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HDQ-FD integrated methodology for nonlinear static and dynamic response of doubly curved shallow shells

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Abstract. The non-linear static and dynamic response of doubly curved thin isotropic shells has been studied for the step and sinusoidal loadings. Dynamic analogues Von Karman-Donnel type shell equations are used. Clamped immovable and simply supported immovable boundary conditions are considered. The governing nonlinear partial differential equations of the shell are discretized in space and time domains using the harmonic differential quadrature (HDQ) and finite differences (FD) methods, respectively. The accuracy of the proposed HDQ-FD coupled methodology is demonstrated by the numerical examples. Numerical examples demonstrate the satisfactory accuracy, efficiency and versatility of the presented approach.

Key words: non-linear dynamic analysis; doubly curved shells; harmonic differential quadrature; coupled methodology.

1. Introduction

The successful solution of the engineering problems begins with an accurate physical model of the problem. In turn, this physical model is transformed into a mathematical model. The mathematical models for a wide range of engineering problems are expressed in terms of linear and non-linear partial differential equations. With the exception of a few simple cases, partial differential equations cannot be solved analytically. Consequently, the solution of the mathematical model is usually obtained by numerical methods (Bathe 1992, Celia and Gray 1992, Zienkiewicz 1977).

The practical importance of vibration analysis of doubly curved shells has been increased in structural, aerospace, submarine hulls, and mechanical applications. Nonlinear static and dynamic analyses of plates and shells of various shapes have been carried out by various researchers (Chia 1980, Leissa 1973, Markus 1988, Soedel 1996, Civalek 1998). Lim and Liew (1994) studied the

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flexural vibration of shallow cylindrical shells of rectangular platform using a pb-2 Ritz method. A Ritz vibration analysis of doubly curved rectangular shallow shells using a first order shear deformation theory has been studied by Liew and Lim (1995). Recent developments on the free vibration analysis of thin, moderately thick and thick shells are documented in the articles by Liew *et al.* (1997). Liew and Lim (1996) have proposed a higher order theory for doubly curved shallow shells. Liew *et al.* (1995a) also studied the effects of initial twist and thickness variation on the vibration behavior of shallow conical shells. Matsunaga (1999) studied the vibration and stability behavior of thick simply supported shells subject to in-plane stresses. Some selected works in this research topic includes those of Leissa *et al.* (1983), Leissa and Kadi (1971), Leissa and Nariata (1984).

Recently, the method of DQ has been extended to solve static and dynamic shell problems by Bert and Malik (1996), Shu (1996), and Civalek (2003, 2004). Various problems in structural mechanics have been solved successfully by this method (Liew *et al.* 1996, 1999, 2001, Shu and Richards 1992, Civalek 2001, 2004a) up to now. Employing the GDQ method, Wu and Liu (2000) have analyzed the static bending of shells of revolution and free vibration of circular plates (2001, 2002). Analysis of conical shells using GDQ method has been carried out by Hua and Lam (2000). The nonlinear static analysis of circular plates by the DQ method has been presented by Striz *et al*. (1988).

Nath *et al*. (1987) presented the finite differences methods for spatial discretization and Houbolt's time marching discretization to study the dynamic analysis of doubly curved shallow shells resting on elastic foundation. For non-linear static analysis of shallow shells, Nath *et al.* (1983) proposed collocation methods. Nath and Jain (1983) have investigated nonlinear dynamic analysis of shallow spherical shells on elastic foundation employing the Chebyshev series and Houbolt technique. Liew and Lim (1995b) proposed the Ritz procedure based on the principle of minimum total energy for doubly curved shallow shells. The objective of this study is to present an approximate numerical solution of the Von Karman-Donnel type governing equations for the geometrically nonlinear analysis of doubly curved shallow shells. For this purpose, DQ and finite differences methods had been used for spatial and temporal discretization of governing differential equations of problem. We use the harmonic functions as the test functions to obtain the weighting coefficients in the method of DQ. To the authors' knowledge, it is the first time the differential quadrature method has been successfully applied to thin, isotropic doubly curved shallow shell problem for the geometrically nonlinear dynamic analysis.

2. Differential quadrature method

In the differential quadrature method, a partial derivative of a function with respect to a space variable at a discrete point is approximated as a weighted linear sum of the function values at all discrete points in the region of that variable. For simplicity, we consider a one-dimensional function $u(x)$ in the $[-1, 1]$ domain, and N discrete points. Then the first derivatives at point i, at $x = x_i$ is given by

$$u_x(x_i) = \left.\frac{\partial u}{\partial x}\right|_{x = x_i} = \sum_{j=1}^{N} A_{ij} u(x_j); \quad i = 1, 2, \ldots, N \tag{1}$$

where x_j are the discrete points in the variable domain, $u(x_j)$ are the function values at these points and A_{ij} are the weighting coefficients for the first order derivative attached to these function values.

Bellman *et al.* (1972) suggested two methods to determine the weighting coefficients. The first one is to let Eq. (1) be exact for the test functions

$$u_k(x) = x^{k-1}; \quad k = 1, 2, \ldots, N \tag{2}$$

which leads to a set of linear algebraic equations

$$(k-1)x_i^{k-2} = \sum_{j=1}^{N} A_{ij} x_j^{k-1}; \quad \text{for} \quad i = 1, 2, \ldots, N \quad \text{and} \quad k = 1, 2, \ldots, N \tag{3}$$

which represents N sets of N linear algebraic equations. As similar to the first order, the second order derivative can be written as

$$u_{xx}(x_i) = \left.\frac{\partial^2 u}{\partial x^2}\right|_{x = x_i} = \sum_{j=1}^{N} B_{ij} u(x_j); \quad i = 1, 2, \ldots, N \tag{4}$$

where the B_{ij} is the weighting coefficients for the second order derivative. Eq. (4) also can be written as

$$u_{xx}(x_i) = \left.\frac{\partial^2 u}{\partial x^2}\right|_{x = x_i} = \sum_{j=1}^{N} A_{ij} \sum_{k=1}^{N} A_{jk} u(x_k); \quad i = 1, 2, \ldots, N \tag{5}$$

Again, the function given by Eq. (2) is used so that the second order derivative is

$$(k-1)(k-2)x_i^{k-3} = \sum_{j=1}^{N} B_{ij} x_j^{k-1} \tag{6}$$

This can be solved in the same manner as indicated for Eq. (3) above. Another way to determine the weighting coefficients is to employ harmonic functions, named the harmonic differential quadrature (HDQ). Harmonic differential quadrature has been proposed by Striz *et al.* (1995). Unlike the differential quadrature that uses the polynomial functions, such as power functions, Lagrange interpolated, and Legendre polynomials as the test functions, harmonic differential quadrature uses harmonic or trigonometric functions as the test functions. Thus, this method is called the HDQ method. Shu and Xue proposed an explicit means of obtaining the weighting coefficients for the HDQ (Shu and Xue 1997). When the $f(x)$ is approximated by a Fourier series expansion in the form

$$f(x) = c_0 + \sum_{k=1}^{N/2} \left(c_k \cos\frac{k\pi x}{L} + d_k \sin\frac{k\pi x}{L} \right) \tag{7}$$

and the Lagrange interpolated trigonometric polynomials are taken as

$$h_k(x) = \frac{\sin\frac{(x-x_0)\pi}{2} \cdots \sin\frac{(x-x_{k-1})\pi}{2} \sin\frac{(x-x_{k+1})\pi}{2} \ldots \sin\frac{(x-x_N)\pi}{2}}{\sin\frac{(x_k-x_0)\pi}{2} \ldots \sin\frac{(x_k-x_{k-1})\pi}{2} \sin\frac{(x_k-x_{k+1})\pi}{2} \ldots \sin\frac{(x_k-x_N)\pi}{2}} \tag{8}$$

for $k = 0, 1, 2, \ldots, N$.

According to the HDQ, the weighting coefficients of the first-order derivatives A_{ij} for $i \neq j$ can be obtained by using the following formula:

$$A_{ij} = \frac{(\pi/2)P(x_i)}{P(x_j)\sin[(x_i - x_j)/2]\pi}; \quad i, j = 1, 2, 3, \ldots, N \tag{9}$$

where

$$P(x_i) = \prod_{j=1, j \neq i}^{N} \sin\left(\frac{x_i - x_j}{2}\pi\right); \text{ for } j = 1, 2, 3, \ldots, N \tag{10}$$

The weighting coefficients of the second-order derivatives B_{ij} for $i \neq j$ can be obtained using following formula:

$$B_{ij} = A_{ij}\left[2A_{ii}^{(1)} - \pi ctg\left(\frac{x_i - x_j}{2}\right)\pi\right]; \quad i, j = 1, 2, 3, \ldots, N \tag{11}$$

The weighting coefficients of the first-order and second-order derivatives $A_{ij}^{(p)}$ for $i = j$ are given as

$$A_{ii}^{(p)} = -\sum_{j=1, j \neq i}^{N} A_{ij}^{(p)}; \quad p = 1 \text{ or } 2; \text{ and for } i = 1, 2, \ldots, N \tag{12}$$

The weighting coefficient of the third and fourth order derivatives can be computed easily from A_{ij} and B_{ij} by (Bert and Malik 1996, Shu 2000):

$$C_{ij} = \sum_{k=1}^{N} A_{ik}B_{kj}; \quad D_{ij} = \sum_{k=1}^{N} B_{ik}B_{kj} \tag{13,14}$$

As it can bee seen from the above equations that the weighting coefficients of the third and fourth-order derivative can be completely determined from those of the first and second-order derivative. The main advantage of HDQ over the differential quadrature is its ease of the computation of the weighting coefficients without any restriction on the choice of grid points.

2.1 Choices of sampling grid points

A decisive factor to the accuracy of the differential quadrature solution is the choice of the sampling or grid points. It should be mentioned that in the differential quadrature solutions, the sampling points in various coordinate directions may be different in number as well as in their type. Two different types of sampling grids are taken into consideration in this study. A natural, and often convenient, choice for sampling points is that of equally spaced grid (ES-G) points. These points are given by,

$$\text{Type-I}: \quad x_i = \frac{i-1}{N_x - 1}; \text{ and } \quad y_i = \frac{i-1}{N_y - 1}; \quad i = 1, 2, \ldots, N_x \text{ and } i = 1, 2, \ldots, N_y \tag{15a,b}$$

in the related directions. Sometimes, the differential quadrature solutions deliver more accurate results with unequally spaced sampling points. A better choice for the positions of the grid points between the first and the last points at the opposite edges is that corresponding to the zeros of orthogonal polynomials such as; the zeros of Chebyshev polynomials (Bert and Malik 1996, Shu

2000, Civalek 2004, 2004a, Liew *et al.* 1999). Another choice that is found to be even better than the Chebyshev and Legendre polynomials is the set of points proposed by Shu and Richards (1992). These points are given as

$$\text{Type-II;} \quad x_i = \frac{1}{2}\left[1-\cos\left(\frac{2i-1}{N_x-1}\right)\pi\right]; \quad \text{and} \quad y_i = \frac{1}{2}\left[1-\cos\left(\frac{2i-1}{N_y-1}\right)\pi\right]$$

$$i = 1, 2, \ldots, N_x \ \text{ and } \ i = 1, 2, \ldots, N_y \tag{16a,b}$$

in the x- and y- directions, respectively. These type grid points are known the Chebyshev-Gauss-Lobatto or non-equally spaced grid (NES-G) points.

3. Governing equations

We consider a doubly curved shallow shell of length a in x- direction, width b in the y-direction and thickness h in the z-direction. The geometry of a typical doubly curved shallow shell is shown in Fig. 1.

Neglecting the in-plane and rotary inertia, the governing differential equations of motion in terms of non-dimensional displacements components U, V, and W for geometrically nonlinear dynamic analysis of doubly curved shallow shells can be expressed as

$$U_{,XX} + \frac{\beta^2}{2}(1-\upsilon)U_{,YY} + \frac{\beta}{2}(1+\upsilon)V_{,XY} + \left[W_{,XX} + \frac{\beta^2}{2}(1-\upsilon)W_{,YY}\right]W_{,X}$$
$$+ \frac{\beta^2}{2}(1+\upsilon)W_{,YY}W_{,XY} - (R_Y + \upsilon R_X)W_{,X} = 0 \tag{17}$$

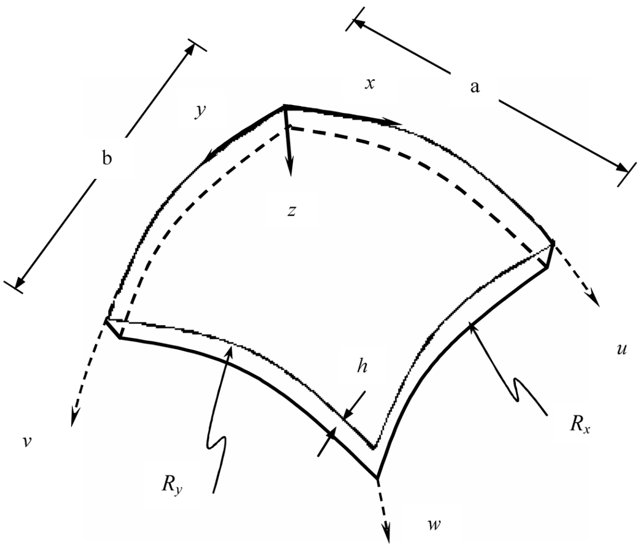


Fig. 1 A schematic diagram of a doubly curved shallow shell

$$\beta^2 V_{,YY} + \frac{1}{2}(1-\upsilon)V_{,XX} + \beta\frac{1}{2}(1+\upsilon)U_{,XY} + \beta\left[\beta^2 W_{,YY} + \frac{1}{2}(1-\upsilon)W_{,XX}\right]W_{,Y}$$

$$+ \beta\frac{1}{2}(1+\upsilon)W_{,X}W_{,XY} - W_{,Y} - (R_X + \upsilon R_Y)W_{,Y} = 0 \tag{18}$$

$$W_{,XXXX} + 2\beta^2 W_{,XXYY} + \beta^4 W_{,YYYY} - 12\left[U_{,X} + \beta\upsilon V_{,Y} + \frac{1}{2}(W_{,X})^2 + \frac{1}{2}\beta^2\upsilon(W_{,Y})^2 - W(R_X + \upsilon R_Y)\right](W_{,XX} + R_X)$$

$$-12\left[\beta V_{,Y} + \upsilon U_{,X} + \frac{1}{2}\upsilon(W_{,X})^2 + \frac{1}{2}\beta^2(W_{,Y})^2 - W(R_Y + \upsilon R_X)\right](\beta^2 W_{,YY} + R_Y)$$

$$-12[\beta U_{,Y} + V_{,X} + \beta W_{,X}W_{,Y}](1-\upsilon)\beta W_{,XY} + 12(1-\upsilon^2)\frac{qa^4}{Eh^4} + W_{,\tau\tau} + CW_{,\tau} = 0 \tag{19}$$

The non-dimensional quantities in the above equations are defined as

$$W = x/h,\ X = x/a,\ Y = y/b,\ \beta = a/b,\ U = ua/h^2,\ V = va/h^2,\ C = c\sqrt{\rho h a^4/D}$$

$$R_X = r_x b^2/h,\ R_Y = r_y b^2/h,\ \tau = t\sqrt{D/\rho a^4 h},\ D = Eh^3/12(1-\upsilon^2),\ P = qa^4/Eh^4 \tag{20}$$

where u, v and w are displacement componens in the x, y, ans z directions, respectively, h is the thickness of the shell, E is Young's modulus, ν is Poisson's ratio, ρ is density, D is the flexural rigidity, a and b are the sides of shell along x and y directions, r_x and r_y are the shell curvatures, t is the time, and C is the damping coefficients. Using the finite difference (central difference approach) method for the time-wise integration, the velocity and acceleration at time i, can be expressed as

$$\dot{u}_i = \frac{1}{2\Delta\tau}[u_{i+1} - u_{i-1}] \tag{21a}$$

$$\ddot{u}_i = \frac{1}{(\Delta\tau)^2}[u_{i+1} - 2u_i + u_{i-1}] \tag{21b}$$

3.1 Boundary and initial conditions

The following two types of boundary conditions are considered in the present study. For all four edges simply supported and immovably constrained against in-plane translation (SSSS).

$$U = V = W = 0 \text{ and } \left(\beta^2\frac{\partial^2 W}{\partial Y^2} + \upsilon\frac{\partial^2 W}{\partial X^2}\right) = 0 \text{ at } Y = 0 \text{ and } Y = 1 \tag{22}$$

$$U = V = W = 0 \text{ and } \left(\frac{\partial^2 W}{\partial X^2} + \upsilon\beta^2\frac{\partial^2 W}{\partial Y^2}\right) = 0 \quad X = 0 \text{ and } X = 1 \tag{23}$$

For all four edges clamped and immovably constrained against in-plane translation (CCCC).

$$U = V = W = 0 \text{ and } \left(\frac{\partial W}{\partial Y}\right) = 0 \text{ at } Y = 0 \text{ and } Y = 1 \tag{24}$$

$$U = V = W = 0 \text{ and } \left(\frac{\partial W}{\partial X}\right) = 0 \quad X = 0 \text{ and } X = 1 \tag{25}$$

In the following the HDQ and FD methods are applied to discretize the derivatives for spatial and time domain in the governing Eqs. (17)-(19) boundary and initial conditions. In the following, $d_1 = (1 - \nu)/2$ and $d_2 = (1 + \nu)/2$ are used for shortening. After spatial and time discretization, governing equations, boundary and initial conditions become (Civalek 2004):

$$\begin{aligned}
&\sum_{k=1}^{N_x} B_{ik}U_{kj} + \beta^2 d_1 \sum_{k=1}^{N_y} B_{jk}U_{ik} + \beta d_2 \sum_{m=1}^{N_y} A_{jm} \sum_{k=1}^{N_x} A_{ik}V_{km} \\
&+ \sum_{k=1}^{N_x} A_{ik}W_{kj}\left[\sum_{k=1}^{N_x} B_{ik}W_{kj} + \beta^2 d_1 \sum_{k=1}^{N_y} B_{jk}W_{ik}\right] + \beta^2 d_2 \sum_{k=1}^{N_y} A_{jk}W_{ik} \sum_{m=1}^{N_y} A_{jm} \sum_{k=1}^{N_x} A_{ik}W_{km} \\
&-(R_x + \upsilon R_y)\sum_{k=1}^{N_x} A_{ik}W_{kj} = 0
\end{aligned} \tag{26}$$

$$\begin{aligned}
&\beta^2 \sum_{k=1}^{N_x} B_{jk}V_{ik} + d_1 \sum_{k=1}^{N_x} B_{ik}V_{kj} + \beta d_2 \sum_{m=1}^{N_y} A_{jm} \sum_{k=1}^{N_x} A_{ik}U_{km} \\
&+ \beta \sum_{k=1}^{N_y} A_{jk}U_{ik}\left[d_1 \sum_{k=1}^{N_x} B_{ik}W_{kj} + \beta^2 \sum_{k=1}^{N_y} B_{jk}W_{ik}\right] + \beta d_2 \sum_{k=1}^{N_x} A_{ik}W_{kj} \sum_{m=1}^{N_y} A_{jm} \sum_{k=1}^{N_x} A_{ik}W_{km} \\
&-(R_y + \upsilon R_x)\sum_{k=1}^{N_y} A_{jk}W_{ik} = 0
\end{aligned} \tag{27}$$

$$\begin{aligned}
&\sum_{k=1}^{N_x} D_{ik}W_{kj} + 2\beta^2 \sum_{m=1}^{N_y} B_{jm} \sum_{k=1}^{N_x} B_{ik}W_{km} + \beta^4 \sum_{k=1}^{N_y} D_{jk}W_{ik} \\
&-12\left(\sum_{k=1}^{N_x} B_{ik}W_{kj} + R_x\right)\left\{\sum_{k=1}^{N_x} A_{ik}U_{kj} + \beta\upsilon \sum_{k=1}^{N_y} A_{jk}U_{ik} + \frac{1}{2}\left[\left(\sum_{k=1}^{N_x} A_{ik}W_{kj}\right)^2 + \beta^2 \upsilon \left(\sum_{k=1}^{N_y} A_{jk}U_{ik}\right)^2\right] - W_{ik}(R_x + \upsilon R_y)\right\} \\
&-12\left(\beta^2 \sum_{k=1}^{N_y} B_{jk}W_{ik} + R_y\right)\left\{\beta \sum_{k=1}^{N_y} A_{jk}V_{ik} + \upsilon \sum_{k=1}^{N_x} A_{ik}W_{kj} + \frac{1}{2}\left[\upsilon\left(\sum_{k=1}^{N_x} A_{ik}W_{kj}\right)^2 + \beta^2\left(\sum_{k=1}^{N_y} A_{jk}U_{ik}\right)^2\right]\right. \\
&\left. - W_{ik}(R_y + \upsilon R_x)\right\} - 12(1-\upsilon)\beta\left[\beta \sum_{k=1}^{N_y} A_{jk}U_{ik} + \sum_{k=1}^{N_x} A_{ik}V_{kj} + \beta \sum_{k=1}^{N_x} A_{ik}W_{kj} \sum_{k=1}^{N_y} A_{jk}W_{ik}\right] \sum_{m=1}^{N_y} A_{jm} \sum_{k=1}^{N_x} A_{ik}W_{km} \\
&-12(1-\upsilon^2)\frac{qa^4}{Eh^4} - \frac{1}{(\Delta\tau)^2}(W_{i+1} - 2W_i + W_{i-1}) + C\left[\frac{1}{2(\Delta\tau)}(W_{i+1} - W_{i-1})\right] = 0
\end{aligned} \tag{28}$$

For SSSS boundary conditions:

$$U_{i1} = V_{i1} = W_{i1} = 0 \quad \text{and} \quad U_{iN} = V_{iN} = W_{iN} = 0 \tag{29}$$

$$U_{1j} = V_{1j} = W_{1j} = 0 \quad \text{and} \quad U_{Nj} = V_{Nj} = W_{Nj} = 0 \tag{30}$$

$$\sum_{k=1}^{N_y} B_{ik} W_{k1} + \upsilon\beta^2 \sum_{k=1}^{N_x} B_{jk} W_{j1} = \sum_{k=1}^{N_y} B_{ik} W_{kN} + \upsilon\beta^2 \sum_{k=1}^{N_x} B_{jk} W_{jN} = 0 \tag{31}$$

$$\beta^2 \sum_{k=1}^{N_y} B_{jk} W_{1k} + \upsilon \sum_{k=1}^{N_x} B_{ik} W_{1j} = \beta^2 \sum_{k=1}^{N_y} B_{jk} W_{iN} + \upsilon \sum_{k=1}^{N_x} B_{ik} W_{kN} = 0 \tag{32}$$

For CCCC boundary conditions:

$$U_{i1} = V_{i1} = W_{i1} = 0 \quad \text{and} \quad U_{iN} = V_{iN} = W_{iN} = 0 \tag{33}$$

$$U_{1j} = V_{1j} = W_{1j} = 0 \quad \text{and} \quad U_{Nj} = V_{Nj} = W_{Nj} = 0 \tag{34}$$

$$\sum_{k=1}^{N_y} A_{ik} W_{k1} = \sum_{k=1}^{N_y} A_{ik} W_{kN} = 0 \tag{35}$$

$$\sum_{k=1}^{N_y} A_{jk} W_{1k} = \sum_{k=1}^{N_y} A_{jk} W_{iN} = 0 \tag{36}$$

For initial conditions:

$$W_1 = 0 \quad \text{and} \quad \sum_{t=1}^{\tau_N} A_{jk} W_{1\tau} = 0 \tag{37}$$

where A_{ij}, B_{ij} and D_{ij} are the weighting coefficients for first, second and fourth order derivatives which can be determined as discussed in section two. The set of non-linear algebraic Eqs. (26)-(37) can be solved for $\{U\}$, $\{V\}$ and $\{W\}$ using non-linear algorithms such as Newton-Raphson method (Bathe 1992, Celia and Gray 1992, Zienkiewicz 1977, Civalek 1998).

4. Numerical results

The title problem is analysed and some of HDQ-FD results are compared with results in the open literature [35,49,50] to show the applicability and efficiency of HDQ-FD coupled methodology. A uniform step load of infinite duration and sinusoidal loading of finite duration $\tau = 0.16$ have been considered (Fig. 2).

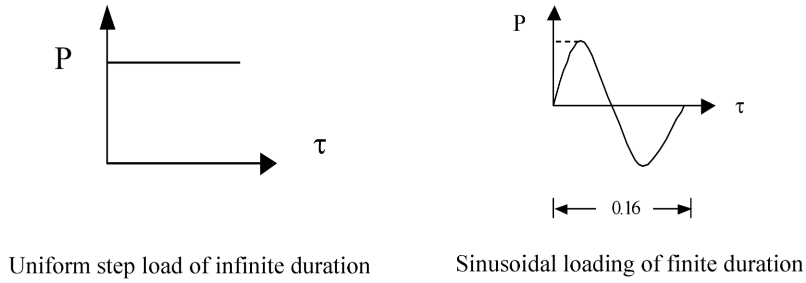


Fig. 2 Dynamic loads in numerical applications

To check whether the purposed formulation and programming are correct, a clamped immovable plates (R_x = R_y = 0) for aspect ratio β = 1 (β = a/b) subject to uniform pressure is analysed first. All numerical results are obtained using Δt = 0.1 for dynamic responses. The obtained results by HDQ for equally sampling grid (ES-G) and non-equally sampling grid (NES-G) points are shown in Fig. 3. The results given by Timoshenko and Woinowsky-Krieger (1959) and Nath and Kumar (1995) are also plotted in this figure. The displacements by Civalek (1998) using the FEM method are also presented in the figure for comparison. It is also concluded that the results obtained with non-equally sampling grid (NES-G) points are more accurate than the values calculated by equally sampling grid (ES-G) points. The numerical solution of the HDQ method using non-equally sampling grid (NES-G) points is equivalent to the Timoshenko and Woinowsky-Krieger's (1959) results. In Table 1, central deflection results of doubly curved shallow shell for different numbers of grid points. When the present HDQ solution is compared with the literature results (Nath and Kumar 1995), the greatest deviation is 5.3% for ES-G points (N = 9). However, this deviation is

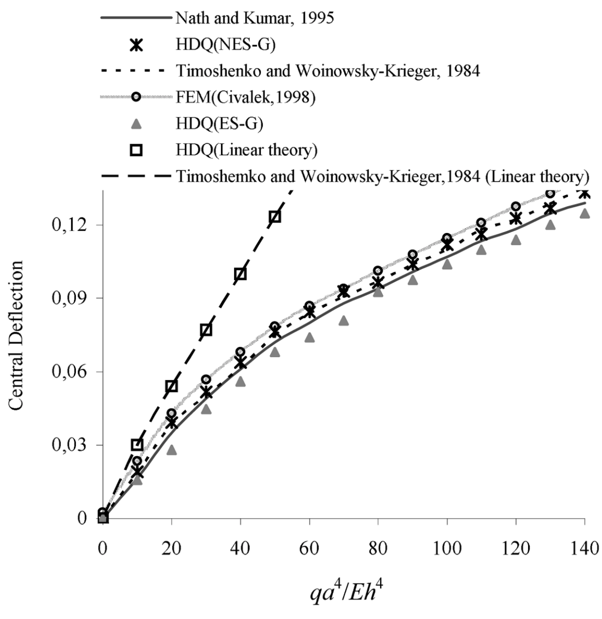


Fig. 3 Nonlinear static analysis results of clamped rectangular (R_x = R_y= 0) plate

Table 1 Static analysis results for doubly curved shell (a/b = 1; R_x = 5.0; R_y = 5.0; P = 42.7)

Support conditions	Central displacement					
	Nath *et al.* (1987)	HDQ (N=9) ES-G	HDQ (N=9) NES-G	HDQ (N=11) ES-G	HDQ (N=11) NES-G	HDQ (N=13) ES-G
CCCC	0.619	0.652	0.631	0.640	0.622	0.620
SSSS	1.616	1.665	1.648	1.651	1.617	1.619

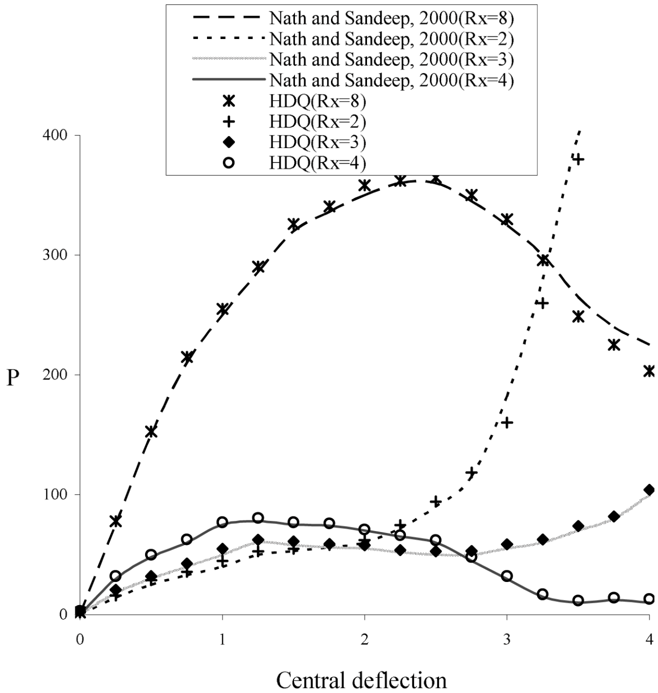


Fig. 4 Load-central deflection curves for curved panels

1.93% for non-equally sampling grid (NES-G) points. It can be seen that the reasonable accurate results are obtained for only 11 grid points (N = 11) by HDQ method for NES-G points. For ES-G points, however, reasonable accurate results are obtined for 13 grid points.

Fig. 4 show the central deflection-load curves of the clamped immovable shell. The obtained results by non-equally sampling grid (NES-G) points are shown in this figure. The results given by Nath and Sandeep (2000) are also plotted in this figure. The numerical solution of the HDQ method using non-equally sampling grid (NES-G) points is equivalent to the Nath and Sandeep's results. Figs. 5(a) and 5(b) show the time-deflection curves of the simply supported and doubly curved shells. For these figures, a uniform step load of infinite duration has been considered. The authors have compared their results with those of Nath and Sandeep (2000) as shown in these figures. The numerical solution of the HDQ-FD methods using non-equally sampling grid (NES-G) points is equivalent to the Nath and Sandeep's (2000) results.

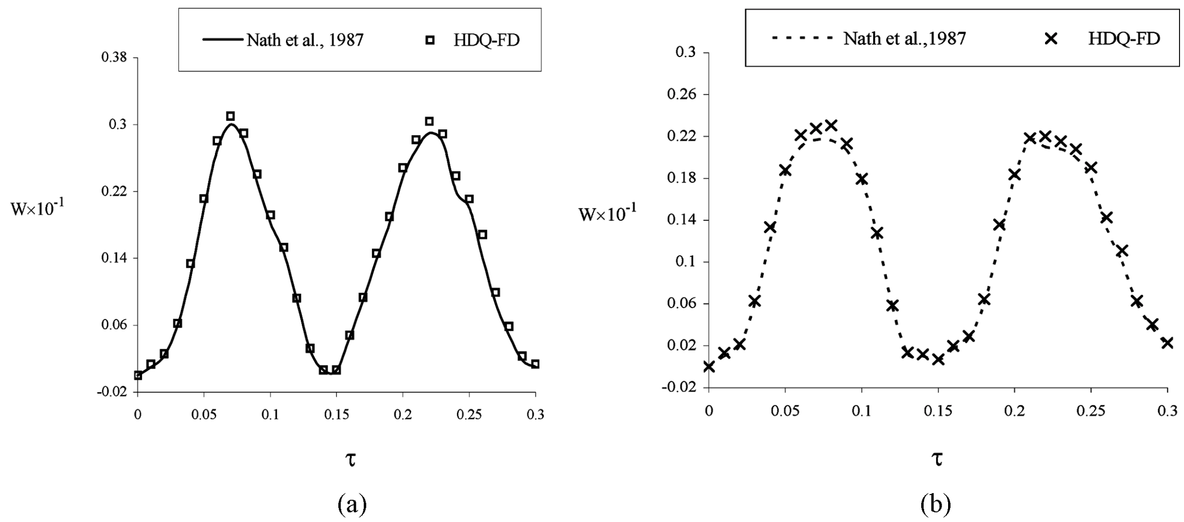


Fig. 5 Time-displacement curve of doubly curved shells for uniform step load of infinite duration (a) simply supported (b) clamped

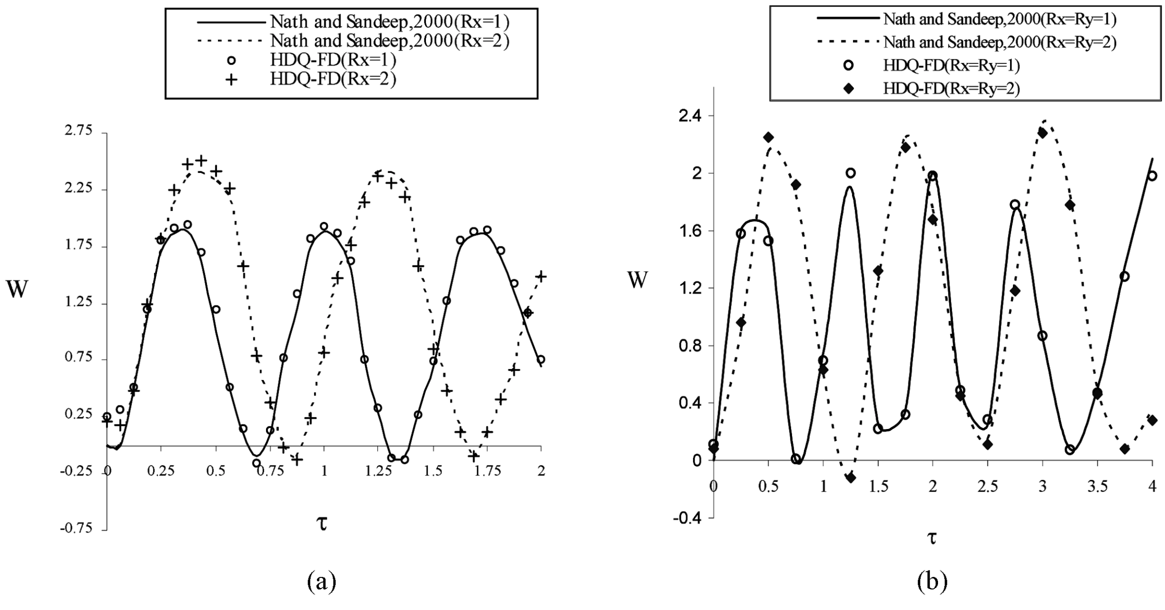


Fig. 6 Dynamic response of shell under the step load of infinite duration (a) clamped (b) simply supported

The effect of R on the response of clamped and simply supported doubly curved shells under the step load of infinite duration is shown in Fig. 6(a) and Fig. 6(b) together with the results of Nath and Sandeep (2000). The present results are in very good agreement with those of Nath and Sandeep (2000) for step load. Fig. 6(a) and Fig. 6(b) show that deflections will increase with increase in radius parameter for shell. The effect of R on the response of clamped immovable

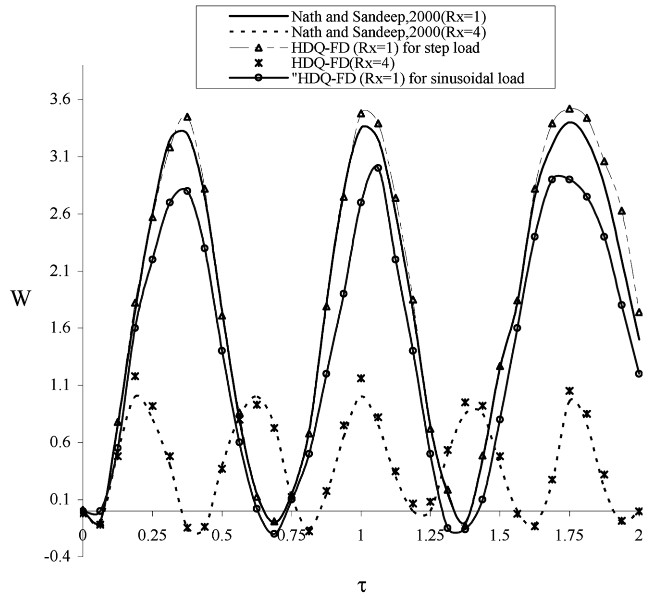
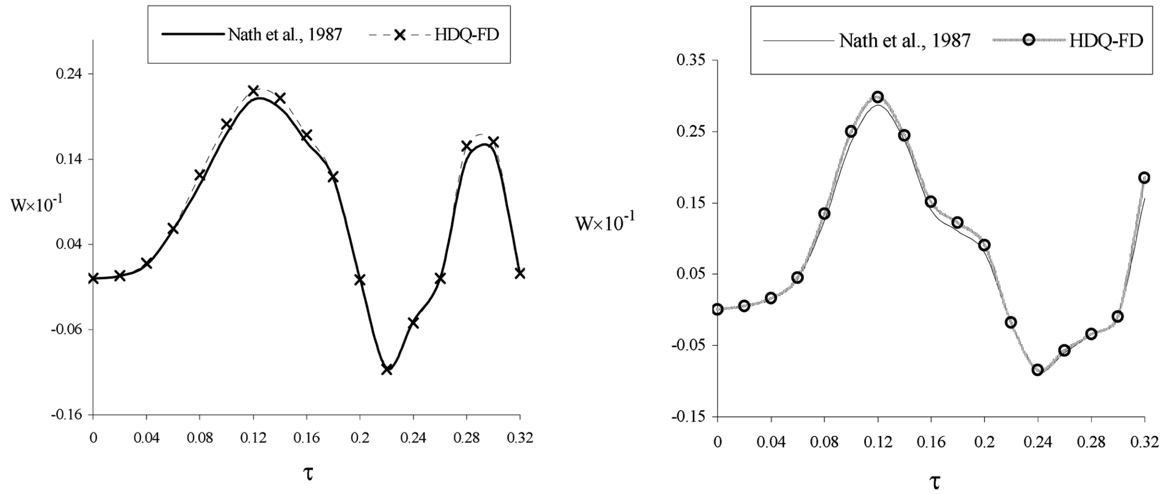


Fig. 7 Dynamic response of clamped shell under the step load of infinite duration and sinusoidal load of finite duration



(a) (b)

Fig. 8 Time-displacement curve of shell for sinusoidal load of finite duration (a) clamped (b) simply supported

supported doubly curved shells under the step and sinusiadal load P = 78 is shown in Fig. 7. The curvature coefficient R has been found to have significant influence on the dynamic response of the shells. It is also interesting to note that the response to a step load is higher than the response to a sinusoidal load. Figs. 8(a) and 8(b) show the time-deflection curves of the immovable clamped and immovable simply supported shell. For these figures, a sinusoidal loading of finite duration have been considered. It is interesting to note, however, that the response to a simply supported is higher

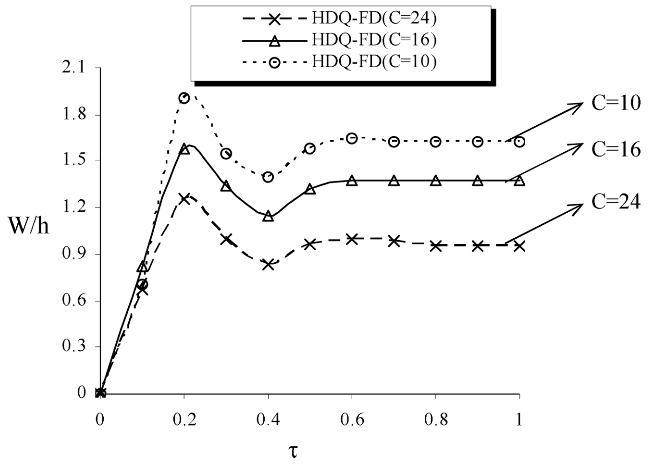
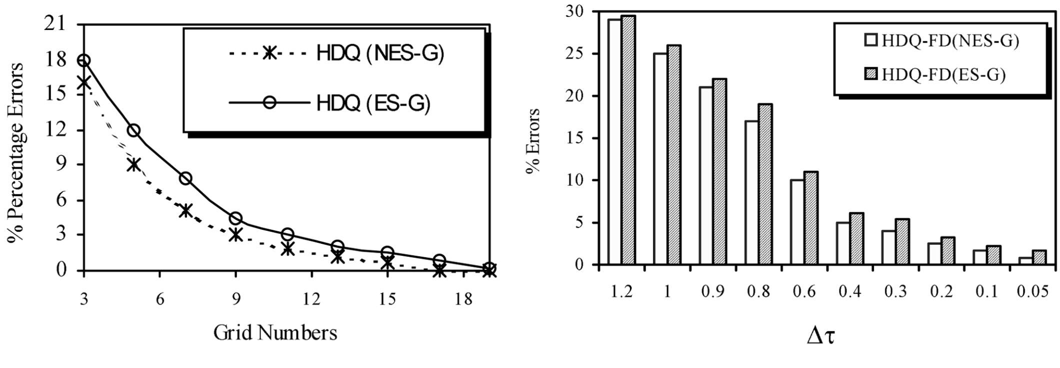


Fig. 9 Time- displacement curve of clamped shells for various damped coefficients



(a) (b)

Fig. 10 (a) Variation of % errors versus grid numbers, (b) Variation of maximum % error with time steps

than the response to a clamped supported. In Fig. 9, three-different damping coefficient (C = 10,16,24) are taken into consideration for clamped shell. The damping coefficient C has been found to have significant influence on the dynamic response of the shells. From this curves given in Fig. 9, it may be concluded that decreasing the damping coefficient, C will always result in increased deflection.

The number of sampling points is taken to be 7,9,11,13,15, and 17. It is observed that $N = 9$ is sufficient to obtain accurate results for linear analysis, and $N = 13$ is sufficient to obtain accurate results for nonlinear analysis. The percentage errors of the displacements between the HDQ solution and the references data of Nath and Sandeep (2000) are displayed in Fig. 10(a). These error value is obtained for dynamic analysis of immovable clamped shell under the infinite duration dynamic step load. The results of this analysis had been given in above in Fig. 8(a). For the displacements of HDQ method provide acceptable results with a maximum discrepancy of 2.94% for ES-G points using 11 grid points. If the grid numbers in each directon are taken as 13, the percentage error is reduced the value 1.92%. For NES-G points the percentage error is obtained as 2.75% using 11 grid points. If the grid numbers in each direction are taken as 13, the percentage error is reduced the value 0.86% for NES-G distribution. For small value of N, the HDQ solutions with the stretched Chebyshev-Gauss-Lobatto grids or non-equally sampling (NES-G) points is much more accurate than those with the conventional equally spaced sampling grid (ES-G) points. This means that the equally spaced grid points are not reliable in the HDQ solution of dynamic problems. The percentage errors of the dynamic response between the HDQ-FD solution and the references data of Nath and Sandeep (2000) are displayed in Fig. 10(b). For the displacements of HDQ method provide acceptable results with a maximum discrepancy of 3.98% for $\Delta t = 0.3$ using NES-G points. If the time steps are taken as $\Delta t = 0.1$, the percentage error is reduced the value 1.66%. It was found that as the time intervals or step sizes increases, the percentage error also increases. For small value of Δt, the HDQ solutions with the stretched Chebyshev-Gauss-Lobatto grids or non-equally sampling grid (NES-G) points is much more accurate than those with the conventional equally spaced sampling grid (E-SG) points. This can be seen clearly in Fig. 10(b).

5. Conclusions

The geometrically non-linear static and dynamic analysis of doubly curved shells has been presented using the HDQ-FD coupled methods. Typical results obtained by HDQ-FD coupled methodolgy are compared with the avaliable results in the literature. It is appeared that the geometric parameter R of the shell has been found to have a significant influence on the dynamic response of the doubly curved shells. It is shown from numerical examples that the non-equally sampling grid (NES-G) points give more accurate results than the results obtained by the equally sampling grid (ES-G) distributions. It is also shown from results that decreasing the damping coefficient, C will always result in increased deflection. It can be concluded from the present study that the HDQ-FD methodology is a simple, efficient, and accurate method for the linear and nonlinear analysis of doubly curved shells. The proposed coupled methodology is quite simple and efficient and can be extended to other nonlinear mechanics problems.

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