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Effects of strain hardening of steel reinforcement on flexural strength and ductility of concrete beams

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Abstract. In the design of reinforced concrete beams, it is a standard practice to use the yield stress of the steel reinforcement for the evaluation of the flexural strength. However, because of strain hardening, the tensile strength of the steel reinforcement is often substantially higher than the yield stress. Thus, it is a common belief that the actual flexural strength should be higher than the theoretical flexural strength evaluated with strain hardening ignored. The possible increase in flexural strength due to strain hardening is a two-edge sword. In some cases, it may be treated as strength reserve contributing to extra safety. In other cases, it could lead to greater shear demand causing brittle shear failure of the beam or unexpected greater capacity of the beam causing violation of the strong column-weak beam design philosophy. Strain hardening may also have certain effect on the flexural ductility. In this paper, the effects of strain hardening on the post-peak flexural behaviour, particularly the flexural strength and ductility, of reinforced normal- and high-strength concrete beams are studied. The results reveal that the effects of strain hardening could be quite significant when the tension steel ratio is relatively small.

Key words: flexural ductility; high-strength concrete; reinforced concrete beams.

1. Introduction

In the ultimate limit state design of reinforced concrete structures, the ultimate failure stress of the steel reinforcement is normally taken as the yield stress of the steel reinforcing bars. However, due to strain hardening, the tensile strength of the steel reinforcement is often substantially higher than the yield stress, as can be seen from the typical stress-strain curves of mild steel and high-yield steel bars shown in Fig. 1. For comparing the tensile strength to the yield stress, the tensile test results of about 2,000 mild steel and high-yield steel bars have been collected from an accredited materials testing laboratory and the tensile strength results of these steel bars are plotted against the corresponding yield stress results in Fig. 2. From the test results plotted, it can be seen that the tensile strength is generally higher than the yield stress by 20 to 50%. Statistical analysis of the test results revealed that 98% and 77% of the steel bars tested had their tensile strength higher than yield stress by at least 20% and 30%, respectively. In fact, some national standards, e.g., the British Standard BS 4449 (1988), require that the tensile strength of any steel bar must be at least 15%

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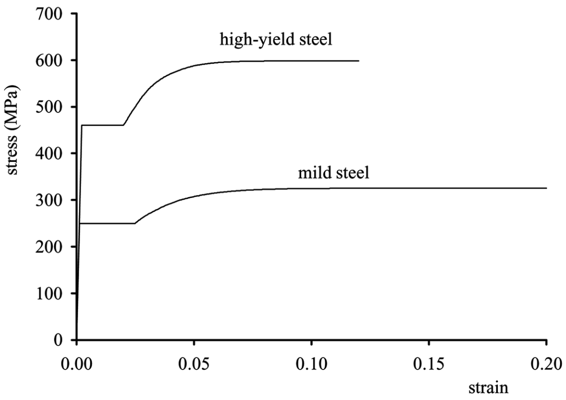


Fig. 1 Typical stress-strain curves of mild steel and high-yield steel bars

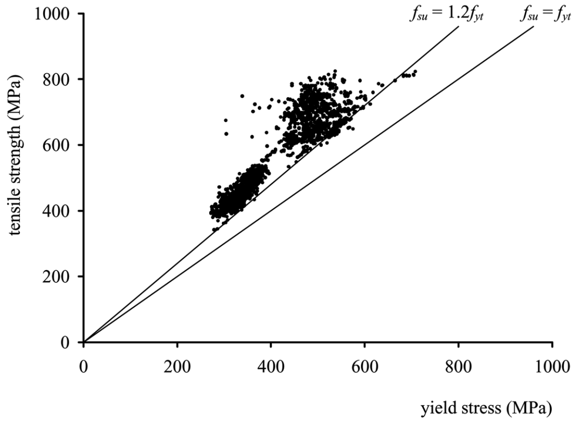


Fig. 2 Comparison between tensile strength and yield stress of steel reinforcing bars

greater than the actual yield stress as part of the specification requirements.

However, the tensile strength is generally not developed until the tensile strain of the steel reinforcement reaches a fairly large value of 50,000 $\mu\varepsilon$ or more, which is many times larger than the yield strain. Hence, although the tensile strength is higher than the yield stress, it might not be developed even when the reinforced concrete structure has failed completely. Since it is generally quite difficult to predict whether the tensile strength would be developed, it is a standard practice to just use the yield stress in the structural design and ignore any possible effects of strain hardening. Nevertheless, some engineers believe that due to the use of the yield stress instead of the tensile strength in the design, there should be a hidden strength reserve contributing to extra structural safety and thus tend to adopt a more lenient attitude towards the use of substandard steel reinforcing bars with slightly lower yield stress than specified. There is also the view that if the contribution of strain hardening to member strength could be quantified, then its incorporation in the design might lead to better economy and a more uniform standard of safety. In any case, the fundamental question is whether the contribution of strain hardening is really significant.

On the other hand, it should be noted that the possible increase in flexural strength of a reinforced concrete beam due to strain hardening of the steel reinforcement is a two-edge sword. When the structure is subjected to static load, the increase in flexural strength may be treated as strength reserve contributing to extra safety. However, when the structure is subjected to earthquake load, the increase in flexural strength could lead to greater shear demand causing brittle shear failure of the beam or unexpected greater load carrying capacity of the beam causing violation of the strong column-weak beam design philosophy. Thus, the necessity to study the effects of strain hardening arises not only from the temptation to exploit strain hardening for better economy but also from the need to avoid brittle beam or column failure for better earthquake survivability.

In this study, the effects of strain hardening of the steel reinforcement on the flexural behaviour of reinforced concrete beams are investigated analytically. Since strain hardening occurs after the steel reinforcement has yielded, it is expected that the effects of strain hardening are more significant at the post-peak stage. Hence, to investigate the effects of strain hardening, nonlinear flexural analysis extended well into the post-peak stage is needed. When studying the numerical results, it should be of particular interest to quantify the effects of strain hardening on the flexural strength and ductility of reinforced concrete beams and find out how the effects of strain hardening would vary with the concrete grade, steel yield strength and steel area ratio etc. The main objective of this study is therefore to evaluate the effects of strain hardening on the flexural strength and ductility of reinforced concrete beams with various structural parameters using nonlinear flexural analysis that covers the post-peak range.

The authors' research team has recently developed an improved method of analysing the post-peak flexural behaviour of reinforced concrete beam sections that uses the actual stress-strain curves of the constitutive materials and takes into account the stress-path dependence of the stress-strain curve of the steel reinforcement (Ho *et al.* 2003, Pam *et al.* 2001). Using this improved method, the effects of various structural parameters on the flexural strength and ductility of reinforced concrete beam sections have been studied (Ho *et al.* 2003, Pam *et al*. 2001, Kwan *et al.* 2004). One major finding from these studies is that although before the beam section reaches the peak resisting moment, the strain in the tension reinforcement keeps on increasing with the curvature of the beam section, at a certain point after the beam section has reached the peak resisting moment, the strain in the tension reinforcement starts to decrease despite monotonic increase of the curvature of the beam section. Because of such strain reversal, the strain in the tension reinforcement would only reach a certain maximum value. The maximum strain reached by the tension reinforcement is dependent mainly on the tension steel ratio, being generally larger at lower tension steel ratio and smaller at higher tension steel ratio. It is anticipated that if the maximum strain is large enough for strain hardening to take place so that the maximum tensile stress developed is higher than the yield stress, then strain hardening could have significant effects on the flexural behaviour of the beam section.

2. Nonlinear flexural analysis

For the concrete, it is assumed to be unconfined and the stress-strain curve model developed by Attard and Setunge (1996), which has been shown to be applicable to a broad range of concrete strength from 20 to 130 MPa, is adopted. The equation of the stress-strain curve is given by:

$$\sigma_c/f_{co} = \frac{A(\varepsilon_c/\varepsilon_{co}) + B(\varepsilon_c/\varepsilon_{co})^2}{1 + (A-2)(\varepsilon_c/\varepsilon_{co}) + (B+1)(\varepsilon_c/\varepsilon_{co})^2} \tag{1}$$

in which σ_c and ε_c are the compressive stress and strain at any point on the stress-strain curve, f_{co} and ε_{co} are the compressive stress and strain at the peak of the stress-strain curve, and A and B are coefficients dependent on the concrete grade. It should be noted that f_{co} is actually the in-situ compressive strength, which may be estimated from the cylinder or cube compressive strength using appropriate conversion factors. Fig. 3(a) shows some of the stress-strain curves so derived. The concrete strength covered in this study ranges from f_{co} = 30 MPa to f_{co} = 90 MPa.

For the steel reinforcement, the stress-strain curve recommended by Mander *et al*. (1984) is used. It comprises of an initial linearly elastic portion, a flat yield plateau portion and a nonlinear strain hardening portion. Since there could be strain reversal in the steel reinforcement at the post-peak stage despite monotonic increase of curvature, the stress-strain curve of the steel is stress-path dependent. It is assumed that when strain reversal occurs, the unloading path of the stress-strain curve is linear and has the same slope as the initial elastic portion of the stress-strain curve. When the strain is increasing, the stress-strain curve is given by the following equations:

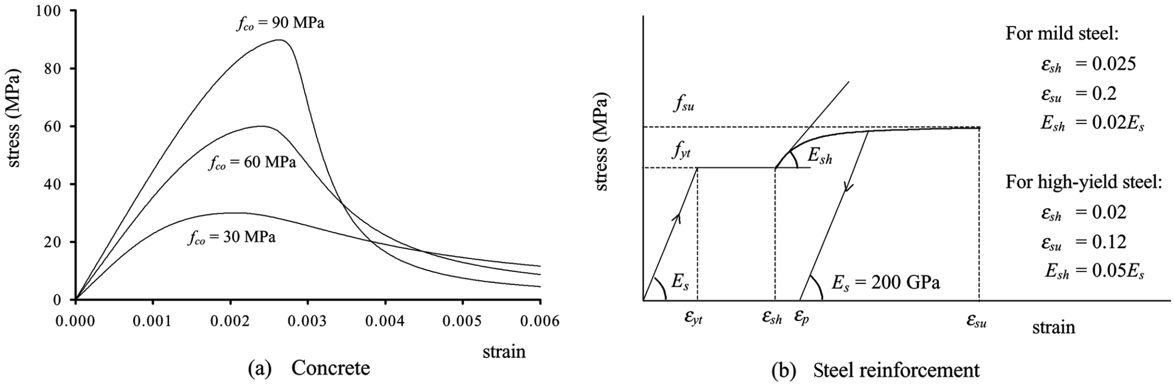


Fig. 3 Stress-strain curves of concrete and steel reinforcement

at elastic stage: $$\sigma_{st} = E_s\, \varepsilon_{st} \tag{2a}$$

within yield plateau: $$\sigma_{st} = f_{yt} \tag{2b}$$

during strain hardening: $$\sigma_{st} = f_{yt} + (f_{su} - f_{yt})\left[1 - \left(\frac{\varepsilon_{su} - \varepsilon_{st}}{\varepsilon_{su} - \varepsilon_{sh}}\right)^n\right] \tag{2c}$$

in which σ_{st} and ε_{st} are the stress and strain in the steel, E_s is the Young's modulus, f_{yt} is the yield stress, f_{su} is the tensile strength, ε_{sh} is the tensile strain when strain hardening starts to take place, ε_{su} is the ultimate tensile strain and n is a parameter determined by the tangent modulus when strain hardening just starts. On the other hand, when the strain is decreasing (after strain reversal), the stress-strain curve is given by:

$$\sigma_{st} = E_s\,(\varepsilon_{st} - \varepsilon_p) \tag{3}$$

where ε_p is the residual strain at the end of the last strain increasing cycle. Fig. 3(b) shows the resulting stress-strain curve of the steel reinforcement. In this study, two types of steel reinforcement, namely: mild steel with f_{yt} = 250 MPa and high-yield steel with f_{yt} = 460 MPa, are considered and the tensile strength is assumed to be 0% (strain hardening ignored), 20% or 30% higher than the yield stress.

Only three basic assumptions have been made in the analysis: (1) plane sections remain plane after bending, (2) the tensile strength of concrete is negligible, and (3) there is no bond-slip between concrete and steel. However it should be pointed out that the assumption of no bond-slip between concrete and steel is only satisfied in an average sense. Nevertheless these assumptions are widely accepted for section analysis in the literature (Park and Paulay 1975). Fig. 4 shows a typical beam section analysed in this study. The moment-curvature behaviour of the beam section is analysed by applying prescribed curvatures to the beam section incrementally starting from zero. At a prescribed curvature, the strain profile is first evaluated based on the above assumptions. From the strain profile so obtained, the stresses developed in the concrete and the steel reinforcement are determined from their respective stress-strain curves. The stresses developed have to satisfy the axial equilibrium condition, from which the neutral axis depth is evaluated by iteration. Having determined the neutral axis depth, the resisting moment is calculated from the moment equilibrium condition. The above

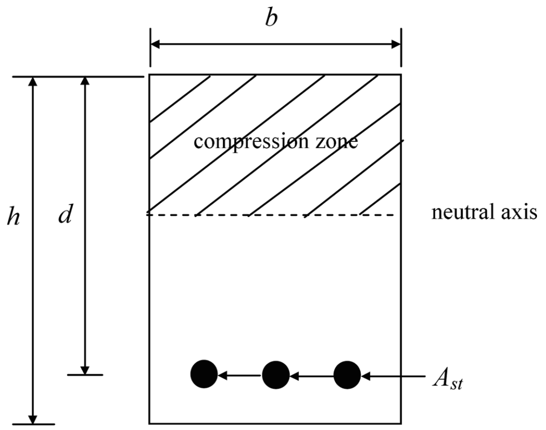
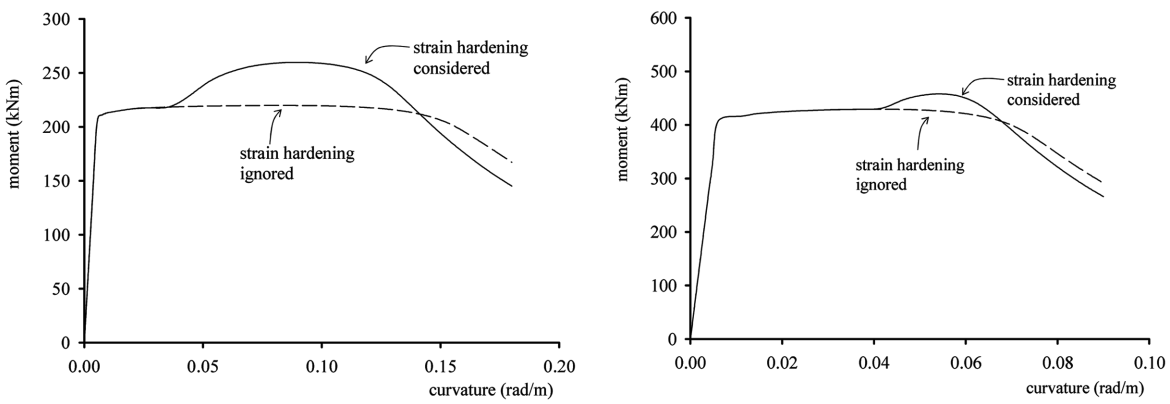


Fig. 4 Typical beam section analysed



(a) f_{co} = 60 MPa, f_{yt} = 460 MPa, f_{su} = 1.2f_{yt} and ρ_t = 0.5%   (b) f_{co} = 60 MPa, f_{yt} = 460 MPa, f_{su} = 1.2f_{yt} and ρ_t = 1.0%

Fig. 5 Moment-curvature curves with and without strain hardening in tension reinforcement

procedure is repeated until the curvature is large enough for the resisting moment to increase to the peak and then decrease to half of the peak moment.

Some selected moment-curvature curves of beam sections with f_{co} = 60 MPa, f_{yt} = 460 MPa and f_{su} = 1.2 f_{yt}, evaluated with and without strain hardening of the tension reinforcement, are plotted in Fig. 5. It is obvious from these moment-curvature curves that when the tension steel ratio ρ_t (ρ_t = A_{st}/bd) is relatively small, strain hardening of the tension reinforcement could have significant effects on the post-peak flexural behaviour of the reinforced concrete beam section. Basically, with strain hardening of the tension reinforcement taken into account in the analysis and when strain hardening occurs, the portion of the moment-curvature curve within the yield plateau range would be lifted upwards but after reaching the peak moment, the curve would also dive downwards more rapidly. In other words, strain hardening of the tension reinforcement could significantly increase the flexural strength and at the same time decrease the flexural ductility of the beam section. Comparing the moment-curvature curves in Figs. 5(a) and 5(b), it can also be seen that the effects of strain hardening are generally larger when the tension steel ratio is less.

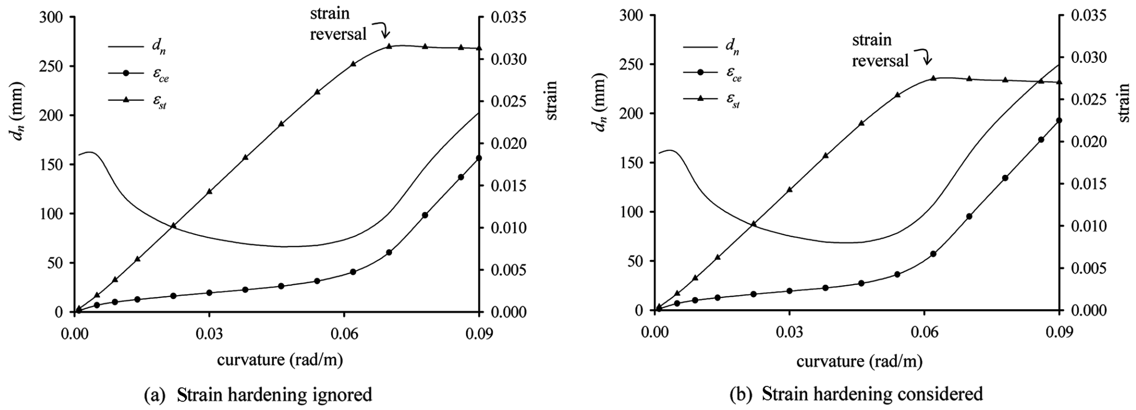


Fig. 6 Variation of neutral axis depth, concrete strain and steel strain with curvature in a beam section with f_{co} = 60 MPa, f_{yt} = 460 MPa, f_{su} = 1.2 f_{yt} and ρ_t = 1.0%

To study in greater details the effects of strain hardening, the variations of the neutral axis depth d_n, the concrete strain at extreme compression fibre ε_{ce} and the steel strain in the tension reinforcement ε_{st} with the curvature ϕ in a typical beam section with f_{co} = 60 MPa, f_{yt} = 460 MPa, f_{su} = 1.2 f_{yt} and ρ_t = 1.0%, evaluated with or without strain hardening, are plotted in Fig. 6. It is seen that initially, the neutral axis depth remains almost constant. As the curvature increases and the concrete becomes inelastic, the neutral axis depth gradually decreases. However, after entering into the post-peak stage, the neutral axis depth starts to increase such that the distance between the tension reinforcement and the neutral axis becomes smaller and smaller and beyond a certain point, the strain in the tension reinforcement stops increasing and starts to decrease causing strain reversal. Such strain reversal occurs regardless of whether strain hardening has been taken into account in the analysis. Because of strain reversal, the strain in the tension reinforcement would only reach a certain maximum value. Comparing the curves in Fig. 6(a) to those in Fig. 6(b), it can be seen that if the strain developed in the tension reinforcement is large enough for strain hardening to take place ($\varepsilon_{st} > \varepsilon_{sh}$), then the strain hardening would result in earlier strain reversal and thus cause the maximum strain reached by the tension reinforcement to be slightly reduced.

3. Maximum strain in tension reinforcement

As can be seen from the above analysis, whether strain hardening of the tension reinforcement would occur is dependent on the maximum strain reached by the tension reinforcement. If the maximum strain reached by the tension reinforcement is larger than the tensile strain value for strain hardening to take place ($\varepsilon_{st} > \varepsilon_{sh}$), then strain hardening of the tension reinforcement would occur and might have significant effects on the flexural behaviour of the beam section. Otherwise, strain hardening would not occur and therefore would have no effects. To predict when strain hardening would occur, it is necessary to study how the maximum strain varies with the various structural parameters.

From the numerical results, it is evident that the major structural parameter determining the maximum strain reached by the tension reinforcement is the tension steel ratio ρ_t or in normalized

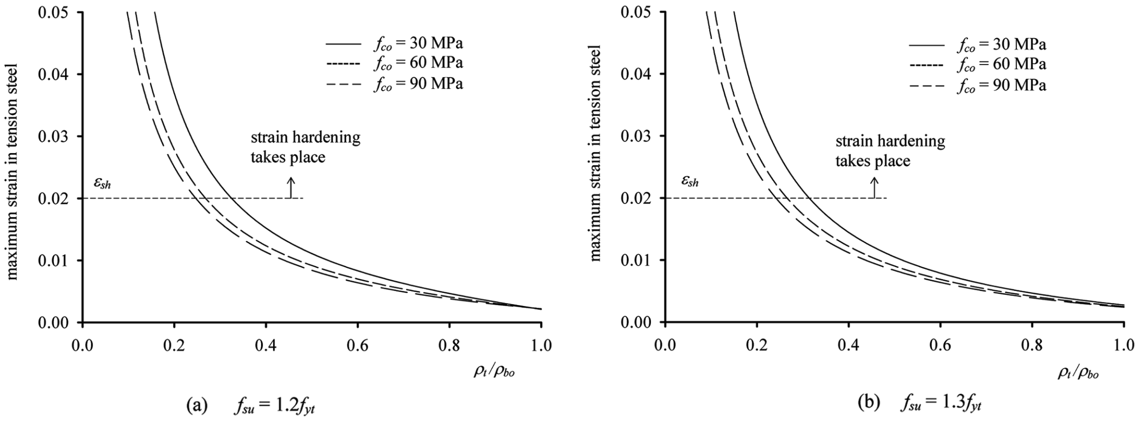


Fig. 7 Variation of maximum strain in tension reinforcement with ρ_t/ρ_{bo} ratio in beam sections with f_{co} = 30, 60 or 90 MPa and f_{yt} = 460 MPa

form the tension steel to balanced steel ratio ρ_t/ρ_{bo}. Fig. 7 shows the variations of the maximum strain with the ρ_t/ρ_{bo} ratio in beam sections with f_{co} = 30, 60 or 90 MPa and f_{yt} = 460 MPa evaluated with strain hardening (f_{su} = 1.2 f_{yt} or 1.3 f_{yt}) taken into account. It can be seen from this figure that in general the maximum strain reached by the tension reinforcement is larger at lower ρ_t/ρ_{bo} ratio and smaller at higher ρ_t/ρ_{bo} ratio. This is the root cause of having generally larger strain hardening effects when the tension steel ratio is relatively small. Comparing the maximum strains in the tension reinforcement at the same ρ_t/ρ_{bo} ratio but at different concrete strength levels, it is evident that at the same ρ_t/ρ_{bo} ratio, the maximum strain reached by the tension reinforcement decreases as the concrete strength increases. Hence, at the same ρ_t/ρ_{bo} ratio, the strain hardening effects should be more significant at lower concrete strength and less significant at higher concrete strength. Comparing the maximum strains in the tension reinforcement at the same ρ_t/ρ_{bo} ratio but at different steel tensile strength levels (at different f_{su}/f_{yt} ratios), it is also evident that at the same ρ_t/ρ_{bo} ratio when strain hardening occurs, the maximum strain reached by the tension reinforcement decreases as the steel tensile strength increases. This is because the higher tensile stress developed in the tension reinforcement during strain hardening would result in earlier strain reversal and thus lower maximum strain reached by the tension reinforcement and that the amount of reduction in the maximum strain reached is more significant when the f_{su}/f_{yt} ratio is higher.

For every given set of material parameters f_{co} and f_{yt}, there is a tension steel ratio below which the maximum strain reached by the tension reinforcement would be large enough for strain hardening to take place (maximum value of $\varepsilon_{st} > \varepsilon_{sh}$). Such tension steel ratios at different given values of f_{co} and f_{yt} have been evaluated and tabulated in Table 1. From the tabulated results, it can be seen that the tension steel ratio ρ_t below which strain hardening would take place increases with the concrete strength but decreases with the steel yield strength. At f_{yt} = 250 MPa, it varies from 1.77 to 2.88% while at f_{yt} = 460 MPa, it varies from 1.11 to 1.88%. However, expressing in terms of tension steel to balanced steel ratios, it is found that the tension steel to balanced steel ratio ρ_t/ρ_{bo} below which strain hardening would take place decreases with the concrete strength but increases with the steel yield strength. At f_{yt} = 250 MPa, it varies from 0.256 to 0.175 while at f_{yt} = 460 MPa, it varies from 0.348 to 0.258. Hence, it may be concluded that strain hardening would not take place when mild

Table 1 Values of ρ_t and ρ_t/ρ_{bo} below which strain hardening would take place

f_{co} (MPa)	f_{yt} = 250 MPa		f_{yt} = 460 MPa	
	ρ_t (%)	ρ_t/ρ_{bo}	ρ_t (%)	ρ_t/ρ_{bo}
30	1.77	0.256	1.11	0.348
40	1.98	0.228	1.26	0.319
50	2.19	0.211	1.40	0.299
60	2.38	0.198	1.53	0.284
70	2.56	0.189	1.65	0.272
80	2.72	0.181	1.77	0.264
90	2.88	0.175	1.88	0.258

steel is used (f_{yt} = 250 MPa) and ρ_t/ρ_{bo} > 0.256 or when high-yield steel is used (f_{yt} = 460 MPa) and ρ_t/ρ_{bo} > 0.348. Strain hardening would take place only when the ρ_t/ρ_{bo} ratio is sufficiently low, i.e., lower than the corresponding value given in Table 1.

Having the maximum strain reached by the tension reinforcement large enough for strain hardening to take place does not necessarily imply that the strain hardening would have significant effects on the flexural behaviour of the beam section. This is because the tensile strain for strain hardening to be fully developed is much larger than the tensile strain for strain hardening to be just initiated. Moreover, the effects of strain hardening are not directly proportional to the increase in tensile stress due to strain hardening. Hence, the actual effects of the strain hardening on the flexural strength and ductility of a reinforced concrete beam cannot be predicted in any simple way and have to be evaluated using a rigorous nonlinear flexural analysis method such as the one used in this study, as presented in the following.

4. Effect of strain hardening on flexural strength

Herein, the effect of strain hardening on the flexural strength of the reinforced concrete beam section is measured in terms of a flexural strength ratio defined as the ratio of the flexural strength evaluated with strain hardening considered to the flexural strength evaluated with strain hardening ignored. Fig. 8 shows the variation of the flexural strength ratio with the tension steel to balanced steel ratio ρ_t/ρ_{bo} in beam sections with f_{co} = 30, 60 or 90 MPa, f_{yt} = 460 MPa and f_{su} = 1.2 f_{yt} or 1.3 f_{yt}. It can be seen from this figure that at a sufficiently low ρ_t/ρ_{bo} ratio, the strain hardening of the tension reinforcement could lead to a significant increase in the flexural strength of the beam section, which is generally larger at lower ρ_t/ρ_{bo} ratio. It is noted that the ρ_t/ρ_{bo} ratios below which the flexural strength ratio would be greater than 1.0 as shown in Fig. 8 are somewhat lower than the corresponding ρ_t/ρ_{bo} ratios below which strain hardening would take place as listed in Table 1. This means that the onset of strain hardening does not always produce an increase in flexural strength. The reason is that in some cases, when strain hardening takes place, the beam section (well into its post-peak stage) might have already lost so much of its flexural capacity that the increase in flexural capacity due to strain hardening might not be able to compensate. It is further noted that the flexural strength ratio is larger when the f_{su}/f_{yt} ratio is larger, but the flexural strength ratio is not directly proportional to the f_{su}/f_{yt} ratio. The flexural strength ratio also varies with the concrete strength and the steel yield strength. Added together, the variations of the flexural strength ratio with the ρ_t/ρ_{bo}

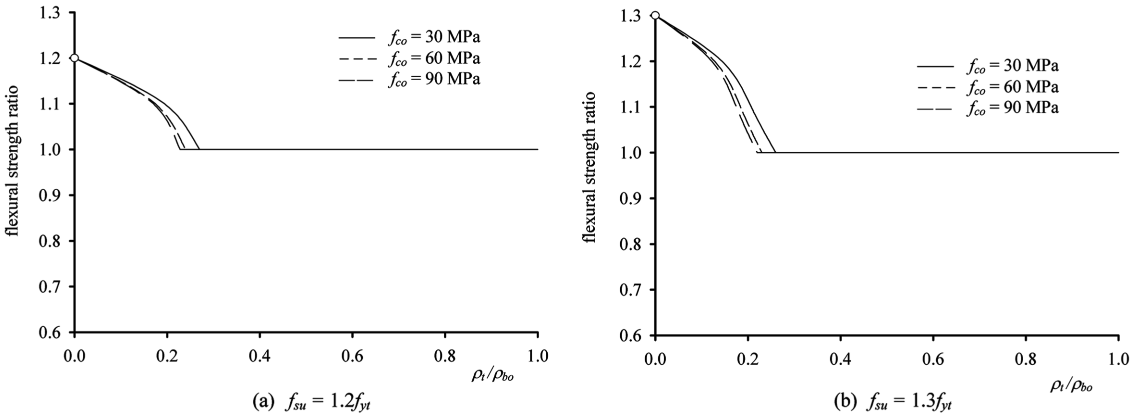


Fig. 8 Effect of strain hardening on flexural strength in beam sections with f_{co} = 30, 60 or 90 MPa and f_{yt} = 460 MPa

ratio, the f_{su}/f_{yt} ratio, the concrete strength and the steel yield strength are fairly complicated. As a result, the increase in flexural strength due to strain hardening cannot be easily predicted.

Because of the difficulty in predicting the increase in flexural strength due to strain hardening, it is suggested that if an increase in flexural strength is on the safe side, it is better and much simpler to neglect any possible increase in flexural strength due to strain hardening and treat such possible increase as a kind of strength reserve. In any case, since strain hardening would not occur until the flexural deformation is very large and the beam is already quite badly damaged, it is not advisable to rely on the possible increase in flexural strength due to strain hardening. However, there are many cases that an increase in flexural strength is not on the safe side and has to be taken into account in the structural design. Fortunately, since the increase in flexural strength is usually of the order of only 20 to 30%, an accurate prediction is not really necessary; a rough but conservative estimation should suffice.

To allow rough estimation of the flexural strength ratio, the values of ρ_t and ρ_t/ρ_{bo} below which the flexural strength ratio would be greater than 1.10 or 1.20 when f_{su} = 1.2 f_{yt} or 1.3 f_{yt} have been evaluated and listed in Tables 2-4. From these tables, it can be seen that in general the tension steel ratio ρ_t below which the flexural strength ratio would exceed a certain value increases with the

Table 2 Values of ρ_t and ρ_t/ρ_{bo} below which the flexural strength ratio would be greater than 1.10 (f_{su} = 1.2 f_{yt})

f_{co} (MPa)	f_{yt} = 250 MPa		f_{yt} = 460 MPa	
	ρ_t (%)	ρ_t/ρ_{bo}	ρ_t (%)	ρ_t/ρ_{bo}
30	0.95	0.137	0.62	0.194
40	1.11	0.128	0.73	0.185
50	1.26	0.121	0.84	0.179
60	1.42	0.118	0.94	0.174
70	1.56	0.115	1.04	0.172
80	1.70	0.113	1.14	0.170
90	1.83	0.111	1.23	0.168

Table 3 Values of ρ_t and ρ_t/ρ_{bo} below which the flexural strength ratio would be greater than 1.10 (f_{su} = 1.3 f_{yt})

f_{co} (MPa)	f_{yt} = 250 MPa		f_{yt} = 460 MPa	
	ρ_t (%)	ρ_t/ρ_{bo}	ρ_t (%)	ρ_t/ρ_{bo}
30	1.02	0.147	0.66	0.207
40	1.18	0.136	0.77	0.195
50	1.34	0.129	0.88	0.188
60	1.50	0.125	0.99	0.184
70	1.63	0.120	1.09	0.180
80	1.77	0.118	1.19	0.178
90	1.90	0.115	1.28	0.175

Table 4 Values of ρ_t and ρ_t/ρ_{bo} below which the flexural strength ratio would be greater than 1.20 (f_{su} = 1.3 f_{yt})

f_{co} (MPa)	f_{yt} = 250 MPa		f_{yt} = 460 MPa	
	ρ_t (%)	ρ_t/ρ_{bo}	ρ_t (%)	ρ_t/ρ_{bo}
30	0.69	0.100	0.46	0.144
40	0.80	0.092	0.54	0.137
50	0.90	0.087	0.61	0.130
60	1.01	0.084	0.69	0.128
70	1.11	0.082	0.76	0.125
80	1.20	0.080	0.83	0.124
90	1.30	0.079	0.89	0.122

concrete strength but decreases with the steel yield strength while the tension steel to balanced steel ratio ρ_t/ρ_{bo} below which the flexural strength ratio would exceed a certain value decreases with the concrete strength but increases with the steel yield strength. These tables may be used for quick estimation of the possible increase in flexural strength due to strain hardening. For instance, in the case of f_{su} = 1.2 f_{yt}, the flexural strength ratio may be estimated as 1.2 when the ρ_t or ρ_t/ρ_{bo} value is lower than the corresponding value listed in Table 2. Likewise, in the case of f_{su} = 1.3 f_{yt}, the flexural strength ratio may be estimated as 1.2 when the ρ_t or ρ_t/ρ_{bo} value is lower than the corresponding value listed in Table 3 but higher than the corresponding value listed in Table 4, and as 1.3 when the ρ_t or ρ_t/ρ_{bo} value is lower than the corresponding value listed in Table 4.

The above method of using Tables 2-4 for the estimation of the flexural strength ratio is a bit too clumsy. Since at a fixed flexural strength ratio, the variations of the tension steel to balanced steel ratio ρ_t/ρ_{bo} with the concrete strength and the steel yield strength are smaller than the corresponding variations of the tension steel ratio ρ_t, it is suggested that the ρ_t/ρ_{bo} value should be used to estimate the flexural strength ratio. In fact, at a fixed flexural strength ratio, the variation of ρ_t/ρ_{bo} with the concrete strength is actually quite small. Hence, the above method of estimating the flexural strength ratio may be simplified by neglecting the variation of ρ_t/ρ_{bo} with the concrete strength. Since the strain hardening effect is generally more significant at lower concrete strength, the ρ_t/ρ_{bo} value at a given flexural strength ratio for any concrete strength may be just taken as that for a concrete strength of f_{co} = 30 MPa. Neglecting the variation of the ρ_t/ρ_{bo} value at a given flexural strength ratio with the concrete strength, the flexural strength ratio may be estimated as

Table 5 Values of ρ_t/ρ_{bo} at different flexural strength ratios when f_{yt} = 250 MPa

flexural strength ratio	ρ_t/ρ_{bo}			
	f_{su} = 1.2 f_{yt}	f_{su} = 1.3 f_{yt}	f_{su} = 1.4 f_{yt}	f_{su} = 1.5 f_{yt}
1.0	≥0.195	≥0.195	≥0.195	≥0.195
1.1	0.137	0.147	0.150	0.152
1.2	0	0.100	0.111	0.117
1.3	-	0	0.076	0.086
1.4	-	-	0	0.061
1.5	-	-	-	0

Table 6 Values of ρ_t/ρ_{bo} at different flexural strength ratios when f_{yt} = 460 MPa

flexural strength ratio	ρ_t/ρ_{bo}			
	f_{su} = 1.2 f_{yt}	f_{su} = 1.3 f_{yt}	f_{su} = 1.4 f_{yt}	f_{su} = 1.5 f_{yt}
1.0	≥0.270	≥0.270	≥0.270	≥0.270
1.1	0.194	0.207	0.211	0.214
1.2	0	0.144	0.159	0.166
1.3	-	0	0.111	0.126
1.4	-	-	0	0.090
1.5	-	-	-	0

follows. In the case of f_{yt} = 460 MPa and f_{su} = 1.2 f_{yt}, the flexural strength ratio may be estimated as 1.2 when ρ_t/ρ_{bo} < 0.194 regardless of the concrete grade. Likewise, in the case of f_{yt} = 460 MPa and f_{su} = 1.3 f_{yt}, the flexural strength ratio may be estimated as 1.2 when 0.144 ≤ ρ_t/ρ_{bo} < 0.207, and as 1.3 when ρ_t/ρ_{bo} < 0.144 regardless of the concrete grade.

Tables 2-4 are applicable only when f_{su} = 1.2 f_{yt} or 1.3 f_{yt}. To allow for more general cases, the values of ρ_t/ρ_{bo} at which the flexural strength ratio would be greater than or equal to 1.0, 1.1, 1.2, 1.3, 1.4 or 1.5 when f_{co} = 30 MPa and f_{su} = 1.2 f_{yt}, 1.3 f_{yt}, 1.4 f_{yt} or 1.5 f_{yt} have been evaluated and listed in Tables 5 and 6. Using Table 5 for the case of f_{yt} = 250 MPa and Table 6 for the case of f_{yt} = 460 MPa, the flexural strength ratio may be estimated directly from the given f_{su}/f_{yt} and ρ_t/ρ_{bo} ratios. These two tables would yield flexural strength ratios slightly higher than the actual values because the flexural strength ratios at different concrete strength levels have been taken as that at the lowest concrete strength of f_{co} = 30 MPa (the strain hardening effect is more significant at lower concrete strength). Use of these two tables is illustrated in the following examples.

Example 1: Consider an example of a beam section of size 350 mm breadth × 650 mm effective depth, cast of concrete with in-situ strength f_{co} = 50 MPa and provided with 2 numbers of 25 mm diameter mild steel reinforcing bars with f_{yt} = 250 MPa and f_{su} = 350 MPa as tension reinforcement. The balanced steel ratio ρ_{bo} is determined as 10.39% while the tension steel ratio ρ_t is evaluated as 0.432%. Hence, the tension steel to balanced steel ratio ρ_t/ρ_{bo} is 0.432/10.39 = 0.042. On the other hand, the tensile strength to yield stress ratio f_{su}/f_{yt} is 350/250 = 1.4. Referring to Table 5, the flexural strength ratio is found to be within 1.3 to 1.4. If high accuracy is not required, the flexural strength ratio may be just taken conservatively as 1.4. If higher accuracy is preferred, the flexural strength ratio may be determined by interpolation as 1.35. To verify the correctness of this predicted flexural strength ratio, the flexural strength ratio has been evaluated using rigorous nonlinear flexural analysis as 1.34.

Example 2: Consider another example of a beam section of size 500 mm breadth × 750 mm effective depth, cast of concrete with in-situ strength f_{co} = 60 MPa and provided with 4 numbers of 25 mm diameter high-yield steel reinforcing bars with f_{yt} = 460 MPa and f_{su} = 598 MPa as tension reinforcement. The balanced steel ratio ρ_{bo} is determined as 5.39% while the tension steel ratio ρ_t is evaluated as 0.524%. Hence, the tension steel to balanced steel ratio ρ_t/ρ_{bo} is 0.524/5.39 = 0.097. On the other hand, the tensile strength to yield stress ratio f_{su}/f_{yt} is 598/460 = 1.3. Referring to Table 6, the flexural strength ratio is found to be within 1.2 to 1.3. If high accuracy is not required, the flexural strength ratio may be just taken conservatively as 1.3. If higher accuracy is preferred, the flexural strength ratio may be determined by interpolation as 1.23. To verify the correctness of these predicted flexural strength ratios, the flexural strength ratio has been evaluated using rigorous nonlinear flexural analysis as 1.22.

5. Effect of strain hardening on flexural ductility

In this study, the flexural ductility of a beam section is evaluated in terms of a curvature ductility factor μ defined by $\mu = \phi_u/\phi_y$ in which ϕ_u and ϕ_y are the ultimate curvature and yield curvature respectively. The ultimate curvature ϕ_u is taken as the curvature of the beam section when the resisting moment of the beam section has, after reaching the peak value of M_p, dropped to 0.8 M_p. On the other hand, the yield curvature ϕ_y is taken as the curvature at the hypothetical yield point of an equivalent linearly elastic-perfectly plastic system with an elastic stiffness equal to the secant stiffness of the section at 0.75 M_p and a yield moment equal to M_p.

The effect of the strain hardening of the tension reinforcement on the flexural ductility of the beam section is illustrated by plotting the values of μ evaluated with or without strain hardening considered against the value of ρ_t/ρ_{bo} for beam sections with f_{co} = 30, 60 or 90 MPa, f_{yt} = 460 MPa and f_{su} = 1.2 f_{yt} or 1.3 f_{yt} in Fig. 9. It is seen from this figure that when the value of ρ_t/ρ_{bo} is sufficiently low for strain hardening to take place, the strain hardening would cause the curvature ductility factor μ of the beam section to be slightly reduced. The amount of reduction in μ is more

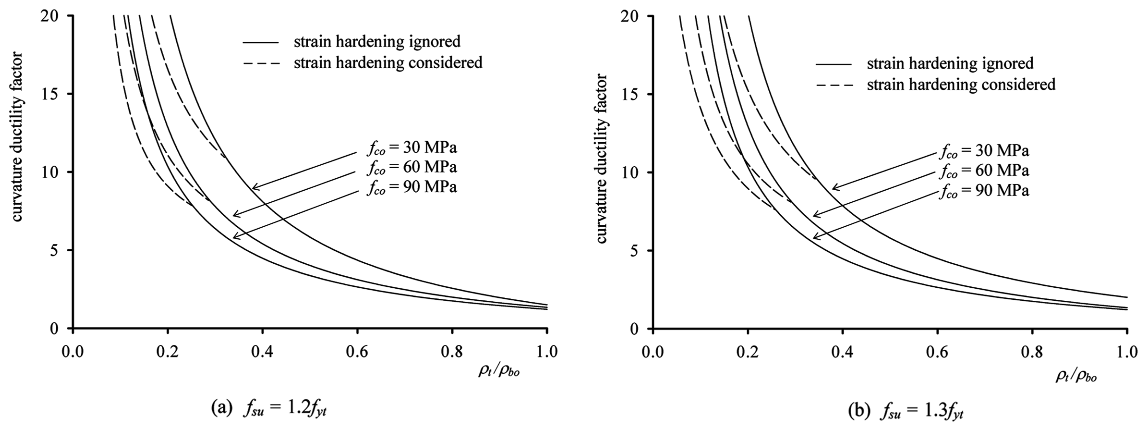


Fig. 9 Effect of strain hardening on flexural ductility in beam sections with f_{co} = 30, 60 or 90 MPa and f_{yt} = 460 MPa

significant when the f_{su}/f_{yt} ratio is larger. It is not easy to quantify the reduction in μ as a function of ρ_t/ρ_{bo}, f_{su}/f_{yt} and the material parameters because of the complicated relationship between μ and these parameters. Nevertheless, since the reduction in flexural ductility occurs when the value of ρ_t/ρ_{bo} is relatively low and the value of μ is relatively high, the reduction in flexural ductility should not be a major concern. Even after reduction due to strain hardening, the flexural ductility is still relatively high. Hence, it is suggested that it is not really necessary to quantify the reduction in flexural ductility of the beam section due to strain hardening of the tension reinforcement.

6. Conclusions

Using a newly developed nonlinear flexural analysis method, which employs the actual stress-strain curves of the constitutive materials and takes into account the stress-path dependence of the stress-strain curve of the steel reinforcement, the effects of the strain hardening of the tension reinforcement on the post-peak flexural behaviour of reinforced normal- and high-strength concrete beams have been studied. It is found that due to strain reversal, the strain in the tension reinforcement would only reach a certain maximum value, which is generally larger at lower tension steel ratio and smaller at higher tension steel ratio. If the maximum strain reached by the tension reinforcement is large enough for strain hardening to take place, then strain hardening of the tension reinforcement could significantly increase the flexural strength and at the same time decrease the flexural ductility of the beam section.

The increase in flexural strength due to strain hardening is found to be larger at lower ρ_t/ρ_{bo} ratio and/or higher f_{su}/f_{yt} ratio. However, the increase in flexural strength is a very complicated function of the various structural parameters and is thus very difficult to predict. Because of the difficulty in predicting the increase in flexural strength due to strain hardening, it is suggested that if an increase in flexural strength is on the safe side, it is better to neglect any possible increase in flexural strength due to strain hardening. However, if an increase in flexural strength is not on the safe side, the increase in flexural strength due to strain hardening should be taken into account in the structural design. For this purpose, tables for quick and conservative estimation of the increase in flexural strength due to strain hardening have been produced. Moreover, examples have been given to illustrate the use of these tables. On the other hand, since the reduction in flexural ductility due to strain hardening occurs only when the tension steel ratio is relatively low and even after reduction due to strain hardening, the flexural ductility is still relatively high, the reduction in flexural ductility is not considered to be of any major concern.

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Notations

A_{st} : area of tension reinforcement
b : breadth of beam section
d : effective depth of beam section
d_n : neutral axis depth
E_s : Young’s modulus of steel reinforcement
E_{sh} : tangent modulus of steel reinforcement when strain hardening just starts
f_{co} : in-situ uniaxial compressive strength of concrete
f_{su} : tensile strength of tension reinforcement
f_{yt} : yield stress of tension reinforcement
h : total depth of beam section
M_p : peak resisting moment of beam section
ε_c : concrete strain
ε_{ce} : concrete strain at extreme compression fibre
ε_{co} : concrete strain at peak stress
ε_{sh} : tensile strain of steel reinforcement when strain hardening just starts
ε_{su} : ultimate tensile strain of steel reinforcement
ε_{st} : steel strain in tension reinforcement
ϕ : curvature of beam section
ϕ_u : ultimate curvature of beam section
ϕ_y : yield curvature of beam section
μ : curvature ductility factor
ρ_{bo} : balanced steel ratio of beam section
ρ_t : tension steel ratio ($\rho_t = A_{st}/bd$)