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Spline function solution for the ultimate strength of member structures

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Abstract. In this paper a spline function solution for the ultimate strength of steel members and member structures is derived based on total Lagrangian formulation. The displacements of members along longitudinal and transverse directions are interpolated by one-order *B* spline functions and three-order hybrid spline functions respectively. Equilibrium equations are established according to the principle of virtual work. All initial imperfections of members and effects of loading, unloading and reloading of material are taken into account. The influence of the instability of members on structural behavior can be included in analyses. Numerical examples show that the method of this paper can satisfactorily analyze the elasto-plastic large deflection problems of planar steel members and member structures.

Key words: member structures; spline function solution; ultimate strength.

1. Introduction

The solution of the ultimate strength of members and member structures is one of the important problems in nonlinear stability theory of steel structures. Numerical integration method (NIM) (Chen, et al. 1977 & Zhang, et al. 1987) is powerful in analyzing the nonlinear behavior of steel members. However, the time-consuming multiple cyclic computation must be carried out in executing NIM program. Finite element methods (FEM) (Aslam 1983, Kam 1983 & Ding, et al. 1992) are usually adopted to analyze the ultimate strength of member structures, in which plastic hinge assumption is used to consider the elasto-plastic behavior of material. Obviously, the accuracy of FEM depends on the reasonableness of the assumptions about plastic hinge distributions.

In this paper, spline fuctions are used to constitute the displacements of steel members, total Lagrangian formulation and Green strain definition are adopted to describe the geometrical nonlinearity of members, and practical constitutive relationships of material are introduced to consider the elasto-plastic loading, unloading and reloading of members. A spline function solution is derived to analyze the ultimate strength and the whole loading procedure of members and member structures, in which all imperfections, different boundaries and instability effects of members are taken into account. The analyzed results show good agreement with those obtained from experiments and other numerical methods.

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2. Basic assumptions

Following assumptions are adopted in deriving the spline function solution:
1. Cross sections of members remain plane after deflecting;
2. Only normal stress parallel to the longitudinal axis of the members are considered;
3. Strains are small compared to the unit;
4. Material is assumed to be ideally elastic plastic and Bauschinger's effect is ignored;

3. Nonlinear geometrical relationship of members

A member divided into m segments is shown in Fig. 1, in which local member and global coordinate systems, $\xi\eta$, xy, XY, are also expressed.

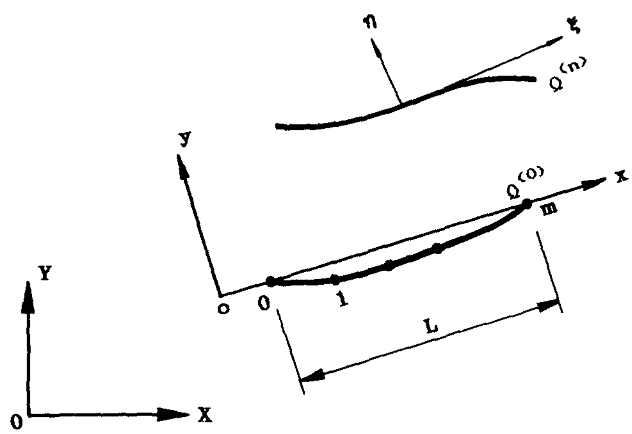

Fig. 1 A member in coordinate systems

Introducing one-order B spline function $[\phi_{1,i}(\xi)]$ and three-order hybrid spline function $[\Phi_{3,i}(\xi)]$ the displacements of the member axis along the x and y directions respectively can be interpolated and expressed as follows:

$$\begin{cases} u_a=[\phi_{1,i}(\xi)]\ \{u\} & (i=0,\ 1,\ 2,\ \cdots,\ m) \\ v_a=[\Phi_{3,i}(\xi)]\ \{v\} & (i=-1,\ 0,\ 1,\ \cdots,\ m,\ m+1) \end{cases} \tag{1}$$

where, $\{u\}=[u_0,\ u_1,\ \cdots,\ u_m]^T$, $\{v\}=[v_{-1},\ v_0,\ \cdots,\ v_{m+1}]^T$.

The hybrid spline function $[\Phi_{3,i}(\xi)]$ in Eq. 1 can be deduced from the three-order B spline function $[\phi_{3,i}(\xi)]$ as follows (Liu 1990):

$$[\Phi_{3,i}]=[\phi_{3,i}]\begin{bmatrix} [T_{30}] & & \\ & [I] & \\ & & [T_{3m}] \\ & & \end{bmatrix}_{(m+3)\times(m+3)} \tag{2}$$

where, for fixed ends:

$$[T_{30}]=\begin{bmatrix} 2h & 0 & 1 \\ -\frac{h}{2} & \frac{3}{2} & -\frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$[T_{3m}]=\begin{bmatrix} 0 & 0 & 1 \\ -\frac{1}{2} & \frac{3}{2} & -\frac{h}{2} \\ 1 & 0 & 2h \end{bmatrix}$$

and for pin-ended ends:

$$[T_{30}]=\begin{bmatrix} \frac{2}{3}h^2 & 2 & -1 \\ -\frac{1}{6}h^2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[T_{30}]=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{6}h^2 \\ -1 & 2 & \frac{2}{3}h^2 \end{bmatrix}$$

$$h=\frac{L}{m}$$

L is the length of this member.

There are three kinds of boundaries

Two fixed ends: $\upsilon_{-1}=\upsilon_0'$, $\upsilon_{m+1}=\upsilon_m'$

Two pin ends: $\upsilon_{-1}=\upsilon_0''$, $\upsilon_{m+1}=\upsilon_m''$

One fixed end and one pin end:

$\upsilon_{-1}=\upsilon_0'$, $\upsilon_{m+1}=\upsilon_m''$ or $\upsilon_{-1}=\upsilon_0''$, $\upsilon_{m+1}=\upsilon_m'$

The displacements of any point of the section can be expressed as:

$$\begin{cases} u_p=u_a-\eta\ [\boldsymbol{\Phi}_{3,i}^{(1)}(\xi)]\ \{\upsilon\} \\ \upsilon_p=\upsilon_a \end{cases} \tag{3}$$

where, $[\boldsymbol{\Phi}_{3,i}^{(1)}(\xi)]=[d\boldsymbol{\Phi}_{3,i}(\xi)/\mathrm{d}\xi]$.

From Eq. 3, we have:

$$\begin{cases} \Delta u_p=[\phi_{1,i}(\xi)]\ \{\Delta u\}\quad -\eta\ [\boldsymbol{\Phi}_{3,i}^{(1)}(\xi)]\ \{\Delta\upsilon\} \\ \Delta\upsilon_p=[\boldsymbol{\Phi}_{3,i}(\xi)]\ \{\Delta\upsilon\} \end{cases} \tag{4}$$

$$\frac{\partial\Delta u_p}{\partial x}=[Dux]\begin{Bmatrix}\{\Delta u\}\\ \{\Delta\upsilon\}\end{Bmatrix}$$

$$\frac{\partial\Delta u_p}{\partial y}=[Duy]\begin{Bmatrix}\{\Delta u\}\\ \{\Delta\upsilon\}\end{Bmatrix}$$

$$\frac{\partial \Delta v_p}{\partial x} = [Dvx] \left\{ \begin{array}{c} \{\Delta u\} \\ \{\Delta v\} \end{array} \right\} \tag{5}$$

$$\frac{\partial \Delta v_p}{\partial y} = [Dvy] \left\{ \begin{array}{c} \{\Delta u\} \\ \{\Delta v\} \end{array} \right\}$$

where,

$$\begin{aligned} &[Dux] = [[\phi_{1,i}^{(1)}(\xi)] J_{11}^{-1};\ -\eta[\Phi_{3,i}^{(2)}(\xi)] J_{11}^{-1} - [\Phi_{3,i}^{(1)}(\xi)] J_{12}^{-1}] \\ &[Duy] = [[\phi_{1,i}^{(1)}(\xi)] J_{21}^{-1};\ -\eta[\Phi_{3,i}^{(2)}(\xi)] J_{21}^{-1} - [\Phi_{3,i}^{(1)}(\xi)] J_{22}^{-1}] \\ &[Dvx] = [[0];[\Phi_{3,i}^{(1)}(\xi)] J_{11}^{-1}] \\ &[Dvy] = [[0];[\Phi_{3,i}^{(1)}(\xi)] J_{21}^{-1}] \end{aligned} \tag{6}$$

In Eq. 6, J_{ki}^{-1} is the element at k line and 1 column in the inversed Jocobi matrix. The jocobi matrix at $\Omega^{(0)}$ state is:

$$[J] = \begin{bmatrix} \dfrac{\partial x}{\partial \xi} & \dfrac{\partial x}{\partial \eta} \\ \dfrac{\partial y}{\partial \xi} & \dfrac{\partial y}{\partial \eta} \end{bmatrix}^T \tag{7}$$

According to the total Lagrangian formulation, Green strain tensor can be expressed as:

$$\Delta \varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial \Delta u_i}{\partial x_j} + \frac{\partial \Delta u_j}{\partial x_i} + \frac{\partial u_k}{\partial x_i}\frac{\partial \Delta u_k}{\partial x_j} + \frac{\partial u_k}{\partial x_j}\frac{\partial \Delta u_k}{\partial x_i} \right) \tag{8}$$

Introducing Eq. 5 into Eq. 8, we can finally obtain the nonlinear geometrical relationship of the member as follows:

$$\{\Delta \varepsilon\} = [B_L] \left\{ \begin{array}{c} \{\Delta u\} \\ \{\Delta v\} \end{array} \right\} \tag{9}$$

where,

$$[B_L] = \begin{bmatrix} \left(1 + \dfrac{\partial u_p}{\partial x}\right)[Dux] + \dfrac{\partial v_p}{\partial x}[Dvx] \\ \dfrac{\partial u_p}{\partial y}[Duy] + \left(1 + \dfrac{\partial v_p}{\partial y}\right)[Dvy] \\ \left(1 + \dfrac{\partial u_p}{\partial x}\right)[Duy] + \left(1 + \dfrac{\partial v_p}{\partial y}\right)[Dvx] + \dfrac{\partial u_p}{\partial y}[Dux] + \dfrac{\partial v_p}{\partial x}[Dvy] \end{bmatrix} \tag{10}$$

$$\{\Delta \varepsilon\} = [\Delta \varepsilon_x,\ \Delta \varepsilon_y,\ 2\Delta \varepsilon_{xy}]^T$$

4. Nonlinear constitutive relationships of material

Denoting $\Delta \sigma'$ and $\Delta \varepsilon'$ as the incremental normal stress and strain parallel to ξ axis of

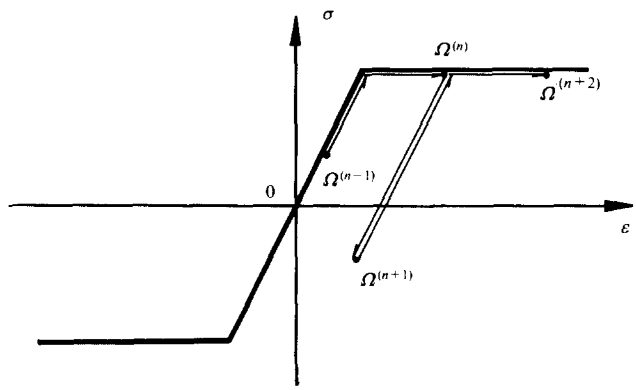

Fig. 2 The stress and strain relationships of material

the member respectively, we can write the nonlinear constitutive relationship of the material shown in Fig. 2 as follows:

$$\Delta\sigma' = E_{ep}\Delta\varepsilon' \tag{11}$$

When the material is loaded in elastic range and unloaded in plastic range E_{ep} takes the elastic modulus, and when loaded in plastic range $E_{ep}=0$.

Denoting $\{\Delta\sigma\}=[\Delta\sigma_x, \Delta\sigma_y, \Delta\tau_{xy}]^T$ as the incremental stress tensor of the member under xy coordinate system, we have

$$\{\Delta\sigma\}=\{T\}\,\Delta\sigma' \tag{12}$$

$$\Delta\varepsilon'=\{T\}^T\{\Delta\varepsilon\} \tag{13}$$

where, $\{T\}=[l^2, m^2, lm]$, l and m are the cosines of the ξ vector to xy coordinate system.

Substituting Eqs. 12 & 13 into Eq. 11, we have

$$\{\Delta\sigma\}=[D]\,\{\Delta\varepsilon\} \tag{14}$$

Here,

$$[D]=\{T\}\;E_{ep}\{T\}^T \tag{15}$$

Eq. 15 shows the nonlinear constitutive matrix of the material.

5. Nonlinear equilibrium equations of members

According to the principle of virtual work, the equilibrium equation of continuum at state $\Omega^{(n+1)}$ can be expressed as

$$\iiint_V[(\sigma_{ij}+\Delta\sigma_{ij})\delta(\Delta\varepsilon_{ij})]dV-\iint_{A\sigma}(f_i+\Delta f_i)\,\delta(\Delta u_i)dA=0 \tag{16}$$

Eq. 16 can also be written as follows

$$\iiint_V[\Delta\sigma_{ij}\,\delta(\Delta\varepsilon_{ij})+\frac{1}{2}\sigma_{ij}\,\delta\Big(\frac{\partial\Delta u_k}{\partial x_i}\frac{\partial\Delta u_k}{\partial x_j}\Big)dV=\iint_{A\sigma}(f_i+\Delta f_i)\delta(\Delta u_i)dA-\iiint_V\sigma_{ij}\,\delta(\Delta\varepsilon_{ij})dV \tag{17}$$

where, σ_{ij} is Kirchhoff stress tensor at state $\Omega^{(n)}$.

The second item in Eq. 17 can be expressed as

$$\frac{1}{2}\sigma_{ij}\delta\left(\frac{\partial\Delta u_k}{\partial x_i}\frac{\partial\Delta u_k}{\partial x_j}\right)$$

$$=\delta\frac{\partial\Delta u}{\partial x}\sigma_x\frac{\partial\Delta u}{\partial x}+\delta\frac{\partial\Delta v}{\partial x}\sigma_x\frac{\partial\Delta v}{\partial x}+\delta\frac{\partial\Delta u}{\partial y}\sigma_y\frac{\partial\Delta u}{\partial y}+\delta\frac{\partial\Delta v}{\partial y}\sigma_y\frac{\partial\Delta v}{\partial y}+$$

$$\delta\frac{\partial\Delta u}{\partial x}\tau_{xy}\frac{\partial\Delta u}{\partial y}+\delta\frac{\partial\Delta u}{\partial y}\tau_{xy}\frac{\partial\Delta u}{\partial x}+\delta\frac{\partial\Delta v}{\partial x}\tau_{xy}\frac{\partial\Delta v}{\partial y}+\delta\frac{\partial\Delta v}{\partial y}\tau_{xy}\frac{\partial\Delta v}{\partial x}$$

$$=\delta\begin{Bmatrix}\{\Delta u\}\\\{\Delta v\}\end{Bmatrix}^T[B_{NL}]^T[S][B_{NL}]\begin{Bmatrix}\{\Delta u\}\\\{\Delta v\}\end{Bmatrix} \tag{18}$$

where,

$$[B_{NL}]=\begin{bmatrix}[Dux]\\ [Dvx]\\ [Duy]\\ [Dvy]\end{bmatrix} \tag{19}$$

$$[S]=\begin{bmatrix}I_2\sigma_x & I_2\tau_{xy}\\ I_2\tau_{xy} & I_2\sigma_y\end{bmatrix} \tag{20}$$

$$I_2=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \tag{21}$$

Introducing Eqs. 9, 14 & 18 into Eq. 17, we can obtain

$$[[k_u]+[k_s]]\begin{Bmatrix}\{\Delta u\}\\\{\Delta v\}\end{Bmatrix}=\begin{Bmatrix}\{f_u+\Delta f_u\}\\\{f_v+\Delta f_v\}\end{Bmatrix}-\{f_R\} \tag{22}$$

where

$$[k_u]=\iiint_V[B_L]^T[D][B_L]\,dV \tag{23}$$

$$[k_s]=\iiint_V[B_{NL}]^T[S][B_{NL}]\,dV \tag{24}$$

$$\{f_R\}=\iiint_V[B_L]^T\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}dV \tag{25}$$

σ_x, σ_y, τ_{xy} are Kirchhoff stresses at state $\Omega^{(n)}$.

Eq. 22 can be written as:

$$[k]\begin{Bmatrix}\{\Delta u\}\\\{\Delta v\}\end{Bmatrix}=\begin{Bmatrix}\{f_u\}\\\{f_v\}\end{Bmatrix}^{(n+1)}-\begin{Bmatrix}\{f_{Ru}\}\\\{f_{Rv}\}\end{Bmatrix}^{(n)} \tag{26}$$

Rearranging Eq. 26, we have

$$\begin{bmatrix} [k_{bb}] & [k_{bi}] \\ [k_{ib}] & [k_{ii}] \end{bmatrix} \begin{bmatrix} \Delta u_0 \\ \Delta v_0 \\ \Delta v_{-1} \\ \Delta u_m \\ \Delta v_m \\ \Delta v_{m+1} \\ \{\Delta u\}' \\ \{\Delta v\}' \end{bmatrix} = \begin{bmatrix} f_{u,0} \\ f_{v,0} \\ f_{v,-1} \\ f_{u,m} \\ f_{v,m} \\ f_{v,m+1} \\ \{f_u\}' \\ \{f_v\}' \end{bmatrix} - \begin{bmatrix} f_{uR,0} \\ f_{vR,0} \\ f_{vR,-1} \\ f_{uR,m} \\ f_{vR,m} \\ f_{vR,m+1} \\ \{f_{uR}\}' \\ \{f_{vR}\}' \end{bmatrix} \tag{27}$$

where, superscript $'$ represents the residual tensor excluded the boundary items.

Eq. 27 can be written as

$$\begin{bmatrix} [k_{bb}] & [k_{bi}] \\ [k_{ib}] & [k_{ii}] \end{bmatrix} \begin{Bmatrix} \{\Delta d_b\} \\ \{\Delta d_i\} \end{Bmatrix} = \begin{Bmatrix} \{f_b\} \\ \{f_i\} \end{Bmatrix}^{(n+1)} - \begin{Bmatrix} \{f_{bR}\} \\ \{f_{iR}\} \end{Bmatrix}^{(n)}$$

Then we obtain

$$[k_m]\ \{\Delta d_b\} = \{f_m\}^{(n+1)} - \{f_{mR}\}^{(n)} \tag{28}$$

where,

$$[k_m] = [[k_{bb}] - [k_{bi}][k_{ii}]^{-1}[k_{ib}]] \tag{29}$$

$$\{f_m\} = \{f_b\} - [k_{bi}]\ [k_{ii}]^{-1}\{f_i\} \tag{30}$$

$$\{f_{mR}\} = \{f_{bR}\} - [k_{bi}]\ [k_{ii}]^{-1}\{f_{iR}\} \tag{31}$$

Eq. 28 is the incremental equilibrium equation of members.

The items in Eq. 28 can be obtained by Gauss numerical integration technique. Members can be divided into m segments and two Gauss points are selected in each segment. The sections at Gauss points can be divided into several small rectangular areas and the magnitudes of whole section can be obtained by accumulating all those of rectangles.

The equilibrium equation aligned in global coordinate system can be written as:

$$[k_m]\ \{\Delta D_m\} = \{F_m\}^{(n+1)} - \{F_{mR}\}^{(n)} \tag{32}$$

where,

$$[K_m] = [T_m]^T\ [k_m]\ [T_m] \tag{33}$$

$$\{F_m\} = [T_m]^T\ \{f_m\} \tag{34}$$

$$\{F_{mR}\} = [T_m]^T\{f_{mR}\} \tag{35}$$

and

$$[T_m] = \begin{bmatrix} L_x & M_x & 0 & 0 & 0 & 0 \\ L_y & M_y & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & L_x & M_x & 0 \\ 0 & 0 & 0 & L_y & M_y & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{36}$$

In Eq. 36, L_x, M_x are the cosine of the x axis to the XY coordinate system, and L_y, M_y are that of y axis, at state $\Omega^{(0)}$.

The total equilibrium equation of member structures can be obtained by assembling all member equations as Eq. 32 of members and can be written as:

$$[K]\ \{\Delta D\}=\{F\}^{(n+1)}-\{F_R\}^{(n)} \tag{37}$$

6. Solving of nonlinear equilibrium equations of structures

Eq. 37 can be written as:

$$[K]_i^{(n+1)}\ \{\Delta D\}_{i+1}=\{\Delta F\}+\{R\}_i^{(n+1)} \tag{38}$$

Here, $\{R\}_i^{(n+1)}=\{F\}^{(n)}-\{F_R\}_i^{(n+1)}$. Superscript $n+1$ represents at $\Omega^{(n+1)}$ state and subscript i represents the i th iteration.

Defining $\{\Delta F\}=\Delta\lambda_{i+1}\{F\}_o$ in which $\Delta\lambda$ is the load parameter, we can express Eq. 38 as:

$$[K]_i^{(n+1)}\ \{\Delta D\}^I=\{F\}_o \tag{39}$$

$$[K]_i^{(n+1)}\ \{\Delta D\}^{II}=\{R\}_i^{(n+1)} \tag{40}$$

and

$$\{\Delta D\}_{i+1}=\Delta\lambda_{i+1}\ \{\Delta D\}^I+\{\Delta D\}^{II} \tag{41}$$

Denoting do as the main controlled displacement, we can write the constraint condition in solving Eq. 38 as:

$$\Delta d_0=\Delta\lambda_{i+1}\Delta d_0^I+\Delta d_0^{II} \tag{42}$$

where, Δd_0 is the given displacement magnitude in the first iteration. And Δd_0 is made zero from the second iteration.

From Eq. 42, we can obtain:

$$\Delta\lambda_{i+1}=\frac{\Delta d_0-\Delta d_0^{II}}{\Delta d_0^I} \tag{43}$$

In computation, the reference load $\{F\}_0$ is first selected and the incremental displacement Δd_0 is given. For $i=0$ from Eqs. 39 & 40 we obtain $\{\Delta D\}^{II}=\{0\}$ ($\Delta d^{II}=0$) and $\{\Delta D\}^I$ (Δd_0^I), and from Eqs. 43 & 41 we can solve $\Delta\lambda_1$ and $\{\Delta D\}_1$. For the following iteration step i, $[K]$ and $\{R\}$ can be formed, then $\{\Delta D\}^I$ and $\{\Delta D\}^{II}$ can be solved from Eqs. 39 & 40, $\Delta\lambda_{i+1}$ (when $\Delta d_0=0$) can be obtained from Eq. 43 and $\{\Delta D\}$ can be found from Eq. 41. Carrying out the cyclic computation we can obtain $\{D\}_{i+1}^{(n+1)}=\{D\}_i^{(n+1)}+\{\Delta D\}_{i+1}$ and $\lambda_{i+1}^{(n+1)}=\lambda_i^{(n+1)}+\Delta\lambda_{i+1}$.

In every iteration i, the incremental deflection vector $\{\Delta D\}$ in global coordinate system must be transformed into those $\{\Delta d_b\}$ in xy system by using Eq. 36. The equations of members can then be re-formed and the changed $[K]$ and $\{R\}$ can be found.

When the iteration converges, the load and deflection increments of member structures under the given displacement increment Δd_0 will be obtained. The computation for every displacement increment needs about three or four iterative times before the structures reach the ultimate strength, and about seven times when the structures are loaded beyond the ultimate points. Fig. 3 shows the illustration of the iterative solution.

The rule for identifying the ultimate strength point of the structures is:

$$\text{Det}\ |K|\leq\varepsilon_1$$

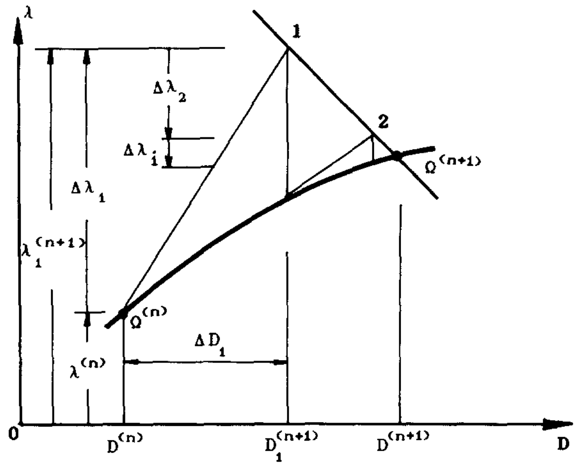

Fig. 3 Illustration of the solving procedures

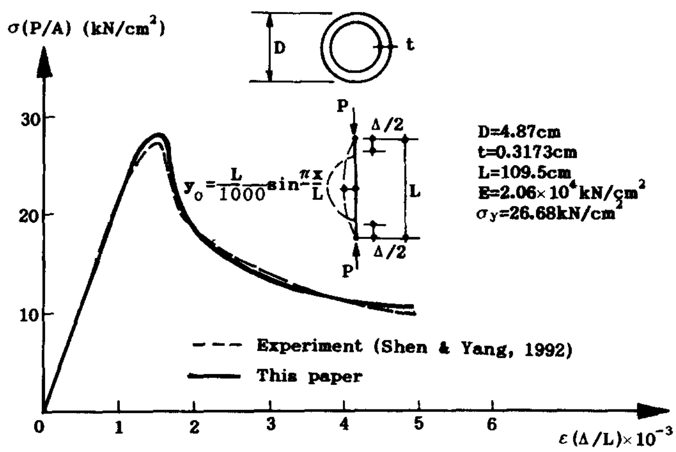

Fig. 4 The whole load-deflection curves of a tubular column

The convergence condition of solution is:

$$\sum_{k=1}^{2m+4} (R_k - F_k)^2 \leq \varepsilon_2$$

where, ε_1 and ε_2 are the controlled accuracy, and R_k and F_k are the k line elements of $\{R\}$ and $\{F\}$ respectively.

7. Numerical examples

7.1. The ultimate strength of single members

Fig. 4 shows the whole load-deflection curve of an axially loaded tubular member. The experiments were conducted in Tongji University (Shen & Yang 1992). The comparison between the experimental and the theoretical results is shown in Fig. 4.

Fig. 5 shows the theoretical results of this paper for eccentrically loaded members. The NIM results are also shown in Fig. 5.

7.2. The elastic large deflection behavior of a two-member truss

The load-deflection relationship of a two-member truss obtained from FEM (Zhang 1991) is shown in Fig. 6. The result of this paper is also shown in the figure.

It is seen from Fig. 6 that the method of this paper can treat the geometrical nonlinearity accurately.

7.3. The ultimate strength of a planar frame

Fig. 7 shows the experimental results about a planar frame obtained by Schilling in 1956

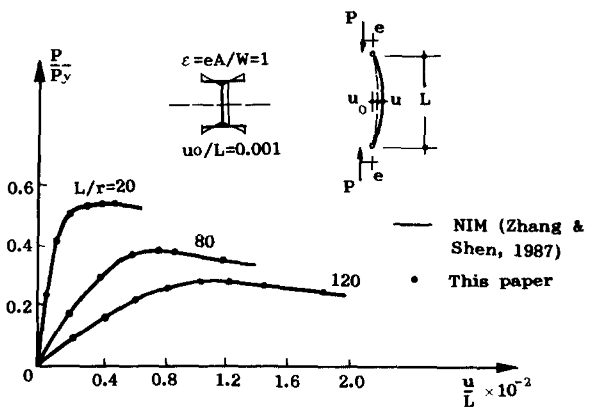

Fig. 5 The whole load-deflection curves of steel members

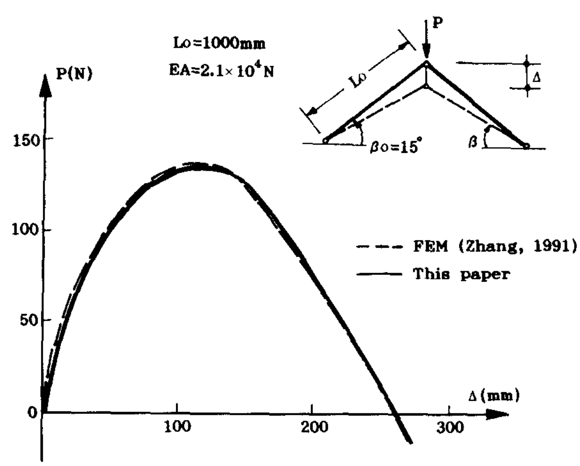

Fig. 6 The elastic large deflection behavior of a two-member truss

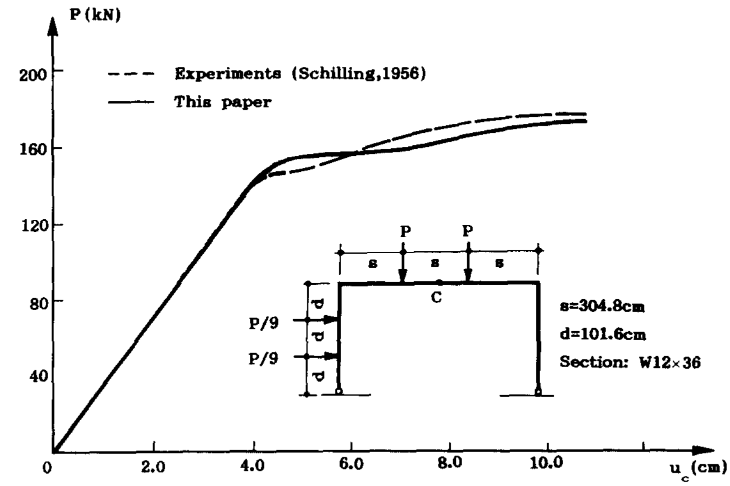

Fig. 7 The ultimate strength of planar frame

(Kam 1983). In this paper the load-deflection relationship of the frame is analyzed, and the results are also shown in Fig. 7.

From Figs. 4-7 we can observe that the spline function method derived in this paper can analyze the whole loading procedures and the ultimate strength of steel members and member structures accurately and effectively.

8. Conclusions

1. The spline function method can be satisfactorily adopted in the analyses of the elasto-plastic large deflection problems of members and member structures with initial imperfections;
2. The plastic hinge assumption for FEM to treat material nonlinearity is not needed in this paper. And the method derived in this paper has higher efficiency compared with NIM solution;
3. Different boundary conditions and joint types of members in structures can be treated easily.

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