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# Toward an accurate effective flange width of composite beams

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**Abstract.** Presented in this paper is the rigorous analysis for the determination of effective flange width for composite beams. To make the solution suitable for routine design, formulas and tables for determining effective flange width for varying load types and geometric shapes are suggested. A variety of effective flange width problems for simple and continuous *T*- and *I*-beams can be solved by these tables and formulas. Although they are derived for *T*- and *I*-beams with symmetrical shapes, flanges and loads, they can be applied for non-symmetrical cases.
Typical numerical examples are given to show how to use the formulas and tables; and their validity and accuracy are assessed by comparison with other known results that are based on the American Codes AISC, AASHTO and ACI.

**Key words:** flange width; composite; design; codes; plates; T-beam; I-beam.

## 1. Introduction

Composite members, consisting of rolled or built-up structural steel shapes and concrete are designed on the assumption that the steel and concrete work together in resisting loads. In contrast with classical structural steel design, which considers only the strength of steel, the inclusion of the contribution of the concrete results in more economical design as the required quantity of steel is reduced.

In recent years, increasing use of the ultimate strength design methods has allowed the structural designer to effect savings of material over traditional elastic techniques. However, available experimental data (Levi 1961) has shown that the determination of the effective flange width *be* of composite beams are more accurately determined using the elastic theory.

The rigorous analysis for effective flange width involves theory of elasticity applied to plates, using an extremely long continuous beam on equidistant supports, with an extremely wide flange having a small thickness compared to the beam depth. To make the solution suitable for routine design, formulas and tables for determining effective flange width, in which section, loading types, and the necessary geometric parameters are taken into account, are suggested in this study.

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A variety of effective flange width problems for simple and continuous $T$- and $I$-beams can be solved by these tables and formulas. Although they are derived for $T$- and $I$-beams with symmetrical beams, flanges and loads, they can be applied approximately for beams with slightly nonsymmetrical flanges and loads. Moreover, these formulas and tables can also be used to study associated problems, since the parameters included appear in a clearly defined form and the effect of each parameters can be estimated and calculated easily.

Some typical numerical examples are given to show how to use the formulas and tables; and the validity as well as the accuracy of these formulas and tables are assessed by comparison with other known results that are based on the American Codes, AISC, AASHTO and ACI.

## 2. Code requirements

As a simplification for design purpose, the American Institute of Steel Construction (AISC 1986) and the American Association of State Highway and Transportation Officials (AASHTO 1973) have adopted the same method of computing effective flange widths as used by the American Concrete Institute Building Code (ACI 1989) for reinforced concrete beams. The permissible effective width *be* is defined in terms of the span length $L$ and of the flange thickness $t$. Also, the effective width can not exceed the given *bo*.

Referring to Fig. 1 the maximum value of effective flange width $be=(2bn+bw)$ permitted by the codes is the least of the values computed by the following relations:

### 2.1. For an interior girder with slab extending on the both sides of girder:

a. $be<1/4$
b. $be<bo$ (for equal beam spacing)
c. $be<bw+16t$

### 2.2. For an exterior girder with slab extending only on one side:

a. $be<L/12+bw$
b. $be<1/2\,(bo+bw)$
c. $be<bw+6t$

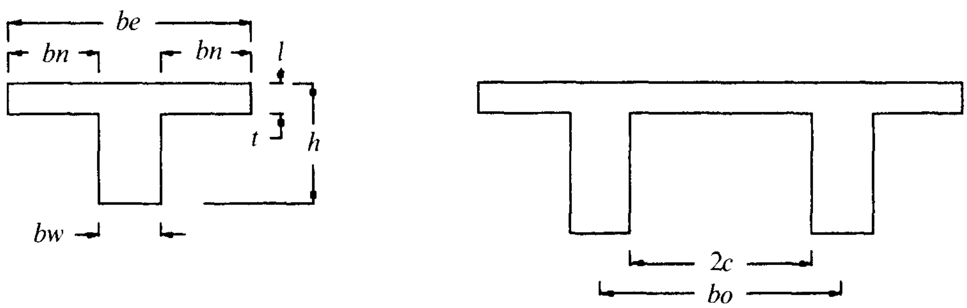


Fig. 1 Dimensions governing effective width *be* on $T$-beam.

Similarly, for highway bridge design, effective width according to AASHTO is identical with that given by AISC except that Eq. c for an interior girder is replaced by

a. $be < 12t$

and for an exterior girder, Eq. a and c are replaced by:

a. $be < L/12$
c. $be < 6t$

In the case of a floor slab built monolithically over floor beams, there is transverse bending in the slab between beams. This tends to reduce effectiveness of the slab in carrying compression at remote points from the beam stem (Salmon and Wang 1985). Thus, there is a valid reason for using a conservatively low effective flange width in the codes. The codes seem properly adequate for certain cases but unduly conservate for most others. From the elastic theory and test results, the ACI, AISC and AASHTO requirements have limited accuracy because the codes consider that the neutral axis (N.A.) remains in the middle of the stem and also some of the compression acting on the system is carried by the stem, thus reducing the effective flange width *be*. In composite beams, the neutral axis (N.A.) usually remains either very close to the flange or inside the flange. Therefore, compression acting on the system is not carried by stem thus resulting in larger effective flange width.

A computer program was developed to calculated the moment-curvature curve for a given composite cross section from the stress-strain curves of steel and concrete used (Barnard 1965). In this calculation, for a given value of strain in the extreme concrete fiber, the neutral axis remained very close to the flange and full compression was carried by the flange. For this reason, the elastic theory is considered to give accurate result in composite beams. However, application of the theory is very difficult and the solution approach is too rigorous. Therefore, for simplication purpose, tables have been developed, based on the rigorous elastic solution, for various loading conditions and beam types. These tables provide improved values for effective width flange.

## 3. Theoretical approach

The approach is to define the effective width *bn* of the compressive flange so that, by replacing the actual stress distribution over this effective width by a uniform stress distribution, the values of ultimate load and of curvature will remain practically unchanged. Thus an equivalent uniform stress distribution (Fig. 3) is created to replace the varying stress along the flange-beam surface (Fig. 2). The resulting flange forces in each system are equated:

$$(\sigma_m)(2be)t = t\int_{flange} \sigma_x \, dy \tag{1a}$$

Solving for *bn* gives

$$bn = t\int_0^{2c} \sigma_x \, dy/2 \ \ \sigma_m t \tag{1b}$$

The problem is to define the expression for $\sigma x$ so that *bn* can be solved. The actual expression for bending moment *Mx* along the length of the beam can be expanded into an odd term

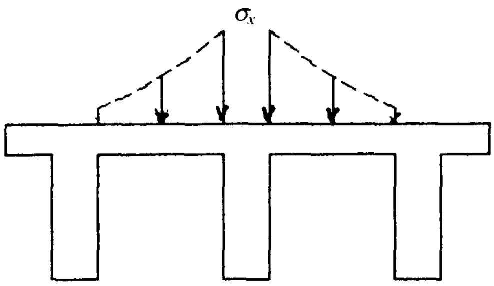


Fig. 2 Original system.

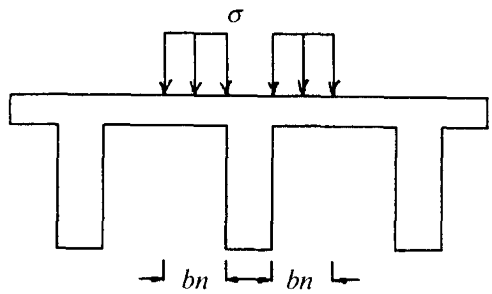


Fig. 3 Equivalent system.

Fourier series of the type (Heins 1976, Schleicher 1955, Sechler 1953):

$$Mx = \sum Mn \ \sin n\,\pi x/L$$

$Mn$ = Fourier coefficient

$2L$ = period

The selection of either sine or cose terms was made for the case of simply supported beam since the moments at the ends will be zero. Thus the sine terms satisfy the boundary conditions at the supports.

Considering the flange as a plate loaded at the junction of the rib and the flange, the stresses in the flange must then satisfy the elastic theory equation (Southwell 1936, Wang 1953):

$$\partial^4\varphi/\partial x^4 + 2\partial^2\varphi/\partial x^2\partial y^2 + \partial^4\varphi/\partial y^4 = 0 \tag{2}$$

where $\varphi$=Airy's stress function, and the stresses as given in the elastic theory (Timoshenko and Woinowsky-Krieger 1959) are:

$$\sigma x = \partial^2\varphi/\partial y^2 \tag{3}$$

$$\sigma y = \partial^2\varphi/\partial x^2 \tag{4}$$

$$\tau xy = -\partial^2\varphi/\partial x\partial y \tag{5}$$

Analyzing the stresses resulting from the $n$th term of Fourier expansion of moment,

$$Mx = Mn \sin n\,\pi x/L$$

According to bending theory the flexural stresses along any longitudinal fiber vary directly as the moment and hence as sin $n\ \pi\ x/L$. The stresses are considered to vary according to some function $Yn$, which is independent of coordinate $x$. Therefore, a stress function $\varphi$ in the product form is used for the solution of the elastic theory equation:

$$\varphi n = Yn \ \sin n\,\pi x/L$$

which is the same equation as used for the multi-material beam solution in elastic theory. The general solution for the stress function is then:

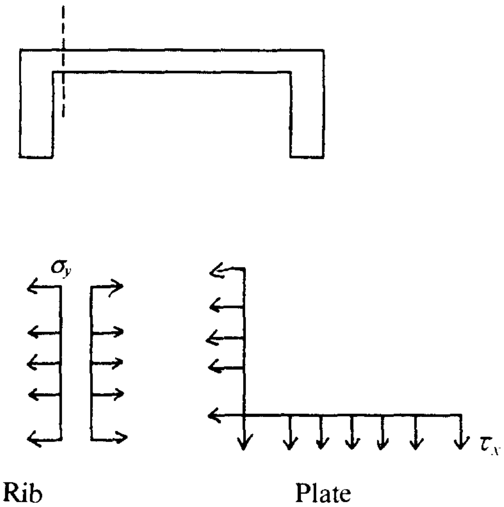


Fig. 4 Joint forces.

$$\varphi n = (\mathrm{A1}\ \ \sinh n\ \pi y/L + A2\cosh n\ \tau y/L + A3\ \ y\ \ \sinh n\ \tau y/L$$
$$+ A4 y \cosh n\ \tau y/L)\ \ \sin n\ \tau x/L \tag{6}$$

where $A1$, $A2$, $A3$ and $A4$ are constants to be determined. Also boundary conditions will be determined for different cases.

## 4. Channel section (or L-section)

The first case that will be examined will be a simply supported channel or $L$-section which may act as spandrel beam to support floor slab. If the stem or rib and plate are now separated, as shown in Fig. 4, compatibility of stresses at the junction gives

$$\sigma x_{rib} = \sigma x_{flange} = \sigma m \sin n\ \pi x/L \tag{c1}$$

$$\sigma y = 0 \tag{c2}$$

$$\tau_{rib} = \tau_{flange} \tag{c3}$$

Taking stress equation (6) in accordance with equations (3) through (5) and applying the boundary conditions (c1) and (c2) at $y=0$ and $y=2c$ gives:

$$y=0: \quad \sigma y = \partial^2\varphi/\partial x^2 = 0 = -\lambda^2(A2);\ A2=0$$

$$y=2c: \quad \sigma y = \partial^2\varphi/\partial x^2 = 0 = -\lambda^2(A1 \qquad \sinh \lambda 2c + A3\,2c$$
$$\sinh \lambda 2c + A4\,2c \cosh \lambda 2c)$$

$$y=0: \quad \sigma x = \partial^2\varphi/\partial x^2 = \sigma m \sin \lambda x = [A3\,\lambda 2]\ \sin \lambda x$$

$$y=2c: \quad \sigma x=\partial^2\varphi/\partial x^2=\sigma m \ \sin\lambda x$$
$$\sigma x=[A1\,\lambda^2\sinh\lambda 2c+A3\,\lambda(2\cosh\lambda 2c+2c\,\lambda\sinh\lambda 2c)$$
$$+A4\,\lambda(2\sinh\lambda 2c+2c\,\lambda\cosh\lambda 2c)]\ \sin\lambda x$$

The solution of these equations gives the following values for the coefficients $A1$, $A2$, $A3$ and $A4$:

$$A1=(\sigma m\ \ c/\lambda)\ [(1-\cosh 2\lambda c)/\sinh^2 2\lambda c]$$
$$A2=0$$
$$A3=\sigma m/2\lambda$$
$$A4=(\sigma m/2\lambda[(1-\cosh 2\lambda c)/\sinh 2\lambda\ c]$$

where $\lambda=n\,\pi/L$
Substituting these coefficients into the stress equation for $\sigma x$,

$$\sigma x=\partial^2\varphi/\partial y^2=[A1\,\lambda^2\sinh\lambda y+A2\,\lambda^2\cosh\lambda y$$
$$+A3\,\lambda\ (2\cosh\lambda y+y\,\lambda\sinh\lambda y)$$
$$+A4\,\lambda\ (2\sinh\lambda y+y\,\lambda\cosh\lambda y)]\ \sin\lambda x$$

and integrating across the flange width gives:

$$R=t\int_0^{2c}\sigma x\,dy=t\ \sigma m/\lambda[(2\lambda c+\sinh 2\lambda c)/(\cosh 2\lambda c+1)]\sinh\lambda x$$

This result represents the numerator of Equation (1b); therefore the effective width is

$$bn=\frac{\left[t\int_0^{2c}\sigma x\ dy\right]}{2\sigma m t}$$

where $\sigma m=\sigma m\sin\ \lambda x$

$$bn=(\sigma m\sin\lambda x/\lambda[(2\lambda c=\sinh 2\lambda c)/(\cosh 2\lambda c+1)]\ \ (1/2\,\sigma\sin\lambda x)$$

substituting the value of $\lambda=n\,\pi/L$

$$bn=(\sigma m\sin n\,\pi/L)/(n\,\lambda/L)\ \ [(2\,n\,\pi/L\,c+\sinh 2\,n\,\pi/L)$$
$$(\cosh 2\,n\,\pi/L\,c+1)]\ \ (1/2\ \sigma m\sin n\,\pi/L\,x)$$

rearrange for each Fourier series term $n$,

$$bn=\sum\frac{L}{2\,n\,\pi}\left[\frac{\sinh(2n\,\pi c/L)+(2n\,\pi c/L)}{\cosh(2n\,\pi c/L)+1}\right]$$

for the extreme case when $c/L$ approaches infinity and $n=1$,

$$bn=L/2\,\pi n=0.159L$$

Table 1 Summary of effective width solutions

| | | |
|---|---|---|
| Case 1: | Two ribs, symmetric loading, lateral edges free.(Channel & $L$-Section). | $bn=\dfrac{\sinh 2\lambda c+2\lambda c}{2\lambda(\cosh 2\lambda c+1)}$ |
| Case 2: | Multiple ribs, symmetric loading, lateral edges continuous. (Multi-Beams) | $bn=\dfrac{2(\cosh 2\lambda c-1)}{\lambda[(3+\mu)\cosh 2\lambda c-(1+\mu)2\lambda c)]}$ |
| Case 3: | One rib, lateral edges free. ($T$-Beams) | $bn=\dfrac{2(\cosh 2\lambda c-1)}{\lambda[(3+\mu)\cosh 2\lambda c+[(1+\mu)/2](2\lambda c)+(5-\mu)]}$ |
| Case 4: | One rib, lateral edges continuous or fixed. | Same as case 2 |
| Case 5: | Two ribs, anti-symmetric loading, lateral edge free. | $bn=\dfrac{(\sinh\lambda c+\lambda c)\ (\cosh\lambda c-1)}{\lambda(\cosh 2\lambda c-1)}$ |
| Case 6: | Multiple ribs, anti-symmetric loading, lateral edges continuous. | $bn=\dfrac{(2)}{\lambda}\ \dfrac{2\cosh\lambda c(\cosh\lambda c-1)+(1+\mu)\lambda c\ \sinh c}{(3+\mu)\sinh 2\lambda c+(1+\mu)2\lambda c}$ |

$-\mu$ is Poisson's ratio, $\lambda=n\pi/L$
−All beams are simply supported at ends.
−Symmetric loading means all ribs are bent in same direction.
−Antisymmetric loading means alternate ribs are bent in same direction.
−Lateral edges free means no lateral stresses $\sigma y$.
−Lateral edges continuous means no lateral displacement of ribs.
−Poisson's ratio does not enter into cases 1 and 5 since the ribs are considered free to rotate axially as well as to deflect laterally, hence there are no stresses resulting from suppression of Poisson's ratio.

The effective flange width is not dependent on flange thickness $t$, stem width $bw$ and overall depth of section $h$, and is constant for each fourier series term.

## 5. Multi-beams

Consider the case of a series of multi-beams. The boundary conditions are: $y=0$, $y=2c: z=0$ or the lateral displacement is zero.
Additionally, at $y=0$, $y=2c: \sigma x=\sigma m\ \sin\lambda x$
The equation of stress $\sigma x$ have previously been defined; the displacement equation, however, is required. This equation is found as described in elastic theory (Sechler 1953) as,

$$[(\partial^3\varphi/\partial y^3+(2+\mu)\partial^3\varphi/\partial x^2\partial y)]=-\partial^2 z/\partial^{x\,2}$$

and for all values of $\sigma x$ the displacement $z$ and thus $\partial^2 z/\partial x^2$ equal zero; therefore it is required to solve

$$[(\partial^3\varphi/\partial y^3+(2+\mu)\partial^3\varphi/\partial x^2\partial y)]=0$$

at $y=0$ and $y=2c$. The solution to this problem has been obtained and the resulting effective width evaluated and given by:

Table 2 Value of effective plate width coefficient *bn*

| | Values of *Bn* | | | | |
|---|---|---|---|---|---|
| | Case No. | | | | |
| *c/Ln* | 1 | 2 & 4 | 3 | 5 | 6 |
| 0.05 | 0.049 | 0.049 | 0.049 | 0.025 | 0.025 |
| 0.10 | 0.094 | 0.094 | 0.093 | 0.049 | 0.049 |
| 0.15 | 0.130 | 0.130 | 0.129 | 0.072 | 0.073 |
| 0.20 | 0.158 | 0.158 | 0.157 | 0.094 | 0.095 |
| 0.25 | 0.176 | 0.178 | 0.175 | 0.113 | 0.116 |
| 0.30 | 0.186 | 0.191 | 0.187 | 0.130 | 0.134 |
| 0.35 | 0.190 | 0.199 | 0.195 | 0.145 | 0.151 |
| 0.40 | 0.191 | 0.204 | 0.199 | 0.158 | 0.165 |
| 0.45 | 0.189 | 0.207 | 0.201 | 0.168 | 0.177 |
| 0.50 | 0.186 | 0.208 | 0.201 | 0.176 | 0.187 |
| | 0.159 | 0.205 | 0.205 | 0.159 | 0.205 |

$Ln = L^*n$
$bn = Bn^*Ln$
*bn* is effective width of overhancing portion of compression flange.

$$bn = \sum \frac{2}{\lambda}\left[\frac{(\cosh 2\lambda c - 1)}{(3+\mu)\sinh 2\lambda c - (1+\mu)\, 2\lambda c)}\right]$$

The evaluation of plate effective width, for other geometry and beam arrangements have also beem computed and listed below. The numerical evaluations of these equations, for each series term $n$, have also been solved and compiled into Table 1. The application of these tables is very simple and extremely accurate.

## 6. Design example

It is required to determine the effective width and ultimate moment capacity of the composite section shown in Fig. 5 according to:

(a) Code requirements
(b) Theoretical approach
Assume $fc' = 3000$ psi.(21 MPa)

a) Code requirements
Determine the effective width:
$be < L/4 = 105$ in.(2625 mm)
$be < bo = 116.14$ in.; $bo = 2c + bw$
$be < bw + 16t = 88.24$ in.(2206 mm)
Effective width *be* for design is the smallest of these three equations.
$be = 88.24$ in.(2206 mm), from the code requirements for composite design.

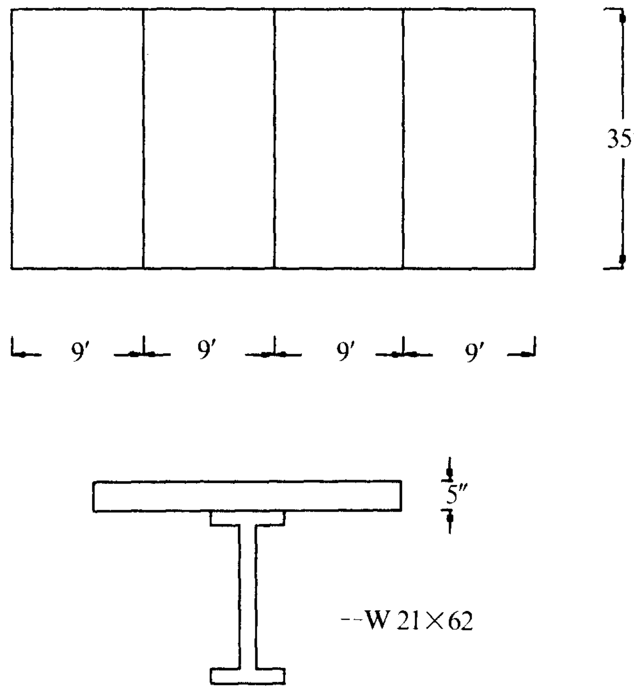


Fig. 5 Composite section for design example.

b) Elastic theory
   Determine the effective width:
   $Ln=L^{*}\ n=420$ in.(10500 mm), consider $n=1$ for the maximum value of the effective width $be$.
   $c/Ln-54/420=0.128$ and using the Table 1, Case 2.
   $Bn=0.11556$ by interpolation.
   $bn=Bn^{*}Ln=0.11556^{*}\ 420=48.54$ in.; and $be=2bn+bw$
   $be=97.09+8.240=105.32$ in.(2633 mm)
   $be=105.32$ in.(2633 mm)

## 7. Determine ultimate moment capacity

The procedure for determining the ultimate moment capacity depends on whether the neutral axis occurs within the concrete slab or within the steel beam. If the neutral axis occurs within the slab, the slab is considered to be adequate i.e., the slab is capable of resisting the total compressive force. If the neutral axis falls within the steel beam, the slab is considered inadequate i.e., the slab is able to resist only a portion of compressive force, the remainder being taken by the steel beam. Referring to Fig. 9, determine the neutral axis and ultimate moment capacity.

a) Code requirements:
   $be=88.24$ in.(2206 mm), $As=18.3$ in.$^2$(11807 mm$^2$), $Fy=36$ ksi.(248.2 MPa)

$a=As\ Fy(0.85\ fc'\ be)=18.3*\ 36/(0.85*\ 3*\ 88.24)$
$a=2.93$ in.$<t=5.0$ in., the neutral axis occurs within the slab. The slab is capable of resisting the total compressive force.
$C=0.85\ fc'\ a\ be=0.85*\ 3*\ 2.93*\ 88.24=658.8$ kips(2951.4 Kn),
$T=As\ Fy=18.3*\ 36=658.8$ kips(2951.4 Kn).
Moment arm is referring the Fig. 9,
$d=hs/2+t-a/2=10.495+5-1.46=14.035$ in.(358.75 mm)
Ultimate moment capacity $Mu$,
$Mu=td-Cd=658.8*\ 14.035=770.52$ ft-kips.(1044.82 KN-M)
$Mu=770.52$ ft-kips.(1044.82 KN-M), from the code requirements for composite design.

b) Theoretical solution:
$be=105.32$ in., $As=18.3$ in.$^2$(11807 mm$^2$), $Fy=36$ ksi.(248.2 MPa)
$a=As\ Fy/(0.85\ fc'\ be)=18.3*\ 36/(0.85*\ 3*\ 105.32)$
$a=2.45$ in.(61.3 mm)$<t=5.0$ in.(125 m), the neutral axis occurs within the slab. The slab is capable of resisting the total compressive force.
$C=0.85\ fc'\ a\ be=0.85*\ 3*\ 2.45*\ 105.32=658.8$ kips(295.4 Kn)
$T=As\ Fy=18.3*\ 36=658.8$ kips(2951.4 Kn).
Moment arm is given by:
$d=hs/2+t=a/2=10.495+5-1.225=14.27$ in.(356.75 mm)
Ultimate moment capacity $Mu$,
$Mu=Td=Cd=658.8*\ 14.27=783.423$ ft-kips.(1062.3 KN-M)
$Mu=783.423$ ft-kips.(1062.3 KN-M), from the elastic theory without the flange stiffness for composite design.

From Example:
The neutral axis (N.A.) remains inside the slab and the slab is capable of resisting the total compressive force. Large effective width is obtained and large ultimate moment capacity results.

$be$(code)$<be$(elastic theory)
88.24 in.(2106 mm)<105.32 in.(2632.5 mm)
$Mu$(code)$<Mu$(elastic theory)
770.52 ft-kips(1044.2 KN-M)<783.423 ft kips(106.23 KN-M3)

## 8. Conclusions

It can be concluded from the results of this research and study that:

1. In composite structures, the neutral axis (N.A.) usually remains either inside the flange or close to the flange. Therefore, compression acting on the structure is not carried by stem and effective width $be$ should not be reduced.
2. The true effective width is dependent on the type of loading and the ratios $t/h$, $L/bw$, $L/bo$ whereas, the effective width is defined in nearly all codes in terms of the beam length and flange thickness. Also the effective width can not exceed the given width $bo$. Effective flange width $bn$ is almost accurately calculated by elastic theory.
3. Using the appropriate effective flange width $bn$ not only increases the ultimate moment capacity but decreases the stress in the flange.

4. Tests results prove that the elastic theory is safe enough to use for calculating the effective flange width *bn*. Without any additional safety factor.
5. Curves clearly show the greater importance of the elastic theory for calculating the effective flange width *bn* (Fig. 6 to Fig. 20 given in the appendix).

## Notations

$t$ : Flange thickness
$bw$ : Stem width (width of web)
$L$ : Span length
$2c$ : Clear distance between stems
$bo$ : Center to center distance between stems
$bn$ : Effective width of overhanging portion of compression flange
$be$ : Total effective width for concrete design $(2bn+bw)$ for T-section flanges
$h$ : Overall depth of section
$F_y$ : Yield stress of steel
$f'_c$ : Compressive strength of concrete

## References

AASHTO (1973), *Standard Specifications for Highway Bridges*, 11th Edition, American Association of State Highway and Transportation Officials, Washington D.C. 1973. Also 1974-1977 Interim Specifications Bridges.

ACI (1977), ACI Committee 318, *Building Code Requirements for Reinforced Concrete*. American Concrete Institute, Detroit, Michigan.

ACI (1989), ACI Committee 381-*89 and Commentary*-ACI 318R-89, *Building Code Requirements for Reinforced Concrete*, American Concrete Institute, Detroit, Michigan.

ACI-ASCE Committee 333. "Tentative recommendations for design of composite beams and girder for buildings", *Journal of the American Concrete Institute*, Proceedings, 57, December, 1960. Also *Journal of the Structural Division*, Proceeding of the American Society of Civil Engineers, **86**(ST12), December, 1960.

AISC (1986), *Specification for the Design, Fabrication and Erection of Structural Steel for Buildings*, American Institute of Steel Construction, 8th Edition, Chicago.

Barnard, Peter R. (1965), "A Series of Tests on Simply Supported Composite Beams", *Journal of the American Concrete Institute*, Proceedings **62**(3), April.

Goodier, J. N. and Timoshenko, S. (1987), *Theory of Elasticity*, 3rd Edition, New York, McGraw-Hill, pp. 262-268.

Heins, C. P. (1976), *Applied Plate Theory for the Engineer*, Lexington Books, pp. 87-107.

Levi Franco (1961), "Work of the European concrete committee", *Journal of the American Concrete Institute*, **67**, March.

Salmon, G. Charles and Wang, Chu-Kia (1985), *Reinforced Concrete Design*, 4th Edition, New York, Harper and Row.

Schleicher, F. Taschenbuck fur Bauingenieure (1955), Springer Verlag, Vienna, Austria.

Scordelis, C. Alexander and Song Qi-Gen (1990), "Formulas for shear-lag effect of T-, I-and box beams", *Journal of the Structural Engineering*, **116**(5), May.

Sechler, E. E. (1953), *Elasticity in Engineering*, New York, Dober Pub. Co.

Southwell, R. V. (1936), *An Introduction to the Theory of Elasticity*, Section 46-55, 164-196, Oxford.

Timoshenko, S. and Woinowsky-Krieger (1959), *Theory of Plates and Shells*, 2nd Edition, New York, McGraw-Hill.

Wang, C. T. (1953), *Applied Elasticity*, New York, McGraw-Hill Book Co.

## Appendix

Fig. 6 to Fig. 20 were developed to show differences between the elastic theory and the code requirements. Referring to the figures, $y$-axis indicates ratio between the effective width of overhanging portion of compression flange and half clear distance between stems, $x$-axis indicates ratio between span length and half clear distance between stems. The curves clearly show that the code requirements (AISC, AASHTO and ACI) are conservative in all cases. Larger values for effective flange width can be used on the basis of theoretical approach and for cases of $L/c$ that are normally encountered. The cases illustrated are only multiple $T$-beam uniformly loaded.

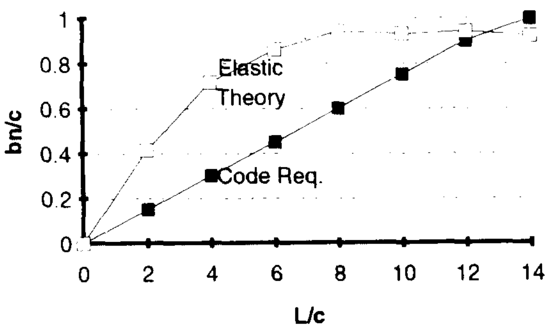


Fig. 6 Comparison of eff. flange width $t/h=0.1$, $L/bw=10$.

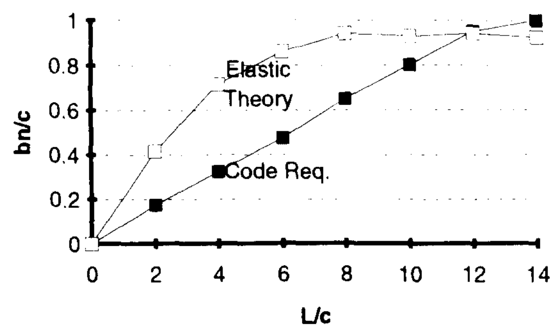


Fig. 7 Comparison of eff. flange width $t/h=0.1$, $L/bw=50$.

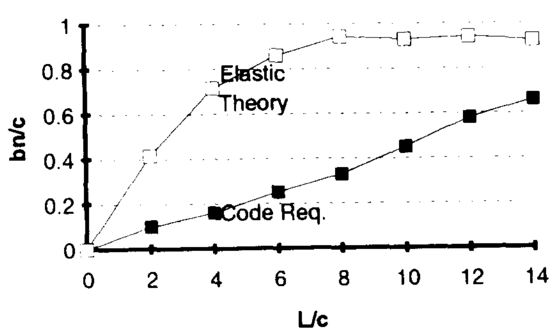


Fig. 8 Comparison of eff. flange width $t/h=0.1$, $L/bw=100$.

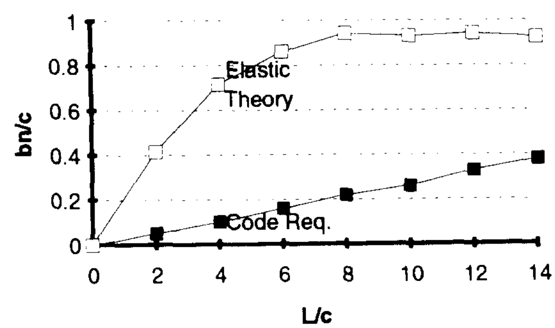


Fig. 9 Comparison of eff. flange width $t/h=0.1$, $L/bw=150$.

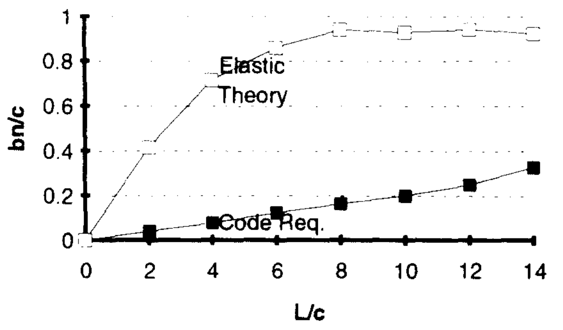


Fig. 10 Comparison of eff. flange width $t/h=0.1$, $L/bw=200$.

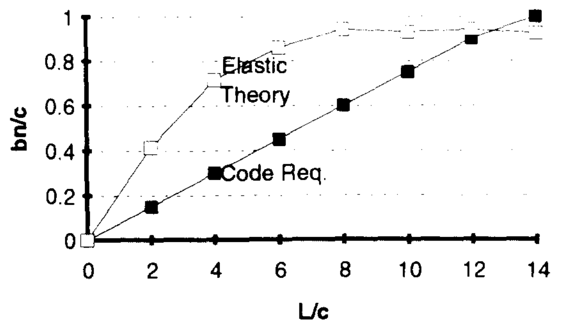


Fig. 11 Comparison of eff. flange width $t/h=0.2$, $L/bw=10$.

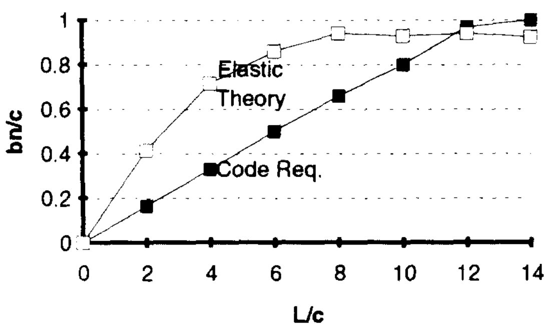


Fig. 12 Comparison of eff. flange width $t/h=0.2$, $L/bw=50$.

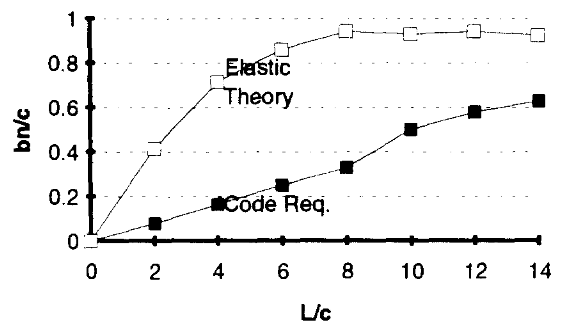


Fig. 13 Comparison of eff. flange width $t/h=0.2$, $L/bw=100$.

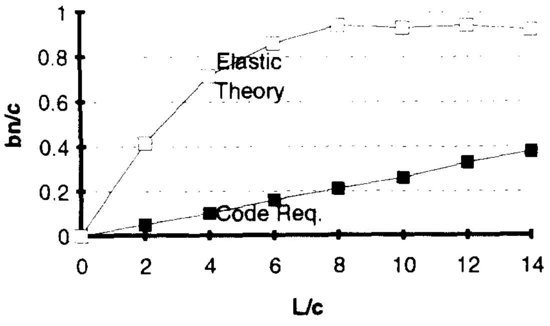


Fig. 14 Comparison of eff. flange width $t/h=0.2$, $L/bw=150$.

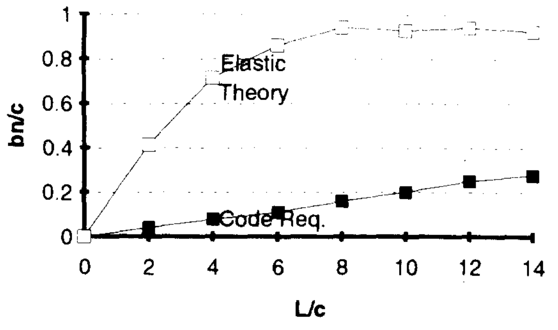


Fig. 15 Comparison of eff. flange width $t/h=0.2$, $L/bw=200$.

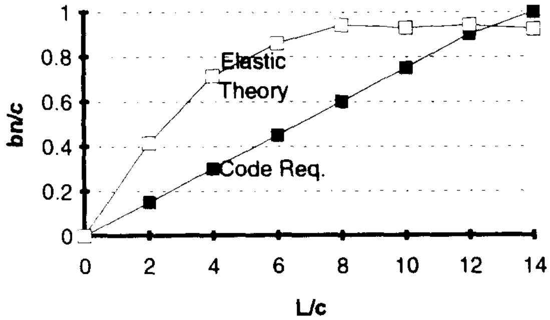


Fig. 16 Comparison of eff. flange width $t/h=0.3$, $L/bw=10$.

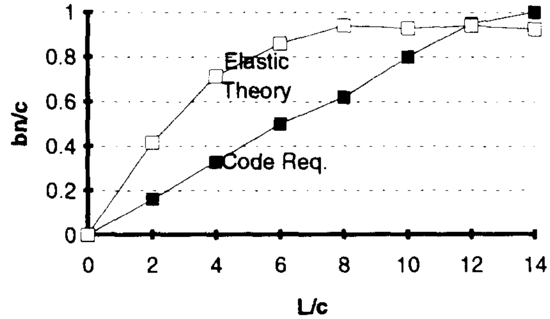


Fig. 17 Comparison of eff. flange width $t/h=0.3$, $L/bw=50$.

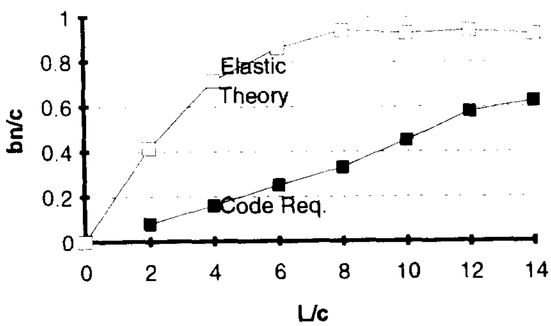


Fig. 18 Comparison of eff. flange width $t/h=0.3$, $L/bw=100$.

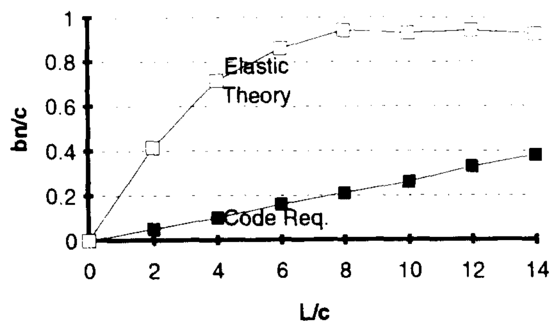


Fig. 19 Comparison of eff. flange width $t/h=0.3$, $L/bw=150$.

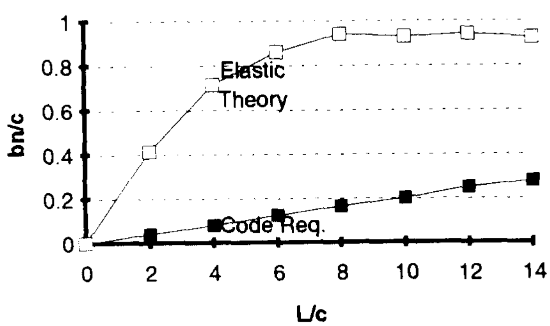


Fig. 20 Comparison of eff. flange width $t/h=0.3$, $L/bw=200$.