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Highway bridge live loading assessment and load carrying capacity estimation using a health monitoring system

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Abstract. The Land Transport Authority of Singapore has a continuing program of highway bridge upgrading, to refurbish and strengthen bridges to allow for increasing vehicle traffic and increasing axle loads. One subject of this program has been a short span bridge taking a busy highway across a coastal inlet near a major port facility. Experiment-based structural assessments of the bridge were conducted before and after upgrading works including strengthening. Each assessment exercise comprised two separate components; a strain and acceleration monitoring exercise lasting approximately one month, and a full-scale dynamic test carried out in a single day. This paper reports the application of extreme value statistics to estimate bridge live loads using strain measurements.

Key words: bridge assessment; extreme value statistics; structural health monitoring; bridge live loading.

1. Introduction

In recent years, there has been growing interest in non-destructive field-testing of bridges to determine representative structural models (Cantieni 1996, Brownjohn and Xia 2000) and to assess their load carrying capacities (Stallings and Yoo 1993, Lake *et al.* 1997). This has been made possible by technological developments in data acquisition hardware and software, sensors and

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system identification procedures. The advantage of field-testing over traditional assessments based on standard axle loads is that realistic structural systems can be obtained via modal testing and finite element modelling (FEM) (Bakht and Csagoly 1980, Darlow and Bettigue 1989, Bakht and Jaeger 1990). In addition, field testing enables collection of valuable information about vehicle loading of a bridge. For example, monitoring strains at selected locations on a bridge can offer in-depth insights about the level of vehicle loading on a bridge. While such information can be obtained with relative ease, challenges of interpreting such data still exist. This paper proposes a methodology for deriving bridge specific live loads from strain measurements. For this purpose, field measurements on a bridge identified by the Land Transport Authority of Singapore for upgrading due to its strategic location will be used.

2. Bridge description

Pioneer Bridge (Fig. 1) was built along Pioneer Road in Western Singapore in 1968-70 and was designed for the loading of that era (Ministry of Transport, UK, 1961). The bridge carries a dual carriageway with two lanes in each direction. The span is 18.16 m between elastomeric bearings (Fig. 2), which were designed as simple supports, and the width is 18.796 m. The bridge crosses the outflow of a storm drain into Jurong Port and due to tidal variations the bed is wet much of the time with clearance of up to 3 m.

The bridge comprises 37 pre-cast pre-tensioned inverted T-beams, shown in section in Fig. 2, tied together by 25 cast in-situ 203 mm thick transverse diaphragms at 762 mm centres. The T-beams carry a deck slab having thickness that varies from 152 mm to 305 mm. Concrete cube strengths assumed in the original design were 42 $MN \cdot m^{-2}$ for pre-tensioned girders and 26 $MN \cdot m^{-2}$ for diaphragms and slab.

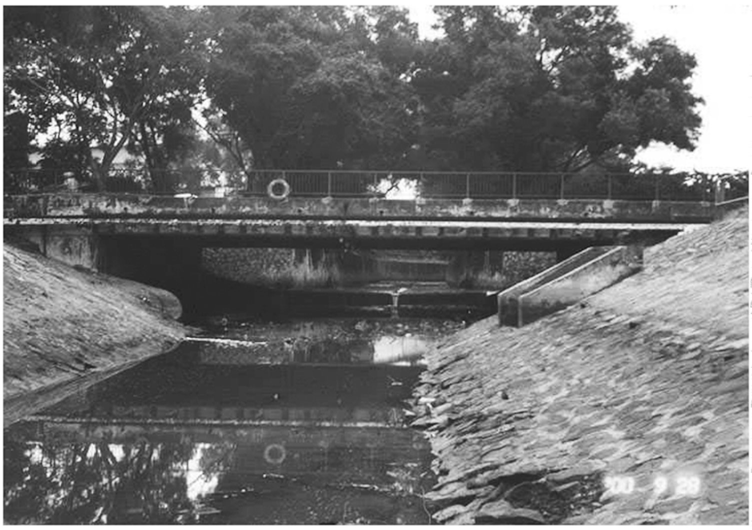

Fig. 1 Pioneer Bridge before upgrading

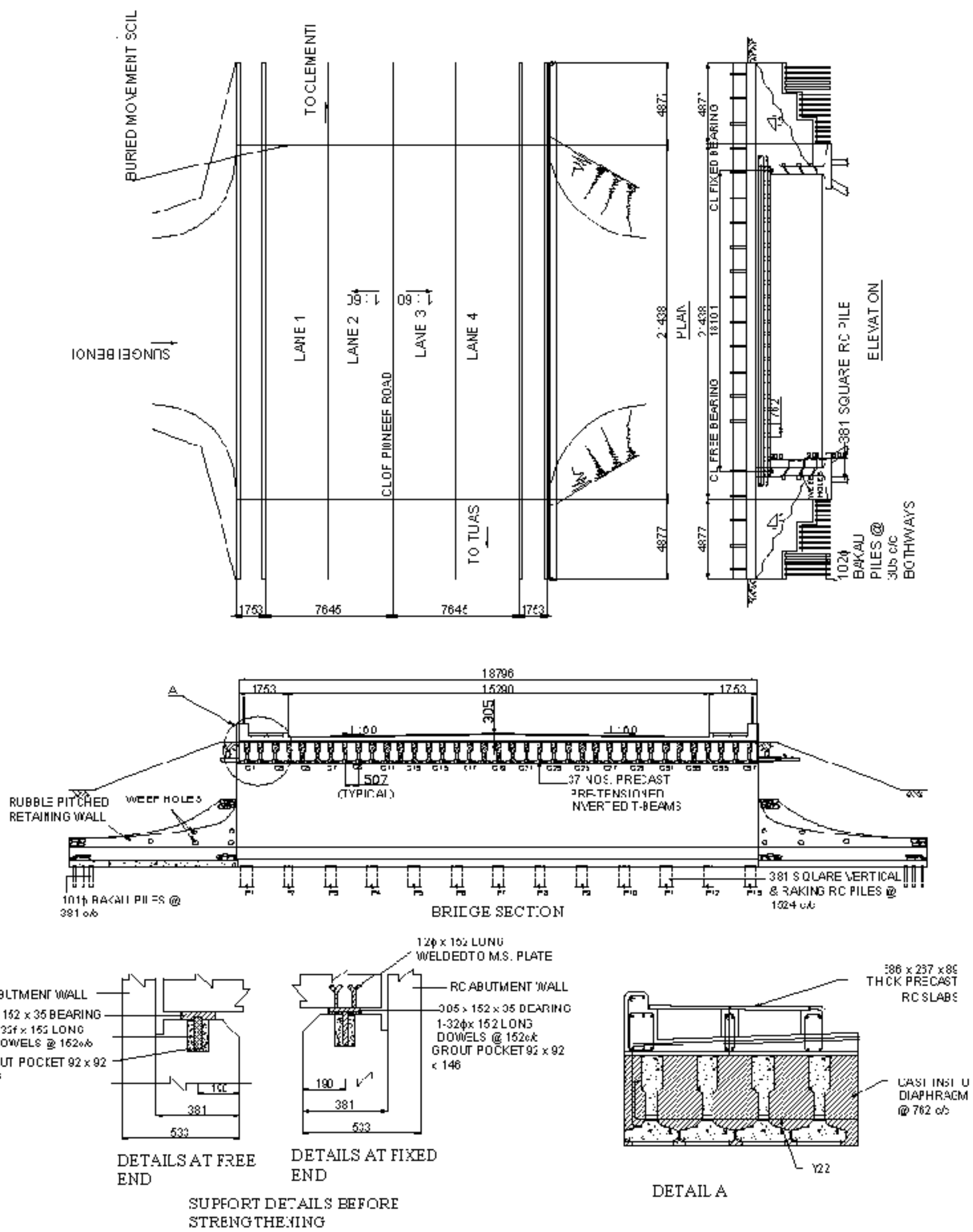

Fig. 2 Bridge details

3. Bridge assessment and strengthening works

As part of the Singapore Land Transport Authority (LTA) bridge management and upgrading program to cater for increased vehicle traffic and loading, the bridge was assessed to evaluate its strength and to identify any defects in the structure. British bridge assessment procedures laid out in documents BD 21/97, BA 55/94 and BD 44/95 (Highways Agency 1997, 1994, 1995) were adopted

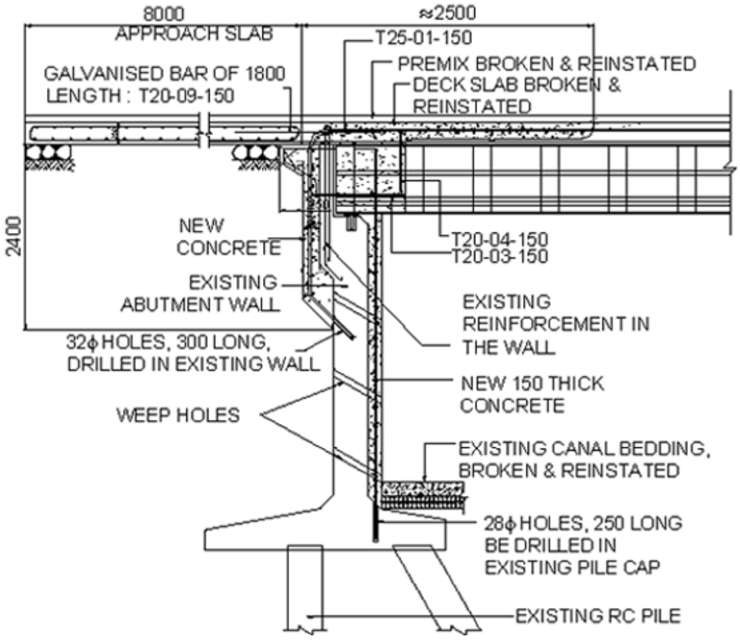

Fig. 3 Bridge upgrading details

for the assessment of the load carrying capacity. BD 44/95 specifies use of ‘worst credible strength’ defined as the worst value of strength that can be obtained in the structural element under consideration, and for concrete elements this was estimated from core tests yielding compressive strengths of 55 MN $\cdot$ m^{-2} and 30 MN $\cdot$ m^{-2} for T-beams and cast in-situ slab respectively.

The analytical and visual assessments revealed that rubber bearings had been overstressed, but that the superstructure was capable of carrying vehicles of up to 44 Mg gross mass as specified in BD21/97. However, to maintain the load carrying capacity over the design life with loading specified by LTA, strengthening works were proposed in which the simply supported system was converted to a jointless structure with the superstructure assumed to be continuous and monolithic with abutments (Fig. 3). This would eliminate the need for bearings, increase redundancy and enhance load distribution at supports. In addition, this type of construction was found to suit site conditions where heavy vehicles had to continue to use the bridge during upgrading works in which one-third width of the bridge was closed and upgraded at a time.

4. Bridge monitoring

The upgrading program provided a perfect opportunity to demonstrate the application of field testing in modal analysis and condition assessment of bridges. Field testing was carried out before and after upgrading works and in each case this consisted of, strain and acceleration monitoring exercise lasting approximately one month, and a full-scale dynamic test carried out in a single day. Details of dynamic testing are reported by Brownjohn *et al.* (2003).

The bridge monitoring program involved measurement of dynamic strain, at the bridge’s mid-span using a purpose made bridge monitoring system. The monitoring system comprised of four

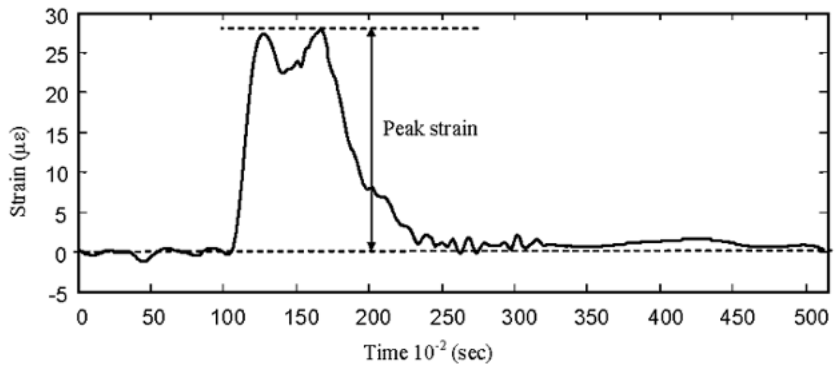

Fig. 4 Typical strain record

demountable strain gauges (DSG), and a data acquisition box with sampling rate up to 500 Hz. A major advantage of the system is that the data acquisition system is powered by a 12 V battery, enabling use in remote sites. Data acquisition was triggered by ambient traffic at selected levels of strain. For each event the data acquisition system captures and records the strain waveform, peak values, date and time (Fig. 4). The strain gauges were mounted on the soffits of girders 7, 15, 24, 33 (on lane 1, lane 2, lane 3 and lane 4 respectively) before and after upgrading works with each monitoring program lasting at least 20 days.

Fig. 5 shows the scatter plot of all peak strains recorded on the four girders and Fig. 6 depicts the

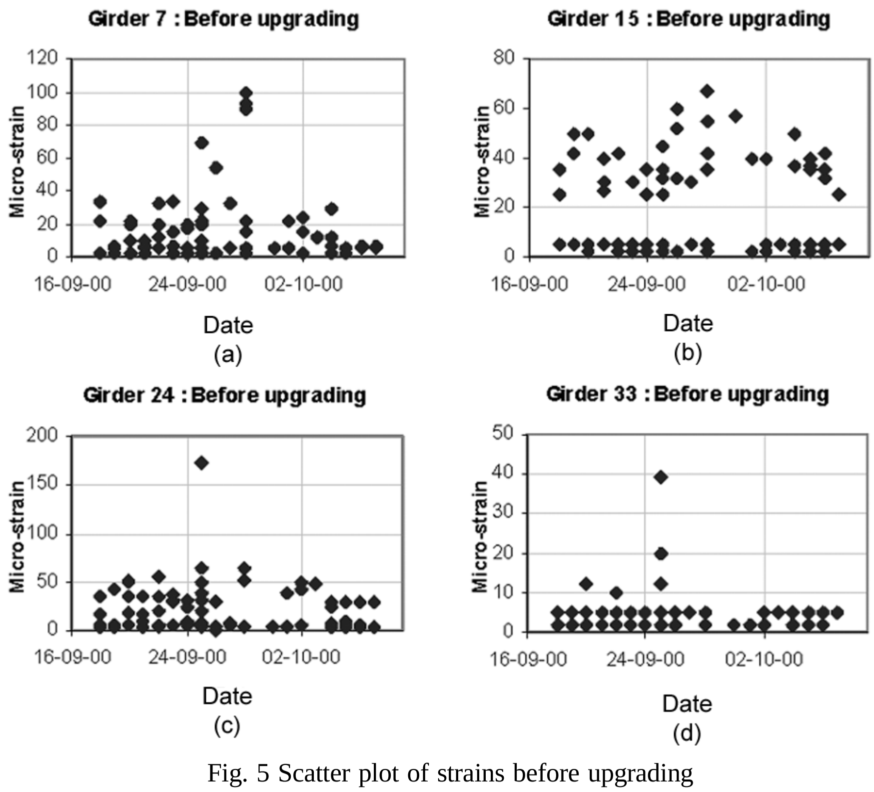

Fig. 5 Scatter plot of strains before upgrading

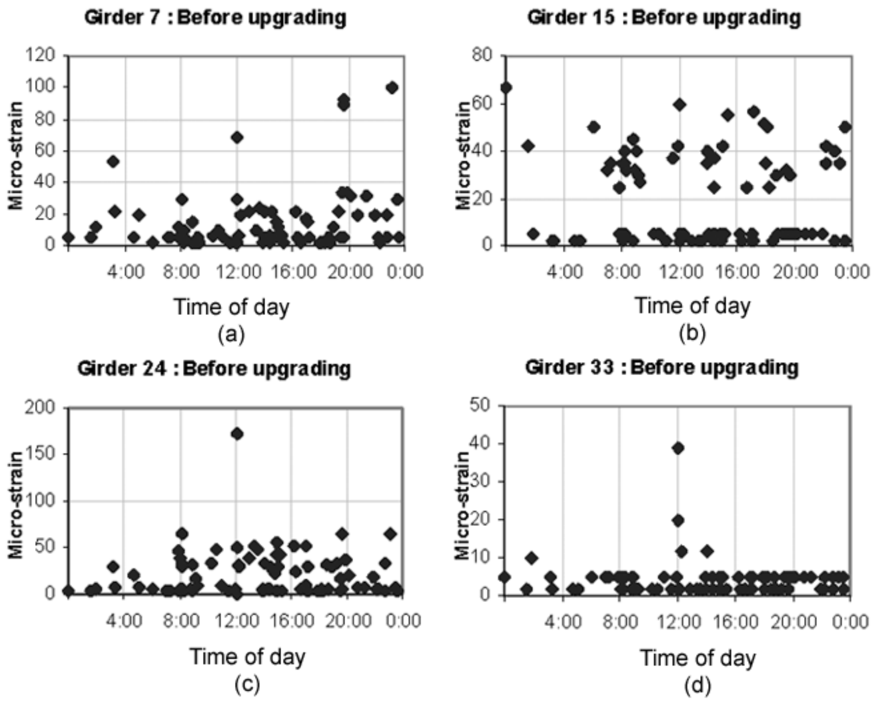

Fig. 6 Pre-upgrading distribution of peak strain during the day

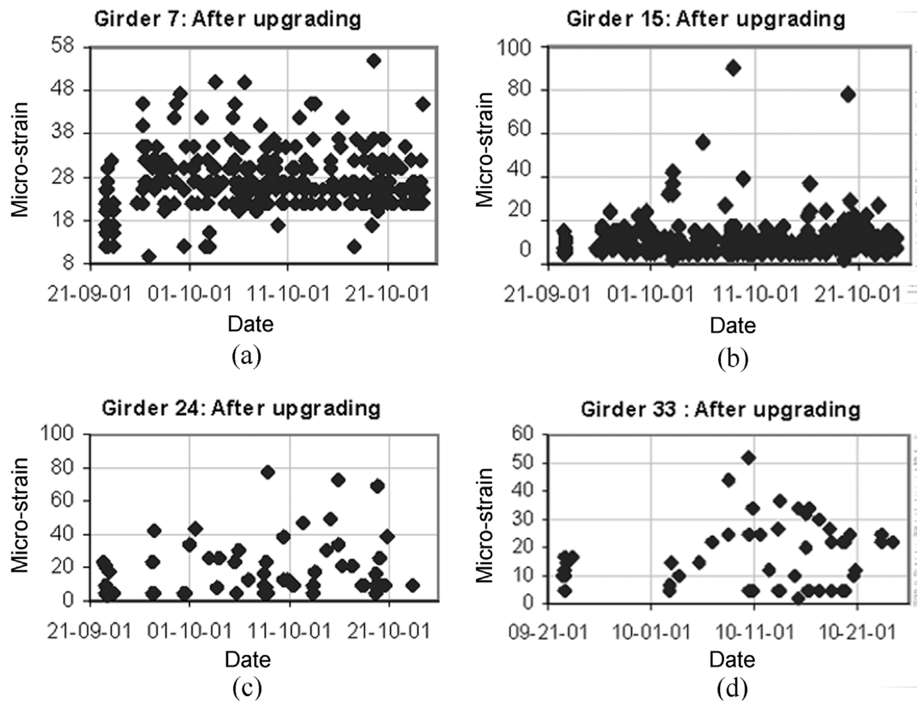

Fig. 7 Scatter plot of strains after upgrading

distribution of peak strains during the day, from 24:01 hrs to 24:00 hrs, for the duration of the monitoring period and prior to strengthening works. Most of the peaks strains are concentrated between 6.00 hrs and 24.00 hrs and extreme events are evenly distributed during the day. The scatter plots clearly show that the majority of heavy vehicles use lane 1 and lane 2. Lane 4 experienced little heavy traffic during the monitoring period. This is expected since a relatively small volume of heavy traffic joining Pioneer Road from Ship Yard Road uses lane 4. The maximum strain recorded prior to upgrading works is 172 $\mu\varepsilon$ on lane 3.

Post-upgrading traffic patterns, depicted in Figs. 7 and 8 show trends similar to traffic patterns before upgrading works, i.e., there are more heavy vehicles using lane 1 and lane 2 and heavy

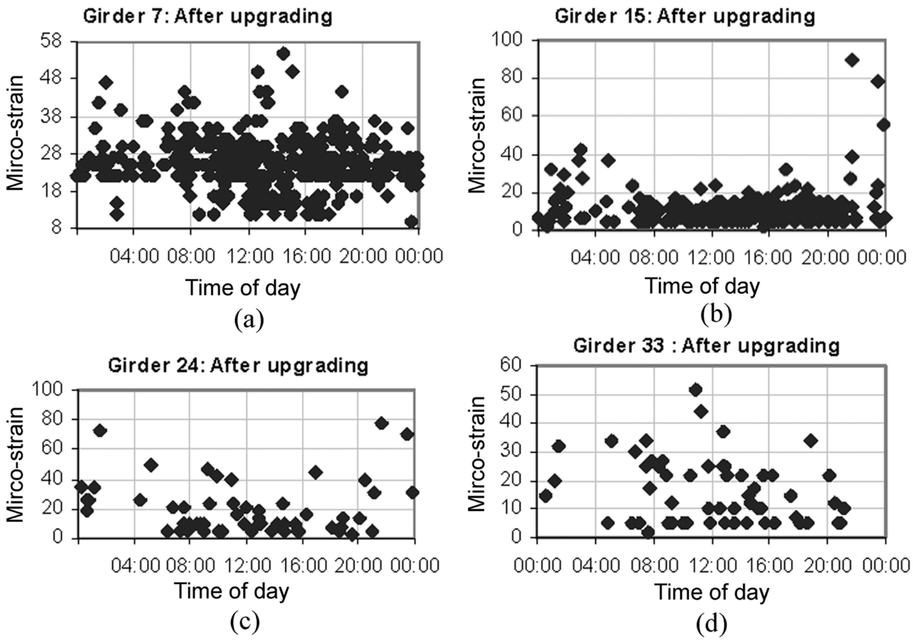

Fig. 8 Post-upgrading distribution of peak strain during the day

Table 1 Statistical summary of recorded strains

		Lane 1 ($\mu\varepsilon$)	Lane 2 ($\mu\varepsilon$)	Lane 3 ($\mu\varepsilon$)	Lane 4 ($\mu\varepsilon$)
Before upgrading	Mean	25	21	21	12
	Maximum	100	67	172	39
	Standard deviation	18.6	7.5	7.9	9.8
	Kurtosis	8.8	5.2	10.6	5.9
	Skewness	3.0	1.9	2.9	2.3
After upgrading	Mean	22	11	20	16
	Maximum	55	90	78	52
	Standard deviation	6.9	6.4	17.6	11.6
	Kurtosis	1.4	13.1	2.7	0.3
	Skewness	0.7	2.9	1.7	0.9

vehicle traffic is concentrated between 600 hrs and 2400 hrs. The maximum strain recorded after upgrading works is 90 $\mu\varepsilon$ on lane 2. Table 1 gives a numerical suumary of strain data obtained during pre-strengthening and post-strengthening monitoring.

5. Live loading assessment

BD21/97 specifies assessment live loading based on a range of vehicles up to 40/44 tonnes gross weight. The standard also states that the ultimate load occurs with a return period of 200,000 years or 0.06% chance in 120 years. In most cases, this loading does not reflect actual live loads on bridges and BD21/97 acknowledges this by recommending a relaxation of the loading requirements for certain bridge situations. The drawback of evaluating bridges following abstract live loads such as those given in BD21/97 is that assessments are often conservative (Bakht and Csagoly 1980, Darlow and Bettigue 1989, Bakht and Jaeger 1990, Shetty and Chubb 2000) and may lead to unwarranted bridge closures and maintenance works. Therefore, alternative procedures to assess bridges' ability perform their function satisfactorily and with adequate reliability should be explored. A rational approach is to apply statistical analysis on bridge specific strain measurements to determine a representative load effect model. In this way, the resulting live loads integrate site specific conditions including traffic volume, bridge natural frequency and damping, proximity to heavy industry, road alignment, vehicle suspension, traffic barrier design, speed environment and traffic mix.

It has been demonstrated (Shetty and Chubb 2000, Das 2001) that maximum traffic loading and the resulting bending moments and shear forces conform to the Type 1 Extreme value distribution i.e., the Gumbel distribution. Therefore, it can be argued that strains resulting from heavy traffic would also be represented by the Gumbel distribution. Hence, the Type 1 Extreme value distribution can be used to estimate ultimate live loads using strain measurements.

The Type 1 Extreme value distribution for maxima takes the form;

$$P(x) = \exp\left\{-\exp\left(\frac{x-\lambda}{\alpha}\right)\right\} \tag{1}$$

α and λ are distribution parameters that are estimated from data. By letting $y = \left(\frac{x-\lambda}{\alpha}\right)$ Eq. (1) can be expressed as;

$$y = -\ln(-\ln(Py)) \tag{2}$$

y is known as the reduced variate of the Gumbel distribution and is related to order statistics $x_1 \le x_2 \le x_3 \ldots \le x_m \ldots \le x_N$ by (Gumbel 1960);

$$y = -\ln\left(-\ln\left(\frac{m}{N+1}\right)\right) \tag{3}$$

Now consider a structural monitoring system such as that described in section 4. The monitoring system records all strain above a predefined threshold. The Gumbel distribution can be used to estimate 200,000 year strains as required by BD21 as follows. First select maximum strain from a day's record. Rank the strain values ascending order of size i.e., assign rank m = 1 to the smallest value and rank $m = N$ to the largest value, where N is the total number of daily maximum values for the monitoring period. Plot each value of the ordered maximum strains against the reduced variate

$y = -\ln\left(-\ln\left(\frac{m}{N+1}\right)\right)$ and obtain the parameters α and λ from the slope and intercept of the straight line graph. Extrapolate the straight line to determine the value of strain for which the probability of being exceeded in 120 years is 0.06. This procedure yields accurate results provided there is sufficient data (Cook 1982, Harris 1999l, Kottegota and Rosso 1997). Where periods of observation are short, a number of data discarded by the above selection procedure, i.e., second, third, … highest daily strains, may be higher than the highest values of other days. Owing to these draw backs Cook (1982) proposed the Method of Independent Storms, which was later modified by Harris (1996, 1999).

6. The method of independent storms

Cook's method of Independent Storms (MIS) is a modification of the standard Gumbel method and was developed to analyse maximum mean wind speed. The procedure essentially selects all values above a predefined threshold from a data set instead of a single maximum value per day selected by the standard Gumbel procedure. This effectively increases the number of observations used in the analysis and captures all extreme values from data. Suppose that N independent maximum strains are identified by Cook's method from a collection of R days of record. This corresponds to a daily rate of occurrence of maximum strains of $r = N/R$. The threshold value is selected such that $r \geq R$ (Cook 1982).

Assuming that all strains are independent, it follows from the theory of order statistics (Gumbel 1960), that the distribution of the largest daily maxima out of r independent maxima per day has a probability distribution given by $[P(z)]^r$. Since maximum strains are assumed to conform to the Gumbel distribution, $[P(z)]^r$ also conforms to the Gumbel distribution. The reduced variate for the distribution $[P(z)]^r$ can be derived from the theory of order statistics.

First consider a variable x with a probability density $f(x)$ and a cumulative density function $F(x)$ (so that $f(x) = dF(x)/dx$). If N independent samples from this distribution are ranked from $m = 1$, for the smallest to $m = N$ for the largest, then the exact probability density for the value of rank m is given by (Gumbel 1960);

$$\varphi_m(x)dx = \frac{N!}{(m-1)!(N-m)!}[F(x)]^{N-m}[1-F(x)]^{m-1}f(x)dx \;\; x \leq \infty \tag{4}$$

Substituting $z = F(x)$ and gives;

$$h_m(z)dz = \frac{N!}{(m-1)!(N-m)!}z^{N-m}[1-z]^{m-1}dz \quad 0 \leq z \leq 1 \tag{5}$$

The expected value of a variable z with probability density $h_m(z)$ is given by (Harris 1996)

$$\bar{z} = \int_0^1 z h_m(z)dz$$

$$= \frac{N!}{(N-m)!(m-1)!}\int_0^1 z^m(1-z)^{N-m}dz$$

$$= \frac{m}{N+1} \tag{6}$$

This result is the standard Gumbel probability ordinate. Therefore the reduced Gumbel variate, $y = -\ln\left(-\ln\left(\frac{m}{N+1}\right)\right)$, for a sample of rank m is a transformed mean value of the parent distribution. Following the above procedure, the reduced MIS variate for a sample of rank m can be defined as a transformed mean value of the distribution $[P(z)]^r$. Harris (1996, 1999), argues that systematic errors are introduced by the log-log transformation and such errors would be minimised if the transformation is performed first and then the mean of the transformed value used as the reduced variate i.e., the reduced variate is $\bar{y}_m = \int_0^1 -\ln(-r\ln(z))h_m(z)dz$. Now, if N samples of data are arranged in descending order, with $v = 1$ being the largest and $v = N$ the smallest, the new reduced variate is given by;

$$\bar{y}_v = \int_0^1 -\ln(-r\ln(z))h_v(z)dz$$

$$= \frac{N!}{(v-1)!(N-v)!}\int_0^1 -\ln(-\ln(z))z^{N-v}(1-z)^{v-1}dz \ -\ln(r) \tag{7}$$

The first expression in Eq. (7) can be solved by numerical integration. Having obtained the plotting positions y_v the least squares method can be used to fit the straight line on the Gumbel plot and the parameters α and λ can be obtained from the graph.

7. Ultimate strains for Pioneer Bridge

The data recorded by the SHM system described above was analysed using both the standard Gumbel method and the MIS. In each case the ultimate strains were estimated by extrapolation. A threshold of 6 $\mu\varepsilon$ was set for each lane for the MIS. This is based on the accuracy of strain gauges which is 2 $\mu\varepsilon$. Figs. 9-11 shows Gumbel plots and MIS plots for data recorded before strengthening works, while Figs. 12-15 depicts Gumbel and MIS plots for data recorded after strengthening works. There were no sufficient strain records above the set threshold of 6 $\mu\varepsilon$ for MIS analysis recorded in lane 4 prior to upgrading and the result from Gumbel analysis will be used for further analysis. Clearly the MIS captures more extreme values and results in better fit than the standard Gumbel method. A summary of the 120 year strains is tabulated in Table 2. Notice the significant decrease in the 120 year strains after upgrading works confirming increased fixity and the abutments.

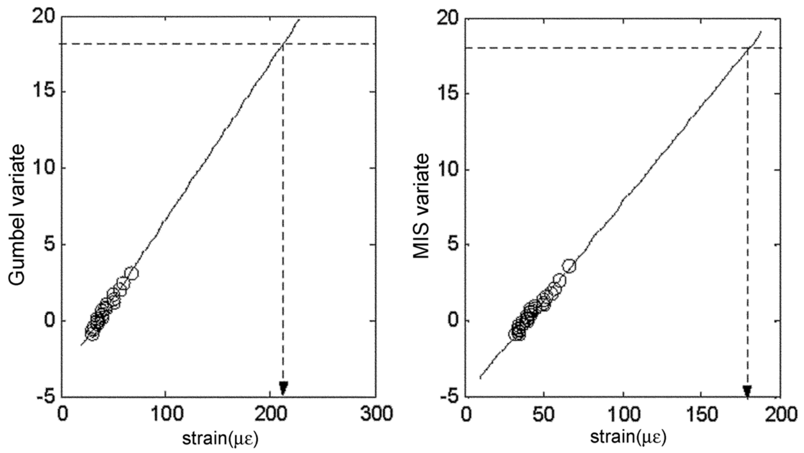

Fig. 9 Lane 1 120 year strains before bridge upgrading

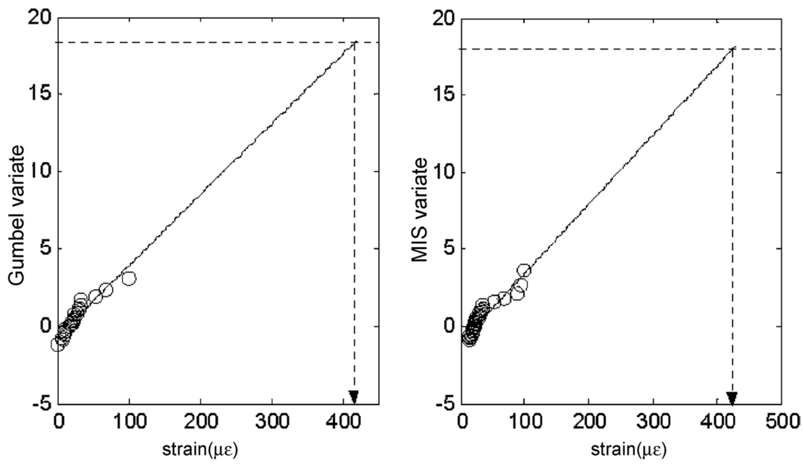

Fig. 10 Lane 3 120 year strains before bridge upgrading

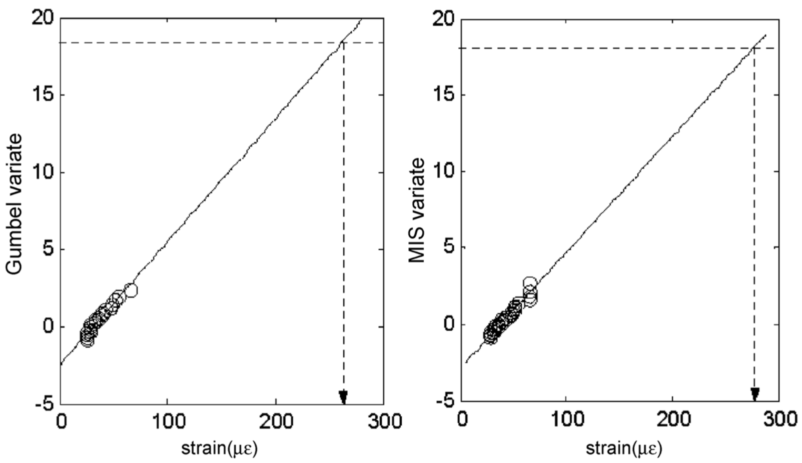

Fig. 11 Lane 4 120 year strains before bridge upgrading

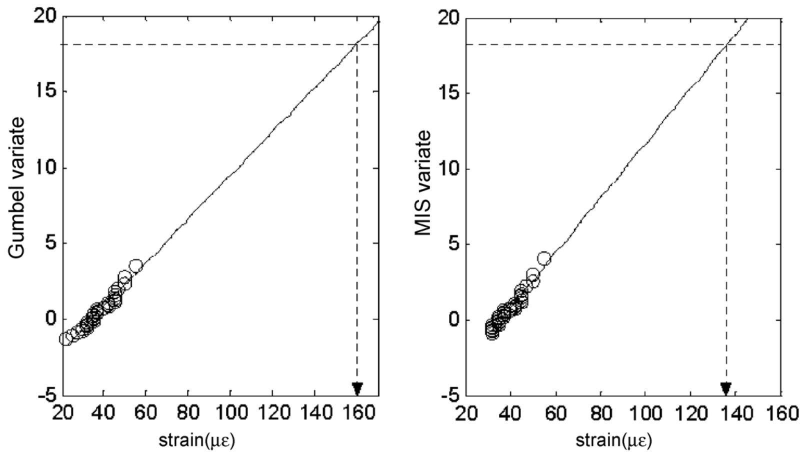

Fig. 12 Lane 1 120 year strains after bridge upgrading

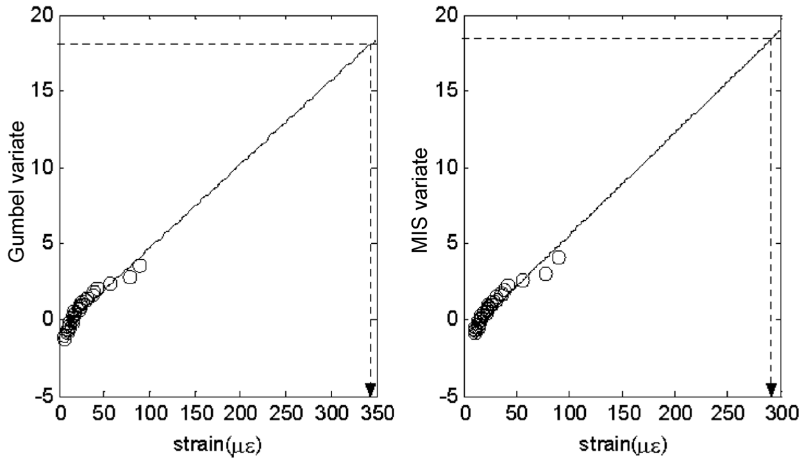

Fig. 13 Lane 2 120 year strains after bridge upgrading

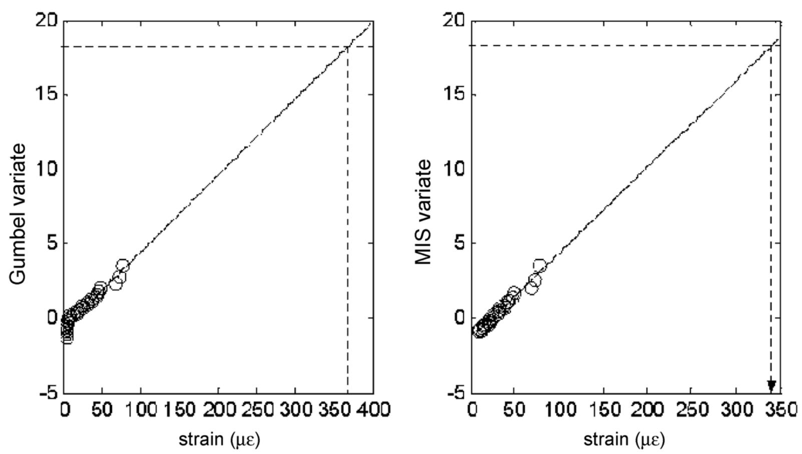

Fig. 14 Lane 3 120 year strains after bridge upgrading

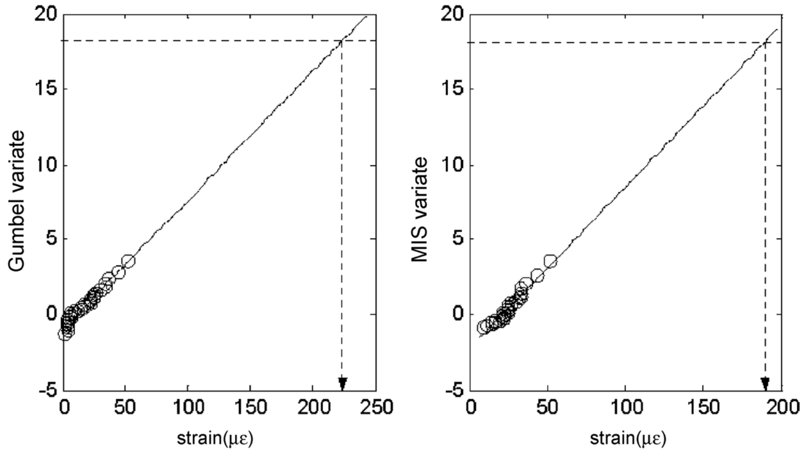

Fig. 15 Lane 4 120 year strains after bridge upgrading

Table 2 120 year strains

		Lane 1 ($\mu\varepsilon$)	Lane 2 ($\mu\varepsilon$)	Lane 3 ($\mu\varepsilon$)	Lane 4 ($\mu\varepsilon$)
Before upgrading	Gumbel	215	388	415	260
	MIS	180	**	415	280
After upgrading	Gumbel	160	345	365	225
	MIS	135	290	340	190

A closer look at the Gumbel and MIS plots for lane 2 and lane 4 shows two distinct clusters of strain values, one below 50 $\mu\varepsilon$ and another above 50 $\mu\varepsilon$. The authors' initial conclusion was that the bridge was exhibiting non-linear behaviour due to overstressed bearings. However, this phenomenon appears after converting the bridge from a simply supported structure to an integral bridge. A study of the road usage revealed that heavy trucks carrying bridge segments for a new semi-expressway (Fig. 16) used the bridge at least three times a month. Therefore the clusters of strains above 50 $\mu\varepsilon$ are most likely due to these heavy trucks.

The next step is to determine the bridge ultimate strain from the lane ultimate strains. The bridge ultimate strain is the most severe combination of 120 year lane values obtained using the MIS. BD21/97 specifies that two loading scenarios should be considered in assessing the effect of vehicles. These are;

i. A single vehicle with impact
ii. A convoy of vehicles (jam situation with no impact)

The first scenario is already included in SHM measurements together with other factors such as road profile, vehicle suspension, road alignment and vehicle speed. This is an important advantage of SHM based assessment. SHM records do not show any possible jam situation. Thus extrapolations in Figs. 9-15 can be assumed to be related to single vehicle events. The ultimate live load will be estimated by considering all lanes to be occupied by 120 year strains.

Fig. 16 Heavy vehicle carrying pre-case bridge segment

The distribution of load between lanes is estimated using weighted distribution factors (Stallings and Yoo 1993, Nowak and Kim 1997), defined for an event as the maximum strain divided by the sum of all the maximum strains in that span for that particular event. Weighting of strains is used to account for the difference in section moduli of the girders. The distribution factor for the *i*th lane is thus given by the equation;

$$DF_i = \frac{\varepsilon_i w_i}{\sum_{i=1}^{n} \varepsilon_i w_i} \tag{8}$$

where ε_i is the maximum strain at the girder in the *i*th lane.

w_i is the ratio of the section modulus of the instrumented girder to that of a typical interior girder. For Pioneer road bridge all instrumented girders have the same section modulus, giving $w_i = 1$.

n is the number instrumented girders

Table 3 Average dynamic distribution factors

		Girder 7	Girder 15	Girder 24	Girder 33
Before upgrading	Lane 1	**0.77**	0.07	0.09	0.07
	Lane 2	0.09	**0.63**	0.19	0.09
	Lane 3	0.07	0.15	**0.70**	0.08
	Lane 4	0.07	0.08	0.15	**0.70**
After upgrading	Lane 1	**0.61**	0.24	0.07	0.08
	Lane 2	0.23	**0.52**	0.17	0.08
	Lane 3	0.07	0.14	**0.58**	0.21
	Lane 4	0.08	0.11	0.25	**0.56**

Table 4 Ultimate live load strains

Ultimate live load strain before upgrading				
	Lane 1 ($\mu\varepsilon$)	Lane 2 ($\mu\varepsilon$)	Lane 3 ($\mu\varepsilon$)	Lane 4 ($\mu\varepsilon$)
Lane 1	**180**	16	21	16
Lane 2	55	**388**	117	55
Lane 3	42	89	**415**	47
Lane 4	28	32	60	**280**
Ultimate strain	305	525	613	398
Ultimate live load strain after upgrading				
	Lane 1 ($\mu\varepsilon$)	Lane 2 ($\mu\varepsilon$)	Lane 3 ($\mu\varepsilon$)	Lane 4 ($\mu\varepsilon$)
Lane 1	**135**	53	15	18
Lane 2	128	**290**	95	45
Lane 3	41	82	**340**	123
Lane 4	27	37	85	**190**
Ultimate strain	328	462	535	376

The average distributions factors for all peak events in each lane are shown in Table 3. The values clearly show that there is little distribution of loads between the girders, confirming findings of modal updating regarding the minor contribution of diaphragms to stiffness. Table 3 shows the ultimate live loads before and after upgrading. Using these factors, the live load ultimate strain before upgrading was found to be, 613 $\mu\varepsilon$ while the ultimate live load strain after upgrading was found to be 535 $\mu\varepsilon$ (Table 4).

7. Structural assessment

BD21/97 specifies verification of structural adequacy according to the relationship,

$$R \geq S \tag{9}$$

where R is the assessment resistance
S is the assessment load effect

This equation can be expressed in terms of strains as follows;

$$\varepsilon_u \geq \varepsilon_L + \varepsilon_D \tag{10}$$

ε_u is the yield strain
ε_L is the ultimate live load strain
ε_D is the dead and superimposed dead load strain

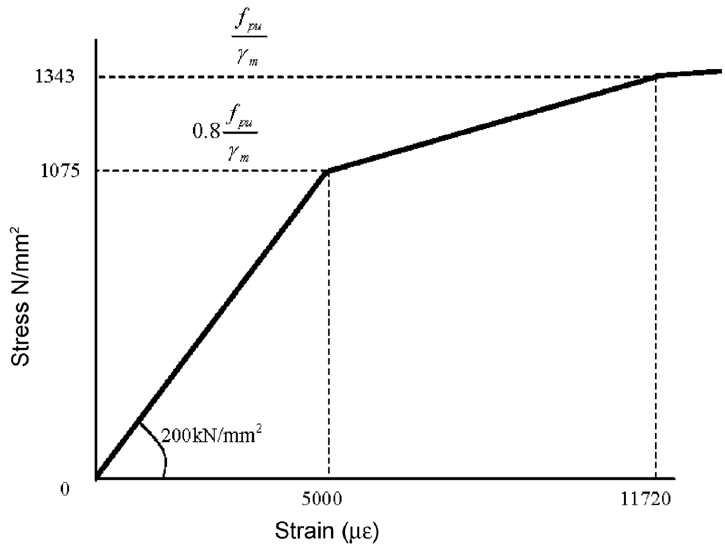

Fig. 17 Stress-strain curve for pre-stressing tendons

Table 5 Summary of load carrying capacity assessment

	Dead load & superimposed dead load strain ε_D	Live load strain ε_L	$\varepsilon_D + \varepsilon_L$	$(\varepsilon_D + \varepsilon_L)/\varepsilon_u$
Prior upgrading	306	613	919	0.73
Post upgrading	200	535	737	0.60

Fig. 17 shows the stress-strain curve for pre-stressing tendons derived following stress-strain curves provided in BD 44/95. First yield of tendons occurs at 5000 $\mu\varepsilon$ and the second yield strain occurs at 11720 $\mu\varepsilon$. Pres-stressing strain, taking into account 30% losses was found to be approximately 3762 $\mu\varepsilon$ giving a capacity of 1238 $\mu\varepsilon$ before first yield.

$\varepsilon_L + \varepsilon_D$ represents the most severe combination of dead and superimposed dead loads with ultimate live load. Ultimate live loads were estimated from monitoring data as presented above. Dead load and superimposed dead load strains were calculated from the updated analytical model described in Brownjohn *et al.* (2003).

The updated model produced a value of ε_D equal to 306 $\mu\varepsilon$ before upgrading and 200 $\mu\varepsilon$ after upgrading, suggesting an approximately 35% increase in flexural strength.

Table 5 summaries the assessment before and after upgrading.

8. Conclusions

The essential tools required to accurately assess the condition of a bridge are; a structural model that reflects the actual structural system in terms of boundary conditions, stiffness and material properties and a representative live load model which is bridge specific. A practical approach to applying field testing to bridge condition assessment has been demonstrated using statistical analysis and signal processing. It has been shown that the modified Gumbel method improves the Gumbel fit and accounts for more extreme conditions than the standard Gumbel method. Further research is underway to determine the optimal monitoring period. The worst combination of vehicle loading has been on a jam situation. The probability of such a condition occurring is small for most bridges and a probabilistic approach, based on structural monitoring, would be a plausible approach.

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