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Fatigue reliability analysis of steel bridge welding member by fracture mechanics method

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Abstract. This paper attempts to develop the analytical model of estimating the fatigue damage using a linear elastic fracture mechanics method. The stress history on a welding member, when a truck passed over a bridge, was defined as a block loading and the crack closure theory was used. These theories explain the influence of a load on a structure. This study undertook an analysis of the stress range frequency considering both dead load stress and crack opening stress. A probability method applied to stress range frequency distribution and the probability distribution parameters of it was obtained by Maximum likelihood Method and Determinant. Monte Carlo Simulation which generates a probability variants (stress range) output failure block loadings. The probability distribution of failure block loadings was acquired by Maximum likelihood Method and Determinant. This can calculate the fatigue reliability preventing the fatigue failure of a welding member. The failure block loading divided by the average daily truck traffic is a predictive remaining life by a day. Fatigue reliability analysis was carried out for the welding member of the bottom flange of a cross beam and the vertical stiffener of a steel box bridge by the proposed model. Results showed that the primary factor effecting failure time was crack opening stress. It was important to decide the crack opening stress for using the proposed model. Also according to the 50% reliability and 90%, 99.9% failure times were indicated.

Key words: fracture mechanics method; fatigue; reliability; stress range frequency.

1. Introduction

The failure probability of the probability variant related to a welding member failure should be precisely calculated in order to predict the life expectancy of the welding member subjected to the loading of fatigue. The basic method used to evaluate directly a failure probability is through adequate repeated simulation as it is so difficult that only an analysis method can calculate exactly it. Reliability analysis can estimate and express the failure possibility quantitatively (or the failure probability). This study utilizes the fatigue reliability analysis model based on the principles of linear elastic fracture mechanics. This approach applies the fatigue analysis to fracture mechanics in order to estimate the failure probability and the fatigue reliability of the block loading that a welding member is subjected to. By applying the proposed fatigue reliability analysis model, the stress history at the welding member of the bottom flange of a cross beam and the vertical stiffener

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of a steel box girder bridge in public use was measured. Estimates of the failure probability and fatigue reliability of a member were also made according to life expectancy.

2. Fatigue reliability analysis model explanation

When a truck passes over a bridge, in the proposed fatigue reliability analysis model, the stress history generated in a member is defined as block loading. A stress range frequency distribution is calculated from the stress history, taking into consideration factors such as dead loading stress and crack opening stress. The Maximum Likelihood Method (MLM) is used to ascertain the parameters of the probability distribution which expresses a stress range frequency distribution. The assessment of probability distribution fitness used a Determinant. The resultant probability distribution with the largest Determinant is adopted. The Monte Carlo Simulation (MCS) uses the probability distribution parameters of the stress range frequency distribution of many block loadings on a steel bridge member. MCS generates the probability variants (stress ranges) with a probability distribution character and uses those in a fatigue crack evolution equation (James 1998). Failure block loadings are calculated by a numerical integration from an initial crack to a limit crack. The parameters of a probability distribution of a failure block loading are calculated using the MLM. The probability distribution of failure block loading is determined by a Determinant. Fatigue reliability is calculated using a failure cumulative probability distribution of a block loading. According to 50%, 90% and 99.9% reliability in terms of a fatigue reliability function, the failure block loading can be estimated and the life expectancy can be predicted. Fig. 1 shows the procedure of the fatigue reliability analysis.

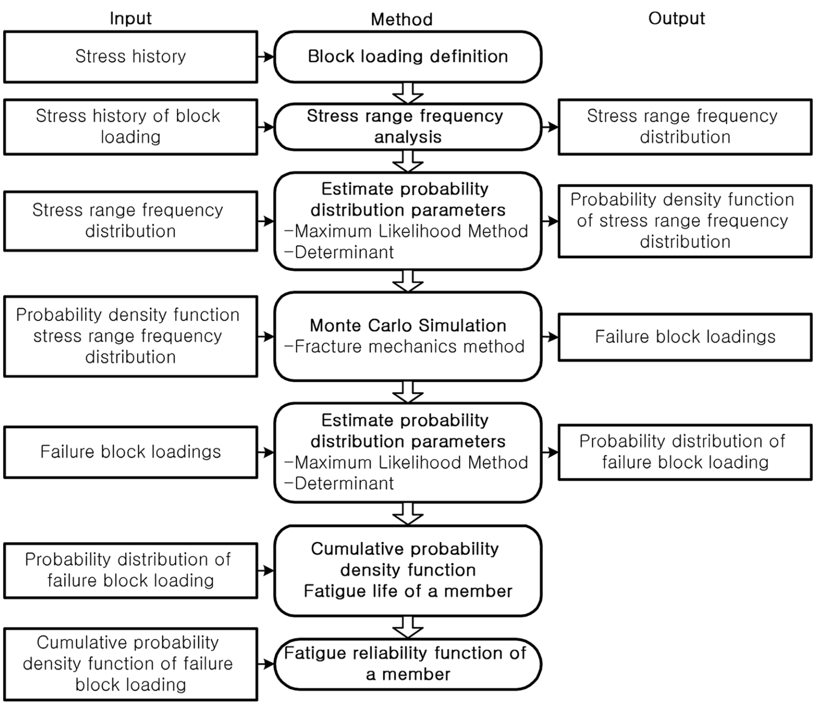

Fig. 1 Procedure of fatigue reliability analysis model

2.1 Effective stress range and crack opening stress

An effective stress range is the difference between the maximum stress and minimum stress and is calculated in following Eq. (1).

$$\Delta f_{eff} = f_{\max} - f_{\min} \tag{1}$$

The $f_{\min}$ is the minimum stress if the minimum stress is larger than the crack opening stress, or the crack opening stress if the minimum stress is smaller than the crack opening stress. In Eq. (2) $f_{\max}$ is defined as the value added the dead loading stress and the maximum stress of a block loading. f_{dead} is the dead loading stress attained in a structural analysis and $f_{\max,\,i}$ is the i-th maximum stress of a block loading.

$$f_{\max} = f_{dead} + f_{\max,\,i} \tag{2}$$

The concept of the crack opening stress was introduced (Hou and Lawrence 1996). This is the state of the working stress (f_{op}) when a crack is open completely. It was proposed that a fatigue crack evolution occurs when a crack is open completely. Crack opening stress can be calculated using the ratio (ρ) of the crack opening stress to the maximum stress of the whole block loading. In Eq. (3) Newman (1981) proposed that the ratio of the crack opening stress to the maximum stress is approximately 0 to 0.7. $(f_{\max})_B$ is the maximum stress of a whole block loading.

$$f_{op} = \rho \cdot (f_{\max})_B \tag{3}$$

Table 1 Likelihood function and parameters of probability distribution

Probability distribution	Likelihood function	Maximun likelihood estimator
Gumbel	$L(x_1,\ldots,x_n;\alpha,\phi) = (1/\phi^n)\prod_{i=1}^{n}\exp\left[-\exp\left(-\frac{x-\alpha}{\phi}\right)\right]\exp\left(-\frac{x-\alpha}{\phi}\right)$	$\alpha = -\phi\ln\left(\sum_{i=1}^{n}\exp(-(x_i/\phi)/n)\right)$, $\phi - n^{-1}\sum_{i=1}^{n}x_i + \sum_{i=1}^{n}(x_i\exp(-x_i/\phi))/\sum_{i=1}^{n}\exp(-x_i/\phi) = 0$
Normal	$L(x_1,\ldots,x_n;\mu,\sigma) = \prod_{i=1}^{n}\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2}\left[\frac{x-\mu}{\sigma}\right]^2\right)$	$\hat{\mu} = \bar{x},\ \hat{\sigma}^2 = \sum_{i=1}^{n}(X_i - \bar{X})^2/n$
Lognormal	$L(x_1,\ldots,x_n;\lambda,\zeta) = \prod_{i=1}^{n}\frac{1}{\sqrt{2\pi}\zeta x_i}\exp\left(-\frac{1}{2}\left[\frac{\ln x_i - \lambda}{\zeta}\right]^2\right)$	$\hat{\lambda} = \sum_{i=1}^{n}\ln x_i/n,\ \hat{\zeta}^2 = \frac{1}{n}\sum_{i=1}^{n}(\ln x_i - \hat{\lambda})^2$
Exponential	$L(x_1,\ldots,x_n;\omega) = \prod_{i=1}^{n}\frac{1}{\omega}\exp\left(-\frac{x}{\omega}\right)$	$\hat{\omega} = \frac{1}{n}\sum_{i=1}^{n}x_i$
Weibull	$L(x_1,\ldots,x_n;\beta,\theta) = \prod_{i=1}^{n}\frac{\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1}\exp\left(-\left[\frac{x}{\theta}\right]^{\beta}\right)$	$\sum_{i=1}^{n}x_i^{\beta}\ln x_i/\sum_{i=1}^{n}x_i^{\beta} - \frac{1}{\beta} - \frac{1}{n}\sum_{i=1}^{n}\ln x_i = 0$, $\theta = \sum_{i=1}^{n}x_i^{\beta}/n$

2.2 Probability distribution parameter estimation

The Maximum Likelihood Method (MLM) is used to determine the point estimator of a parameter directly. The parameter estimation of a stress range frequency distribution, and a probability distribution of failure block loadings, were made using the MLM. Table 1 shows the likelihood function and a maximum likelihood estimator for each of probability distributions.

2.3 Fatigue reliability of a member

In a fatigue analysis before the definition of fatigue reliability, if X is a failure block loading and x is a block loading, the cumulative probability density of a failure block loading can be expressed like a following Eq. (4).

$$F_X(x) = P[X \le x] \tag{4}$$

Here, x means the block loading used when a crack evolves from an initial crack size to a limit crack size. The probability of no failure (the fatigue reliability) is defined as a following Eq. (5).

$$R_X(x) = 1 - F_X(x) = 1 - P[X \le x] \tag{5}$$

$R_X(x)$ is a fatigue reliability function. For example if the cumulative probability function of a failure block loading in Eq. (6) is a Weibull probability density, the fatigue reliability function is the same as a following Eq. (7).

$$F_X(x) = 1 - \exp(-(x/\theta)^{\beta}) \tag{6}$$

$$R_X(x) = 1 - F_X(x) = \exp(-(x/\theta)^{\beta}) \tag{7}$$

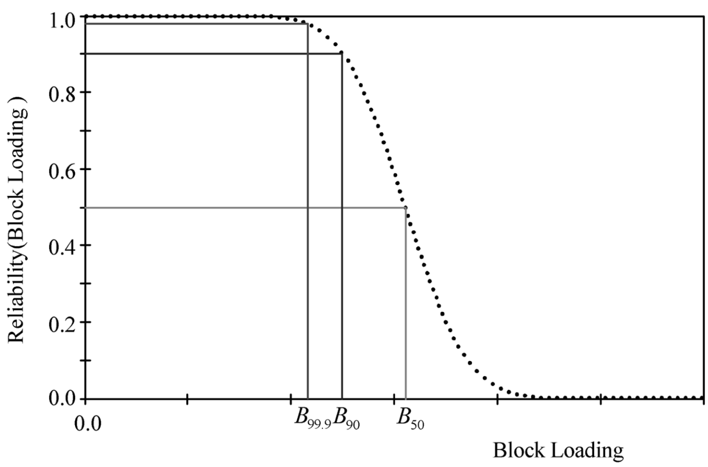

Fig. 2 Fatigue reliability function

Using a fatigue reliability function, the failure block loading according to reliability (or a failure probability in an opposite concept) can be calculated on the curve of a cumulative probability density function. Fig. 2 shows the curve of a cumulative probability density function of a failure block loading calculated by MCS. With a reliability of 99.9%, the failure block is founded to be $B_{99.9}$, 90% B_{90} and 50% B_{50} (Fig. 2).

3. Fatigue reliability analysis of a welding member

3.1 Stress history measurement

Fig. 3 shows the procedure of fatigue reliability analysis of a member in the proposed model. First, stress history measurement was made on a welding member on a steel bridge which is at an optimal location for fatigue damage. Particularly the stress intensity between a vertical stiffener and

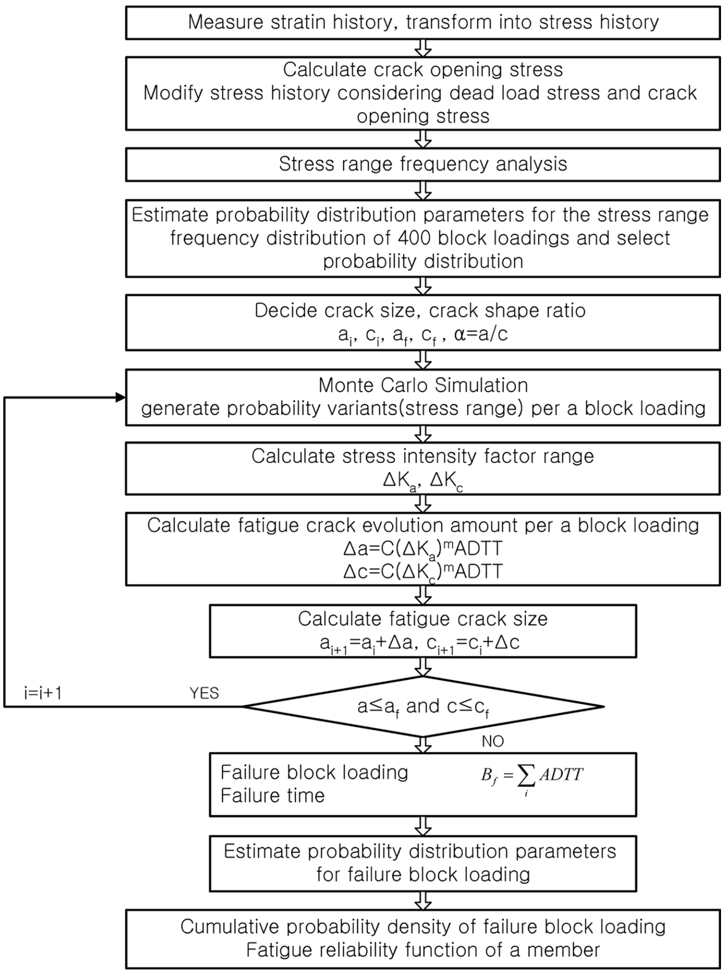

Fig. 3 Procedure of fatigue reliability analysis of a member

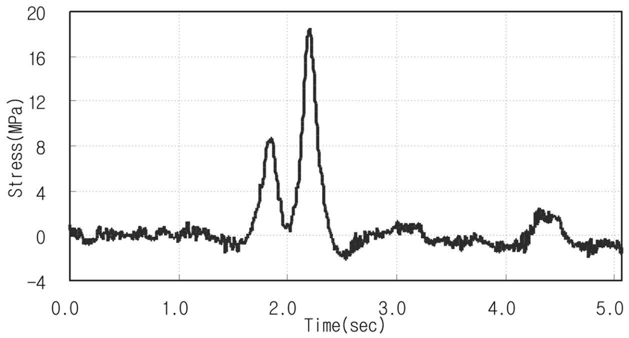

Fig. 4 Stress history of a block loading

the bottom flange of a cross beam exists, as tensional stress is created in terms of the deflection generated by truck crossings. Also there may be fatigue damage possibility due to flaws in welding or by the effect of residual stress on the welding. Strain history data was obtained using a strain gauge on this part. Strain history by Young’s modulus illustrates a stress history as exemplified in Fig. 4.

3.2 Stress range frequency analysis

Stress history was modified by starting a maximum or minimum point, so that the half cycle of the stress range may not be counted. A Rainflow Cycle Counting Method was used for stress range frequency analysis after considering dead loading stress and crack opening stress. Rainflow Cycle Counting Algorism does not take into account the loading history sequences. An excessive loading cycle in a block loading model determines a crack opening stress. It is not clear from the Rainflow Cycle results, when a cycle appears before or after an excessive loading. Therefore, the time location of a stress cycle in a block loading, was determined and analysis including the loading history sequence was undertaken in order to rearrange the Rainflow Cycle Counting results, as a post process. Fig. 5 shows the stress range sequence of a block loading. Stress range frequency analysis was performed by drawing up a program.

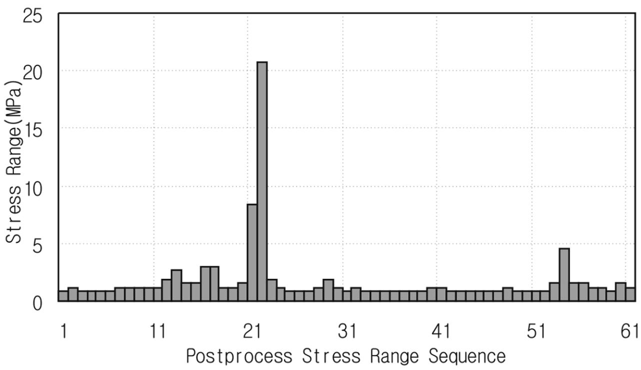

Fig. 5 Stress range considering loading sequence of a block loading

Table 2 Determinants and parameters of each probability distribution for 400 block loadings (ρ = 0.3)

	Gumbel		Normal		Lognormal		Exponential	Weibull	
Parameters	α	φ	μ	σ	λ	ξ	ω	β	θ
	0.661	0.875	1.309	2.086	–0.483	1.251	0.764	0.820	1.122
Determinant (r^2)	0.927		0.737		0.992		0.974	0.987	

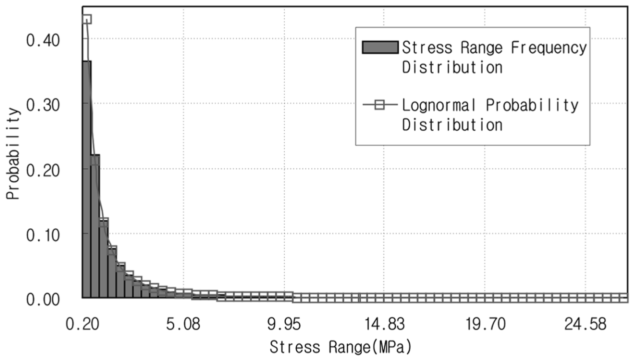

Fig. 6 Stress range frequency distribution & lognormal probability density for 400 block loadings

3.3 Probability distribution parameter estimation of stress range frequency distribution

The stress range frequency analysis was performed for 400 block loadings measured the structural detail of a steel highway bridge and a probability method was applied to the stress range frequency distribution. The probability distribution parameters were established using MLM to find a particular probability distribution that adequately expressed a stress range frequency distribution. Consequently, Lognormal probability distribution sufficiently expressed the stress range frequency distribution of 400 block loadings. Table 2 shows each of probability distribution parameter and Determinant. Fig. 6 shows the stress range frequency distribution of 400 block loadings and the Lognormal probability distribution curve.

3.4 Fatigue crack evolution equation

The welding member of an analysis is the same as in Fig. 7 with a semi-elliptical crack length c and a crack depth a existing at a welding member between a vertical stiffener and the bottom flange of a cross beam on a steel box girder bridge. The evolution equation for this fatigue crack could be expressed as in Eq. (8) derived from block loading and the Paris' raw. Table 3 shows the modulus for a fatigue crack evolution. The material parameters are m and C. a_i and c_i are a crack depth and a crack length, respectively. a_f and c_f are a limit crack depth and a limit crack length, respectively.

$$B_f = \int_{a_i}^{a_f} \frac{da}{C[f(g)\sqrt{\pi a}]^m \sum_{i=1}^{n} (\Delta f_{eff})_i^m} \tag{8}$$

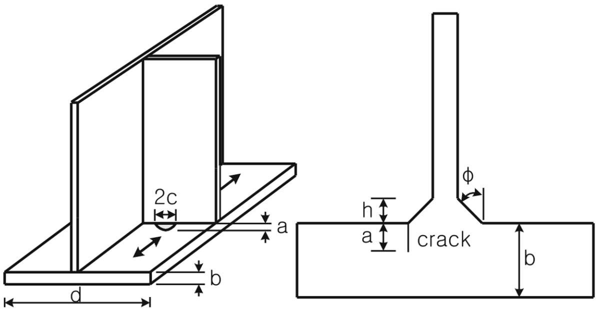

Fig. 7 Welding member of the bottom flange of a cross beam and vertical stiffener

Table 3 Coefficients required for fatigue crack evolution

C	m	a_i (m)	a_f (m)	c_i (m)	c_f (m)	d (m)	b (m)
2.7×10^{-11}	3	0.0001	0.01	0.0002	0.009	0.3	0.01

In case of the fatigue crack generated by the inner flaw or the surface flaw of a welding joint exhibited, the ratio of the block loading employed with a plate thickness penetration to that of a whole life is large. As the crack evolution velocity accelerates after the penetration of the plate, the crack dimensions at the plate thickness penetration was used as a limit crack size. A limit crack length was assumed to be 30% of a flange width.

The crack length change ($\Delta a/\Delta B$) per a block loading is the summation of crack length change as a result of loading cycles. The stress intensity factor range can be expressed using $(\Delta K_{eff})_i = (\Delta f_{eff})_i \sqrt{\pi a}\, Y(a) \cdot M_K(a)$. Where $Y(a)$ is the stress intensity correction factor, a function of the crack depth a as well as the other crack half-length c, the flange thickness b, and the flange width d. Eq. (9) adopted in this study was from Newman and Raju (1983) at the bottom flange with a semi-elliptical surface crack.

$$Y(a) = \frac{1}{\sqrt{1 + 1.464(a/c)^{1.65}}}\left[M_1 + M_2\left(\frac{a}{b}\right)^2 + M_3\left(\frac{a}{b}\right)^4\right] f_w \tag{9}$$

Where, $M_1 = 1.13 - 0.09a/c$, $M_3 = 0.5 - \dfrac{1}{0.65 + a/c} + 14\left(1 - \dfrac{a}{c}\right)^{24}$, $M_2 = \dfrac{0.89}{0.2 + a/c} - 0.54$

$$f_w = 1/\sqrt{\cos\left(\frac{\pi}{d/b}\frac{1}{a/c}\sqrt{\left(\frac{a}{b}\right)^3}\right)}$$

In Eq. (10), the $M_K(a)$ is stress concentration factor, depends not only on the crack depth a but also on other geometric parameters such as the flange thickness b, the weld height h and the weld angle θ. The stress concentration factor represents the magnification factor taking into account the stress concentration due to specific structural detail. The classical solution presented by Sedlacek *et al.* (1997) was adopted for this factor.

$$M_k = v\left(\frac{a}{b}\right)^w \tag{10}$$

Where, the parameter v and ω in Eq. (11), Eq. (12) are calculated according to Hobbacher (1993).

$$v = 0.8086 - 0.1554(h/b) + 0.0429(h/b)^2 + 0.0784(h/b)\tan\varphi \tag{10}$$

$$w = -0.1993 - 0.1839(h/b) + 0.0495(h/b)^2 + 0.0815(h/b)\tan\varphi \tag{11}$$

3.5 Probability distribution and fatigue reliability of failure block loading with MCS

MCS can compute the crack length per a block loading by generating probability variables (stress ranges) of a Lognormal probability distribution. The numerical repetition from an initial crack size to a limit crack size in a crack equation per a block loading can attain a failure block loading. The probability distribution for failure block loadings was estimated and adequately explained a Lognormal probability distribution. Table 4 shows the Lognormal probability distribution parameters according to ADTT, and simulations. The probability distribution parameters of the failure block loadings were calculated using the MLM. Also, a Determinant was also used to evaluate the fitness degree of the probability distribution of failure block loadings. The MCS program coded by Visual Basic 6.0, was implemented. Fig. 8 shows the Lognormal probability distribution curve and the probability distribution of failure block loading according to simulations.

Table 4 Lognormal probability distribution parameters according to simulations (ρ = 0.3)

ADTT	RUN	Lognormal probability distribution parameters		
		λ	ξ	r^2
1000	400	16.25480	0.00582	0.94973
1000	800	16.25479	0.00564	0.97464
1000	1000	16.25474	0.00569	0.99071
1000	2000	16.25476	0.00581	0.99139
1000	5000	16.25478	0.00585	0.99800
1000	10000	16.25475	0.00617	0.99699

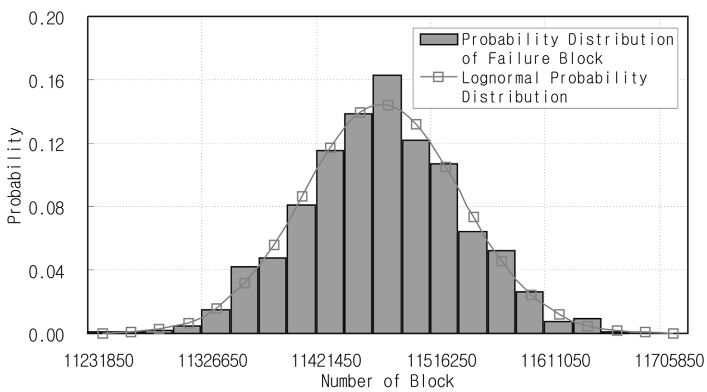

Fig. 8 Failure probability distribution of ADTT = 1000, Run = 1000

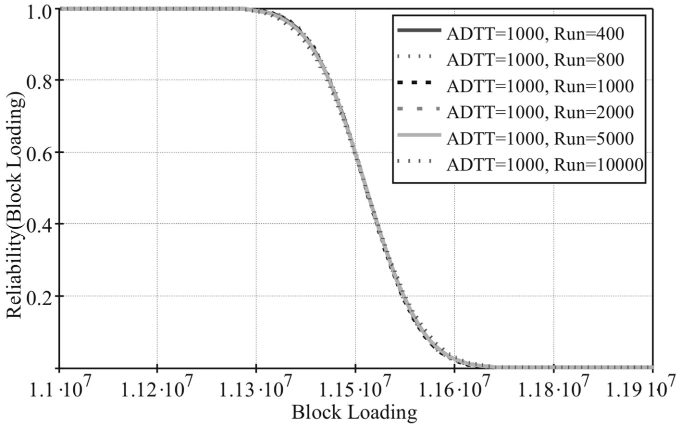

Fig. 9 Fatigue reliability according to simulations

The fatigue reliability of a welding member was obtained in terms of the cumulative probability density of failure block loading. Fig. 9 shows the fatigue reliability curves of Lognormal cumulative probability density according to ADTT = 1000 and the simulations.

A block loading with 50% reliability is 1.1464×10^7, when run = 1000 and ρ = 0.3. Like the failure cumulative probability density curves, the more simulations increase, the wider the block loading range of the reliability curve becomes.

4. Results and discussions

4.1 Fitness of probability distribution and effect on simulations

The fitness degree between a stress range frequency distribution and a theory probability distribution was judged by the Determinant (r^2). In the case of 400 block loadings, the Determinant of the Gumbel probability distribution was r^2 = 0.927, Normal 0.737, Lognormal 0.992, Exponential 0.974, and Weibull 0.987. The Determinant of the Lognormal probability distribution of a stress range frequency distribution was the largest, so it was adopted as the probability distribution of a stress range frequency distribution. Utilizing the same methodology, a Lognormal probability distribution was adopted as the specific probability distribution of a failure block loading. As for scale factors (λ) and shape factors (ξ) of a Lognormal probability distribution, that they converge into a value was unknown, as the simulation increases. It was known, however that the Determinant converges into a value as the simulation increases. Namely, the Determinant converged into a value over 1000 of the simulations.

4.2 Peak analysis method

To validate the proposed model, peak analysis method and RMS methods were executed. The crack evolution equation of a block loading using the peak analysis method is the same as that of Eq. (12). The single stress range $\Delta f_{\max}$, is taken as a probability variable, and is considered to be the largest stress range in a block loading.

$$\Delta a / \Delta B = C(f(g)\sqrt{\pi a}\Delta f_{\max})^m \tag{12}$$

When using the peak analysis method, the probability variable (stress range) is generated by MCS using Lognormal probability distribution parameters. A failure block loading was obtained by numerically integrating the crack evolution amount per a block loading from an initial crack to a limit crack for average daily truck traffic.

4.3 RMS (Root-Mean-Square) method

Bosoms (1973)' RMS method was used in Eq. (13). This is an analytical model that is based on the root mean square of a stress intensity factor range. The crack evolution equation for a block loading is derived in Eq. (14).

$$\Delta K_{RMS} = \left[\sum_{i=1}^{n} (\Delta K_i^2)/n\right]^{1/2} \tag{13}$$

$$\Delta a / \Delta B = C(f(g)\sqrt{\pi a}\Delta f_{RMS})^m = C\left(f(g)\sqrt{\pi a}\left[\sum_{i=1}^{n} (\Delta f_i)^2/n\right]^{1/2}\right)^m \tag{14}$$

Each block loading Δf_{RMS} from 400 block loadings measured on a steel highway bridge was calculated. It was used to establish a probability distribution parameter. When performing MCS, the daily average truck traffic and the probability distribution parameters were considered to compute a failure block loading by integrating the numerical crack evolution amount per a block loading from an initial crack to a limit crack.

4.4 Comparison of analysis results

Table 5 shows the failure times corresponding to ρ = 0.3 and 0.5 (50%) reliability, that was calculated using the fatigue reliability analysis model and that of the peak analysis method. The failure time of proposed fatigue analysis model, which was smaller than that of the peak analysis method, was considered to be a conservative result. The failure time of the proposed fatigue analysis model differed according to ρ value. The larger ρ is, the larger the failure time is like Table 6. A crack grows when stress is larger than the crack opening stress in the proposed analytical model, while the stress smaller than the crack opening stress does not effect crack growth. When ρ is large, the crack opening stress is large, and failure time increases, because the stress range is small.

Table 5 Failure times according to analysis method

Analysis method	ADTT	RUN	Failure times (year)
RMS	-	-	102.57
Peak analysis	1000	1000	72.73
Proposed model	1000	1000	26.17

Table 6 Failure times according to ρ (Reliability 50%)

Analysis method	ρ	ADTT	RUN	Failure time (year)
Proposed model	0.3	1000	1000	26.17
	0.4			83.56
	0.5			253.22

Table 7 Failure times according to reliability (ρ = 0.3)

Analysis method	Reliability (%)	ADTT	RUN	Failure times (years)
Proposed model	50	1000	1000	26.17
	90			25.98
	99.9			25.71

All stresses in the domain where dead loading stress is large contribute to crack growth in the proposed analysis model. Where, if dead loading stress is small, the small stress does not contribute the crack growth even after extreme loading. If the dead loading stress is small and the crack closure effect exists, the failure probability of the proposed model would be smaller than that of the peak analysis method, or the reliability of the peak analysis method may be less than that of the proposed model. In the peak analysis method, certain characteristics are not derived from the domain where the crack closure exists (in the domain where dead loading stress is small) or where crack closure does not exist (in the domain where dead loading stress is large). In the dead loading effect, the whole stress ranges contribute crack growth in the domain where dead loading stress is large. After extreme loading stress works, crack closure phenomenon occurs in the domain where dead loading stress is small. Thus, the proposed model clearly explains the basic behavior of the delay of crack growth.

As the failure time of RMS method was larger than that of any other method of analysis, RMS method was found to be a questionable practicable estimation. It did not explain the loading cycle sequence effect, crack delay or acceleration. When the loading effect is at a minimum, the RMS method was found to be applicable to only a short spectrum. It was found that the RMS methodology ignores the influence of the peak loading cycle in the tail end of the probability distribution.

Table 7 shows the failure times (ρ = 0.3) at 50% reliability and 90%, 99.9% according to the fatigue reliability.

5. Conclusions

After the stress history was measured at the welding member of the bottom flange of a cross beam and vertical stiffener of a steel box girder bridge, the analysis was fulfilled using the fatigue reliability analysis model based on a linear elastic fracture mechanics method. The following conclusions were obtained.

1. The probability method was applied to a stress range frequency distribution for a stress history. The fatigue reliability analysis model that could compute the fatigue failure Probability and the

reliability of the failure block loading estimated by Monte Carlo Simulation, was brought forward.

2. The probability distribution parameters of the stress range frequency distribution of 400 block loadings were estimated by MLM. Consequently, the probability distribution of a stress range frequency distribution and failure block was a Lognormal probability distribution. Then, Determinant, as a means of the judgment criterion of fitness degree, was larger than that of any other probability distribution.
3. It was well known that the failure time of the proposed analysis method varied in accordance with the ratio of the crack opening stress and maximum stress. Determining crack opening stress was important so as to use the proposed analysis method.
4. Results did not confirm that Lognormal probability distribution parameters of a failure block loading converged into a value. It was however, established that the Determinant converged into single value over 1000 simulations.
5. Results showed that the failure times according to 50, 90, 99.9 % reliability were presented respectively.

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References

Barsom, J.M. and Rolfe, S.T. (1999), *Fracture and Fatigue Control in Structures*, America Society for Testing and Materials, 222-225.

Fernand, Ellyin (1997), *Fatigue Damage, Crack Growth and Life Prediction*, Chapman & Hall, pp25.

Herbert, J. Sutherland and Paul, S. Veers (1995), “Effect of cyclic stress distribution models on fatigue life prediction”, ASME, SED-**16**, 83-90.

Hobbacher, A. (1993), “Stress intensity factors of welded joints”, *Engrg. Fracture Mech*., **46**(2), 173-182.

Hou, C.Y. and Lawrence, F.V. (1996), *A Crack-Closure Model for the Fatigue Behavior of Notched Components*, Advances in Fatigue Lifetime Predictive Techniques: 3rd Volume, ASTM STP 1292, M.R. Mitchell and R.W. Landgraf, Eds., American Society for Testing and Materials, 116-135.

James, E. Gentle (1998), *Random Number Generation and Monte Carlo Methods*, Springer, 85-119.

Julie, A. Bannantine and Jess, J., Comer, James L. Handrock (1990), *Fundamentals of Metal Fatigue Analysis*, Prentice Hall, 178-221.

Newman, J.C. Jr. (1981), “A crack closure model for predicting fatigue crack growth under aircraft spectrum loading, in methods and models for predicting fatigue crack growth under random loading”, ASTM STP 748, American Society for Testing and Materials, Philadelphia, 53-84.

Newman, J.C. Jr. and Raju, I.S. (1983), “Stress-intensity factor equations for cracks in three-dimensional finite bodies”, *Proc., Fracture Mech. : 14th Symp*, J.C. Lewis and G. Sines, eds., ASTM, West Conshohocken, Pa., I-238~I-265.

Sedlacek, G. *et al.* (1997), “Design of steel structures”, Part 2 - Bridges, for Chapter 3 - Materials, Choice of Steel Material to avoid Brittle Fracture., Background Documentation to Eurocode 3, Draft, Rheinische Westfael Hochschule, Aachen, Germany.