3 Ductility of CRCB

It is known that the ratio between ultimate strength P_u and cracking load P_{cr}, P_u/P_{cr}, can be used to describe the ductility of CRCBs. The higher the value of P_u/P_{cr} is, the more ductile the beam will be. We can introduce the change of the ratio $\Delta P_u/P_{cr}$ as ductile index k, which can be plotted against d_f for CRCBs tested in this study in Fig. 9. A 3-D diagram illustrating the relationship among

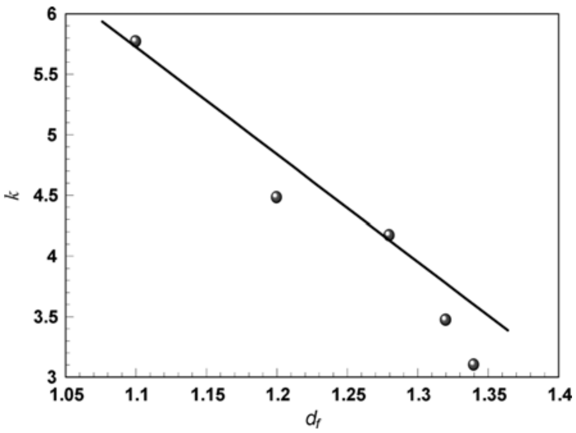


Fig. 9 Relation between fractal dimension and strength degradation of CRC beams in this paper

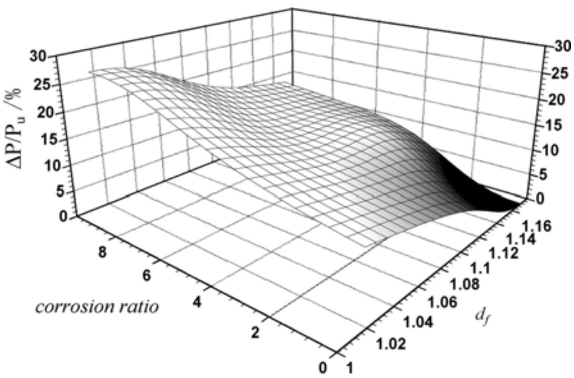


Fig. 10 Relation between fractal dimension and strength degradation of CRC beams in Yuan and Yu (1997)

ductile index, fractal dimension and strength degradation of CRCBs is given in Fig. 10 based on the test results provided by Yuan and Yu (1997).

Fig. 9 and Fig. 10 indicate that with the increase of fractal dimension d_f, ductility of the CRCB will be deteriorated. That is, the beams will change from a ductile failure, a favorable failure mode in design, to a brittle failure that should be avoided in practice. Once the relation between d_f and k is determined, the present ductility of the CRCB can be drawn from the fractal dimension of the surface cracks, and the damage condition can be evaluated effectively.

6.4 Failure mode of CRCB

According to experimental results reported by Tachibana and Maeda (1990), the fractal dimension is plotted against the ratio of P (ultimate load of corroded beams) to P_u (ultimate load of uncorroded beams) in Fig. 11, where failure modes of corroded beams are also provided.

From Fig. 11, it can be seen that beams corroded for 3 days fail in the same mode as those

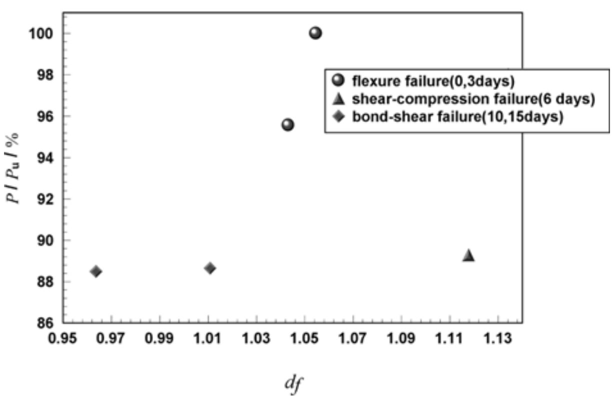


Fig. 11 Relation between d_f and P/P_u of CRC beams in Tachibana and Maeda (1990)

uncorroded, and the fractal dimension of the surface cracks decreases. However, if the corrosion process lasts 6 days, the failure mode changes from flexure to shear-compression failure, and the fractal dimension increases significantly. With the corrosion process continues, the failure mode becomes bond-shear or bond failure, and the fractal dimension decreases gradually. Therefore, it can be concluded that the fractal dimension can reflect the change of failure mode. In summary, at initial corrosion stage, the fractal dimension decreases and the failure mode is the same as uncorroded ones. The fractal dimension will increase significantly and the failure mode will change if the corrosion accumulates to a certain degree. When the corrosion becomes worse, the fractal dimension will decrease and be less than the dimension in the first corrosion stage, and the failure mode will change to bond failure.

6.5 Condition assessment of CRCB

At present, the damage condition of CRCB can be evaluated from strength of concrete core that is drilled from the member and corrosion ratio of the rebar in concrete by NDT method. However, both the strength of corroded concrete and corrosion ratio of the rebar are widely scattered, making it difficult to determine the mechanical property and safety condition of the CRCB solely dependent on the two sets of information. Since the surface cracking can be easily measured, and relations between the fractal dimension and the mechanical properties have been established through the analysis of experimental results on CRCB as shown above, fractal dimension is proposed as a corrosion index to evaluate the condition of the CRCB as follows.

6.5.1 Loading capacity assessment

According to Eq. (2), the loading capacity of CRCB can be expressed as

$$p = (d_f - A)/B \cdot p_u \tag{5}$$

If the load is less than p_u, CRCB is in a safe state; if the present load equals to p_u, CRCB is in a critical stage; and if the present load exceeds p_u, CRCB fails.

6.5.2 Damage state assessment

Similarly, other mechanical properties of the CRCB, such as ductility, material corrosion state and the strength after being corroded, failure mode, etc., can all be assessed by the fractal dimension, and a synthetic evaluation can be obtained for the current state of the CRCB.

7. Conclusions

The fact that corrosion is essentially a stochastic process affected by many factors adds the difficulty of defining a reliable index to describe the corrosion state of a CRCB. A novel method is presented in this study to study the damage state. Based on test results for corroded beams both from actual and accelerated corrosion, it is found that surface cracks on the CRCB generally have highly irregular surface that are fractal in character during the entire loading process. Through the fractal analysis of the cracks, relationship between fractal dimension and mechanical properties of the beams (including strength of concrete, rebar corrosion ratio, and beam ductility, etc.) are defined. The fractal dimension is proposed to be a synthetic quantitative index to describe the corrosion state of CRCB, which can connect the overall geometry characteristics and mechanical properties of the CRCB. This corrosion index can efficiently describe the influential factors of load carrying capacity, such as the corrosion sate of concrete and rebar, bond strength deterioration, etc., which are difficult to obtain in field by NDT method.

Acknowledgements

This paper presents some of the results of a National Funded project, and is partially funded by a Liaoning Provincial Funded project (Grant No.20022135) and a National Doctoral Funded project "Study on Failure Mechanism of Submarine Pipelines Under Corrosion and Loadings". Contributions from Prof. Z.G. Huang (Zhengzhou University, China) and Prof. J.M. Li (Zhengzhou University, China) for acting as co-investigators of the first project are acknowledged. Contributions from Dr. A. Chen (West Virginia University, USA) for reviewing of the paper are acknowledged as well.

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