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Probabilistic determination of initial cable forces of cable-stayed bridges under dead loads

Jin Cheng† and Ru-Cheng Xiao‡

Department of Bridge Engineering, Tongji University, Shanghai, 200092, China

Jian-Jing Jiang‡

Department of Civil Engineering, Tsinghua University, Beijing, 100084, China

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Abstract. This paper presents an improved Monte Carlo simulation for the probabilistic determination of initial cable forces of cable-stayed bridges under dead loads using the response surfaces method. A response surface (i.e. a quadratic response surface without cross-terms) is used to approximate structural response. The use of the response surface eliminates the need to perform a deterministic analysis in each simulation loop. In addition, use of the response surface requires fewer simulation loops than conventional Monte Carlo simulation. Thereby, the computation time is saved significantly. The statistics (e.g. mean value, standard deviation) of the structural response are calculated through conventional Monte Carlo simulation method. By using Monte Carlo simulation, it is possible to use the existing deterministic finite element code without modifying it. Probabilistic analysis of a truss demonstrates the proposed method's efficiency and accuracy; probabilistic determination of initial cable forces of a cable-stayed bridge under dead loads verifies the method's applicability.

Key words: cable-stayed bridges; initial cable forces; probabilistic; parametric uncertainties; response surface; Monte Carlo simulation; statistics.

1. Introduction

Determination of initial cable forces under dead loads is a major prerequisite for carrying out structural analyses of completed cable-stayed bridges. Since the early 1980s, various methods have been developed for determining initial cable forces (Kasuga *et al.* 1985, Furukawa *et al.* 1987, Wang *et al.* 1993, Chen *et al.* 2000). These methods are based on the assumption of complete determinacy of structural parameters. This is usually referred to as deterministic analysis. In reality, however, there are uncertainties in design variables. These uncertainties include geometric properties (cross-sectional properties and dimensions), material mechanical properties (modulus and strength, etc), load magnitude and distribution, etc. Thus deterministic analysis cannot provide complete information regarding initial cable forces of cable-stayed bridges under dead loads.

† Research Assistant Professor
‡ Professor

To determine the initial cable forces of cable-stayed bridges with parametric uncertainties, geometric nonlinearity has to be considered. This is because cable-stayed bridges become more flexible as spans become longer. Two methods have been used to predict the response of geometrically nonlinear structures with parametric uncertainties, namely, the Monte Carlo simulation, and the first-order approximation. Imai and Frangopol (2000) used the two methods to analyze probabilistically the mean and variance of member axial forces of a truss and a suspension cable. The results show that the first-order approximation method and Monte Carlo simulation are in close agreement. However, the above study is limited to simple structures. More complex structures such as cable-stayed bridges are not considered. In addition, the above-mentioned two methods have drawbacks. First, for accurate results, Monte Carlo simulation needs numerous repetitions of deterministic analysis, thus consuming an enormous large amount of computation time. Second, the first-order approximation method needs the computation of response gradients for geometrically nonlinear structures with parametric uncertainties. Unfortunately, the existing deterministic finite element code available to design engineers cannot compute response gradients. Therefore, to use the method, it is necessary to modify the existing deterministic finite element code. To the best of the authors' knowledge, no procedure considering geometrically nonlinear behavior and uncertainty in the determination of initial cable forces of cable-stayed bridges under dead loads is currently available.

The aims of this paper are to propose an efficient method to determine the initial cable forces of cable-stayed bridges with parametric uncertainties, and use the method to investigate the effects of various parameters on the initial cable forces of cable-stayed bridges.

2. Proposed method

2.1 Principle

The proposed method is a Monte Carlo simulation based on the response surface method. The idea of the method is to improve the computation efficiency of conventional Monte Carlo simulation and to effectively take advantage of the existing deterministic finite element code.

Conventional Monte Carlo simulation is the most common and traditional method for a probabilistic analysis. Extensive reviews of the method are found in (Melchers 1999, Haldar 2000). In brief, the method uses randomly generated samples of the input variables for each deterministic analysis, and estimates response statistics after numerous repetitions of the deterministic analysis (Haldar 2000). The main advantages of the method are: (1) engineers with only a basic working knowledge of probability and statistics can use it (Haldar 2000); (2) it always provides correct results when a very large number of simulation cycles are performed (one simulation cycle represents a deterministic analysis). However, the method has one drawback: it needs an enormously large amount of computation time.

To overcome the drawback of conventional Monte Carlo simulation, we use Monte Carlo simulation together with a response surface method. The main idea of the response surface method is to represent the structural response by using a suitable approximation function. Once the approximation function is found, we can directly use the approximation function instead of deterministic finite element analysis. To perform a finite element analysis can require minutes to hours of computation time; in contrast, evaluation of a quadratic function requires only a fraction of a second. Hence, if using the approximate function, we can efficiently use conventional Monte Carlo simulation to calculate the statistics of the structural response. Second, this method makes it possible to use an existing deterministic finite element code without modifying it. For more details concerning the method, the reader is referred to (Khuri and Cornell 1987, Bucher and Bourgund 1990, Zheng and Das 2000, Soares *et al.* 2002).

The unique feature of the proposed method is the combination of the advantages of conventional Monte Carlo simulation and the response surface method. The efficiency and accuracy of the proposed method are verified in Sec.3 of this paper.

2.2 Procedure for the proposed method

The procedures of the proposed method are:

1. Use the response surface method to find an approximation function that can represent the structural response. The approximation function is the so-called response surface function (RSF). The RSF commonly takes the form of polynomials of the random basic input variables. In many cases of engineering analysis, the RSF takes three polynomial forms: (1) linear function; (2) quadratic function without cross-terms; (3) quadratic function with cross-terms. They can be expressed as:

$$\hat{g}(X) = b_0 + \sum_{i=1}^{k} b_i X_i \tag{1}$$

$$\hat{g}(X) = b_0 + \sum_{i=1}^{k} b_i X_i + \sum_{i=1}^{k} b_{ii} X_i^2 \tag{2}$$

$$\hat{g}(X) = b_0 + \sum_{i=1}^{k} b_i X_i + \sum_{i=1}^{k} b_{ii} X_i^2 + \sum_{i=1}^{k-1} \sum_{j>i}^{k} b_{ij} X_i X_j \tag{3}$$

   where $X_i(i = 1, 2, ..., k)$ = the ith random variable; k = number of random variables; and b_0, b_i, b_{ii}, and b_{ij} = unknown coefficients to be determined. All three RSF forms above-mentioned are used in the paper. To obtain the response surface function (RSF), the following steps are necessary: (1) choose a technique to determine the location of the sampling points. Here, we chose the central composite design sampling method (CCDSM); A central composite design consists of a center point, a complete 2^k factorial design, and two axial points on the axis of each random variable at a distance α from the center point, where $\alpha = \sqrt[4]{2^k}$ in order to make the design rotatable; (2) choose a technique to determine the RSF based on the results obtained at the sampling points. Here, we chose the least-squares method (LSM). More details about the CCDSM and the LSM may be found in (Montgomery 1991, Neter *et al.* 1985).

2. Based on the obtained response surface, apply conventional Monte Carlo simulation to obtain the probabilistic results for structural response.

3. Probabilistic analysis of a truss

To demonstrate the efficiency and accuracy of the proposed method, the truss shown in Fig. 1 was analyzed probabilistically. Note that the vertical dimension of 1 m shown in Fig. 1 is unchanged. The statistical parameters of random variables of the truss are listed in Table 1. Five methods were considered: (1) Monte-Carlo simulation on linear response surface (MCS-LRS); (2) Monte-Carlo simulation on quadratic response surface without cross-terms (MCS-QRS1); (3) Monte-Carlo simulation on quadratic response surface including cross-terms (MCS-QRS2); (4) conventional

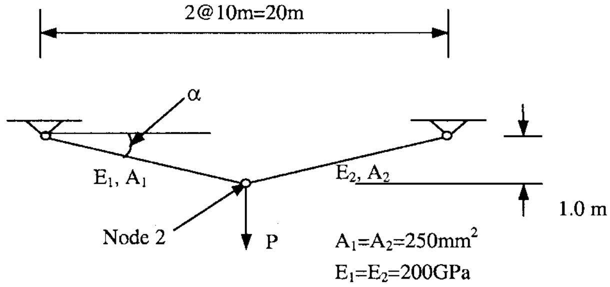

Fig. 1 Two-bar truss

Table 1 Statistical parameters of random variables of a two-bar truss

Variable	μ	σ	Dimension	Distribution
E_1	200	4	GPa	Normal
E_2	200	4	GPa	Normal
P	18	3.6	KN	Normal

Table 2 Comparison of various methods for the means, standard deviations and coefficients of variation (COVs) of the member axial force (a two-bar truss - $\alpha = 0.5$ degrees)

Method	Number of simulations	μ (KN)	σ (KN)	COV
MCS-LRS	10	124.3721	15.0957	0.121375
	1000	125.2763	16.7023	0.133324
	10000	125.2792	16.6932	0.133248
MCS-QRS1	10	125.1267	15.1710	0.121245
	1000	125.9491	16.7148	0.132711
	10000	125.9528	16.7022	0.132607
MCS-QRS2	10	125.1689	15.1078	0.120699
	1000	125.9527	16.7097	0.132666
	10000	125.9536	16.7037	0.132618
MCS-1	50000	125.8658	16.9261	0.134477
MCS-2	50000	125.2750	17.2290	0.137529

Table 3 Comparison of the means and standard deviations of the member axial force obtained by various methods (a two-bar truss - $\alpha = 0.5\sim60.0$ degrees)

α (Degrees)	MCS-QRS1 (10000)		MCS-2 (50000)	
	μ (KN)	σ (KN)	μ (KN)	σ (KN)
0.5	125.9528	16.7022	125.275	17.229
2.5	112.7403	16.9591	111.967	16.932
5.0	85.5150	14.9902	85.467	14.976
7.5	64.2112	12.1166	64.204	12.137
10.0	50.1792	9.7549	50.178	9.783
15.0	34.4318	6.8255	34.435	6.847
30.0	17.9798	3.5929	17.982	3.604
45.0	12.7246	2.5442	12.726	2.553
60.0	10.3916	2.0782	10.393	2.085

Note: the value in the parenthesis is the number of simulations

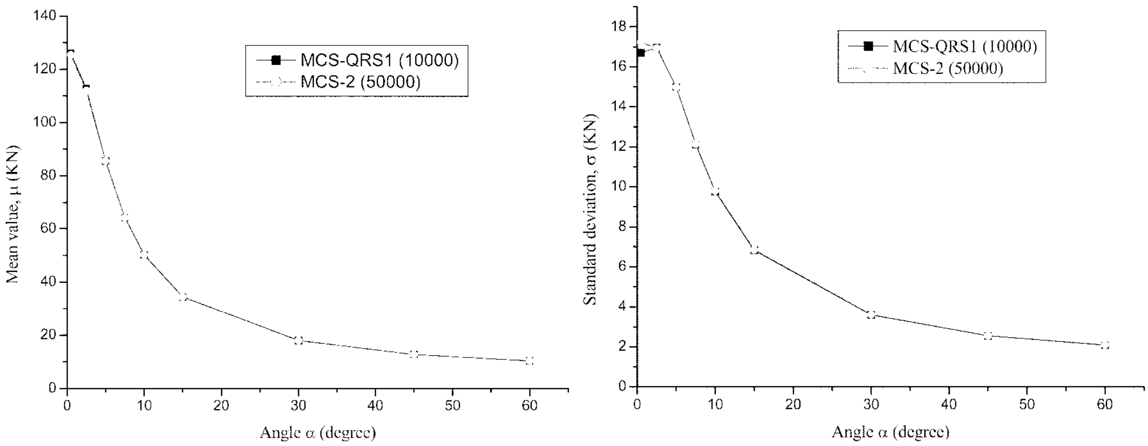

Fig. 2(a) Comparison of the means value - angle curves for various methods, (b) Comparison of the standard deviations - angle curves for various methods

Monte-Carlo simulation used in the present paper (MCS-1); (5) conventional Monte-Carlo simulation used in (Imai and Frangopol 2000) (MCS-2). The comparison of various methods for the means, standard deviations and coefficients of variation (COVs) of the member axial force ($\alpha = 0.5$ degrees) is shown in Table 2. The means, standard deviations of the member axial force obtained by both MCS-QRS1 and MCS-2 for different values of the angle α are listed in Table 3. Fig. 2 presents a graphical comparison of the results of both MCS-QRS1 and MCS-2 for different values of the angle α. The comparison of various methods for number of FEM computations and computation time ($\alpha = 0.5$ degrees) is listed in Table 4.

It is seen from Table 2 that when the number of simulations was relatively large, e.g., more than 1000, the Monte-Carlo simulation methods based on various response surfaces provided good

Table 4 Comparison of various methods for number of FEM computations and computation time (a two-bar truss - $\alpha = 0.5$ degrees)

Method	Number of FEM computations	Computation time (sec.)
MCS-LRS	9	
MCS-QRS1	9	About 75
MCS-QRS2	9	
MCS-1	50000	About 14400

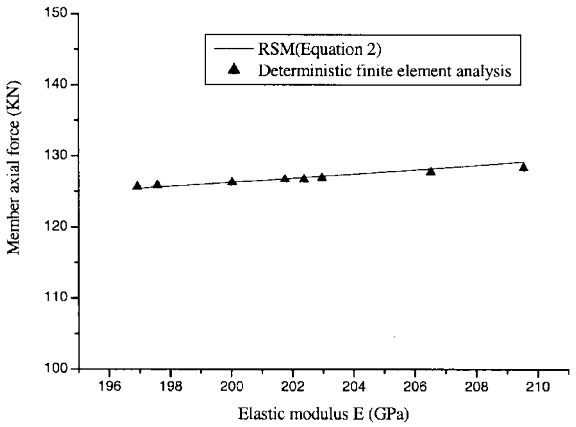

Fig. 3 Member axial force of a truss ($\alpha = 0.5$ degrees) versus modulus E (Load P = 18 KN)

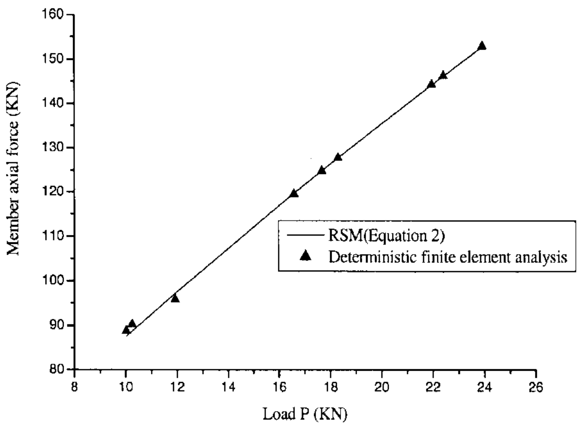

Fig. 4 Member axial force of a truss ($\alpha = 0.5$ degrees) versus Load P (modulus E = 200 GPa)

results. This indicates that the forms of response surface have a minor effect on the results. The number of simulations about 10,000 gives even better results. The number of simulations is significantly less than the number of simulations used in (Imai and Frangopol 2000) (50,000). But, it should be pointed out that the personal computer running time arising from the increase in the number of simulations is absolutely negligible by using the proposed method. This is because the proposed method uses the RSF instead of deterministic finite element analysis.

It may be further observed from Table 3 and Fig. 2 that the Monte-Carlo simulation method based on MCS-QRS1 is accurate for the number of simulations about 10,000. In addition, the form of response surface in the MCS-QRS1 is also simple. Therefore, Monte-Carlo simulation based on MCS-QRS1 with the number of simulations about 10,000 was used in the subsequent study.

It is observed from Table 4 that by using the proposed method, the number of FEM calculations is greatly reduced, thus significantly saving the computation time.

To check the accuracy of the probabilistic analysis of a truss ($\alpha = 0.5$ degrees) by Eq. (2), the variations of the member axial force constructed in Eq. (2) versus the variables, E and P, are shown in Figs. 3 and 4, along with the 'measured' data points from the deterministic finite element analyses. From these figures, it can be seen that the response surfaces constructed by Eq. (2) has a very good agreement with the numerical solutions using deterministic finite element analyses.

In view of its efficiency and accuracy, the proposed method is considered to be a suitable probabilistic analytical tool for complex structures.

4. Probabilistic determination of initial cable forces of cable-stayed bridges

To further demonstrate the applicability of the proposed method, initial cable forces of a cable-stayed bridge shown in Fig. 5 were probabilistically determined. Note that all towers are moveable hinge (roller) supports as shown in Fig. 5. The statistical parameters of random variables of the bridge are listed in Table 5.

Wang *et al.* (1993) presented a finite element computation procedure for determining the initial cable forces of cable-stayed bridges with deterministic structural parameters. In this paper, the computation procedure is used to perform a deterministic analysis of the bridge.

In the following analyses, the proposed method was used to obtain the probabilistic initial cable forces of cable-stayed bridges under dead loads. In all but some of Sec.4.1 cases, all the geometric nonlinearities in the cable-stayed bridges are considered.

4.1 Effect of geometric nonlinearities

Geometric nonlinear behavior of cable-stayed bridges originates from three primary sources: (1) cable sag effects; (2) combined axial load and bending moment interaction for the girder and towers; (3) large displacement, which is produced by the geometry changes of the structure. An equivalent modulus approach proposed by Ernst (1965) is used in the paper to account for the

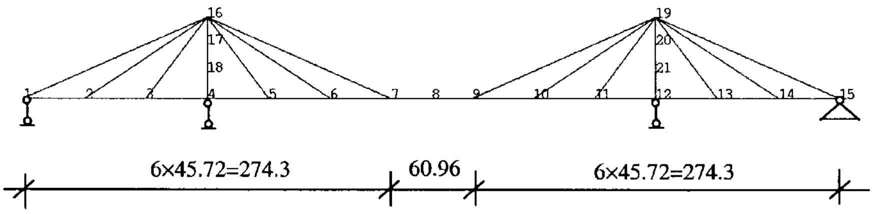

Fig. 5 Symmetric radiating cable-stayed bridge (unit: m)

Table 5 Statistical parameters of random variables of the symmetric radiating cable-stayed bridge

Variable	μ	σ	Dimension	Distribution
E	2.067e8	2.067e7	kN/m^2	Normal
IGIRD	1.13066	0.05653	m^4	Normal
AGIRD	0.31959	0.01598	m^2	Normal
ACABE	0.04199	0.0021	m^2	Normal
ACABI	0.01617	8.08256E-4	m^2	Normal
WGIRD	87.6	7.008	kN/m	Normal
WCABE	3.2266	0.258128	kN/m	Normal
WCABI	1.241	0.09928	kN/m	Normal

(Note: E: Elastic modulus of the girder and the cables; IGIRD: In-plane moment of inertia of the girder; AGIRD: Area of the girder; ACABE: Area of the exterior cables; ACABI: Area of the interior cables; WGIRD: Dead load of the girder; WCABE: Dead load of the exterior cables; WCABI: Dead load of the interior cables) sagging of inclined cables. In order to investigate the effect of geometric nonlinearities on the probabilistic results for the initial cable forces of cable-stayed bridges under dead loads, the following cases are considered:

1. CA=1-All these geometric nonlinearities are considered.
2. CA=2-Only (1) of the geometric nonlinearities above-mentioned is considered.
3. CA=3-Only (2) and (3) of the geometric nonlinearities above-mentioned are considered.

The comparison of the means (μ), standard deviations (σ) and coefficients of variation (Cov) of the cable forces for different cases is listed in Table 6. Figs. 6 and 7 show the comparison of the cumulative distribution function (CDF) and the probability density function (PDF) of the cable forces for different cases, respectively. It can be seen from Table 6 that the difference in the means of cable forces between the two cases of CA=1 and CA=2 is larger than that between the two cases of CA=1 and CA=3. This implies that, for the accurate mean values of cable forces, (2) and (3) in the geometric nonlinearities above-mentioned need to be considered. However, the difference in the standard deviations of cable forces for all cases above-mentioned is small. This implies that the geometric nonlinearities have a minor effect on the standard deviations of cable forces. It can be seen from Figs. 6 and 7 that omission of the cable sag effects does not influence both the CDF curve and the PDF curve. But the effects of (2) and (3) in the geometric nonlinearities above-mentioned cannot be ignored.

Table 6 Comparison of the means, standard deviations and coefficients of variation (COVs) of the cable forces with different cable elements for different cases (the symmetric radiating cable-stayed bridge)

Cable No.	CA=1			CA=2			CA=3		
	μ (kN)	σ (kN)	Cov	μ (kN)	σ (kN)	Cov	μ (kN)	σ (kN)	Cov
1-16	11212.89	912.2055	0.081	11051.58	904.7295	0.082	11257.7	891.1125	0.079
2-16	7583.023	598.659	0.079	7775.53	611.0295	0.079	7479.249	620.597	0.083
3-16	5222.075	432.718	0.083	5069.04	422.8835	0.083	5256.607	413.0935	0.079

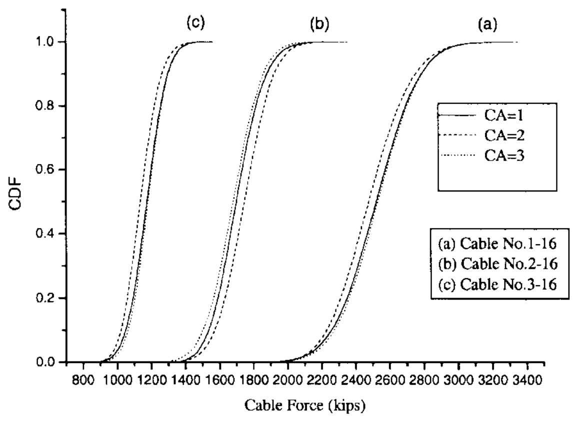

Fig. 6 Comparison of CDF curves with different cable elements for different cases

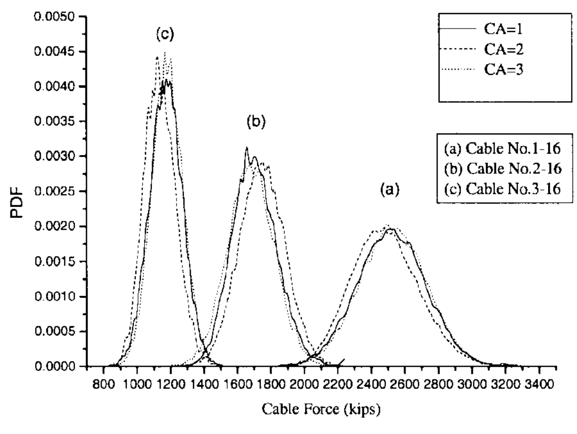

Fig. 7 Comparison of PDF curves with different cable elements for different cases

For accurate cable forces, it is necessary that the analysis technique incorporate the effect of structural parameters randomness. This is of special importance for accurate determination of cable forces in cable-stayed bridges, which exhibit wide dispersion in structural parameters. This problem can be solved by the proposed method (MCS-QRS1). The proposed method does offer a significant improvement over deterministic analysis.

4.2 Effect of probability distribution of random input variables

The means (μ), standard deviations (σ) and coefficients of variation (Cov) of the cable forces are computed for the lognormal distributed input variables with the same mean and coefficient of variation as the normal distributed input variables. The computational results are listed in Table 7. As shown, the results associated with the lognormal distribution are almost the same as those associated with the normal distribution.

Table 7 Comparison of the means, standard deviations and coefficients of variation (COVs) of the cable forces with different cable elements for normal and lognormal distributed input variables (the symmetric radiating cable-stayed bridge)

Cable No.	Normal distribution			Lognormal distribution		
	μ (kN)	σ (kN)	Cov	μ (kN)	σ (kN)	Cov
1-16	11212.89	912.2055	0.081	11211.73	911.538	0.0813
2-16	7583.023	598.659	0.079	7585.248	601.2395	0.0793
3-16	5222.075	432.718	0.083	5220.74	432.1395	0.0828

4.3 Effect of variation of dead loads

Seven cases listed in Table 8 are considered to investigate the effects of variation of dead loads on the probabilistic results for the initial cable forces of cable-stayed bridges under dead loads. The variations of the mean value (μ) and standard deviations (σ) of cable forces for different cable elements with different cases (Cases I-VII) are listed in Table 9. For a better visualization, these values are plotted in Figs. 8 and 9. From these figures and the table, it can be seen that the variation of dead loads has a minor effect on μ values, while an opposite effect is observed for σ values.

Table 8 Illustration of the seven cases used in Sec.4.3 of the paper

Case	Numbers of random input variable	Random input variable
I	2	E, IGIRD
II	3	E, IGIRD, AGIRD
III	4	E, IGIRD, AGIRD, ACABE
IV	5	E, IGIRD, AGIRD, ACABE, ACABI
V	6	E, IGIRD, AGIRD, ACABE, ACABI, **WGIRD**
VI	7	E, IGIRD, AGIRD, ACABE, ACABI, WGIRD, WCABE
VII	8	E, IGIRD, AGIRD, ACABE, ACABI, WGIRD, WCABE, WCABI

Table 9 Comparison of the means and standard deviations of the cable forces with different cable elements for different cases (the symmetric radiating cable-stayed bridge)

Case	Cable No.	μ (kN)	σ (kN)
I	1-16	11198.65	103.9965
	2-16	7598.909	185.3425
	3-16	5220.117	101.905
II	1-16	11239.45	116.4565
	2-16	7522.28	209.2835
	3-16	5221.541	110.182
III	1-16	11222.81	71.645
	2-16	7538.879	117.7915
	3-16	5260.612	67.3285
IV	1-16	11190.95	73.603
	2-16	7597.663	133.9005
	3-16	5225.858	84.728
V	1-16	11223.26	918.0795
	2-16	7556.145	599.9045
	3-16	5211.351	436.6785
VI	1-16	11228.46	916.433
	2-16	7551.739	599.86
	3-16	5212.152	435.6995
VII	1-16	11212.89	912.2055
	2-16	7583.023	598.6585
	3-16	5222.075	432.718

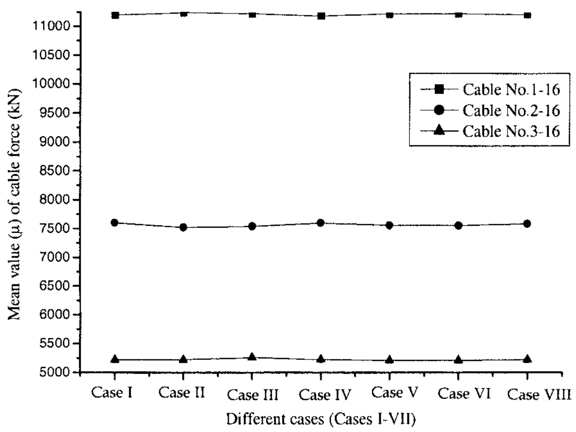

Fig. 8 Variations of the mean value (μ) of cable forces for different cable elements with different cases (Cases I-VII)

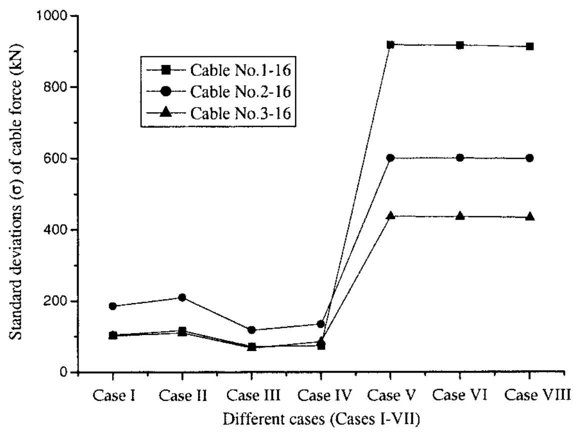

Fig. 9 Variations of the standard deviations (σ) of cable forces for different cable elements with different cases (Cases I-VII)

4.4 Effect of dead loads of girder

The variations of the mean value of cable forces with the mean value of dead loads of girder are shown in Fig. 10. Also shown in Fig. 10 is the variation of the mean value of cable forces with the standard deviation of dead loads of girder. It can be seen that the mean value of dead loads of girder has a significant effect on the mean value of cable forces. The mean value of cable forces increases with the increase in the mean value of dead loads of girder. However, the mean value of cable forces does not change with the increase of the standard deviation of dead loads of girder.

The variations of the standard deviation of cable forces with the mean value of dead loads of girder are shown in Fig. 11. Also shown in Fig. 11 is the variation of the standard deviation of cable forces with the standard deviation of dead loads of girder. It can be seen that the increase in the mean value of dead loads of girder almost does not influence the standard deviation of cable forces. But, the increase of the standard deviation of dead loads of girder does increase the standard deviation of cable forces. For example, the standard deviation of cable forces for Cable No.1-16 with the standard deviation of dead loads of girder $\sigma = 7.008$ kN/m and the mean value of dead loads of girder $\mu = 87.6$ kN/m is 912.2055 kN, while the standard deviation of cable forces for Cable No.1-16 with the standard deviation of dead loads of girder $\sigma = 10.512$ kN/m and the mean value of dead loads of girder $\mu = 87.6$ kN/m increases to 1367.3515 kN.

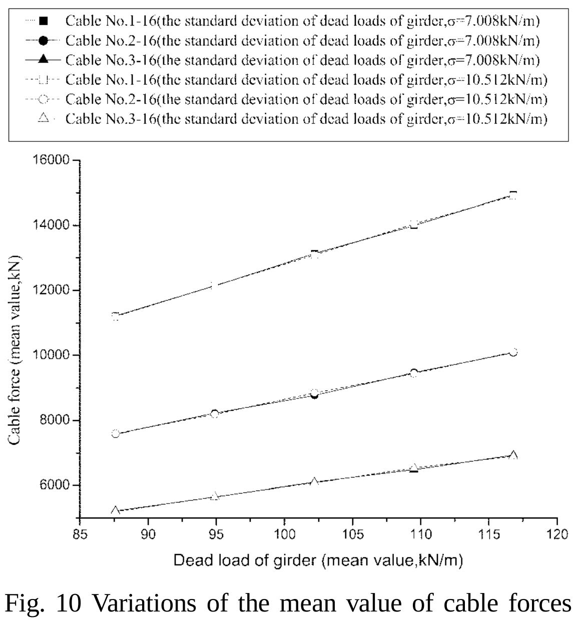

Fig. 10 Variations of the mean value of cable forces with the mean value of dead loads of girder for different cable elements

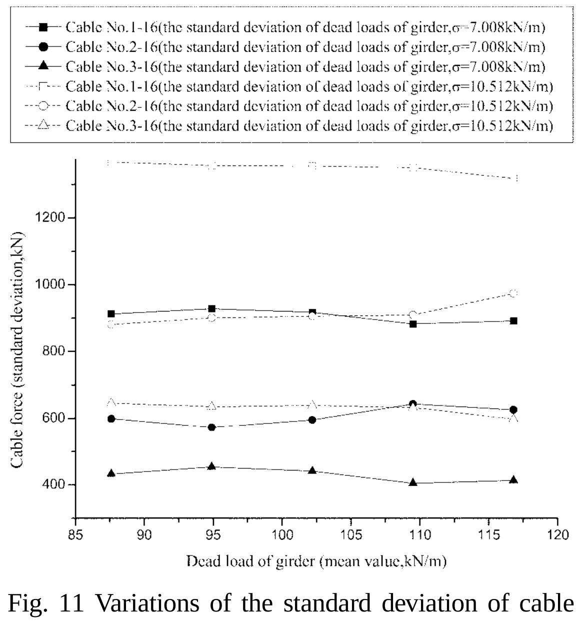

Fig. 11 Variations of the standard deviation of cable forces with the mean value of dead loads of girder for different cable elements

5. Conclusions

An improved Monte Carlo simulation method (MCS-QRS1) is proposed for the probabilistic determination of the initial cable forces of cable-stayed bridges under dead loads incorporating the response surfaces method. In the proposed method, a response surface (i.e. a quadratic response surface without cross-terms) is used to approximate structural response. The use of the response surface eliminates the need to perform a deterministic analysis in each simulation loop. In addition, use of the response surface requires fewer simulation loops than conventional Monte Carlo simulation. Thereby, the computation time is saved significantly. By using Monte Carlo simulation, it is possible to use the existing deterministic finite element code without modifying it. This is advancement over the first-order approximation method. The accuracy and reliability of the proposed method are verified through comparison with conventional Monte Carlo simulation. Comparative study on the probabilistic analysis of a truss confirms the accuracy and economy of the proposed method.

The proposed method has been applied to determine the initial cable forces of cable-stayed bridges with parametric uncertainties under dead loads. It is found that the proposed method can obtain more information about the initial cable forces than the commonly used deterministic method (e.g. Wang *et al*. 1993). Using the proposed method, a parametric study was conducted to investigate the effect of various parameters on the initial cable forces of cable-stayed bridges. The results of the parametric study lead to the following conclusions:

1) The combined axial load and bending moment interaction for the girder and towers and large displacement in the geometric nonlinearities of cable-stayed bridges generally affect the mean value of initial cable forces. The geometric nonlinearities of cable-stayed bridges have a minor effect on the standard deviations of initial cable forces.
2) The variation of dead loads has a minor effect on the mean value of initial cable forces, but a significant effect on the standard deviations of initial cable forces.
3) The mean value of dead loads of girder has a significant effect on the mean value of initial cable forces. The mean value of initial cable forces increases with the increase in the mean value of dead loads of girder. However, the mean value of initial cable forces does not change with increase in the standard deviation of dead loads of girder.
4) The standard deviation of initial cable forces is greatly influenced by the standard deviation of dead loads of girder. As the standard deviation of dead loads of girder increases, the standard deviation of initial cable forces also increases.

It should be pointed out that the application of the proposed method is not limited to the probabilistic determination of initial cable forces of cable-stayed bridges under dead loads. Wider application of the proposed method is being explored.

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