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# A semi-active acceleration-based control for seismically excited civil structures including control input impulses

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**Abstract.** Structural acceleration regulation is a means of managing structural response energy and enhancing the performance of civil structures undergoing large seismic events. A quadratic output regulator that minimizes a measure including the total structural acceleration energy is developed and tested on a realistic non-linear, semi-active structural control case study. Suites of large scaled earthquakes are used to statistically quantify the impact of this type of control in terms of changes in the statistical distribution of controlled structural response. This approach includes the impulses due to control inputs and is shown to be more effective than a typical displacement focused control approach, by providing equivalent or better performance in terms of displacement and hysteretic energy reductions, while also significantly reducing peak story accelerations and the associated damage and occupant injury. For earthquake engineers faced with the dilemma of balancing displacement and acceleration demands this control approach can significantly reduce that concern, reducing structural damage and improving occupant safety.

**Key words:** structural acceleration; quadratic regulator; semi-active control; civil structures; near-field earthquakes; LQRy.

## 1. Introduction

High-rise buildings are an effective way of utilizing limited ground space within cities. Although modern tall structures have been proven to retain integrity under normal loading conditions, their

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increased flexibility makes them prone to failure when subjected to extreme environmental loadings that are outside their linear, damage-free design envelope. Active and passive control methods have been proposed as a means of mitigating damage due to seismic excitation. However, active methods are dependent on large amounts of power, and the need for very specific tuning and a narrower performance band limit the performance of passive methods. As a result, low-power semi-active control methods and actuators have been developed to provide the wider performance band of active control approaches with pratical power requirements.

In recent years, research in the development of control systems has made significant progress in reducing civil structural response due to seismic excitations, as most recently reviewed by Housner *et al*. (1997). The recent research focus has been on active and semi-active control systems using hydraulics or "smart fluids" (e.g., Patten 1994a,b, Spencer *et al*. 1996). In either case, some form of optimal control method, such as LQR or H-infinity, is often employed as a basis for controlling structural response. For semi-active cases, some types of actuators are controlled to provide the commanded optimal control input, as with clipped optimal control using magneto-rheological (MR) dampers (Dyke *et al*. 1996a,b,c,d). Hence, regardless of the type of control, semi-active or active, optimal control methods play an important role in the management of seismic and other large environmental inputs.

The primary goal of civil structural control is to minimize damage, often interpreted as minimizing the inter-story drift values directly associated with permanent structural damage. As a result, a great deal of prior research has focused on control methods and analysis that are aimed at this single metric. Given that internal damage and occupant safety are approximately proportional to floor accelerations, the selection of a suitable performance metric is not necessarily straightforward (Spencer *et al*. 1994, Barroso *et al*. 2000). In particular, the link between acceleration and the permanent deformation remaining after an earthquake, especially for near-field seismic events that are characterized by a large pulse in the ground acceleration, provides a potentially very different metric and approach for managing structural response.

Near field seismic events are characterised by one or more large, sudden changes in the ground acceleration and can result in significant permanent deformation and structural damage. Given that displacement focused structural control methods can result in greater peak floor accelerations for such events, the ability to control the total acceleration is potentially important for mitigating damage and managing response for these seismic events (Barroso *et al*. 2002a,b). In addition, displacement focused structural control methods focus on managing the vibratory behaviour common to far-field seismic events. However they can perform less ably for near-field events due to their large magnitude acceleration pulses. This research examines optimal regulator control strategies that minimize the total structural acceleration energy of the structure as a function of control cost. The use of total structural acceleration, including acceleration contributions from ground motion and control inputs, leads to a straightforward design approach using well-accepted, optimal linear quadratic regulator design equations.

A great deal of research in civil structural control has been conducted, but very little of it has focused on controlling the structural acceleration directly. Acceleration feedback control has been investigated by several researchers, most notably Spencer *et al.* (1993) and Dyke *et al.* (1996e), and has been used within optimal control frameworks including optimal regulators and robust H-infinity control (Jabbari *et al.* 1995, Chase *et al*. 1999). However, these methods focus on using measured acceleration as a feedback signal to minimize displacements, velocities, or an objective function representing some form of potential or kinetic energy norm of the structural response. In contrast, this research examines the use of displacement and velocity feedback measurements to minimize the rms energy norm of the total structural acceleration within a standard linear quadratic regulator (LQR) approach.

## 2. Model definition and control

The basic model for developing quadratic optimal control designs starts with a basic second order linear model for the structural system defined:

$$\mathbf{M}\ddot{\mathbf{v}}(t) + \mathbf{C}\dot{\mathbf{v}}(t) + \mathbf{K}\mathbf{v}(t) = -\mathbf{M}\mathbf{S_g}\ddot{\mathbf{x}}_\mathbf{g}(t) + \mathbf{F_{ext}}(t) \tag{1}$$

where v, $\dot{\mathrm{v}}$, $\ddot{\mathrm{v}}$ are the relative displacement, velocity, and acceleration of the structure respectively, and **M**, **K**, and **C**, are the assembled global mass, stiffness, and damping matrices. For this research the stiffness matrix, **K**, includes provision for P-delta effects modelled by geometric stiffness reductions in flexural stiffness due to axial loading from gravity. The terms on the right hand side of Eq. (1) account for the external loading applied by the earthquake and actuator forces. The vector, $\mathbf{S_g}$, maps the earthquake ground acceleration, $\ddot{\mathbf{x}}_\mathbf{g}(t)$, to the horizontal degrees-of-freedom for each floor. Finally, the external inputs, $\mathbf{F_{ext}}(t) = \mathbf{S_{act}}\mathbf{F_{act}}$, consist of the control input vector, $\mathbf{F_{act}}$, and it's mapping matrix, $\mathbf{S_{act}}$, although it can include other additive terms for inputs such as wind loading. This system is easily converted to a linear, first order state space system defined:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} \mathbf{0} \\ -\mathbf{S_g} \end{bmatrix}\ddot{\mathbf{x}}_\mathbf{g}(t) + \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1} \end{bmatrix}\mathbf{F_{ext}}(t) \\ &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}_1\mathbf{F_{ext}}(t) + \mathbf{B}_2\ddot{\mathbf{x}}_\mathbf{g}(t) \end{aligned} \tag{2}$$

where the state vector $\mathbf{x}(t) = [\mathbf{v}(t)^\mathbf{T} \ \dot{\mathbf{v}}(t)^\mathbf{T}]^\mathbf{T}$ is a column vector comprised of the structural displacements and velocities respectively, **A** is the plant matrix, $\mathbf{B}_1$ is the mapping matrix for external forces or control inputs and $\mathbf{B}_2$ is the mapping matrix for the ground motion input.

The most straightforward means of regulating structural acceleration would be to use the relative acceleration term for the structural degrees-of-freedom as a regulated output signal, $\mathbf{y}(t)$, to be minimised, as has been done in some of the Benchmark Problem descriptions (Ohtori *et al*. 2000). However, contributions to structural acceleration can also include impulses from control input forces, in actively or semi-actively controlled structures, as well as the ground motion acceleration. The result is a system that is less tractable for control design and does not include these effects.

The ground motion contribution is important because to truly reduce the acceleration experienced by a given structural story would require allowing the structure to respond in a fashion that accounts for the ground motion as well. More specifically, if the relative structural acceleration is minimized, or eliminated, the structure would still experience accelerations that result from control inputs and the ground motion itself. Similarly, mathematically including impulses due to control forces as part of the regulated variables would account for this effect. Hence, a sudden semi-active resisting force can reduce displacement, but also adds an impulsive acceleration to the structure.

Within standard optimal control methods it is mathematically possible to regulate all three variables by defining a modified regulated output variable. Using Eqs. (1) and (2), the relative structural acceleration, $\ddot{\mathbf{v}}(t)$, can be defined:

$$\ddot{\mathbf{v}}(t) = [-\mathbf{M}^{-1}\mathbf{K} \quad -\mathbf{M}^{-1}\mathbf{C}]\mathbf{x}(t) - \mathbf{S_g}\ddot{\mathbf{x}}_\mathbf{g}(t) + \mathbf{M}^{-1}\mathbf{S_{act}}\mathbf{F_{act}}(t) \tag{3}$$

Eq. (3) defines the relative structural acceleration, in units of distance per second squared, and contains contributions from the unknown ground motion and the control input. Rearranging Eq. (3) results in the following regulated output signal, $\mathbf{y}(t)$, definition:

$$\mathbf{y}(t) = \ddot{\mathbf{v}}(t) + \mathbf{S_g}\ddot{\mathbf{x}}_\mathbf{g}(t) - \mathbf{M}^{-1}\mathbf{S_{act}}\mathbf{F_{act}}(t) = [-\mathbf{M}^{-1}\mathbf{K} \quad -\mathbf{M}^{-1}\mathbf{C}]\mathbf{x}(t) = \mathbf{C_1}\mathbf{x}(t) \tag{4}$$

where the output matrix $\mathbf{C_1} = [-\mathbf{M}^{-1}\mathbf{K} \; -\mathbf{M}^{-1}\mathbf{C}]$ maps the state vector, $\mathbf{x}(t)$, to this modified acceleration metric, or variable. As a result, Eq. (4) defines a metric for structural acceleration in a form easily amenable to the development of optimal control gains for semi-active, clipped optimal control implementations, such as those performed by Dyke *et al.* (1996b,c,d).

It is important to note that the signal, $\mathbf{y}(t)$, in Eq. (4) is not a measured output signal but a mathematical metric of acceleration terms to be regulated by the control system. Hence, this approach is about regulating a variable that includes the relative structural and ground motion accelerations, as well as the acceleration impulses related to potential control inputs. As such, it is a mathematically formulated design metric to be investigated in this research.

The typical optimal control approach minimizes a signal defined as in Eq. (4) in an optimal tradeoff with control effort using a standard quadratic cost function.

$$J = \int_0^\infty (\mathbf{y}^T(t)\mathbf{Q}\mathbf{y}(t) + \mathbf{u}^T(\mathbf{t})\mathbf{R}\mathbf{u}(t))dt \tag{5}$$

where $\mathbf{Q}$ and $\mathbf{R}$ are positive definite, symmetric, user defined weighting matrices, and $\mathbf{u}(t)$ is the control input. Standard derivations found in a variety of references lead to a state feedback control input defined: $\mathbf{u} = -\bar{\mathbf{R}}^{-1}\mathbf{B}_1^\mathrm{T}\mathbf{P}\mathbf{x} = -\mathbf{K}_{\mathbf{g}ain}\mathbf{x}$, where the matrix $\mathbf{P}$ is the symmetric, positive-definite solution to an algebraic Riccati equation (ARE) (Kwakernaak and Sivan 1972, Soong 1988, Meirovitch 1990). This optimal ARE is defined:

$$\mathbf{A}^\mathbf{T}\mathbf{P} + \mathbf{P}\mathbf{A} + \mathbf{C}_1^\mathbf{T}\mathbf{Q}\mathbf{C}_1 - \mathbf{P}\mathbf{B_1}\mathbf{R}^{-1}\mathbf{B}_1^\mathbf{T}\mathbf{P} = 0 \tag{6}$$

where $\mathbf{P} = \mathbf{P}^\mathbf{T} > \mathbf{0}$, is the desired symmetric, positive-definite matrix solution for the equation that minimizes the cost function in Eq. (5) subject to the symmetric, user defined weighting matrices $\mathbf{Q}$ and $\mathbf{R}$.

This LQRy approach is unique for its definition and use of total acceleration as a regulated control variable. Another specific difference from typical civil structural control approaches is the inclusion of the acceleration contributions due to control inputs as well as those from the ground motion. This last term enables the controller to control the structure so that any acceleration experienced by occupants and equipment is regulated. Therefore, the control design is not restricted only to structural metrics, such as relative displacements or accelerations.

## 3. Case study definition

The following sections describe the structure, earthquake suites and non-linear models used to obtain an accurate simulation of true structural behavior and a complete evaluation of the semi-active total acceleration control method presented. Recent studies have shown that for civil structural applications the appropriate non-linear behaviours must be included (Barroso *et al*. 2000). Therefore, to obtain a practical evaluation of structural performance two key factors must be included in the modelling and simulation. The first is the inclusion of structural non-linear behaviour, as both the structural damage and large-motion hysteretic structural damping processes are inherently non-linear. In addition, as the response of a structure, and damage that may result, is highly dependent on the nature of the excitation, the second factor is the use of a wide variety of realistic ground motion excitations that are representative of broad ranges of potential earthquakes scaled to have similar probability of occurrence. Using suites of such seismic inputs, each suite with specific different probabilities of occurrence, rather than one or a few, enables the use of statistical methods to completely characterize the structural response and determine the true impact of the structural control.

### 3.1 Structure

The SAC3 structure is a 3-story steel moment-resisting frame building (SMRF) designed as part of the SAC steel project for the Los Angeles area. It is also being utilized for the Third Generation of Structural Control Benchmark problem (Ohtori *et al*. 2000). The building conforms to local code requirements and is designed for gravity, wind, and seismic loads, with a basic live load of 2.4 kPa (50 psf). The structural system consists of steel perimeter-moment-resisting-frames and interior gravity-frames tied to longitudinal elements by rigid in-plan floor diaphragms. The columns are fixed at the base and extend the full height of the structure. For detailed information on the structural system and its uncontrolled behavior, the reader is referred to the work of Krawinkler and Gupta (Gupta 1999, Krawinkler and Gupta 1998). For this research the floor slabs are assumed to be rigid in-plane, and 2% Rayleigh damping is enforced at the first mode and at a period of 0.2 seconds. The resulting modal properties for this structure are given in the Table 1.

Table 1 Modal properties of the SAC3 elastic structure model

| Mode | SAC3 Frame | |
|---|---|---|
| | Period | $\xi$ |
| 1st | 1.02 sec | 2.0% |
| 2nd | 0.33 sec | 1.5% |
| 3rd | 0.17 sec | 2.2% |

### 3.2 Earthquake suites

Suites of ten different earthquake time histories, with two orthogonal directions for each history, were generated (Sommerville *et al*. 1997) for the SAC project to represent ground motions having probabilities of excedance of 50% in 50 years, 10% in 50 years, and 2% in 50 years in the Los Angeles region. These sets of ground motions are referred to as the 50 in 50 or Low Set, the 10 in 50 or Medium Set, and the 2 in 50 or High Set, respectively. The High Set are primarily near-field ground motions, while the Low Set are primarily vibratory far-field excitations, and the Medium Set represent a mid-range combination. The acceleration histories have been scaled to conform to the 1997 NEHRP design spectrum for firm soil and the specified return periods. Finally, these sets correspond to return periods of 72 years, 474 years and 2474 years, respectively. Hence, the 50 in 50 Low Set are still relatively “large” (magnitude) earthquakes given the approximately once in a lifetime likelihood of occurring.

### 3.3 Modeling

In selecting a nonlinear evaluation method for structural dynamics, the level of detail needs to be matched with the quantities that are of interest. This study is concerned with response values and demands on the global or story level, for which a lumped plasticity model of the frame elements with centerline dimensions is appropriate. The frame members are modeled as linear elastic beam-column elements. Nonlinear behavior is added by nonlinear torsional spring connection elements at the ends of the beam-column elements to add hysteretic and yielding behaviours.

Various hysteretic models for the restoring force of an inelastic structure have been developed in recent years. The model chosen for the nonlinear rotational spring is the smoothly-varying Bouc-Wen hysteretic model (Wen 1976). This model includes a number of parameters, enabling a mathematically tractable state-space representation capable of expressing several hysteretic properties. The force-deformation curve for a given element is described by a dimensionless parameter, $z(t)$:

$$\dot{z}(t) = r(\dot{t})\left[1 - 0.5(1 + \mathrm{sgn}(r(\dot{t})z(t)))\left|\frac{z(t)}{Y}\right|^{n}\right] \tag{8}$$

where $r(t)$ is the relative element deformation, $Y$ is the yield displacement, and $n$ is a shaping parameters (Barroso 1999, Wen 1976). These terms can be assembled and added to the state space system of Eq. (2) where the stiffness broken into linear and non-linear, hysteretic components based on Eq. (8). The equations of motion can then be written in a nonlinear state-space format.

$$\dot{\mathbf{x}}_s(t) = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} & -\mathbf{M}^{-1}\mathbf{K}_{\mathbf{H}} \\ \mathbf{0} & \left[\frac{d\mathbf{z}}{d\mathbf{v}}\right] & \mathbf{0} \end{bmatrix}\mathbf{x}_s(t) - \begin{bmatrix} \mathbf{0} \\ \mathbf{S}_g \\ \mathbf{0} \end{bmatrix}\ddot{\mathbf{x}}_g(t) + \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1} \\ \mathbf{0} \end{bmatrix}\mathbf{F}_{\mathbf{ext}}(t) \tag{10}$$

Where $\dot{\mathbf{z}}(t) = [d\mathbf{z}/d\mathbf{v}]\dot{\mathbf{v}}(t)$ relates the hysteretic model and structural motion and is a non-square matrix (Barroso 1999), and $\mathbf{x}_s(t) = [\mathbf{v}(t)^{\mathbf{T}}\ \dot{\mathbf{v}}(t)^{\mathbf{T}}\ \mathbf{z}(t)^{\mathbf{T}}]^{\mathbf{T}}$ is an augmented state vector that includes the hysteretic vector $\mathbf{z}(t)$.

The procedure utilized to account for $P$-$\Delta$ effects is derived directly from the $P$-$\Delta$ effect on an individual axially loaded cantilever column, resulting in an effective reduction in flexural stiffness. This relationship between axial forces and the lateral flexibility of the column can be put into matrix form using an additional geometric stiffness matrix (White and Hajjar 1991).

Finally, the hysteretic model is extended to include yielding using a bi-linear plasticity model incorporated with the Bouc-Wen model. Plastic moments are assumed to form at, or near, the joint connections utilizing non-linear connections as justified by strong column, weak girder (SCWG) assumptions within the building design codes. The plastic yielding limit is defined using the plastic moment capacity of the beams. Hence, under strong motion simulation, yielding will occur following the bi-linear elastic-plastic definition, with a post-yielding stiffness ratio of 0.03, when this limit is exceeded.

### *3.4 Performance metrics*

Given the structural models and excitations defined, simple measures of the performance of the controlled structure must be defined to measure efficacy. As a representative value of the response, the primary statistic of interest is a “best” or central estimate of the data, along with larger or maximum response values. From this concept, the results obtained can be statistically quantified over an entire suite of inputs using appropriate lognormal statistics to find the 50th and 84th percentile values for these structural responses (Kennedy *et al.* 1980).

A wide consensus exists in the earthquake engineering community that for moment-resisting frames the inter-story drift demand, expressed in terms of the inter-story drift ratio, is the best indicator of expected damage (Krawinkler and Gupta 1998). Residual drift is often used as an indicator of inelastic damage to the structure; however the peak permanent drift may occasionally be larger if the record contains more than one large pulse. Acceleration demands are of concern for the nonstructural components of the building. While peak values provide a good indication of performance, the resulting information is incomplete, as it does not take into account the cumulative damage to the structure. Experimental investigations have demonstrated that structural damage is a function of both peak as well as cumulative values (Krawinkler *et al.* 1975, Tussi and Yao 1983, Yao and Munze 1968, Bannon and Veneziano 1982). Normalized hysteretic energy (NHE) provides a good indication of cumulative damage in steel structures, and the mean values of NHE are evaluated for each set of ground motions.

### *3.5 Statistical analysis*

By presenting the 50th and 84th percentile levels the change in lognormal distribution shape for the different controlled cases may be observed. The changes between the uncontrolled and controlled cases may be classed into three general categories:

i) Case 1 – the geometric mean decreases while the 84th percentile increases. This type of movement is a result of an increase in distribution variance, which is likely due to the extreme outliers at the tail of the distribution. A performance improvement may be observed for these outliers; however, due to a potentially greater decrease for the majority of earthquakes and the subsequent lowering of the geometric mean, the distribution is stretched and the 84th percentile level increases. Hence, for this case the numerical average (~65th percentile) may give a clearer indication of the response improvement (Limpert *et al.* 2001).

ii) Case 2 – the geometric mean increases while the 84th percentile decreases due to a decrease in distribution variance. This result is potentially due to effective controller performance for the larger excitations that produce outliers where the level of actuation is high, with smaller improvements in performance during the bulk of the excitations.

iii) Case 3 – both the geometric mean and 84th percentile levels decrease indicating generally consistent improvements in all of the responses in the suite.

It should be noted that the structural hysteretic energy does not follow a lognormal distribution, unlike peak drift, permanent drift, and peak acceleration (Breneman 2000). Instead, the standard "counted" mean and its 84th percentile are presented. The counted mean is the data point at which half of the response values fall below, meaning that for a sample of 20 values linear interpolation is used to obtain the value between the 10th and 11th highest data points. Similarly, the counted 84th percentile is the data point below which 84% of the response values fall.

### *3.6 Control architecture and implementation*

The SAC3 structure was modeled using non-linear finite elements for the beam and column joint connections. The entire model consists of 50 degrees of freedom as all three bays are modeled, although the lateral deflections are slaved to a single node within each story to maintain the rigid in-plane floor assumption. A maximum semi-active actuator force of 2.0 MN (450 kips) was selected, representing 13.8% of the total building weight of 14.5 MN (3255 kips). This level of semi-active actuation represents the peak dissipative force allowed, and is much lower than that used in many prior investigations.

Actuator authority is distributed via an actuator on the floor with angled tendons to the first story, as shown in Fig. 1, and the lateral force applied is the total force reduced by the cosine of the angle. The distribution of actuator authority up the structure to other floors has little impact on this squat, shear structure, however the best approximation is roughly proportional to the shear force diagram for the structure at that story, a method which has been shown to be the most effective for passive and semi-active systems (Pekcan *et al*. 2000a,b, Hunt 2002, Lin *et al*. 1998, 2000). The control gains are obtained for this model by solving Eq. (6) for a reduced order 3 degree-of-freedom-model that accounts only for lateral motion, using full state feedback. The gains are then applied to the full non-linear finite element simulation of Eq. (10) using only those lateral deflections and velocities as feedback. Note that the semi-active nature of the system means that only forces that resist motion are applied and the actuator is essentially open otherwise. Therefore, the semi-active control system as simulated is essentially applied to the full, non-linear model as a direct output feedback system.

As the actuator force is dependent on the floor displacements relative to the equilibrium position of the structure, the definition of the equilibrium point is critical (Hunt *et al*. 2002). When using a linear, or largely linear, structural model, as in many previous structural control investigations, the

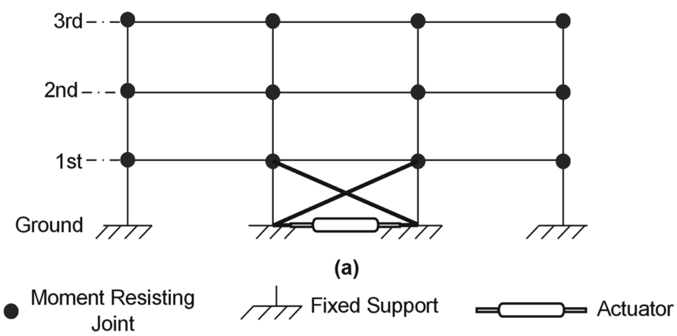


Fig. 1 SAC3-A1 actuator architectures

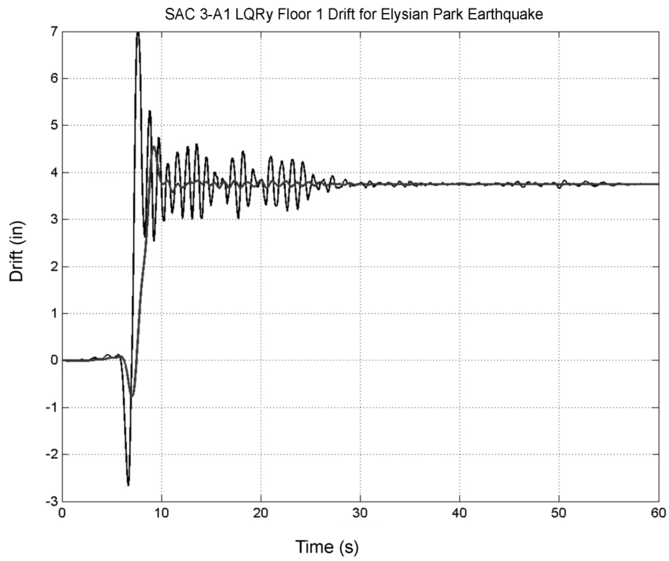


Fig. 2 Example of time-varying equilibrium tracking for the SAC-3 uncontrolled floor 1 drift – Elysian Park earthquake

equilibrium point is clearly the original building position. However, when a memory of non-linear structural damage is included in the model, the permanent deformations that result from strong motions lead to an equilibrium position that changes with time.

If a semi-active controller allows such large responses, the structure can re-yield back toward its original position, resulting in an apparent improvement in performance through decreased permanent drift. However, when a beam-column joint is damaged through large joint rotations, the flexibility of the joint is increased dramatically. Hence, if large forces act during the same event to restore the building back to its un-yielded position, the decreased structural stiffness can result in very large structural deformations and more extreme structural damage as reflected in the NHE metric.

The time-varying equilibrium point, called the moving-zero hereafter, is obtained using a moving average of the floor displacements. At each time step a new displacement value is added to a column vector that is then averaged to provide a simple low-pass filtered mean displacement. It was determined that a 2 second long filter provides a moving-zero sufficiently insensitive to transient peaks while still providing adequate tracking with acceptable lag. An example of the moving-zero tracking is shown in Fig. 2 for the uncontrolled SAC3 first floor drift subject to the large, near-field Elysian Park earthquake from the 2 in 50 Set, where the bold line represents the time-varying equilibrium point. This figure is also a good example of the large pulse and resulting permanent deflection that occurs from this type of near-field event.

Note that in Fig. 2, following the large pulse all displacement is positive valued and without correction the proportional control forces based on displacement feedback would act in a destabilizing direction, augmenting the motion rather than opposing it for half of each vibration cycle. As a result, displacement values used in the control law are computed relative to the instantaneous moving-zero value for the relevant lateral degrees-of-freedom, rather than to the absolute zero. The resulting LQRy state feedback control law for the commanded control input is defined as:

$$u_i = \begin{cases} -\mathbf{K}_{\mathrm{LQRy}}(i,:)\mathbf{v}_{\mathrm{temp}} & (\text{if } u_i < F_{i\max}) \\ -\mathbf{F}_{i\max}\,\mathrm{sgn}(v_i) & (\text{if } u_i < F_{i\max}) \end{cases} \tag{11}$$

Where $\mathbf{K}_{\mathrm{LQRy}}(i, :)$ is the $i$th row of the optimal state feedback gain matrix, $\mathbf{v}_{\mathrm{temp}}$ is a reduced state vector containing displacements and velocities of the lateral degrees-of-freedom relative to the moving-zero, $F_{i\max}$ is the $i$th actuator saturation force, $v_i$ is the displacement of the floor to which the actuator is attached, relative to the moving-zero, and sgn( ) is the sign function. Eq. (11) provides the control input for a clipped optimal input so that when the peak actuator authority is exceeded the control input is clipped to ensure a realistic simulation and test of the LQRy total structural acceleration control systems developed.

## 4. Simulations and analysis

The LQRy clipped optimal controller tries to optimise structural performance in terms of the total structural acceleration, providing a very different approach from typical displacement focused methods of structural control. Numerical simulations were performed for all 20 earthquakes in each seismic input suite and the results statistically quantified using lognormal distributions. For each of the structural response performance metrics, the values at the 50th and 84th percentiles as well as the numerical average, which is approximately the 65th percentile of a lognormal distribution, were obtained.

The LQRy clipped optimal controller resulting from Eq. (6) for the SAC3-A1 control architecture in Fig. 1 produces state feedback gains designed to minimise the structural acceleration metric defined in Eq. (4). The controller designed employs only the lateral displacements and velocities as measured feedback signals and is essentially a direct output feedback controller. The specific controller design for the 3-story linear shear model uses an equal performance weighting of $Q = 5$ on each of the three floors, while the control cost weighting was set at $R = 1.6 \times 10^{-6}$.

Table 2 presents results for the high earthquake suite, where the impact of designing for structural accelerations is striking. In this table F1, F2, and F3 refer to the response of floors 1-3 respectively and Gr refers to the ground motion response. Regardless of which statistical performance measure is assessed, the peak and permanent drift demands are significantly reduced for the first floor, showing an overall Case 3 distribution shift. For floors 2 and 3, a Case 1 distribution shift is observed, with reductions seen at the 50th and 65th percentiles. The average reductions in the geometric mean of the peak and permanent drifts are 13.0% and 23.8% respectively, while the average geometric mean peak acceleration is reduced by 15.8%. This last result includes a reduction observed even on the

Table 2 Results for high suite with LQRy controller - percent change from uncontrolled

| | Numerical average | | | | Geometric-mean/50th Percentile | | | | 84th Percentile | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Gr | F1 | F2 | F3 | Gr | F1 | F2 | F3 | Gr | F1 | F2 | F3 |
| Peak drift (%) | | –22.35 | –9.40 | –7.18 | | –22.62 | –8.66 | –7.58 | | –22.99 | –10.02 | –5.10 |
| Permanent drift (%) | | –17.68 | –14.02 | –12.42 | | –15.79 | –26.83 | –28.87 | | –23.38 | 18.10 | 14.07 |
| Peak acceleration (%) | | –15.90 | –21.12 | –9.71 | | –16.31 | –21.66 | –9.54 | | –14.32 | –19.79 | –10.88 |
| Hysteretic energy (%) | –47.79 | –23.11 | –10.31 | –20.78 | –56.75 | –20.38 | –9.65 | –21.65 | –47.88 | –20.16 | –16.69 | –22.02 |

Table 3 Results for medium suite with LQRy controller - percent change from uncontrolled

| | Numerical average | | | | Geometric-mean/50th Percentile | | | | 84th Percentile | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Gr | F1 | F2 | F3 | Gr | F1 | F2 | F3 | Gr | F1 | F2 | F3 |
| Peak drift (%) | | –22.01 | –7.59 | –7.83 | | –20.81 | –7.97 | –9.55 | | –24.65 | –6.32 | –5.24 |
| Permanent drift (%) | | –42.44 | –16.03 | –5.53 | | –64.19 | –19.01 | –24.87 | | –21.81 | 8.82 | 94.19 |
| Peak acceleration (%) | | –23.24 | –30.21 | –13.32 | | –25.04 | –30.28 | –13.07 | | –20.77 | –30.67 | –14.85 |
| Hysteretic energy (%) | –69.37 | –38.34 | –24.76 | –40.42 | –76.21 | –40.85 | –30.00 | –57.61 | –62.47 | –36.25 | –21.41 | –26.07 |

Table 4 Results for low suite with LQRy controller - percent change from uncontrolled

| | Numerical average | | | | Geometric-mean/50th Percentile | | | | 84th Percentile | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Gr | F1 | F2 | F3 | Gr | F1 | F2 | F3 | Gr | F1 | F2 | F3 |
| Peak drift (%) | | –23.48 | –14.94 | –18.78 | | –24.22 | –18.30 | –22.22 | | –21.93 | –10.92 | –17.36 |
| Permanent drift (%) | | –44.81 | –39.42 | –38.79 | | –73.37 | –84.02 | –87.76 | | –72.45 | –81.55 | –85.63 |
| Peak acceleration (%) | | –41.09 | –45.00 | –24.91 | | –45.48 | –45.70 | –25.25 | | –38.93 | –45.21 | –25.05 |
| Hysteretic energy (%) | –71.91 | –61.14 | –45.25 | –57.77 | –99.25 | –98.13 | –99.96 | –100.00 | –93.24 | –54.25 | –44.77 | –77.30 |

first floor to which the actuator is attached, where typical displacement focused control methods usually result in increased floor acceleration as a tradeoff for reduced displacement.

The results for the medium earthquake suite in Table 3 show the same general trend as the high suite, with differences attributable to the different characteristics of the earthquakes within each suite. Each of the performance measures show decreased structural demand on all three floors, other than the 84th percentile permanent drifts for floors 2 and 3. It should be noted that the absolute value of these increases are relatively small when compared to the uncontrolled case. Average acceleration reductions are approximately 23% across each of the statistical levels and floors.

The results for the low earthquake suite in Table 4 clearly show large reductions in each of the measures of structural demand for these relatively large ground motions. For the high and medium suites an increase in the 84th percentile was observed for the permanent drifts of floors 2 and 3, however for the low suite large reductions of over 80% are seen, indicating a Case 3 distribution shift of all performance measures. The average geometric-mean peak acceleration is reduced by 38.8%, while hysteretic energy is reduced by 99.3%. The average geometric-mean peak and permanent drifts are reduced by 21.7% and 81.7% respectively.

The same simulations were run using a semi-active LQR clipped optimal controller that weighted displacement reduction 10x higher than velocity in the weighting matrix, **Q**, to compare displacement focused design to this new approach. The results for each controller are normalized to ensure a simple, fair comparison. The normalized results, or normalized performance values (NPVs) are defined as the average geometric mean (50th percentile) response across all three floors divided by the absolute change in counted-mean normalised hysteretic energy (NHE) between the controlled and uncontrolled cases (a negative number). NHE was employed because for semi-active systems the reduction in structural hysteretic energy in the response is proportional to the control effort or energy removed via the dissipative control system.

Positive NPVs indicate that a response reduction is obtained from the control input, whereas a negative value indicates that control expenditure results in an increase of the average response

Table 5 Hysteretic energy normalized average geometric-mean results for LQRy and LQR

| | High suite | | Medium suite | | Low suite | |
|---|---|---|---|---|---|---|
| | % Reduction | NPV | % Reduction | NPV | % Reduction | NPV |
| LQR | | | | | | |
| Average peak drift | −11.60 | **0.35** | −16.30 | **0.97** | −28.20 | **6.83** |
| Average permanent drift | −19.61 | **0.59** | −31.92 | **1.89** | −88.07 | **21.32** |
| Average peak acceleration | 8.83 | **−0.27** | 11.39 | **−0.68** | −18.80 | **4.55** |
| LQRy | | | | | | |
| Average peak drift | −12.95 | **0.49** | −12.78 | **0.96** | −21.58 | **5.29** |
| Average permanent drift | −23.83 | **0.91** | −36.02 | **2.71** | −81.72 | **20.04** |
| Average peak acceleration | −15.84 | **0.60** | −22.80 | **1.72** | −38.81 | **9.51** |

metric. The more positive the NPV the greater the response reductions per unit of control dissipated energy. Hence, this normalization technique essentially provides a cost-benefit comparison. Table 5 presents the NPV data for the LQRy and LQR simulations.

The general trend in the NPVs across the three earthquake suites in Table 5 is an increasingly positive value as the magnitude of the suite decreases. This trend shows that for each unit of control input expended, the improvements in structural response increase as the magnitude of the earthquake suite decreases, as is intuitively expected when the actuator authority is saturated at a practical, relatively small level that is likely to be saturated throughout the strong motion portion of the response. The approximately exponential relationship between ground excitations of the three earthquake suites is mirrored by the NPVs. As the medium and low earthquake suites represent 72- and 474-year mean return period events this trend is beneficial, as extremely large performance reductions are obtained for these more likely, yet still sizable, seismic events.

More specifically, the LQRy clipped optimal controller has positive NPVs across each of the performance measures for each of the three earthquake suites, unlike the LQR controller that trades off displacement reduction with increased acceleration. For the high earthquake suite consisting of large magnitude, near-field earthquake ground motions, the LQRy controller has the higher NPVs for each of the four performance measures. For the high and medium suites the LQRy NPVs are significantly higher with the exception of peak drift for the medium suite, which is of comparable value. Similar trends hold true for the average percentage reductions indicating that the higher NPVs are not merely an artefact of an efficient controller with little impact on the structural response, as all the reductions are quite sizable for earthquakes of these magnitudes given the relatively low total actuator authority. For the low suite both controllers do extremely well in both NPV and average percentage reduction in each of the performance metrics. While the focus of the LQRy control design is in the minimisation of total structural accelerations in an attempt to increase occupant safety, these results highlight the benefit of acceleration control as a means of increasing internal occupant safety and also reducing external structural damage.

Overall, there are two primary results from the data presented. The first is the ability of this structural acceleration focused control design to significantly improve both displacement and peak acceleration performance metrics. This approach breaks accepted compromises between increased accelerations and reduced displacements. As a result, a control designer does not have to choose between mitigating structural damage, and mitigating damage to equipment or improved occupant safety. The second is the effectiveness and interpretive ability conferred by statistically quantifying the impact of the control methods applied, using several performance metrics associated with damage, in terms of how they modify the shape of the response distribution. This last result is enabled through the use of scaled suites of ground motions, rather than a few selected inputs.

## 5. Conclusions

This paper presents optimal linear quadratic regulators designed to control a modified structural acceleration metric for a semi-actively controlled structure subjected to ground motion. An novel acceleration-focused regulated variable suitable for the LQRy optimal quadratic output regulator design is defined and includes acceleration contributions due to control inputs and the ground motion. This approach is employed to design and simulate clipped optimal, semi-active control systems based on regulating total structural acceleration to mitigate damage to the structure as well as the occupants or equipment inside.

The application side of the research focuses on controlling total structural acceleration to minimize structural response and damage in civil structures subjected to large seismic events, particularly large magnitude, near field earthquakes. A realistic, non-linear structural model that includes hysteresis and permanent structural deformation is presented as part of a case study for this new control approach along with a set of response metrics that are representative of several different accepted measures of damage. The inputs for this case study consist of three suites of 20 seismic records from which lognormal statistical quantification of the structural response, and hence control efficacy, is developed. In particular, control efficacy is measured based on the changes in the statistical distribution of the structural responses at the 50th, 84th and approximately 65th percentiles, rather than direct measures of improvement for individual or small groups of ground motions. As a result, the statistical distributions can be generalised to determine broad ranges of controlled performance for the return period of each suite of ground motions and its probability of occurrence. Therefore, the statistical quantification presented is suitable for use in standard seismic hazard analyses and design.

Comparisons are made between the LQRy control method and a typical displacement focused LQR control approach for civil structural control, with performance measured for each against the uncontrolled response. The comparison is enabled by a normalization scheme that measures control performance versus its control effort cost. The results show that while the two controllers have similar displacement reduction performance the LQR controller tends to increase peak story accelerations in the attempt to regulate displacement, while the LQRy controller regulates and reduces both displacement metrics and peak story accelerations. Hence, the acceleration focused control approach presented does not compromise between mitigating structural damage due to displacements and reducing internal equipment damage or occupant injury due to high peak accelerations.

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