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Suboptimal control strategy in structural control implementation

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Abstract. The suboptimal control rule is introduced in structural control implementation as an alternative over the optimal control because the optimal control may require large amount of processing time when applied to complex structural control problems. It is well known that any time delay in structural control implementation will cause un-synchronized application of the control forces, which not only reduce the effectiveness of an active control system, but also cause instability of the control system. The effect of time delay on the displacement and acceleration responses of building structures is studied when the suboptimal control rule is adopted. Two examples are given to show the effectiveness of the suboptimal control rule. It is shown through the examples that the present method is easy in implementation and high in efficiency and it can significantly reduce the time delay in structural control implementation without significant loss of performance.

Key words: suboptimal control; optimization; structural control; time delay; vibration; earthquake simulation.

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1. Introduction

As large civil engineering structures such as tall buildings, high-rise towers and long-span bridges become lighter and more flexible, their dynamic responses due to wind, earthquake and other extraordinary loads are of great concern. To keep these structures functioning safely, effective vibration control devices have been developed to reduce the dynamic responses of these kinds of dynamic sensitive structures to environmental loads.

There are three major classes of control systems:

1. Passive control system that does not require external power source to provide the control forces in reduction of structural dynamic responses (Han *et al.* 2003, Li *et al.* 1999, Li *et al.* 2002a, 2002b).
2. Active control system that requires large external power sources (such as electro-hydraulic or electromechanical actuators) to supply control forces in reduction of structural dynamic responses (Soong 1990, Polak *et al.* 1994, Wang *et al.* 1994, Li *et al.* 2001a, Fang *et al.* 2003).
3. Semi-active control system that typically requires small external power source (such as batteries) to develop the control forces in reduction of structural dynamic responses (Abe 1996, Dupont *et al.* 1997, Symans and Constantinou, 1997a, 1997b, 1999, Ricciardelli *et al.* 2000).

The optimal control has been the subject of study in the field of structural control for many years (Li *et al.* 2001b, 2002). However, time delay is one of the major obstacles in the application of the optimal control to complex and large-scale structures, especially those in civil engineering. The delay can cause un-synchronized application of the control forces, which not only reduce the effectiveness of an active control system, but also cause instability of the control system. Many researchers have studied the time delay problem in the application of vibration control. However, the previously developed methods are generally suitable for small time delay for the case of simple harmonic excitations. The control system stability is not guaranteed as the time delay becomes larger (Xu *et al.* 2002). An algorithm was proposed by Cai and Huang (2002) to transform the continuous time differential equation with time delay into a standard discrete time form. And the time performance index is used in the design of the optimal controller. The control system stability is prone to be guaranteed through the proposed control method. Furthermore, this method is applicable to the case of a large time delay. Cai *et al.* (2003) also proposed an approach to reform the motion equation of the time-delay control system into a standard form of a first-order differential equation to incorporate the time-delay effect into the motion equation throughout the derivation of the control algorithm. Normally the classical optimal control method requires all the state variables of the control system to be measurable and available for control feedback. However, for a high-order or complex system, the dimension of the degree-of-freedom of the system is usually high (Li 2003). Furthermore, some state variables of the system are usually unmeasurable and some state variables in control feedback have only a small effect on control performance. Therefore, the classical optimal control method requires large amounts of computation time. One feasible way to overcome this problem is to adopt the discrete sub-optimal control method with partial-state feedback, only the state variables which have a significant effect on the control performance are determined. The sub-optimal controller is finally designed using only the determined stated variables (Levine and Athans 1970, Choi and Sirisena 1974, Cai and Sun 2003). Another efficient and effective method is to simplify the solution process of the Riccati equation by reducing the order of the Riccati matrix and appropriate selection of the weighting functions (Xu *et al.* 2002). Thus, it simplifies the design of a structural control system and reduces the time delay in structural control implementation dramatically.

Literature review reveals that the majority of the previous research works generally concentrated on the application of the optimal control methods to structural control. It has been gradually recognized that the suboptimal control strategy is simple in terms of the controller's structure and is easy to realize in practices (Cai and Sun 2003). However, comprehensive studies on examining the performance of the suboptimal control methods in structural control implementation have been seldom reported. In this paper, a suboptimal control rule is proposed by appropriate selection of the weighting functions, and the detailed comparison of the performances of the suboptimal control rule and the classical optimal control methods is made. The effects of time delay on the proposed suboptimal control algorithm are studied. It is found that the suboptimal control is a viable alternative to the optimal control without significant loss of performance.

2. Simulation of earthquake excitations

When the vibration control techniques are applied to civil engineering structures under earthquake excitations, it is often necessary to simulate the earthquake actions for the design of the structural control systems. Generally speaking, it is over-simplified to treat earthquake excitation as a white noise. It is more reasonable to dissolve an earthquake wave as a sum of a deterministic disturbance and a white noise, that is

$$\ddot{\boldsymbol{x}}_t = \boldsymbol{a}_t + \boldsymbol{\varepsilon}_t \qquad (t = 1, 2, \ldots, N) \tag{1}$$

where $\ddot{\boldsymbol{x}}_t$ is the ground acceleration of earthquake, $\boldsymbol{\varepsilon}_t$ is the white noise and $\boldsymbol{a}_t$ is a deterministic disturbance. The simulation of earthquake excitation can be divided into two parts. The first deterministic part $\boldsymbol{a}_t$, also known as trend part, can be extracted from the original earthquake record sequence by the GM (2,1) method (Yang and Wu 1992). This method will be briefly introduced in the following section. While the second term (stationary random part $\boldsymbol{\varepsilon}_t$) can be described by the ordinary Auto Regressive Moving Average (ARMA) model, the GM (2,1) method is introduced into this study for extracting the deterministic part $\boldsymbol{a}_t$ from the original earthquake record $\ddot{\boldsymbol{x}}_t$.

In the GM (2,1) method, first of all, the Accumulated Generating Operation (AGO) is conducted for the original earthquake record sequence $\ddot{\boldsymbol{x}}_t$, thus the newly built sequence $\boldsymbol{x}_t^{(1)}$ $\left(\boldsymbol{x}_t^{(1)} = \sum_{j=1}^{t-1} \ddot{x}_j\right)$ by the AGO process can be obtained; then the grey model is used to describe the new sequence $\boldsymbol{x}_t^{(1)}$, and the deterministic part $\boldsymbol{a}_t$ of the original earthquake record $\boldsymbol{x}_t$ can be obtained indirectly by derivation of the new sequence $\boldsymbol{x}_t^{(1)}$. The detailed description of this algorithm can be found in Yang and Wu (1992). Only the brief introduction of this method is given below:

After the newly built sequence $\boldsymbol{x}_t^{(1)}$ is obtained by means of the AGO process, a second-order differential equation is established as follows:

$$\begin{cases} \ddot{x}^{(1)} + b_1\dot{x}^{(1)} + b_2 x^{(1)} = b_3 \\ x_0^{(1)} = \dot{x}_0^{(1)} = 0 \end{cases} \tag{2}$$

Eq. (2) is called as the GM (2,1) model, where b_1, b_2 and b_3 can be obtained as follows,

$$(b_1, b_2, b_3)^T = (\boldsymbol{B}_1^T \boldsymbol{B}_1)^{-1} \boldsymbol{B}_1^T \boldsymbol{B}_{1a},$$

$$\boldsymbol{B}_1 = \begin{bmatrix} -\ddot{x}_2 & -0.5(x_1^{(1)} + x_2^{(1)}) & 1 \\ -\ddot{x}_3 & -0.5(x_2^{(1)} + x_3^{(1)}) & 1 \\ \cdots & & \cdots \\ -\ddot{x}_N & -0.5(x_{N-1}^{(1)} + x_N^{(1)}) & 1 \end{bmatrix}, \qquad \boldsymbol{B}_{1a} = \begin{bmatrix} \ddot{x}_2 - \ddot{x}_1 \\ \ddot{x}_3 - \ddot{x}_2 \\ \cdots \\ \ddot{x}_N - \ddot{x}_{N-1} \end{bmatrix} \tag{3}$$

Letting the characteristic roots of Eq. (2) be $\lambda_1 = a + b\mathrm{i}$ and $\lambda_2 = a - b\mathrm{i}(\mathrm{i} = \sqrt{-1})$, one has

$$x_{t+1}^{(1)} = e^{at}[c_1 \cos bt + c_2 \sin bt] + \frac{b_3}{b_2} \tag{4}$$

where c_1 and c_2 are constants determined from the initial conditions in Eq. (2). From Eq. (4), the deterministic part $\boldsymbol{a_t}$ of the original earthquake record $\ddot{\boldsymbol{x}}_{\boldsymbol{t}}$ can be obtained indirectly as follows:

$$a_{t+1} = e^{at}[(ac_1 + bc_2)\cos bt + (ac_1 - bc_2)\sin bt] \tag{5}$$

Once the deterministic part $\boldsymbol{a_t}$ is obtained, the residual part $\boldsymbol{\varepsilon_t}$ can be simulated by ARMA model. In order to verify the reliability and effectiveness of the proposed method, twenty earthquake records (including El Centro 1940, Olympia 1949, Taft 1952, Parkfield 1966, San Fernando 1971 etc.) listed in Naeim (2001) were adopted in this study for the simulation of earthquake excitations and it was found that the residual components are very close to the white noise. Thus, it seems that the procedures described in Eqs. (1) to (5) are applicable to the simulation of earthquake actions.

Let
$$x = \mathrm{sign}[a_t(l_1\Delta t)x(l_1\Delta t)]\frac{y_{\max}}{|a_t(l_1\Delta t)|^{-1}}a_t + \varepsilon_t \tag{6}$$

where, $y_{\max} = |x(l_1\Delta t)| = \max_i x(i\Delta t)$, Δt is usually taken as 0.02 second. The adoption of Eq. (6) makes the residual component ε_t in Eq. (6) be almost zero at $l_1\Delta_t$.

For the El Centro earthquake record (Naeim 2001), it is found from the simulation results with the above method that $a = -0.0526$, $b = 0.079$, $c_1 = 150.1$ and $c_2 = -99.86$.

3. Optimal control force

For a linear system including a structure with n degrees of freedom and n_1 control devices, its equation of motion can be expressed as,

$$\boldsymbol{M}\ddot{\boldsymbol{X}} + \boldsymbol{C}\dot{\boldsymbol{X}} + \boldsymbol{K}\boldsymbol{X} = \boldsymbol{F}(t) + \boldsymbol{D}\boldsymbol{U} \tag{7}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are, respectively, the mass matrix, damping and stiffness matrices; $\boldsymbol{X}$ is the $(n + n_1)$-dimensional displacement vector, $\boldsymbol{F}(t)$ is a $(n + n_1)$-vector representing the applied load or external excitation, and $\boldsymbol{U}$ is the n-dimensional control force vector. The $(n + n_1) \times n_1$ order matrix $\boldsymbol{D}$ is the location matrix that defines the locations of the control forces.

To facilitate discussion, let $\boldsymbol{Z} = (\boldsymbol{X}^T, \dot{\boldsymbol{X}}^T)^T$, $N = n + n_1$, then

$$\dot{\boldsymbol{Z}} = \boldsymbol{AZ} + \boldsymbol{BU} + \boldsymbol{V}, \quad \boldsymbol{Z}(0) = 0 \tag{8}$$

where

$$\boldsymbol{A} = \begin{bmatrix} 0 & \boldsymbol{I}_{N\times N} \\ -\boldsymbol{M}^{-1}\boldsymbol{K} & -\boldsymbol{M}^{-1}\boldsymbol{C} \end{bmatrix} \quad \boldsymbol{B} = \begin{bmatrix} 0_{N\times n_1} \\ \boldsymbol{M}^{-1}\boldsymbol{D} \end{bmatrix} \quad \boldsymbol{V} = \begin{bmatrix} 0_{N\times 1} \\ \boldsymbol{M}^{-1}\boldsymbol{F} \end{bmatrix}$$

The optimal control vector $\boldsymbol{U}$ can be obtained as follows;

$$\boldsymbol{U} = -\boldsymbol{R}^{-1}\boldsymbol{BPZ}, \quad \boldsymbol{DU} = -\boldsymbol{DR}^{-1}\boldsymbol{B}^T\boldsymbol{PZ} = -\Delta\boldsymbol{KX} - \Delta\boldsymbol{C}\dot{\boldsymbol{X}} \tag{9}$$

where $\boldsymbol{R} = r\boldsymbol{I}_{n_1\times n_1}$, (matrix $\boldsymbol{I}$ is called the unit matrix), $r > 0$,

$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{P}_1 & \boldsymbol{P}_2 \\ \boldsymbol{P}_3 & \boldsymbol{P}_4 \end{bmatrix}$ is obtained from the Raccati equation

$$\boldsymbol{PA} + \boldsymbol{A}^T\boldsymbol{P} - \boldsymbol{PBR}^{-1}\boldsymbol{B}^T\boldsymbol{P} + \boldsymbol{Q} = 0 \tag{10}$$

where $\boldsymbol{P}_3 = \boldsymbol{P}_2^T$, $\Delta\boldsymbol{C} = \boldsymbol{DR}^{-1}\boldsymbol{D}^T\boldsymbol{M}^{-1}\boldsymbol{P}_4$, $\Delta\boldsymbol{K} = \boldsymbol{DR}^{-1}\boldsymbol{D}^T\boldsymbol{M}^{-1}\boldsymbol{P}_2^T$, ,

$\boldsymbol{Q}$ and $\boldsymbol{R}$ are referred to as weighting matrices, whose magnitudes are assigned according to the relative importance of the state variables and the control forces in the minimization procedure. By varying the relative magnitudes of $\boldsymbol{Q}$ and $\boldsymbol{R}$, one can synthesize the controllers to achieve a proper trade off between the control effectiveness and control energy consumption.

Eq. (9) is known as the optimal control rule. Normally, the optimal control force is often delayed by l time step, denoting as $l = \left[\dfrac{\Delta t}{\delta t}\right]$, in which $[\bullet]$ is the symbol for integer truncation. Therefore $\boldsymbol{U}$ is specified at time step $(t - l)$, that is

$$\left.\begin{aligned} \boldsymbol{DU}_{t-l} &= \Delta\boldsymbol{KX}_t - \Delta\boldsymbol{C}\dot{\boldsymbol{X}}_t \\ \boldsymbol{DB}_2^l\boldsymbol{U}_t &= -(\Delta\boldsymbol{CB}_2 + \Delta\boldsymbol{K})\boldsymbol{X}_t \\ \boldsymbol{X}_{i-1} &= \boldsymbol{B}_2\boldsymbol{X}_i \end{aligned}\right\} \tag{10a}$$

Let the time delay of the control force be δt, in the time interval $[t_{k-1}, t_{k-1} + \Delta t]$, and let $\boldsymbol{F}(t) = -\boldsymbol{M}\{\boldsymbol{1}\}\ddot{x}_g$, then for Eq. (7) we have

$$\dot{\boldsymbol{X}}^T[\boldsymbol{M}\ddot{\boldsymbol{X}}\Delta t + \boldsymbol{C}\dot{\boldsymbol{X}}\Delta t + \boldsymbol{KX}\Delta t] = \Delta t\dot{\boldsymbol{X}}^T\boldsymbol{DU} - \Delta t\dot{\boldsymbol{X}}^T\boldsymbol{M}\{\boldsymbol{1}\}\ddot{x}_g \tag{11}$$

If the linear acceleration method is adopted, then $\dot{\boldsymbol{X}} = \dot{\boldsymbol{X}}_{k-1} + \ddot{\boldsymbol{X}}\Delta t$; substituting it into Eq. (11), one has

$$\dot{\boldsymbol{X}}^T[\Delta t\boldsymbol{KX} + (\boldsymbol{M} + \Delta t\boldsymbol{C})\dot{\boldsymbol{X}}] = \Delta t\dot{\boldsymbol{X}}^T\boldsymbol{DU} - \boldsymbol{M}\{\boldsymbol{1}\}\ddot{x}_g + \boldsymbol{M}\dot{\boldsymbol{X}}_{k-1} \tag{12}$$

Let

$$\boldsymbol{Z}_{k-1} = (\boldsymbol{X}_{k-1}^T, \dot{\boldsymbol{X}}_{k-1}^T)^T, \quad \boldsymbol{Q}_2 = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \boldsymbol{K}\Delta t & \boldsymbol{M} + \Delta t\boldsymbol{C} \end{bmatrix}, \quad \boldsymbol{W}_1 = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{M} \end{bmatrix},$$

$$\boldsymbol{W} = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{M}\{\mathbf{1}\}\Delta t \end{bmatrix}, \quad \boldsymbol{V}_1 = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{D}\Delta t \end{bmatrix}$$

Then Eq. (12) can be written in the following form

$$\boldsymbol{Z}^T(\boldsymbol{Q}_2\boldsymbol{Z} + \boldsymbol{W}\ddot{x}_g - \boldsymbol{V}_1\boldsymbol{U} - \boldsymbol{W}_1\boldsymbol{Z}_{k-1}) = \mathbf{0} \tag{13}$$

Letting the Hamilton function (Yang *et al.* 1995) as

$$H = \boldsymbol{Z}^T\boldsymbol{Q}_1\boldsymbol{Z} + \boldsymbol{U}^T\boldsymbol{R}\boldsymbol{U} + \lambda\boldsymbol{Z}^T(\boldsymbol{Q}_2\boldsymbol{Z} + \boldsymbol{W}\ddot{x}_g - \boldsymbol{V}_1\boldsymbol{U} - \boldsymbol{W}_1\boldsymbol{Z}_{k-1}) \tag{14}$$

Solving the extreme value of H in Eq. (14), one obtains

$$\left.\begin{aligned} \boldsymbol{Q}_1\boldsymbol{Z} + \lambda\boldsymbol{Q}_2\boldsymbol{Z} &= 0 \\ 2\boldsymbol{R}\boldsymbol{U} - \lambda\boldsymbol{V}^T\boldsymbol{Z} &= 0 \end{aligned}\right\} \tag{15}$$

Let

$$\boldsymbol{Y} = \boldsymbol{K}\boldsymbol{X}\Delta t + (\boldsymbol{M} + \Delta t\boldsymbol{C})\dot{\boldsymbol{X}} \tag{16}$$

Then

$$U = \begin{cases} 0 & \boldsymbol{Y} = 0 \\ -\dfrac{[0 \;\; \boldsymbol{Y}^T]\boldsymbol{Q}_1\boldsymbol{Z}\Delta t\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{Z}}{\boldsymbol{Y}^T\boldsymbol{Y}} & \boldsymbol{Y} \neq 0 \end{cases} \tag{17}$$

Let

$$a' = \frac{[0 \;\; \boldsymbol{Y}^T]\Delta t\boldsymbol{Q}_1 Z}{\boldsymbol{Y}^T\boldsymbol{Y}}, \quad Q_1 = \begin{bmatrix} \boldsymbol{Q}_{11} & \\ & \boldsymbol{Q}_{22} \end{bmatrix} \tag{18}$$

Then a' is a function of $\boldsymbol{Z}$, and $\boldsymbol{Y}^T\boldsymbol{Y} = \boldsymbol{Z}^T\boldsymbol{Q}_2^T\boldsymbol{Q}_2\boldsymbol{Z}$, therefore we have

$$a' = \frac{\Delta t\boldsymbol{Y}^T(\boldsymbol{Q}_1\boldsymbol{X} + \boldsymbol{Q}_2\dot{\boldsymbol{X}})}{\boldsymbol{Y}^T\boldsymbol{Y}} \tag{19}$$

Solving Eqs. (17) and (19) is not an easy task since $\boldsymbol{U}$ is a non-linear function of $\boldsymbol{Z}$. Therefore, the suboptimal control strategy is introduced in this paper for suboptimal solution of $\boldsymbol{Q}_1$ to simplify the calculation. Let

$$\boldsymbol{Q}_{11} = \alpha \boldsymbol{K}, \quad \boldsymbol{Q}_{22} = \frac{\alpha}{\Delta t}(\boldsymbol{M} + \Delta t \boldsymbol{C}), \quad \alpha > 0$$

$$\boldsymbol{U} = \begin{cases} \boldsymbol{0} & \boldsymbol{Y} = 0 \\ -\alpha \boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{Z} & \boldsymbol{Y} \neq 0 \end{cases} \tag{20}$$

Selecting $\boldsymbol{R}$ as identity matrix and letting $\alpha \boldsymbol{R}^{-1} = \beta \boldsymbol{I},\ \beta > 0$, then

$$\boldsymbol{U} = \begin{cases} 0 & \boldsymbol{Y} = 0 \\ -\beta \boldsymbol{D}^T\dot{\boldsymbol{X}} & \boldsymbol{Y} \neq 0 \end{cases} \tag{21}$$

Eq. (21) is the suboptimal control rule. It is noted that $\boldsymbol{U}$ is a function of $\dot{\boldsymbol{X}}$, which means it would increase the damping of the system. Also, once one large value of β is selected, the damping of the system increases dramatically. However, the selection of β should be appropriate, for example the selection should make the damping ratio of the system less than 1. In fact, the selection is restricted by the energy provided by the control system.

Using Eq. (21), the motion equation of a structure becomes

$$\boldsymbol{M}\ddot{\boldsymbol{X}} + (\boldsymbol{C} + \beta \boldsymbol{D}\boldsymbol{D}^T)\dot{\boldsymbol{X}} + \boldsymbol{K}\boldsymbol{X} = -\boldsymbol{M}\{\boldsymbol{1}\}\ddot{x}_g \tag{22}$$

Eq. (22) can be rewritten as

$$\boldsymbol{M}\ddot{\boldsymbol{X}} + \boldsymbol{C}\dot{\boldsymbol{X}} + \boldsymbol{K}\boldsymbol{X} = \boldsymbol{D}\boldsymbol{U} - \boldsymbol{M}\{\boldsymbol{1}\}\ddot{x}_g \tag{23}$$

or

$$\boldsymbol{M}\ddot{\boldsymbol{X}} + (\boldsymbol{C} + \Delta \boldsymbol{C})\dot{X} + (\boldsymbol{K} + \Delta \boldsymbol{K})\boldsymbol{X} = -\boldsymbol{M}\{\boldsymbol{1}\}\ddot{x}_g \tag{24}$$

$$\ddot{\boldsymbol{X}}_t = \frac{1}{(\Delta t)^2}(1 - 2B_2 + B_2^2)\boldsymbol{X}_t \tag{25}$$

$$\dot{\boldsymbol{X}}_t = \frac{1}{\Delta t}(1 - B_2)\boldsymbol{X}_t \tag{26}$$

$$\left.\begin{aligned} \Phi(B_2)X_t &= -\boldsymbol{M}\{\boldsymbol{1}\}\ddot{x}_g \\ \Phi(B_2) &= \frac{1}{(\Delta t)^2}\boldsymbol{M}(1 - 2B_2 + B_2^2) + \frac{1}{\Delta t}(\boldsymbol{C} + \Delta \boldsymbol{C})(1 - B_2) + \boldsymbol{K} + \Delta \boldsymbol{K} \\ \boldsymbol{X}_t &= -\Phi^{-1}(B_2)\boldsymbol{M}\{\boldsymbol{1}\}\ddot{x}_g \end{aligned}\right\} \tag{27}$$

From Eq. (27), we have,

$$\boldsymbol{X}_t = \Phi_1 \boldsymbol{X}_{t-1} + \Phi_2 \boldsymbol{X}_{t-2} + \Phi_3 \ddot{x}_{gt} \tag{28}$$

where

$$\Phi_1 = \Psi^{-1}\left(\frac{2\boldsymbol{M}}{\Delta t^2} + \frac{\boldsymbol{C} + \Delta\boldsymbol{C}}{\Delta t}\right), \quad \Phi_2 = \Psi^{-1}\boldsymbol{M}\frac{1}{(\Delta t)^2}$$

$$\Phi_3 = -\Psi^{-1}\boldsymbol{M}\{\boldsymbol{1}\}, \quad \Psi = \frac{1}{(\Delta t)^2}\boldsymbol{M} + \frac{1}{\Delta t}(\boldsymbol{C} + \Delta\boldsymbol{C}) + \boldsymbol{K} + \Delta\boldsymbol{K}$$

From Eq. (28), the control force and displacement response of the structure can be predicted. From Eq. (1), the response of a linear structure can be divided into two parts: one is the response Y_t under the deterministic function α_t; the other is the response $\boldsymbol{Z}_t$ under the near white noise ε_t. $\boldsymbol{Z}_t$ can be predicted using the method described below.

The best prediction of Z_t for the first step is:

$$\bar{Z}_t(1) = E[Z_{t+1}] = \Phi_1 Z_t + \Phi_2 Z_{t-1} \tag{29}$$

The best prediction of Z_t for the second step is:

$$\bar{Z}_t(2) = E[Z_{t+2}] = \Phi_1 Z_t + \Phi_2 Z_t \tag{30}$$

The best prediction of Z_t for the l-th ($l > 2$) step is:

$$\bar{Z}_t(l) = \sum_{i=1}^{2} \Phi_i Z_t(l-i) \tag{31}$$

$$\bar{X}_t(l) = Y_{t+l}\bar{Z}_t(l) \tag{32}$$

Therefore, the best prediction of the control force for the l-th step is,

$$\bar{U}_t(l) = -\left(\frac{\Delta C}{\Delta t} + \Delta K\right)\bar{X}_t(l) + \frac{\Delta C}{\Delta t}\bar{X}_t(l-1) \tag{33}$$

where $l = \dfrac{\delta t}{\Delta t}$.

It is noted that the prediction should be combined with the observation values. This is shown from Eqs. (29)-(33) that the observation values Z_t, Z_{t-1} appear in these equations.

4. Time delay

Active structural control systems have been installed into many high-rise buildings in the world. Relevant information, including structural responses, external disturbances and control forces etc. need to be known for the implementation of an active control system. In other words, a certain amount of time is required for the application of active control forces. Time delay also exists in exerting the active control forces. As the degrees of freedom of the system increase, the time delay will increase rapidly. The more complex the structural control system is, the more the time delay may be.

For structural control of a 20-story building, in general, the time delay δt is between 0.02~0.05 second. Usually, for $\delta t < 0.1\ T_1$ (T_1 is the period of the first mode of the structure), an efficient computation processing is required. Otherwise, the structure may be destroyed before the control force is applied, if the structural response is too large.

On the other hand, if the structural system is under the earthquake excitation, the value of δt should be controlled within a certain range. Let the characteristic period of soil where the building located be T_g, δt should be controlled under the range of 0~0.2 T_g.

To simplify the discussion, let

$$\boldsymbol{U}(t-\delta t) = \begin{cases} 0 & t < \delta t \\ -\beta \boldsymbol{D}^T \dot{\boldsymbol{X}}(t-\delta t) & t \geq \delta t \end{cases} \tag{34}$$

The above equation is the suboptimal control rule.

Under the earthquake excitation, the motion equation of the system becomes,

$$\boldsymbol{M}\ddot{\boldsymbol{X}}_z + \boldsymbol{C}\dot{\boldsymbol{X}}_z + \boldsymbol{K}\boldsymbol{X}_z = -\boldsymbol{M}\{\boldsymbol{1}\}\ddot{x}_g - \beta \boldsymbol{D}\boldsymbol{D}^T \dot{\boldsymbol{X}}_z(t-\delta t) \tag{35}$$

Assuming

$$\dot{\boldsymbol{X}}_z(t) = \dot{\boldsymbol{X}}_z(t-\delta t) + \ddot{\boldsymbol{X}}_z(t)\delta t + \varepsilon(\delta t^2) \tag{36}$$

Then,

$$(\boldsymbol{M} - \beta \boldsymbol{D}\boldsymbol{D}^T \delta t)\ddot{\boldsymbol{X}}_z + (\boldsymbol{C} + \beta \boldsymbol{D}\boldsymbol{D}^T)\dot{\boldsymbol{X}}_z + \boldsymbol{K}\boldsymbol{X}_z = -\boldsymbol{M}\{\boldsymbol{1}\}\ddot{x}_g + \beta \boldsymbol{D}\boldsymbol{D}^T \varepsilon(\delta t^2) \tag{37}$$

From Eq. (37), it can be seen that $\boldsymbol{U}$ reduces the inertial force, increases the damping of the system and changes a little in the external disturbance.

For a simple system (a cantilever structure with mass m, stiffness k, damping c), we have

$$(m - \beta\delta t)\ddot{x}_z + (c+\beta)\dot{x}_z + kx_z = -m\ddot{x}_g + \beta\varepsilon(\delta t^2) \tag{38}$$

Ignoring the high order term in the right hand side of Eq. (38), letting

$$m_z = m - \beta\delta t, \quad \omega_z = \sqrt{k/m_z}, \quad c_z = c + \beta = 2\xi_z\omega_z m_z, \quad f_z = \omega_g/\omega_z \tag{39}$$

in which ω_z, ξ_z are the damping ratio and circular natural frequency of the structural system described by Eq. (38), respectively; ω_g is the characteristic circular natural frequency of the earthquake excitation.

Then the displacement amplification factor can be deduced from Eqs. (38) and (39) as

$$\mu_z = [(1-f_z^2)^2 + (2\xi_z f_z)^2]^{-0.5} \tag{40}$$

Taking differentiation for Eq. (40) with respect to the parameter β, we have

$$\frac{\partial \mu_z}{\partial \beta} = -\frac{\omega_g^2}{k}[(1-f_z^2)^2 + (2\xi_z f_z)^2]^{-1.5}\left[\frac{c+\beta}{k} - (1-f_z^2)\delta t\right] \tag{41}$$

Table 1 Displacement amplification factors

δt \ $\mu_z(\mu)$ \ β	100	500	1000	2000
0.01	0.5792 (0.578)	0.5789 (0.576)	0.5791 (0.575)	0.5792 (0.570)
0.03	0.5792	0.5834 (0.575)	0.5885 (0.576)	0.5796 (0.570)
0.05	0.5801	0.5880	0.5980 (0.575)	0.6173 (0.570)

Note: The data in parenthesis are displacement amplification factors obtained by the optimal control algorithm. The units for β and δt are kN · s/m and second, respectively.

Letting the right side of Eq. (41) be zero, then one has

$$f_z^2 = \frac{\omega_g^2}{\omega_z^2} = 1 + \frac{(c+\beta)}{k\delta t} \tag{42}$$

Therefore, it can be proved that when $\frac{\omega_g^2}{\omega_z^2} < 1 + \frac{(c+\beta)}{k\delta t}$, μ_z decreases as β increases.

4.1 Example 1. Evaluation of dynamic amplification factors

To demonstrate the effectiveness of the suboptimal control algorithm developed in this paper, a cantilever structure with only one degree of freedom is considered herein as a numerical example. The characteristic period of the soil on which the cantilever structure is located is specified by the China Code for Seismic Design of Buildings (GB50011-2001) as: T_g = 0.35 second, ω_g = 17.95 rad/s. Also assuming the cantilever structure has the following structural parameters: ω = 10 rad/s, m = 20 kN · s^2/m, ξ = 0.05. The displacement amplification factors for different time delays and β are determined by the optimal and suboptimal control algorithms, respectively, as shown in Table 1, while the comparisons of displacement amplification factors under different cases are shown in Figs. 1, 2 and 3.

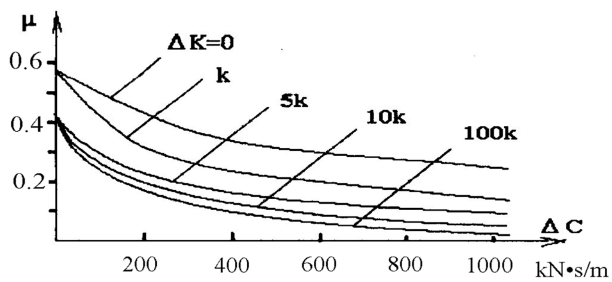

Fig. 1 Displacement amplification factors for the optimal control algorithm

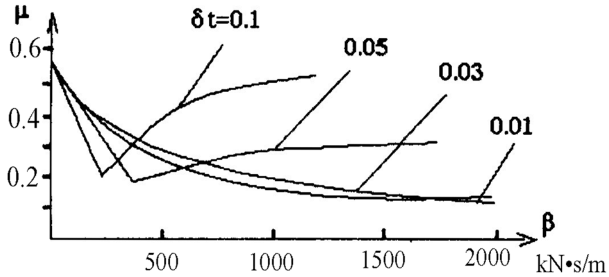

Fig. 2 Displacement amplification factors for the suboptimal control algorithm

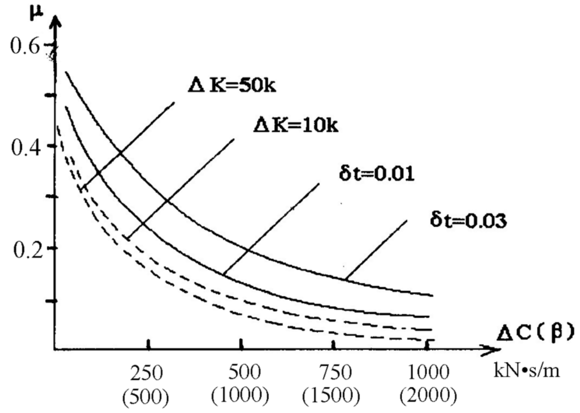

Fig. 3 Comparison of the displacement amplification factors (the solid lines are for the suboptimal control algorithm and the dotted lines are for the optimal algorithm)

Fig. 1 demonstrates that displacement amplification factors decrease as ΔK and ΔC increase, therefore, the dynamic response of the system reduces with the use of the optimal control method. It is also shown that the trend of reduction on displacement amplification is very fast for small values of ΔK and ΔC, but the trend of reduction is very slow for large values of ΔK and ΔC. Therefore, it is not necessary to select large values of ΔK and ΔC for obtaining certain control effects.

From Fig. 2, it can be seen that when δt is selected to be small than 0.03 second, the suboptimal control rule makes the displacement amplification factors decrease as β increases. If δt is selected to be relatively large (e.g. $\geq$0.05 second), the displacement amplification factors reduce rapidly as β increases for small values of β. However the displacement amplification factors increase slowly as β increases for relatively large values of β. Therefore, the time delay of the control force should be as small as possible, and the selection of β should match that of δt.

From Fig. 3, it can be seen that the displacement amplification factors for the optimal control are less than those of the suboptimal control. However, for a suitable β, a large value of ΔC and a small value of ΔK, the displacement amplification factors for these two control algorithms are very close. It can also be seen that when δt is relatively small, the results are very close for the two control methods. These results verify that the performance of the suboptimal control rule developed in this paper is acceptable in structural control implementation.

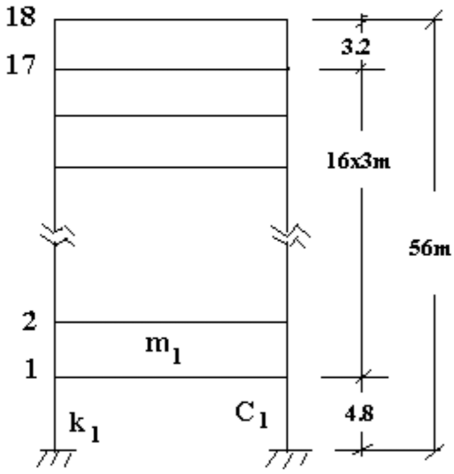

Fig. 4 Analytical model of an 18-story residential building

Table 2 The design parameters of the 18-story residential building

Floor level	Mass M (ton)		Stiffness ($K_l + K_f$) (10^5 kN/m)	
	Control	No control	Control	No control
1	672	670	5.86	3.25
2-12	568	550	4.56	3.05
13-17	556	545	4.56	2.90
18	750	740	3.74	2.50

4.2 Example 2. Comparison of structural responses with and without vibration control

The simplified analytical model of an 18-story residential building is shown in Fig. 4. Its design parameters are given in Table 2.

The structural damping ratio is selected as $\xi_i = 0.05$, and in Table 2, K_l and K_f are the lateral shear and bending stiffness, respectively; $K_i = K_{li} + K_{fi}$.

The El Centro earthquake record (Naeim 2001) is selected as the earthquake excitation, and it is simulated by the method developed in this paper with the following results:

$$\ddot{x}_{gt} = \alpha_t + \varepsilon_t, \quad \alpha_{t+1} = e^{at}[(ac_1 + bc_2)\cos bt + (ac_2 - bc_1)\sin bt]$$

$$a = -0.0526, \quad b = 0.079, \quad c_1 = 150.1, \quad c_2 = -99.86 \tag{43}$$

Control device: Active Tuned Mass Damper (ATMD) will be installed at the top floor, the associated parameters are

$$k = 3400 \text{ kN/m},\ m = 210 \text{ kN} \cdot \text{s}^2\text{/m},\ c = 265 \text{ kN} \cdot \text{s/m},\ \beta = 1.5\times10^5 \text{ kN} \cdot \text{s/m}$$

Table 3 Seismic responses of the 18-story building

Level	Displacement (mm)			Acceleration (mill-g)		
	No control	Optimal	Suboptimal	No control	Optimal	Suboptimal
10	10.82	6.31	6.73	10.85	6.14	6.57
12	12.51	7.53	8.12	12.23	7.03	7.68
16	14.22	8.87	9.53	13.65	7.68	8.33
18	16.35	9.76	10.84	15.35	8.23	9.15

The calculated responses from the two control rules (the optimal and suboptimal) and for the case without structural control are shown in Table 3 for comparison purposes.

From Tables 2 and 3, it can be seen that

1. When there is no control system installed in this building, the maximum displacement Δ = 16.35 mm < H/1000. Under the suboptimal and optimal control system, Δ = 10.84 mm = H/5166 and Δ = 9.76 mm = H/5738, respectively. The maximum displacement responses under the optimal control and suboptimal control systems are only about 66% and 60% of that without any control system installed.
2. When there is no control system installed on the building, the maximum acceleration is 15.35 mill-g. With the use of the optimal or suboptimal control systems, the maximum accelerations are 8.23 or 9.15 mill-g, suggesting that the effectiveness of the control systems is satisfactory.

5. Conclusions

A sub-optimal control strategy has been presented in this paper for structural control implementation and it can be a viable alternative to the optimal control rule. It requires less amount of processing time when applied to complex structural control systems in comparison with the optimal control method. The effect of time delay on the reduction of structural response was studied for the suboptimal control algorithm. The major conclusions of this study are drawn as follows

1. The GM (2,1) method was adopted in this paper for extracting the deterministic part $\boldsymbol{a}_t$ from original earthquake records. To the authors' best knowledge, this is the first time to apply the GM (2,1) method in the simulation of earthquake action. From the simulation results with twenty earthquake record waves, it has been verified that the GM (2,1) method is effective for the simulation of earthquake excitations.
2. If the suboptimal control strategy is applied to a simple system (a cantilever structure with mass m, stiffness k, damping c) under earthquake excitation, the displacement amplification factor μ_z decreases as β increases when $\dfrac{\omega_g^2}{\omega_z^2} < 1 + \dfrac{(c+\beta)}{k\delta t}$.
3. Normally the optimal control rule is more effective than the suboptimal control strategy. However, if a suitable β, a large value of ΔC and a small value of ΔK are selected for both optimal and suboptimal control algorithms, the reduction of structural responses for these two control algorithms is very close.

4. Numerical results of example 2 show that the reduction percentage of the displacement and acceleration responses can reach to 30%-40% when the suboptimal control strategy was adopted for the vibration control of an 18-story building.
5. Compared with the optimal control algorithm, the proposed suboptimal control strategy is easy in implementation and is satisfactory in efficiency. It can significantly reduce the time delay in structural control implementation without significant loss of performance.

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