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A high precision shear flexible element for bending analysis of thick/thin triangular plate

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Abstract. A high precision shear deformable triangular element has been proposed for bending analysis of triangular plate. The element has twelve nodes at the three sides and four nodes inside the element. Initially the element has thirty-five degrees of freedom, which has been reduced to thirty by eliminating the degrees of freedom of the internal nodes through static condensation. Plates having different boundary conditions, side ratios (*b/a*) and thickness ratios (*h/a* = 0.001, 0.1 and 0.2) have been analyzed using the proposed shear locking free element. Concentrated and uniformly distributed transverse loads have been used for the analysis. The formulation is made based on first order shear deformation theory. For validation of the present element and formulation few results of thin triangular plate have been compared with the analytical solutions. Results for thick plate have been presented as new results.

Key words: shear locking free element; first order shear deformation theory; static condensation; triangular.

1. Introduction

The finite element method (Zienkiewicz and Taylor 1988) is regarded as one of the most versatile analysis tool specifically in structural analysis problems. The plate bending is one of the first problem where finite element was applied in early sixties. The initial attempt was made with thin plate based on Kirchhoff's hypothesis where a number of difficulties were encountered. These are mostly concerned with the satisfaction of normal slope continuity along the element edges. Subsequently, the method has been applied to thick plates based on Reissner-Mindlin's hypothesis where the above problem has been avoided by considering the transverse displacement (w) and

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rotations of normal (θ_x and θ_y) as independent displacement components. Amongst the thick plate elements developed so far, the most prominent elements are the isoparametric elements, which became very popular. Though these elements are quite elegant, they involve certain problems such as shear locking, stress extrapolation, spurious modes etc. Keeping these aspects in view some research workers have tried to develop an element, which will be free from the above problems. The necessity has been geared up further with the wide use of fibre reinforced laminated composite which is weak in shear due to its low shear modulus compared to elastic modulus. As an outcome of these facts, some elements have been proposed by Petrolito (1989), Yuan and Miller (1989), Batoz and Katili (1992), Zhongnian (1992), Wanji and Cheung (2000) and a few others.

Exact thin plate solutions for triangle plates are available only for certain boundary conditions (Timoshenko and Woinowsky-Krieger 1959). For complex boundary conditions a numerical method must therefore be used. Simple polynomials in two directions were proposed to study the vibration of right isosceles triangular (Kim and Dickinson 1990) plate problem. Vibration of cantilevered triangular plates was studied by Bhat (1987) using the boundary characteristic orthogonal polynomials in two variables. Plates of arbitrary shaped subjected to static load was analysed by K. M. Liew (1992) using the principle of minimum potential energy with admissible *pb*-2 Ritz functions. A majority of the arbitrarily shaped plate problems in bending was solved numerically by finite element method (Gallaghar 1975 and Zienkiewics 1971). Wilt *et al*. (1990) analysed a composite right angle triangular plate by four-node quadrilateral plate/shell element.

The aim of the present paper is to propose a thick plate bending element for the analysis of triangular plates. It is a high precision element and it has the advantage that plates of any shapes can be modelled by this element, as it has a triangular geometry. In this element a fourth order complete polynomial has been used to express transverse displacement w while rotations of the normal (θ_x and θ_y) have been expressed with complete cubic polynomials. Thus the interpolation function of w is one order higher than those of θ_x and θ_y, which has helped to make this element free from locking in shear and other relevant problems.

2. Formulation

The formulation is based on Reissner-Mindlin's plate theory. It has been assumed that the transverse displacement is small compared to plate thickness and the plate material will follow Hooke's law. The middle plane of the plate has been considered as the reference plane.

A typical element shown in Fig. 1, has sixteen nodes. The location of the nodes 3, 7 and 11 are at the midpoint of the corresponding sides while nodes 2, 4, 6, 8, 10 and 12 are located at a distance of one third of the length of the corresponding sides from their nearest corner. The co-ordinates of the nodes 13, 14, 15 and 16 are (1/2, 1/4, 1/4), (1/4, 1/2, 1/4), (1/4, 1/4, 1/2) and (1/3, 1/3, 1/3) respectively. The degrees of freedom at nodes 1 to 12, (except 3, 7 and 11) are w, θ_x and θ_y. It is only w at nodes 3, 7, 11, 13, 14 and 15. The centroidal node has θ_x and θ_y as degrees of freedom.

The displacement component (w) and rotations of the normal (θ_x and θ_y) have been taken as independent field variables. A complete fourth order polynomial has been used to approximate w while other independent field variables have been approximated with third order polynomials. These are as follows

$$w = [P_1]\{\gamma\} \tag{1a}$$

$$\theta_x = [P_2]\{\mu\} \tag{1b}$$

and
$$\theta_y = [P_2]\{\lambda\} \tag{1c}$$

where

$$[P_2] = [L_1^3 \;\; L_2^3 \;\; L_3^3 \;\; L_1^2L_2 \;\; L_2^2L_1 \;\; L_2^2L_3 \;\; L_3^2L_2 \;\; L_3^2L_1 \;\; L_1^2L_3 \;\; L_1L_2L_3],$$

$$[P_1] = [L_1^4 \;\; L_2^4 \;\; L_3^4 \;\; L_1^3L_2 \;\; L_2^3L_1 \;\; L_2^3L_3 \;\; L_3^3L_2 \;\; L_3^3L_1 \;\; L_1^3L_3 \;\; L_1^2L_2^2 \;\; L_2^2L_3^2 \;\; L_3^2L_1^2 \;\; L_1^2L_2L_3 \;\; L_1L_2^2L_3 \;\; L_1L_2L_3^2],$$

$$\{\gamma\} = [\alpha_1 \;\; \alpha_2 \;\; \alpha_3 \;\; \alpha_4 \;\; \alpha_5 \;\; \alpha_6 \;\; \alpha_7 \;\; \alpha_8 \;\; \alpha_9 \;\; \alpha_{10} \;\; \alpha_{11} \;\; \alpha_{12} \;\; \alpha_{13} \;\; \alpha_{14} \;\; \alpha_{15}]^T,$$

$$\{\mu\} = [\alpha_{16} \;\; \alpha_{17} \;\; \alpha_{18} \;\; \alpha_{19} \;\; \alpha_{20} \;\; \alpha_{21} \;\; \alpha_{22} \;\; \alpha_{23} \;\; \alpha_{24} \;\; \alpha_{25}]^T,$$

and $\{\lambda\} = [\alpha_{26} \;\; \alpha_{27} \;\; \alpha_{28} \;\; \alpha_{29} \;\; \alpha_{30} \;\; \alpha_{31} \;\; \alpha_{32} \;\; \alpha_{33} \;\; \alpha_{34} \;\; \alpha_{35}]^T$.

Now the above Eqs. (1a-1c) may be substituted appropriately at the different nodes of the element (Fig. 1) with corresponding values of L_i of the nodes, which will give the relationship between the unknown coefficients of the above polynomials (Eqs. 1a-1c) and the nodal degrees of freedom as

$$\{\delta_e\} = [A]\{\alpha\} \quad \text{or}$$

$$\{\alpha\} = [A]^{-1}\{\delta_e\} \tag{2}$$

where

$$\{\alpha\} = [\alpha_1 \;\; \alpha_2 \;\ldots\; \alpha_{35}]$$

$$\{\delta_e\}^T = [w_1 \;\; \theta_{x1} \;\; \theta_{y1} \;\; w_2 \;\; \theta_{x2} \;\; \theta_{y2} \;\; w_3 \;\; w_4 \;\; \theta_{x4} \;\; \theta_{y4} \;\; w_5 \;\; \theta_{x5} \;\; \theta_{y5} \;\; w_6 \;\; \theta_{x6} \;\; \theta_{y6} \;\; w_7 \;\; w_8$$
$$\theta_{x8} \;\; \theta_{y8} \;\; w_9 \;\; \theta_{x9} \;\; \theta_{y9} \;\; w_{10} \;\; \theta_{x10} \;\; \theta_{y10} \;\; w_{11} \;\; w_{12} \;\; \theta_{x12} \;\; \theta_{y12} \;\; w_{13} \;\; w_{14} \;\; w_{15} \;\; \theta_{x16} \;\; \theta_{y16}]$$

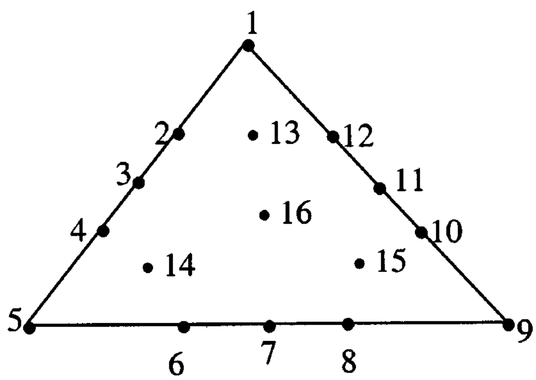

Fig. 1 A typical element with nodes

and the matrix [A] having an order of 35 × 35 contains the coordinates of the different nodes. As the rotations of the normal θ_x and θ_y are independent field variables and they are not derivatives of w, the effect of shear deformation can be easily incorporated as follows

$$\begin{Bmatrix} \phi_x \\ \phi_y \end{Bmatrix} = \begin{Bmatrix} \theta_x - \partial w/\partial x \\ \theta_y - \partial w/\partial y \end{Bmatrix} \tag{3}$$

where ϕ_x and ϕ_y are the average shear strain over the entire plate thickness and θ_x and ϕ_y are components of rotation of the normal.

Now the generalized stress strain relationship of a plate may be written as

$$\{\sigma\} = [D]\{\varepsilon\} \tag{4}$$

In the above equation, the generalized stress vector $\{\sigma\}$ is

$$\{\sigma\}^T = [M_x \;\; M_y \;\; M_{xy} \;\; Q_x \;\; Q_y] \tag{5}$$

the generalized strain vector $\{\varepsilon\}$ in terms of displacement fields is

$$\{\varepsilon\} = \begin{Bmatrix} -\partial\theta_x/\partial x \\ -\partial\theta_y/\partial y \\ -\partial\theta_x/\partial y - \partial\theta_y/\partial x \\ -\theta_x + \partial w/\partial x \\ -\theta_y + \partial w/\partial y \end{Bmatrix} \tag{6}$$

and the rigidity matrix [D] is

$$[D] = \begin{bmatrix} D_{11} & D_{12} & 0 & 0 & 0 \\ D_{21} & D_{22} & 0 & 0 & 0 \\ 0 & 0 & D_{33} & 0 & 0 \\ 0 & 0 & 0 & D_{44} & 0 \\ 0 & 0 & 0 & 0 & D_{55} \end{bmatrix} \tag{7}$$

The different elements of the rigidity matrix [D] are

$D_{11} = D_{22} = Eh^3/12(1.0 - \nu^2)$, $D_{21} = D_{12} = \nu D_{11}$, $D_{33} = (1.0 - \nu)D_{11}/2$ and $D_{44} = D_{55} = Eh/2k(1.0 + \nu)$,

where k is the warping constant, taken as 1.2.

Now, the field variables as defined in Eqs. (1a-1c) may be substituted in the generalized strain vector $\{\varepsilon\}$ as expressed in Eq. (6), which leads to

$$\{\varepsilon\} = [C]\{\alpha\} \tag{8}$$

where the matrix [C] having an order of 5 × 35 contains L_i conform to Eqs. (1a-1c) and their derivatives with respect to x and y. The derivative of any element, say $f(L_i)$ with respect to a variable, say x may be carried out as follows

$$\partial f(L_i)/\partial x = [\partial f(L_i)/\partial L_1][\partial L_1/\partial x] + [\partial f(L_i)/\partial L_2][\partial L_2/\partial x] + [\partial f(L_i)/\partial L_3][\partial L_3/\partial x] .$$

Substituting Eq. (2) in Eq. (8), the generalized strain vector $\{\varepsilon\}$ may be expressed as

$$\{\varepsilon\} = [B]\{\delta_e\} \tag{9}$$

where $[B] = [C][A]^{-1}$.

Once the matrices $[B]$ and $[D]$ are obtained, the element stiffness matrix $[K_e]$ can be easily derived with the help of Virtual work technique and it may be expressed as

$$[K_e] = \int_A [B]^T[D][B]dxdy \tag{10}$$

In a similar manner, the consistent load vector for a distributed transverse load of intensity q can be derived with the help of Eqs. (1a-1c) and Eq. (2) and it may be expressed as

$$[R_e] = [A]^{-T}\int_A q[[Q_0]\ \ [Q_0]\ \ [Q_2]\ \ [Q_0]\ \ [Q_0]]^T dxdy\,, \tag{11}$$

where $[Q_0]$ is a null matrix of order 1×10.

The integration in the above Eqs. (10) and (11) are carried out numerically following Gauss quadrature integration formula.

In this stage, the order of $[K_e]$ and $\{R_e\}$ is thirty-five, which is reduced to thirty by eliminating the degrees of freedom of the internal nodes (w_{13}, w_{14}, w_{15}, u_{16}, v_{16}, θ_{x16} and θ_{y16}) through static condensation.

The stiffness corresponding to the degrees of freedom at the inclined edge has been transformed and obtained by pre and post multiplication by an element transformation matrix $[T]$ of order 30×30 which is as follows.

$$[T] = \begin{bmatrix} [\lambda_1] \\ & [\lambda_2] \\ && [I_1] \\ &&& [\lambda_4] \\ &&&& [\lambda_5] \\ &&&&& [I_2] \\ &&&&&& [I_1] \\ &&&&&&& [I_2] \\ &&&&&&&& [I_2] \\ &&&&&&&&& [I_2] \\ &&&&&&&&&& [I_1] \\ &&&&&&&&&&& [I_2] \end{bmatrix}$$

In the above transformation matrix,

$$[\lambda_1] = [\lambda_2] = [\lambda_4] = [\lambda_5] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}, \ [I_1] = 1 \ \text{and} \ [I_2] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

In the above expression $[\lambda_1]$, $[\lambda_2]$, $[\lambda_4]$ and $[\lambda_5]$ are the transformation matrix corresponding to the nodes 1, 2, 4 and 5 situated on the inclined edge and φ is the angle of inclined edge with vertical.

These matrices have been computed for all the elements and assembled together to form the stiffness matric of the whole structure, which are stored in a single array following the skyline storage technique.

2.1 Numerical examples

In this section a number of examples on triangular plate having different plate side ratio (*b/a*), boundary conditions and thickness ratios (*h/a*) have been analyzed. Attempt has been made to compare the results obtained by the present element with those available in literature. The following boundary conditions have been used:

Simply supported: transverse displacement (w) and tangential rotation (θ_x) have been restrained.

Clamped supported: All degrees of freedom (w, θ_x, θ_y) have been restrained.

2.2 For equilateral triangular plate

First, a simply supported equilateral triangular plate as shown in Fig. 2 has been analyzed. As the plate is symmetry with respect to the *y*-axis, half of the plate is modelled. The deflection and bending moment at different locations of the plate have been presented in Table 1 with the thin plate solutions of Timoshenko and Woinowsky-Krieger (1959). The results have been presented with different mesh divisions. There is an excellent agreement between the results. The present results indicate that the rate of convergence of the proposed element is very fast. The results obtained by the proposed element for thick (h/a = 0.1 and 0.2) plate has been given as new results.

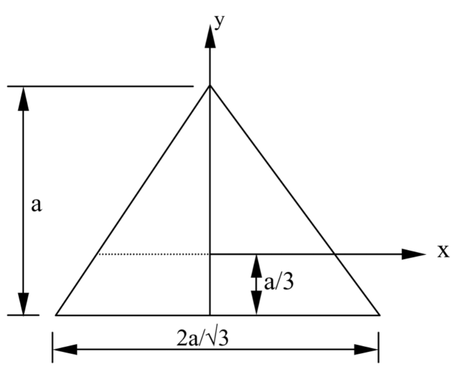

Fig. 2 Equilateral triangular plate

Table 1 Deflections ($w^* = wqa^4/D$) and bending moment ($M_x^* = qa^2 M_x$) of a simply supported equilateral triangular plate under uniform distributed load. $\nu = 0.3$, $a = 1.0$

h/a	Sources	Deflection (w^*)		Bending Moment $M_x^* = M_y^*$
		$x = 0, y = 0$	$x = 0, y = a/3$	$x = 0, y = 0$
0.001	Present (3 × 3*)	0.001036	0.000388	0.02467
	Present (6 × 6)	0.001034	0.000387	0.02418
	Present (9 × 9)	0.001034	0.000387	0.02408
	Present (12 × 12)	0.001034	0.000387	0.02405
0.01	Present (3 × 3)	0.001029	0.000386	0.02449
	Present (6 × 6)	0.001029	0.000386	0.02410
	Present (9 × 9)	0.001030	0.000386	0.02408
	Present (12 × 12)	0.001030	0.000386	0.02408
Thin plate solution		0.001029	0.000386	0.024
0.1	Present (3 × 3)	0.001133	0.000438	0.02426
	Present (6 × 6)	0.001134	0.000438	0.02406
	Present (9 × 9)	0.001134	0.000438	0.02407
	Present (12 × 12)	0.001134	0.000438	0.02407
0.2	Present (3 × 3)	0.001450	0.000596	0.02431
	Present (6 × 6)	0.001452	0.000597	0.02408
	Present (9 × 9)	0.001452	0.000597	0.02407
	Present (12 × 12)	0.001452	0.000597	0.02407

*Represent mesh division.

Table 2 Deflection ($w^* = wP/D$) at different points of a simply supported equilateral triangular plate under concentrated load (at $x = 0$, $y = 0$). $\nu = 0.3$, $a = 1.0$

h/a	Source	Deflection (w^*)	
		$x = 0, y = 0$	$x = 0, y = a/3$
0.001	Present (3 × 3)	0.00558	0.001417
	Present (6 × 6)	0.00568	0.001427
	Present (9 × 9)	0.00570	0.001428
	Present (12 × 12)	0.00570	0.001428
0.01	Present (3 × 3)	0.00562	0.001415
	Present (6 × 6)	0.00571	0.001429
	Present (9 × 9)	0.00573	0.001429
	Present (12 × 12)	0.00573	0.001429
Thin plate solution		0.00575	
0.1	Present (3 × 3)	0.007580	0.001527
	Present (6 × 6)	0.007942	0.001533
	Present (9 × 9)	0.008134	0.001533
	Present (12 × 12)	0.008268	0.001533
0.2	Present (3 × 3)	0.013367	0.001834
	Present (6 × 6)	0.014672	0.001848
	Present (9 × 9)	0.015418	0.001848
	Present (12 × 12)	0.015944	0.001848

In this example a simply supported triangular plate (Fig. 2) under concentrated load has been analyzed. The concentrated load P is applied at $x = 0$ and $y = 0$ as shown in Fig. 2. The deflections at different locations have been presented in Table 2 for different mesh sizes with the thin plate solutions of Timoshenko and Woinowsky-Krieger (1959). The present results are very close to the analytical solutions.

2.3 For right angle triangular plate

A right angle triangular plate as shown in Fig. 3, clamped along all the edges and subjected to uniformly distributed transverse load has been analyzed. The deflection and bending moments have been calculated at points A, B and C for b/a = 1.0 and 1.5 of the plate. The analysis has been performed considering h/a = 0.001, 0.1 and 0.2. The results obtained by the proposed element has been presented in Table 3 with those of Ganga Rao and Chaudhury (1988) in non-dimensional form. Ganga Rao and Chaudhury (1988) have solved the problem by a method based on series solution and compared their results with finite element results obtained by analyzing the plate using STRUDL. An excellent agreement has been found between the present results and the analytical solutions of Ganga Rao and Chaudhury (1988) for thin plate. Solutions of thick plate has been given as new results.

The same plate has been analyzed considering different boundary conditions. The non-dimensional deflection, bending moment and twisting moment have been calculated at point A (a/4, b/4) and presented in Table 4.

Finally a right angle triangular plate with b/a = 1.0 and 2.0 has been analyzed considering different boundary conditions under concentrated load. The concentrated load (P) is applied at different locations as shown in Table 5. The non-dimensional deflections at different points (A, B, C, D and E) as shown in Fig. 3 have been presented in Table 5 for different values of b/a and h/a ratios.

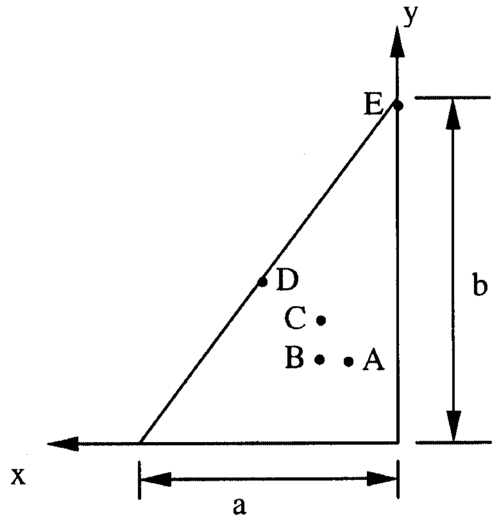

Fig. 3 Right angle triangular plate

Table 3 Deflection ($w^* = 10000wD/qa^4$) and bending ($M^* = 1000M_x/qa^2$) moment at three different points of a clamped right angle triangular plate with different h/a and b/a ratios. $\nu = 0.3$

Points	h/a	References	DOF*	w^*=10000wD/qa^4		$M^* = 1000M_x/qa^2$	
				b/a = 1.0	b/a = 1.5	b/a = 1.0	b/a = 1.5
A ($x=a/4$, $y=b/4$)	0.001	Present (4 × 4)	315	1.6400	3.2953	7.8303	11.768
		Present (8 × 8)	1107	1.6560	3.2956	7.7688	11.600
		Present (12 × 12)	2379	1.6575	3.2955	7.6205	11.556
		Analytical		1.66	3.31	7.61	11.56
		FEM		1.63	3.28	8.08	11.76
B ($x=3a/8$, $y=b/4$)		Present (4 × 4)	315	1.6507	3.7270	6.9642	13.321
		Present (8 × 8)	1107	1.6729	3.7579	6.7438	12.933
		Present (12 × 12)	2379	1.6738	3.7656	6.6012	12.821
		Analytical		1.67	3.76	6.56	12.72
		FEM		1.68	3.78	8.24	13.01
C ($x=3a/8$, $y=3b/8$)		Present (4 × 4)	315	1.4001	2.5932	6.1193	10.487
		Present (8 × 8)	1107	1.4277	2.6372	5.9422	10.324
		Present (12 × 12)	2379	1.4302	2.6451	5.8809	10.256
		Analytical		1.43	2.64	5.86	10.39
		FEM		1.39	2.63	4.93	10.29
A ($x=a/4$, $y=b/4$)	0.1	Present (4 × 4)	315	2.5195	4.5314	7.8509	11.696
		Present (8 × 8)	1107	2.5200	4.5339	7.7721	11.639
		Present (12 × 12)	2379	2.5201	4.5338	7.7606	11.606
B ($x=3a/8$, $y=b/4$)		Present (4 × 4)	315	2.5508	5.0770	6.9876	13.072
		Present (8 × 8)	1107	2.5526	5.0839	6.9603	13.032
		Present (12 × 12)	2379	2.5527	5.0840	6.9518	13.010
C ($x=3a/8$, $y=3b/8$)		Present (4 × 4)	315	2.2437	3.7635	6.2138	10.361
		Present (8 × 8)	1107	2.2456	3.7666	6.0583	10.343
		Present (12 × 12)	2379	2.2458	3.7664	6.0501	10.335
A ($x=a/4$, $y=b/4$)	0.2	Present (4 × 4)	315	4.9811	8.0740	8.0831	11.682
		Present (8 × 8)	1107	4.9818	8.0728	7.9603	11.630
		Present (12 × 12)	2379	4.9818	8.0729	7.9519	11.602
B ($x=3a/8$, $y=b/4$)		Present (4 × 4)	315	5.0587	8.8717	7.5842	13.357
		Present (8 × 8)	1107	5.0597	8.8681	7.5028	13.319
		Present (12 × 12)	2379	5.0598	8.8679	7.5004	13.296
C ($x=3a/8$, $y=3b/8$)		Present (4 × 4)	315	4.5785	7.0261	6.5232	10.397
		Present (8 × 8)	1107	4.5796	7.0240	6.4108	10.382
		Present (12 × 12)	2379	4.5796	7.0239	6.4012	10.371

*Represent degrees of freedom corresponding to mesh division

Table 4 Deflection ($w^* = 10000wD/qa^4$), bending moments ($M_x^* = 1000M_x/qa^2$ and $M_y^* = 1000M_y/qa^2$) and twisting moment ($M_{xy}^* = 1000M_{xy}/qa^2$) at point A ($a$/4, b/4) of a right angle triangular plate for different *h/a* ratio, *b/a* ratio and boundary conditions. ν = 0.3

Boundary condition	*b/a*	*h/a*	w^*	M_x^*	M_y^*	M_{xy}^*
S-S-S	1.0	0.001	6.0643	18.062	18.062	–0.14593
		0.1	6.8572	18.046	18.046	–0.15337
		0.2	9.2370	18.045	18.045	–0.15295
	1.5	0.001	12.410	29.292	22.025	–0.77219
		0.1	13.536	29.285	21.981	–0.77752
		0.2	16.916	29.284	21.982	–0.77757
F-S-S*	1.0	0.001	42.405	26.561	26.561	38.702
		0.1	54.516	27.815	27.815	50.053
		0.2	59.564	27.815	27.816	51.588
	1.5	0.001	94.739	42.061	34.281	58.022
		0.1	122.89	43.163	37.709	75.678
		0.2	131.43	43.336	37.461	78.131
F-F-C**	1.0	0.001	59.545	–21.201	–101.59	–19.920
		0.1	64.846	–18.546	–103.65	–20.102
		0.2	77.974	–13.820	–106.85	–21.015
	1.5	0.001	278.27	–29.811	–212.46	–40.660
		0.1	290.76	–25.085	–215.62	–41.609
		0.2	319.94	–17.941	–219.67	–43.414

*Inclined edge is free and other two edges are simply supported.
**Inclined and vertical edges are free and base is free.

Table 5 Deflection (w^*=1000wP/D) at different points of a right angle triangular plate for different *h/a* ratio, *b/a* ratio and boundary conditions. ν = 0.3

Boundary conditions	*b/a*	*h/a*	Deflection ($w^* = wP/D$) at				
			$x = a/4$, $y = b/4$	$x = 3a/8$, $y = b/4$	$x = 3a/8$, $y = 3b/8$	$x = a/2$, $y = b/2$	$x = 0$, $y = b$
			Load (P) is at point A ($x = a/4$, $y = b/4$)				
S-S-S	1.0	0.001	4.1211	3.3708	2.8153		
		0.1	6.5026	3.8249	3.1312		
		0.2	13.636	5.1897	4.0781		
	2.0	0.001	6.6684	6.3395	3.7336		
		0.1	8.9677	6.9017	3.9604		
		0.2	15.806	8.6046	4.6430		

Table 5 Continued

Boundary conditions	b/a	h/a	Deflection ($w^* = wP/D$) at $x = a/4$, $y = b/4$	$x = 3a/8$, $y = b/4$	$x = 3a/8$, $y = 3b/8$	$x = a/2$, $y = b/2$	$x = 0$, $y = b$
			Load (P) is at point D ($x = a/2$, $y = b/2$)				
F-S-S	1.0	0.001	22.972	33.570	49.945	86.208	
		0.1	29.380	43.122	64.189	114.73	
		0.2	31.254	45.970	68.912	134.26	
	2.0	0.001	43.459	63.455	99.463	176.42	
		0.1	57.385	84.286	129.21	231.19	
		0.2	60.084	88.180	136.14	255.71	
			Load (P) is at point E ($x = 0$, $y = b$)				
F-F-S	1.0	0.001	45.617	37.840	82.169	114.86	737.58
		0.1	48.187	39.715	85.284	117.53	784.06
		0.2	53.189	43.400	90.926	122.25	869.93
	2.0	0.001	308.27	288.29	632.78	1040.2	4809.1
		0.1	313.99	293.76	642.25	1054.0	4906.6
		0.2	323.92	302.76	656.85	1073.8	5068.7
			Load (P) is at point A ($x = a/4$, $y = b/4$)				
C-C-C	1.0	0.001	1.8758	1.2640	0.91719		
		0.1	4.3287	1.7718	1.2785		
		0.2	11.544	3.1986	2.2745		
	2.0	0.001	2.8335	2.4712	0.94149		
		0.1	5.2399	3.1159	1.1999		
		0.2	12.248	4.9484	1.9469		

3. Conclusions

A high precision shear deformable triangular thick plate bending element is proposed. The effect of shear deformation is incorporated in the formulation and it is done in such a manner, which has made the element free from locking in shear. The element is used for analysis of very thin plate (h/a = 0.001) to sufficiently thick plate (h/a = 0.2). There is no shear locking problem even for very thin plate. The element is called high precision because convergence of the proposed element is very fast and the results obtained by the present element are very close to the analytical results. A number of new results for thick plate are presented. It is expected that the new results will be useful in future research.

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