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Numerical study of dynamic buckling for plate and shell structures

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Abstract. A numerical approach combining the finite element method with two different stability criteria namely the Budiansky and the phase-plane buckling criteria is used to study the dynamic buckling phenomena of plate and shell structures subjected to sudden applied loading. In the finite element analysis an explicit time integration scheme is used and the two criteria are implemented in the Finite Element analysis. The dynamic responses of the plate and shell structures have been investigated for different values of the plate and shell imperfection factors. The results indicate that the dynamic buckling time, which is normally considered in predicting elasto-plastic buckling behavior, should be taken into consideration with the buckling criteria for elastic buckling analysis of plate and shell structures. By selecting proper control variables and incorporating them with two dynamic buckling criteria, the unique dynamic buckling load can be obtained and the problems of ambiguity and contradiction of dynamic buckling load of plate and shell structure can be resolved.

Key words: buckling time; dynamic buckling; dynamic buckling criteria; dynamic buckling load; plate and shell structure; finite element method.

1. Introduction

One of the most important questions arising specifically in nonlinear mechanics is that of structural instability or buckling. The problem essentially concerns slender bodies i.e., columns, plates and shells that are subjected to axial compression and lateral loading, and these not only challenge the strength of the structure but could also cause deformations of unacceptably large amplitudes and could lead to loss of stability and collapse of the whole structure. The buckling in structures can occur in several ways due to various kinds of static and dynamic loading. Consequently, the application of the general concept of stability to various problems of instability or buckling of structures, has given rise to numerous approaches that provide several criteria for static and dynamic buckling. Geometrically nonlinear plates and shells are used in steadily broadening applications in almost all branches of modern industry, from aerospace, and ship structures to building construction. The behavior of these plates and shells under loading is accompanied by

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essentially nonlinear instability effects due to their flexibility and slender form. Thus the problem of buckling of nonlinear plates and shells is very diverse and complex, and require serious consideration.

The understanding of nonlinear effects such as buckling is absolutely necessary for the design of plate and shell structures. However, until now, well-founded recommendations for the design and control of the buckling of plate and shell structures have not been available. Therefore, the studies of buckling of structures have received increasing attention in recent decades. In spite of an impressive amount of research and tenacious effort, many problems about buckling remain open, largely owing to great theoretical difficulties, hard numerical and experimental verifications and finally unavoidable physical uncertainties.

Buckling has been normally classified into static buckling and dynamic buckling, which are two types of structural instability due to static and dynamic loadings. Static buckling has been studied extensively, assuming static and even conservative loadings. The released energy during the transition from a state of static instability to a new stable state, can result in disastrous structural responses. Static buckling has been traditionally analyzed as a mode of failure. In the static buckling analysis, two approaches are traditionally adopted, i.e., eigenvalue buckling analysis (linear buckling analysis) and nonlinear buckling analysis. The eigenvalue buckling analysis predicts the theoretical buckling strength of an ideal elastic structure that maintains its shape up to buckling. It computes the structural eigenvalues for the given loading and constraints. This is known as classical Euler buckling analysis. However, structural imperfections and nonlinearities prevent most structures from reaching their buckling strength predicted by this eigenvalue analysis. In fact, the problem of imperfections (Budiansky 1967, Lee 1995) was discovered, to be a decisive explanation of classical discrepancies between theoretical and experimental results in buckling studies. Thus, nonlinear buckling analysis has been developed and considers a load-dependent prebuckling deformation during loading, up to the state of structural instability. Nonlinear buckling analysis is more accurate than eigenvalue analysis because it employs non-linear, and large-deflection, to predict buckling loads. The non-linear nature of this analysis thus permits the modeling of geometric imperfections, load perturbations and material nonlinearities.

It should not be forgotten that the loading systems of external forces are rarely static and that the structures, particularly those made of thin plates and shells, can easily excite dynamic behavior, with complex responses, time and phase shifts, and are highly subjected to dangerous instabilities. Therefore the dynamic buckling instability of plates and shells has been evoked due to dynamic loading and sensitivity to instability phenomena. In spite of its considerable importance, the dynamic buckling of plate and shell has not prompted as much research as might be thought so.

The studies of dynamic buckling of structures have been performed by many researchers. However, it was found in the studies that different numerical approaches tended to yield different buckling loads. For example, Meier (1945) demonstrated that an imperfect column subjected to a suddenly applied axial impulsive load, may withstand compressive stress much in excess of the static buckling critical stress. However, others, in some cases, found that the column will dynamically buckle under the level of load which is smaller than the static critical load for viscoelastic perfect columns under step loads (Dost and Glockner 1982). In order to derive a recommended approach for predicting buckling loads, extensive investigations have been conducted. Lindberg and Florence (1987) published summarized results from the research on dynamic buckling of structures under transient dynamic loads. Ari-Gur *et al.* (1982, 1997a, 1997b) performed a series of studies on the dynamic buckling of column and plate structures. The dynamic response of columns subject to an axial impact was investigated by experimental and theoretical methods. A criterion was defined for determining the dynamic buckling loads based on analytical and experimental results for columns with initial geometric imperfection under an axial impact. In these studies, axial displacement and inertia were considered, while the rotary inertia of cross-section was neglected. Further, Ari-Gur and Elishakoff (1997) studied the dynamic instability of a transversely isotropic pinned-end column subjected to a compressive pulse by a numerical method. Ari-Gur and Simonetta (1997) constructed the analytical dynamic pulse buckling model of rectangular composite plates based on the Kirchhoff thin-plate deformation theory. It was found that the dynamic buckling loads were not always higher than the static ones, in some cases there is a range of loading frequencies near the fundamental frequency of the plate where dynamic buckling occurs for lower loads. Lee (1995a, 1995b, 1997) also performed theoretical and experimental studies on the dynamic buckling phenomena of beams. In his study, the linearized equation of motion in matrix form of an Euler-Bernoulli inextensible beam with initial curvature and a tip mass subjected to an axial pulsating load, was formulated based on a Lagrangian approach and the assumed mode method. The initial curvature of the beam was found to have no effect on the dynamic stability of the beam in the absence of a tip mass. The equation of motion in matrix form of a tapered cantilever Euler beam subjected to a follower force at the free end was also formulated based on the Lagrangian approach and the assumed mode method (Lee *et al.* 1997).

Weller *et al.* (1989) performed a numerical study with the ADINA computer code. They showed that both the maximum initial imperfection and load duration, affect the dynamic buckling properties of structures. The dynamic load amplification factor for beams and plates was determined. The study also showed that for elastic dynamic buckling, resonance effects in combination with large imperfections could reduce the buckling load. It was demonstrated that the dynamic load amplification factor was usually higher than unity. However, in some cases, in the presence of certain magnitudes of initial geometric imperfection and for loading durations close to the first natural period in bending, the value was smaller than unity. Cui *et al.* (1999a, 1999b) investigated the dynamic buckling of thin imperfect rectangular plates subjected to intermediate-velocity impact loads by numerical simulations and experimental studies. The dynamic buckling and dynamic yielding critical conditions are defined, and the corresponding critical dynamic loads are estimated.

It is noted that from the aforementioned, dynamic buckling loads are commonly determined by considering the stability criterion of Budiansky (1962, 1967). A dynamically critical condition is defined if some characteristic value increases rapidly with the loading amplitude. That is the critical conditions under which dynamic buckling occurs, which involves a large change in the response due to a small change in the loading. The critical load is that for which the slope of the buckling curve abruptly changes. In this work the quotient of the dynamic buckling load and the load of bifurcation is defined as the dynamic load-amplification factor. Nevertheless, the practical application of this dynamic buckling criterion shows that it is not always decisive. The disadvantage of this criterion is caused by the fact that the load-carrying capacity of the structure is not taken into account.

Aboudi *et al.* (1990), Cederbaum *et al.* (1991) and Gilat and Aboudi (1995, 2002) adopted the Lyapunov exponents in their approach to investigate dynamic buckling phenomena for homogeneous and composite plates. This approach is based on the evaluation of a set of numbers, the Lyapunov exponents, the signs of which characterize the nature of the dynamical system. The investigation showed that the dynamic stability of plates subjected to suddenly thermal or mechanical loading can be efficiently analyzed by evaluating the largest Lyapunov exponents, the sign of which characterizes the nature of the response. The disadvantage is that the formulation and implementation of Lyapunov exponents is awkward and tedious, and it is quite complex to accurately calculate the largest Lyapunov exponents for complex structures.

For discrete systems, the non-linear dynamic buckling of discrete dissipative/nondissipative autonomous potential systems under step loads of constant magnitude with infinite or finite duration has been extensively studied by Kounadis and his associates (Kounadis *et al.* 1988, 1990, 1991, 1996, 1999, 2004). From the studies, it can be found that the major problem in dealing with these discrete systems is the intractability of the non-linear initial-value problems (Kounadis *et al.* 1999, 2004). In order to solve the problem, a more reliable and efficient approach based on energy geometric consideration has been presented (Kounadis 1999). Furthermore, in order to overcome the disadvantage, stress failure criteria should also be used. For example, Pety and Fahlbusch (2000) investigated the dynamic stability behavior of imperfect simply supported plates subjected to in-plane pulse loading, in which a stress failure criterion is used to calculate dynamic buckling loads. Karagiozova and Jones (1992a, 1992b) investigated the dynamic elastic-plastic buckling of thin-walled plates and shells from the viewpoint of elastic-plastic stress wave propagation. The influences of initial imperfection, dynamic load shape and duration, on the dynamic buckling behavior of the model, and the influence of the impact velocity and the striking mass on the development of the buckling shape were examined. These studies have provided some insight into the dynamic buckling phenomenon of thin-walled structures in the elasto-plastic case.

As reviewed above, the dynamic buckling analysis of plate and shell structures under in-plane load is a problem of dynamic response, in which imperfections are necessary to cause out-of-plane motion. The critical conditions for defining a dynamic buckling load can be found by using different approaches. However, there does not exist any standard criterion for the investigation of structures with dynamic buckling behavior. Therefore it is necessary to establish critical conditions for finding a dynamic buckling load.

This paper presents the results from numerical investigations of the behavior of plates and shells subjected to suddenly applied in-plane pressure loads. A numerical approach is used combining the finite element method with two different stability criteria: the Budiansky buckling criteria and the phase-plane buckling criteria. Results are presented for different values of the imperfection factors of plate and shell structures. It is shown that the approach forms an efficient tool which provides a quantitative and unequivocal answer to the question of dynamic buckling of plates and shells subjected to sudden dynamic loading.

2. Dynamic buckling phenomenon and criteria of plate and shell structures

It is important to first define dynamic stability or buckling. As previously discussed, at buckling, the structure responds with large deflections with maintained loading parameter values. If the developing deflection pattern is basically orthogonal to the pre-buckling pattern, then the event may be called buckling or bifurcation. For plate and shell structures, the instability or buckling may be connected with relatively large displacement amplitudes without a significant change of the pattern i.e., snap-through. In the case of dynamic buckling, the buckling pattern develops and the plates and shells are accelerated to exhibit dynamic deflections into the post-buckling state. This behavior can be observed on actual testing of engineering structures. Although many critical conditions for defining dynamic buckling load have been proposed, but there is no standard criterion for detecting dynamic buckling of plate and shell structures. Therefore, it is imperative to thoroughly review the current buckling criteria and to establish a common acceptable criterion to find the dynamic buckling loads for plate and shell structures.

2.1 Budiansky criterion

For the Budiansky criterion (Budiansky and Roth 1962, Budiansky 1967), the dynamically critical condition is defined if some characteristic value increases rapidly with the loading amplitude. The Budiansky criterion involves a set of equations of motion of dynamic of systems that are solved for various values of loading and the value for which there is a significant jump in the response is assumed critical. It should be noted that selecting a suitable characteristic value of response is very important. That is when monitoring the system response through displacements of selected points for small values of the loading parameter, small oscillations are observed. The amplitudes of which gradually increases as the loading is increased; when the loading reaches its critical value, the maximum amplitude experiences a large jump. Hence the implementation of this criterion requires solving the equations of motion for different values of the loading parameter, then plotting the displacement amplitude versus the loading curve to determine the critical loading value.

In the literature, the quotient of the dynamic buckling load P_{cr}^D and the load of bifurcation P_{cr} may be defined as the dynamic load amplification factor (DLF)

$$DLF = \frac{P_{cr}^D}{P_{cr}} \tag{1}$$

This load-amplifying quotient describes the dynamic behavior of the plate and shell structure under the impact loading case.

The concept of DLF is of practical interest for the designer, since it provides a direction indication of the load carrying capacity of the structural elements, exposed to rapidly applied dynamic load relative to statically applied load.

2.2 Phase-plane criterion

The dynamic buckling of a system from the point of view of nonlinear stability is defined as the smallest load for which an unbounded motion is initiated. Therefore the dynamic criterion can also be observed by the existence of an inflection point on the displacement response curve (Jones and dos Reis 1980, Tabiei and Tanov 1998a, 1998b, Chien and Palazotto 1992). The phase-plane is the plane in which the phase trajectories lie, i.e., the plots of the first time derivative of displacement with respect to time versus displacement if the displacement is a parameter used to monitor the system response. For loads smaller than the critical load, the system simply oscillates about the static equilibrium point. At loading equal to or greater than the critical load escaping motion, indicating buckling, occurs through the unstable static equilibrium point.

The subsequent analysis concerns a structural system which can be discretized by the finite element method. Applying the finite element procedure to the variational equation (Bathe 1996), the governing Lagrange equations of motion of non-linear plate and shell structures in finite element formulation can be expressed as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}_{nl}\mathbf{x} = \mathbf{g}(\mathbf{x}, \dot{\mathbf{x}}, t) \tag{2}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the material damping coefficient matrix, $\ddot{\mathbf{x}}, \dot{\mathbf{x}}$ and $\mathbf{x}$ are the displacement, velocity and acceleration vectors, respectively. The stiffness matrix includes large displacement/rotation and/or large strain non-linear stiffness. The external force vector function is $\mathbf{g}$ which has an n vector function of $\mathbf{x}$ and $\dot{\mathbf{x}}$.

The set of n second-order multi-degree-of-freedom Lagrangian equation of motion (2) can be rewritten as a system of $2n$ first-order Hamiltonian equations subject to prescribed initial equations, i.e.,

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{y} \\ \dot{\mathbf{y}} &= \mathbf{M}^{-1}[\mathbf{g}(\mathbf{x}, \dot{\mathbf{x}}, t) - \mathbf{C}\dot{\mathbf{x}} - \mathbf{K}_{nl}\mathbf{x}] \end{aligned} \tag{3}$$

It can be clearly shown using (3) that the stability for an equilibrium state depends on the loading function $\mathbf{g}$. Let $\mathbf{z} = \begin{Bmatrix} \mathbf{x} \\ \mathbf{y} \end{Bmatrix}$, then $\mathbf{z} = \{\mathbf{x} \quad \mathbf{y}\}^{\mathbf{T}}$ the system equation can further be rewritten as follows:

$$\begin{aligned} \dot{\mathbf{z}} &= f(\mathbf{z}, \lambda) \\ \mathbf{z}(\mathbf{0}) &= \mathbf{z}_0 \end{aligned} \tag{4}$$

where λ is the main control parameter. This can be treated as the state equation of a $2n^{th}$ order autonomous nonlinear dynamical system with vector field $\mathbf{z}$. Such a system can be comprehensively classified in terms of its steady state solutions and limit-sets as shown in Sophianopoulos (1999). For autonomous Hamiltonian systems similar to (3) the only type of steady state behavior and the simplest limit set case is that of the equilibrium fixed point $\mathbf{z_E}$, given by

$$f(\mathbf{z_E}) = 0 \tag{5}$$

which constitutes a necessary and sufficient criterion for static equilibrium. The aforementioned nonlinear dynamic equations can have several distinct steady-state solutions depending upon the particular initial conditions, and it is often of interest to investigate these possibilities. In order to solve this dynamic system, different methods may be used, such as “Poincaré like simple cell mapping” Levitas *et al.* (1994) and “mapping trajectory pursuit” Ding *et al.* (2002). In this study, the Poincaré Phase-plane criterion will be used to study the nonlinear stability of the plate and shell system through numerical analysis. All the previous theoretical qualitative findings will be verified through the comprehensive fully nonlinear static and dynamic stability analysis of plate and shell structures under dynamic loading.

3. Numerical examples of plate and shell structures

The problems of dynamic buckling of plate and shell structures are numerically solved using the finite element code ABAQUS in which the user subroutine is used to implement previous concepts and the expression. The criteria for the choice of one of implicit or explicit time integration methods are dependent on the type of problem. In this study, the performance of both implicit and explicit time integration methods are implemented and compared to establish the computational efficiency and accuracy of the methodology. From a convergence study, it is found that both time integration schemes predict essentially the same response results. However, for the dynamic buckling problem of plate and shell structures, explicit time integration is more efficient than the implicit time integration scheme which has been shown by Tabiei *et al.* (1998). To assess the dynamic stability of the plate and shell, two types of buckling criteria, namely the Budiansky and phase-plane approaches are adopted. In most cases when applying the above buckling criteria displacements and velocities of certain points of the structure are traced. To be able to get good results from the analyses, the points for which displacements are to be monitored are to be carefully chosen; otherwise the plots produced may be rather obscure and confusing. For the Budiansky buckling criteria, the transverse deformations of plate and shell structures are selected as the monitoring parameters to check the numerical results and to detect the bifurcation of the plates and shells. For the phase-plane approach, the shortening displacement of the plates and shells along the edge of the loading direction, is selected as a monitoring condition in which the Poincaré Phase-plane criterion is adopted by the implementing mapping trajectory pursuit method (Ding *et al.* 2002).

This study has applied both the aforementioned buckling criteria. All results have showed that for the problems studied both criteria have predicted equal values for the critical loading within reasonable accuracy.

3.1 Dynamic buckling analysis of a square plate

In the first example, some characteristic features of the dynamic elastic buckling behavior of a square plate subjected to uniformly distributed impact loads along one edge in one direction are investigated. The plate is clamped on the load acting side and on its opposite side, but the constraint on load acting edge in load direction is set free. The other two opposite edges of the plate are set free. Here, a stationary square plate is subjected to a suddenly applied step load which simulates the impact of the structure. The finite element model, boundary conditions and step load shape are shown in Fig. 1. In this model, the side length of the square plate L = 100 mm and the thickness h = 0.5 mm. It is assumed that the plate is made of an aluminum alloy and the material properties of

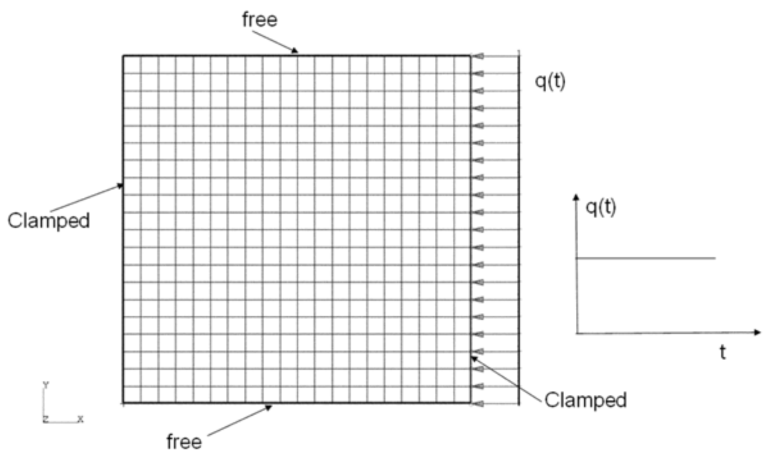


Fig. 1 The finite element mesh of a square plate and its loading

Table 1 The material properties of aluminum alloy

Young's modulus of elasticity (MPa)	70×10^3
Poisson's ratio	$\nu = 0.33$
Mass density (g/mm^3)	0.0027

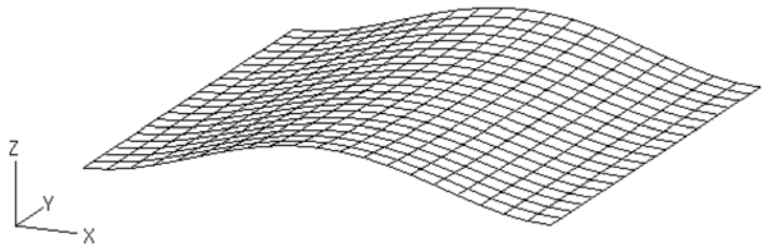


Fig. 2 First static buckling mode shape of a square plate

the aluminum alloy are shown in Table 1. In the analysis, the dynamic load factor

$$L_f = q(t)/21 \tag{6}$$

The initial imperfections are introduced in the geometry of the model to trigger buckling. A linear stability analysis on the geometrically perfect structure is performed first in order to establish probable collapse modes. The imperfection was then introduced by adding the scaled modes to the geometry of the perfect structure. A geometrically nonlinear dynamic analysis is then performed on this imperfect structure. In this example, the initial imperfection of the plate is assumed to follow the same shape as its first static buckling mode. The first static buckling mode shape of the square plate is shown in Fig. 2.

Imperfection factors, defined as Ψ are used to investigate the effect of the initial imperfection of the plate due to the static and dynamic buckling critical loads. The imperfection factors are introduced and defined as

$$\Psi = (\lambda_o(x,y)|_{\max})/h \tag{7}$$

where $\lambda_o(x, y)$ is the imperfection of the plate or shell and follows the same shape of the first static buckling mode, and h is plate thickness.

Ten plate models with different imperfection factors are initially analyzed for the static buckling analysis. Fig. 3 shows the static buckling load factors versus imperfection factors. It can be seen that the imperfections (specified by the imperfection factors) have insignificant effect on the static buckling load for this plate. In the dynamic buckling analysis, the square plates, which are subjected to different dynamic loads $q(t)$, or different load factors L_f, are calculated with five types of imperfection factors. The maximum transverse deflections at the monitoring point are obtained. The ratios of maximum deflections to plate thickness versus different load factors for different imperfection factors are depicted in Fig. 4. It can be demonstrated that the dynamic response of the plate is sensitive to the imperfection factors. The ratio of the maximum deflection with plate thickness increases with an increase in the imperfection factor under the same dynamic loading. The

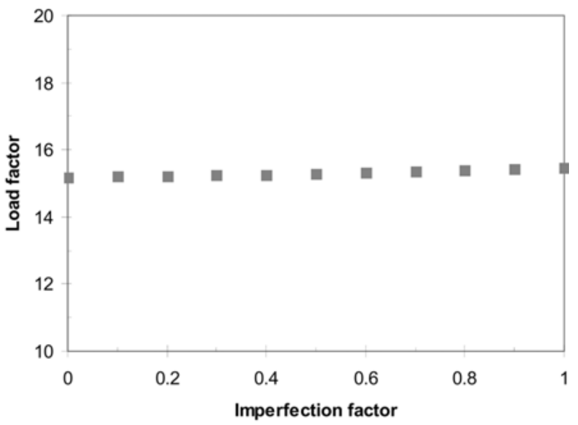


Fig. 3 Static buckling load factors vs imperfection factors

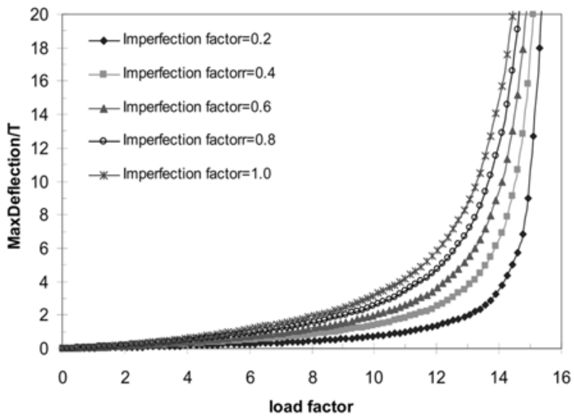


Fig. 4 Maximum transverse displacement of plate versus dynamic load factor with respective imperfection factors

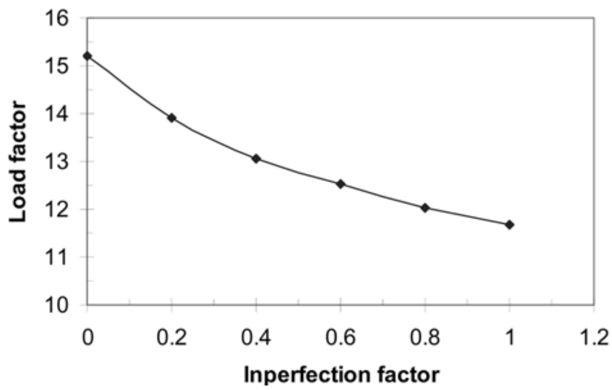
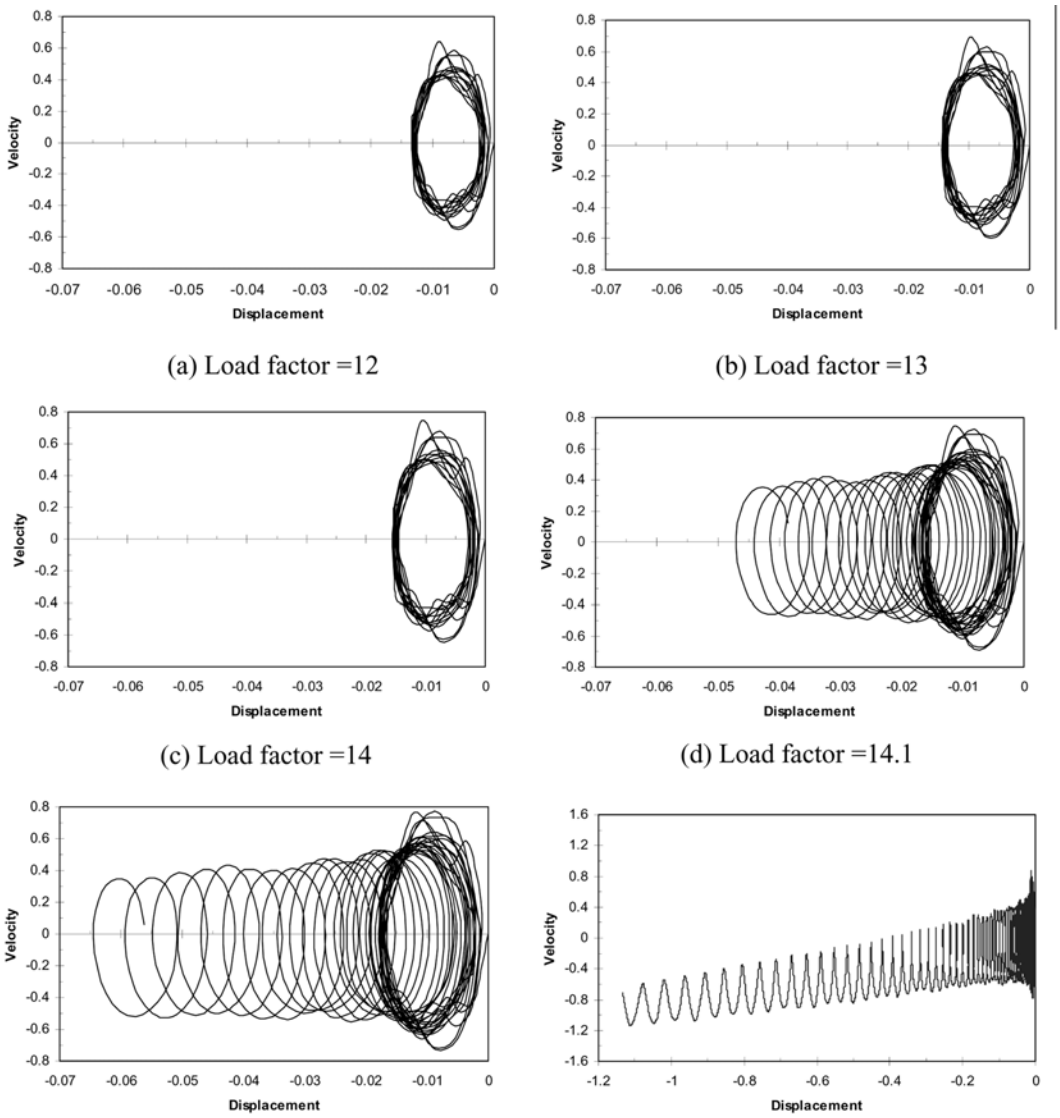


Fig. 5 Dynamic buckling load factors versus imperfection factors for square plate

dynamic buckling load factors are determined from Fig. 4 for different imperfect factors using the Budiansky buckling criteria. Fig. 5 shows the curve of the dynamic buckling load factor versus imperfection factor. It can be clearly seen that the dynamic buckling loads decrease with an increase of imperfection factors.

In order to investigate the phase plane criteria of dynamic buckling, the shortening displacement and the corresponding velocity at the load acting edge are traced. The phase trajectories of the monitored point of the plate with imperfection factor of 0.2 for different load factors are shown in Fig. 6. For the load factor smaller than the critical load factor, the plate simply oscillates about a static equilibrium point as shown in Figs. 6(a) to 6(c); when the loading factors are greater than or equal to the critical loading factor, escaping motion occurs and the phenomenon indicates that the dynamic buckling of the plate takes place as shown in Figs. 6(d) to 6(f). From Figs. 6(d) to 6(f), it also can be seen that even the loading factor has a small increase; the oscillation equilibrium position can have a significant change with an increase in time. According to phase-plane criteria of dynamic buckling, it can be concluded that the dynamic buckling load factor for the plate is about 14.2 which is consistent with the value obtained using the Budiansky buckling criteria.



(a) Load factor =12

(b) Load factor =13

(c) Load factor =14

(d) Load factor =14.1

(e) Load factor =14.2

(f) Load factor =15

Fig. 6 Phase trajectory of square plate for different load factor values

3.2 Dynamic buckling analysis of cylindrical shell

Here some characteristic features of the dynamic elastic buckling behavior of an aluminum cylindrical shell subjected to axial dynamic loads are discussed. A stationary cylindrical shell subjected to an axial suddenly applied step load simulating impact modeled as shown in Fig. 7. In the analysis, the shell is clamped at two ends but not at the axial displacement direction at pressure acting end of the cylindrical shell. The shell has a diameter $d = 100$ mm, a thickness $h = 0.2$ mm and the length $L = 300$ mm. The material properties of the aluminum alloy are the same as for the

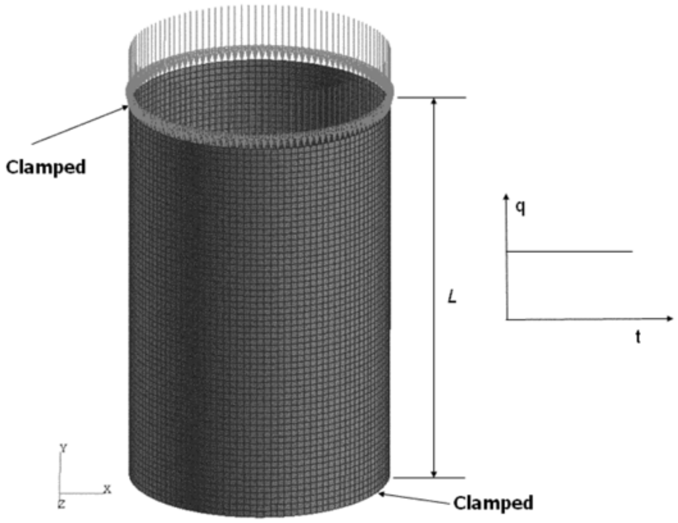
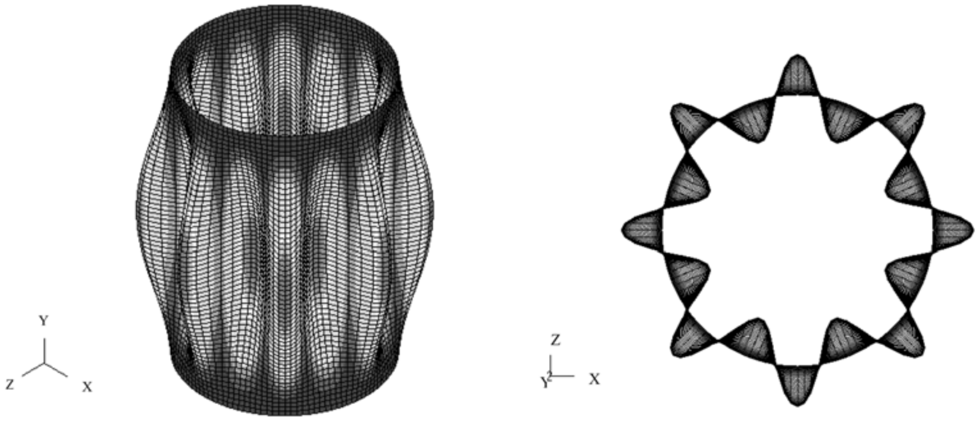


Fig. 7 Finite element mesh of a cylindrical shell subject to axial dynamic step load



(a) Isotropic view (b) Top view

Fig. 8 First mode shape of clamped cylindrical shell

previous example and are shown in Table 1.

Similar to example one, the initial imperfections of the cylindrical shell are introduced in the geometry of the model to trigger buckling. The imperfection shape is the same as the first shell model shape with a natural frequency of 378 Hz. The first model shape of cylindrical shell is shown in Fig. 8. The imperfection factors are introduced and are defined as the ratio of the maximum initial imperfections of the cylindrical shell to the shell thickness.

For the dynamic buckling analysis, the cylindrical shell subjected to different dynamic pressure loads $q(t)$ is calculated with different imperfection factors. The maximum transverse deflection of the shell is selected as a monitoring variable for adopting the Budiansky criteria. In order to use the phase-plane approach the cylinder end shortening displacement is chosen as a monitoring variable.

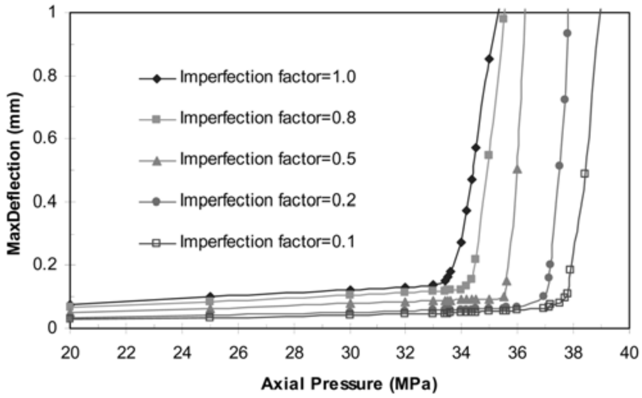
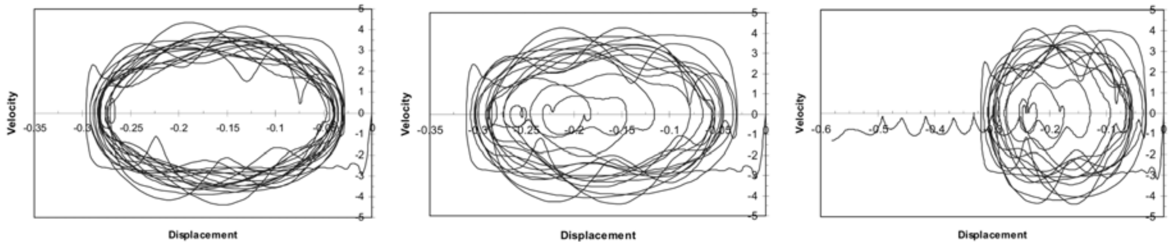


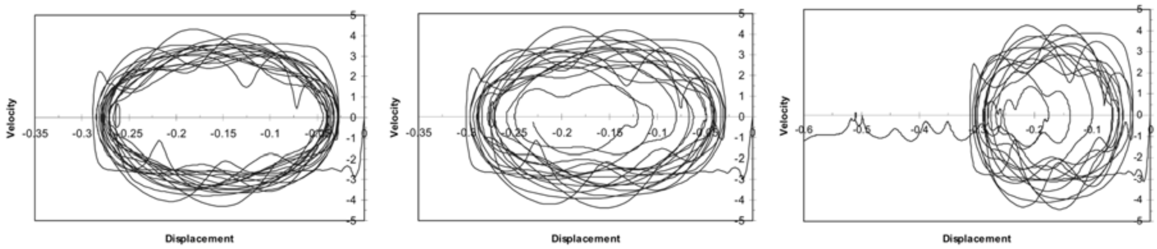
Fig. 9 Maximum transverse deflection of cylindrical vs dynamic axial pressure for respective imperfection factors

The maximum transverse deflection of the cylindrical shell versus the dynamic axial pressure for the respective imperfection factors is shown in Fig. 9. According to the Budiansky dynamic buckling criteria, the dynamic buckling load (pressure) can be retrieved from this figure. It can be observed that the dynamic buckling loads decrease with an increase of imperfection factors. The shortening displacement and the corresponding velocity at end of cylinder at applying pressure edge are traced using the phase plane criteria of dynamic buckling. The phase trajectories of the monitoring point of the cylinder with imperfection factor of 0.1 to 1.0 are shown in Figs. 10 to 14. As expected, for dynamic pressure smaller than the critical pressure load, the cylindrical shell simply oscillates under bounded orbits. The trajectories of the phase plane become unbounded when the pressure loads equal or are greater than dynamic buckling critical loads, and the routes of oscillation rapidly deviate from bounded orbits. In this case, the escaping motions occur and the phenomenon indicates that there is dynamic buckling of the cylindrical shell, as shown in Figs. 10 to 14. It can be observed that even for a small increase of the pressure load; the oscillation equilibrium position can have a significant change with an increase in time. The dynamic buckling loads are derived according to the phase-plane criteria of dynamic buckling, and the dynamic buckling pressure load versus imperfection factors are shown in Fig. 15; the values are consistent with the values determined from the Budiansky buckling criteria. The results are also consistent



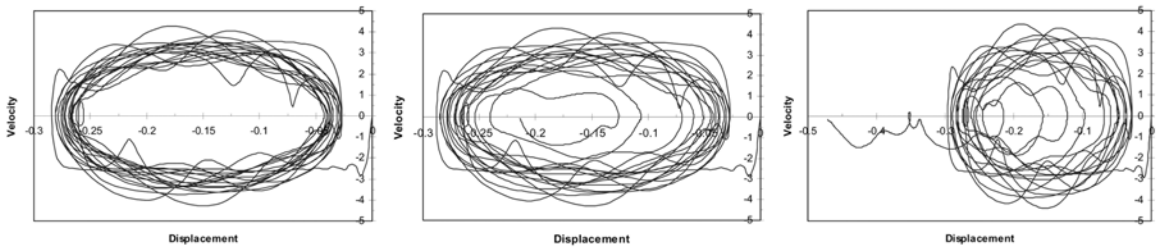
(a) Pressure=37.8 (b) Pressure=37.9MPa (c) Pressure=39MPa

Fig. 10 Phase trajectory of cylindrical shell plate for different dynamic pressure values (Imperfection factor = 0.1)



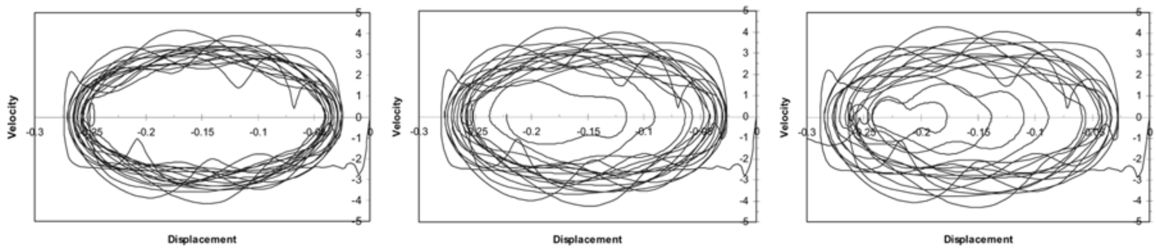
(a) Pressure=37.1 (b) Pressure=37.2MPa (c) Pressure=38MPa

Fig. 11 Phase trajectory of cylindrical shell plate for different dynamic pressure values (Imperfection factor = 0.2)



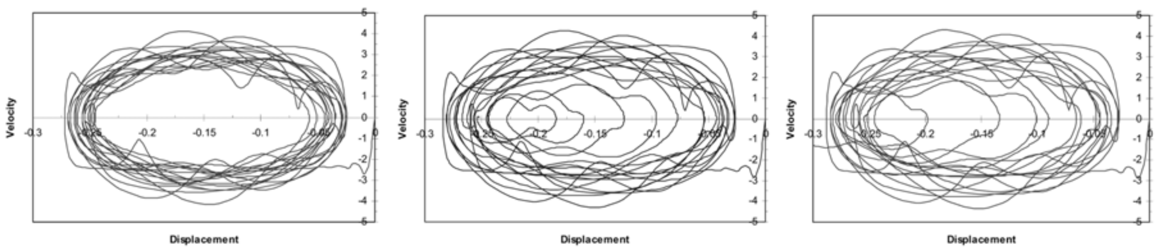
(a) Pressure=35.5MPa (b) Pressure=35.6MPa (c) Pressure=36MPa

Fig. 12 Phase trajectory of cylindrical shell plate for different dynamic pressure values (Imperfection factor = 0.5)



(a) Pressure=34.2MPa (b) Pressure=34.3MPa (c) Pressure=35.0MPa

Fig. 13 Phase trajectory of cylindrical shell plate for different dynamic pressure values (Imperfection factor = 0.8)



(a) Pressure=33.3MPa (b) Pressure=33.4MPa (c) Pressure=34.0MPa

Fig. 14 Phase trajectory of cylindrical shell plate for different dynamic pressure values (Imperfection factor = 1.0)

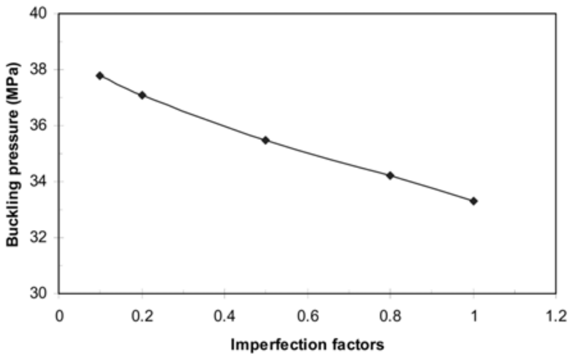


Fig. 15 Dynamic buckling pressure loads vs imperfection factors for cylindrical shell

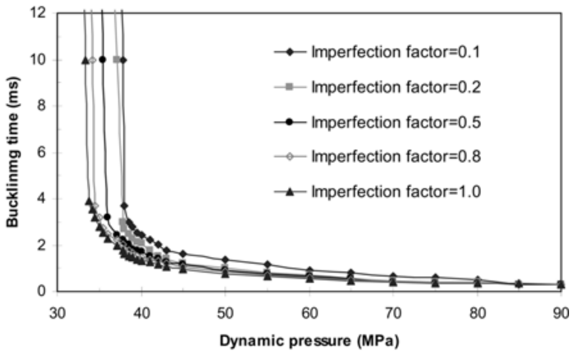


Fig. 16 Dynamic buckling time vs dynamic load pressure for cylindrical shell with different imperfection factors

with Jones and Reis's observation (1980) for an idealized model with initial geometrical imperfections using a phase-plane method.

For the dynamic buckling of a cylindrical shell, the dynamic buckling time should also be considered. In the previous discussion of this example, the dynamic buckling pressure of the cylindrical shell corresponds to an absence of buckling over a long time. The dynamic buckling time versus the dynamic pressure load for different imperfection factors are shown in Fig. 16. It can be observed that the dynamic buckling times decreases with an increase in dynamic pressure. The same phenomenon was shown in Fig. 8 of the paper presented by Jones and Reis (1980). If the impact load is applied to the cylindrical shell very quickly and the duration of the dynamic pressure acting on the cylindrical shell is very small compared to the period of the stress wave reflecting from the other end, the dynamic buckling of the cylindrical shell may not take place even if the dynamic pressure is higher than the static buckling load. It should be noted that Jones and Reis (1980) found that the dynamic buckling load is larger than the static one for small imperfections which is due largely to the different elastic-plastic deformation histories during static and dynamic response for idealized model. Hartzman (1974) observed that the dynamic buckling pressure of a geometrically perfect elastic-plastic spherical dome was larger than the corresponding static buckling pressure. However, Jones and Reis (1980) also revealed that the dynamic loads for idealized model are smaller than the associated static one when the initial imperfections are larger than the corresponding static ones when the initial imperfections are larger than a critical value. For dynamic buckling of plates and shells, the dynamic buckling time may be introduced as a supplemental critical parameter. Actually, the same observation was shown in Jones and Reis (1980) paper Fig. 11, ever though they did not discuss the phenomenon. Hence, this phenomenon may be used to explain why some papers conclude that the dynamic load is higher than the static buckling load and others state that the dynamic load of the cylindrical shell is smaller than the static buckling load. If the buckling time is considered in the analysis, this contradiction should be elucidated. For relatively long standing dynamic loads, the dynamic buckling load is smaller than the static buckling load, while for the case of a relatively short duration of dynamic pressure, the dynamic buckling load of the cylindrical shell is greater than static dynamic load. Therefore, the dynamic progressive buckling concept should be considered even for the elastic dynamic buckling analysis. It should be noted that for simplicity of the problem, it is assumed that the imperfections of plate and shell structures lie in the static buckling mode shape in the present study. However, the mode shapes of dynamic buckling are usually in the form of higher modes due to the inhibiting effects of radial inertia at dynamic loadings. This effect will reduce the dynamic buckling load. The detailed dynamic buckling effects on different initial buckling modes can be further studied using the finite element simulations. Since the stress wave propagation will make the response even more complex, the stress wave effect on the dynamic buckling of plates and shells may be considered using the Smooth Particle Hydrodynamics (SPH) methodology. For high speed impact load, the stress wave propagation effect on dynamic buckling can be treated using SPH method Liu *et al.* (2002).

4. Conclusions

The dynamic elastic buckling phenomena of plate and shell structures subjected to suddenly applied uniformed pressure loads are investigated. In this analysis, the Budiansky buckling criteria and the phase plane dynamic buckling criteria have been adopted to study the dynamic buckling phenomena for plate and shell structures under axial dynamic loads. The numerical simulations are performed using the finite element code ABAQUS incorporating with two dynamic buckling criteria implemented in ABAQUS user subroutine. The effects of initial imperfections of the plate and shell structures on the dynamic buckling load are investigated. It is revealed from numerical simulation results that the dynamic buckling loads of plate and shell structures can be uniquely determined through selecting proper control or monitoring variables and adopting the Budiansky and phase plane buckling criteria. To determine the dynamic buckling load, the buckling time which is normally considered in the elasto-plastic dynamic buckling analysis of plate and shell structures, should also be taken into account for elastic dynamic buckling analysis. The contradiction that the dynamic buckling load is larger or smaller than static buckling load, can be resolved by using the present buckling criteria incorporated with the buckling time concept.

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