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J-integral and fatigue life computations in the incremental plasticity analysis of large scale yielding by *p*-version of F.E.M.

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Abstract. Since the linear elastic fracture analysis has been proved to be insufficient in predicting the failure of strain hardening materials, a number of fracture concepts have been studied which remain applicable in the presence of plasticity near a crack tip. This work thereby presents a new finite element model to predict the elastic-plastic crack-tip field and fatigue life of center-cracked panels(CCP) with ductile fracture under large-scale yielding conditions. Also, this study has been carried out to investigate the path-dependence of *J*-integral within the plastic zone for elastic-perfectly plastic, bilinear elastic-plastic, and nonlinear elastic-plastic materials. Based on the incremental theory of plasticity, the *p*-version finite element is employed to account for the accurate values of *J*-integral, the most dominant fracture parameter, and the shape of plastic zone near a crack tip by using the *J*-integral method. To predict the fatigue life, the conventional Paris law has been modified by substituting the range of *J*-value denoted by ΔJ for ΔK. The experimental fatigue test is conducted with five CCP specimens to validate the accuracy of the proposed model. It is noted that the relationship between the crack length a and ΔK in LEFM analysis shows a strong linearity, on the other hand, the nonlinear relationship between a and ΔJ is detected in EPFM analysis. Therefore, this trend will be depended especially in the case of large scale yielding. The numerical results by the proposed model are compared with the theoretical solutions in literatures, experimental results, and the numerical solutions by the conventional *h*-version of the finite element method.

Key words: large scale yielding; *J*-integral; fatigue life; HRR-singularity; EPFM; CCP specimen; *p*-version of F.E.M.; modified Paris Law.

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1. Introduction

The fracture analysis can be based on linear-elastic or more complex elastic-plastic (nonlinear) models. The linear-elastic fracture mechanics (LEFM) assume the theory of small scale yielding that may be explained by the stress intensity factor K and the contour integral J characterizing the crack tip condition. At a certain distance of the crack tip, the stress distribution is proportional to $1/\sqrt{r}$, and this area is called K-dominant area. On the other hand, the nonlinear elastic-plastic fracture mechanics(EPFM) provide more realistic measures of fracture behavior of cracked structures with high toughness and low strength materials compared with LEFM. In EPFM, the crack-driving force is frequently described in terms of J-integral. The J-integral is an appropriate fracture parameter that represents the crack-tip stress and strain fields adequately when there are no constrain effects. Since a J-dominant area occurs in the plastic region ahead of the crack tip, the theory of large scale yielding should be considered. This region is limited by another region where finite deformations take place. A rough estimate of the size and shape of the plastic zone was proposed by Irwin (1971) and Dugdale (1960). However, the application of a plastic zone correction is doubtful because of the limited validity of the expressions for the stress intensity factor K which are based on elastic solutions. Therefore, these hypothetical solutions showed large differences from the works by Miller (1974) and Gdoutos (1986).

The asymptotic analysis, called the HRR solution, was proposed by Hutchinson (1968), Rice and Rosengren (1968) to characterize the elastic-plastic field near the crack tip. The intensity coefficient of the **HRR** singularity, J-integral, can be taken as a single parameter dominating crack initiation.

Therefore, the value of J-integral is very important fracture parameter in both LEFM and EPFM analysis. A huge amount of papers have been published concerning the J-integral in theoretical, experimental and numerical investigations, since the J-integral method was proposed by Rice (1968). The acceptance of the J-integral method was promoted by the fact that the J-integral can be determined directly from numerical analyses, especially by the finite element method. Several authors (Feng and Zhang 1993, Schmitt and Kienzler 1989, Kim and Orange 1988, Dadkhah and Kobayashi 1989) have shown that the J-integral is path independent for EPFM problems in their papers. Such calculations are commonly performed with either an incremental plasticity theory or a deformation plasticity theory. However, it should be noted that the J-integral usually was calculated in the region where the selected integration path or domain was located far from the crack tip.

Nevertheless, it is worth noting that the work done by McMeeking (1977) used an incremental plasticity approach with large-deformation kinematics and showed that the J-integral is path dependent in the close vicinity of the crack tip. Stump and Zywicz (1993) also showed that the J-integral is path dependent within the plastic zone. The Moire' interferometer results measured by Sivaneri *et al*. (1991) also indicated that the J-integral is integration-path dependent, very near to the crack tip. Recently, Kuang and Chen (1996) have conducted the experimental and numerical study of the strain hardening materials like 7075-T651 aluminum alloy and HY-130 steel. In their results, the J-integral is path dependent for an integration contour within the plastic zone.

In this paper, the values of J-integral across or within the plastic zone have been investigated by using the p-version of the finite element method on the basis of incremental theory of plasticity. The size and shape of the plastic zone in the vicinity of a crack tip have been compared with those available in literatures. Also, the crack growth study is considered by using the p-version of the finite element method on the basis of the accurate J-integral for elastic-plastic materials. The experiments were performed with five CCP specimens to validate the accuracy of the proposed

EPFM approach in this work for prediction of fatigue life. For this purpose, the *p*-version finite element program has been developed by modifying the source program written by Owen and Fawkes (1983) that is based on the *h*-version of the finite element method.

2. Elastic-plastic behavior of crack-tip field

The material model considering the strain hardening effect is adopted by the well-known Ramberg-Osgood form to represent the material's uniaxial stress-strain ($\sigma - \varepsilon$) response, which is given by (Rahman 2001, Wei and Wang 1995);

$$\frac{\varepsilon}{\varepsilon_o} = \frac{\sigma}{\sigma_o} + \alpha\left(\frac{\sigma}{\sigma_o}\right)^n \tag{1}$$

where σ_o is the reference stress that is usually assumed to be the yield stress, E is the modulus of elasticity, $\varepsilon_o = \sigma_o/E$ is the associated reference strain, and α and n are parameters chosen to fit experimental data. n is the strain hardening exponent.

The increment relation of multiaxial stress and strain by J_2-flow theory of plasticity is expressed by (Wei and Wang 1995);

$$d\sigma_{ij} = \frac{E}{1+\nu}\left\{\delta_{im}\delta_{jn} + \frac{\nu}{1-\nu}\delta_{ij}\delta_{mn} - \frac{9\mu\Omega}{(6\mu + 2\hat{H})\sigma_e^2}S_{ij}S_{mn}\right\}d\varepsilon_{mn} \tag{2}$$

in which, S_{ij} is the deviatoric stress, $\sigma_e = \sqrt{3S_{ij}S_{ij}/2}$ is the effective stress, ν is Poisson's ratio, μ is shear modulus and $\hat{H}$ is tangential modulus of plasticity or called strain hardening parameter, which by Eq. (1) is denoted by;

$$\hat{H} = \frac{d\sigma_e}{d\varepsilon_p} = \frac{\sigma_o}{n\alpha\varepsilon_o}\left(\frac{\sigma_e}{\sigma_o}\right)^{1-n} \tag{3}$$

which parameter Ω is defined by identity on the loading surface and for $S_{ij}d\varepsilon_{ij} > 0$, otherwise zero. If the plastic strain increment in the direction of loading is $d\varepsilon_p$, then $(d\varepsilon_1)_p = d\varepsilon_p$ and since plastic straining is assumed to be incompressible, Poisson's ratio is effectively 0.5 and $(d\varepsilon_2)_p = -\frac{1}{2}d\varepsilon_p$ and $(d\varepsilon_3)_p = -\frac{1}{2}d\varepsilon_p$. Then the effective plastic strain $d\bar{\varepsilon}_p$ becomes the plastic strain increment $d\varepsilon_p$ shown in Eq. (3) (Owen and Fawkes 1983).

The plastic strain increment $d\varepsilon_p$ can be calculated by the Prandtl-Reuss equation.

$$(d\varepsilon_{ij})_p = d\lambda\frac{\partial f}{\partial\sigma_{ij}} = d\lambda\frac{\partial J_2}{\partial\sigma_{ij}} \tag{4}$$

Here, $d\lambda$ is a proportionality constant termed the plastic multiplier, f is the yield function that is same as plastic potential Q, and J_2 is the second deviatoric stress invarient. Generalizing Eq. (1) to multiaxial states by J_2 flow theory of plasticity, the stress-strain relation can be derived as;

$$\varepsilon_{ij} = \frac{1+\nu}{E}S_{ij} + \frac{1-2\nu}{3E}\sigma_{kk}\delta_{ij} + \frac{3}{2}\alpha\varepsilon_o\left(\frac{\sigma_e}{\sigma_o}\right)^{n-1}\frac{S_{ij}}{\sigma_o} \tag{5}$$

In the basis of the small strain J_2 deformation theory, the asymptotic solution ahead of a stationary crack could be developed. The first order asymptotic solution was given by Hutchinson (1968), Rice and Rosengren (1968) which was called for short the **HRR** singularity field;

$$\frac{\sigma_{ij}}{\sigma_o} = \left(\frac{J}{\alpha\varepsilon_o I_n r}\right)^{\frac{1}{n+1}}\tilde{\sigma}_{ij}(\theta) \tag{6}$$

where r and θ are polar coordinates, and $\tilde{\sigma}_{ij}(\theta)$ is angular distribution function that may be determined by matching two term solution with finite element solution. This **HRR** equation represents the crux of the basis for "*J*-controlled" crack growth behavior. The different type of stress fields in the vicinity of a crack tip is shown in Fig. 1. Recently, the high order asymptotic solutions are studied to explain the elastic-plastic behavior precisely near a crack tip that may be denoted by two parameter solutions such as *J*-*Q* and *J*-*T* characterization (Wei and Wang 1995). Regardless of solution with one parameter or two parameters, it should be noted that the accuracy of *J*-integral has directly effect on the asymptotic solution for stress field ahead of a crack tip.

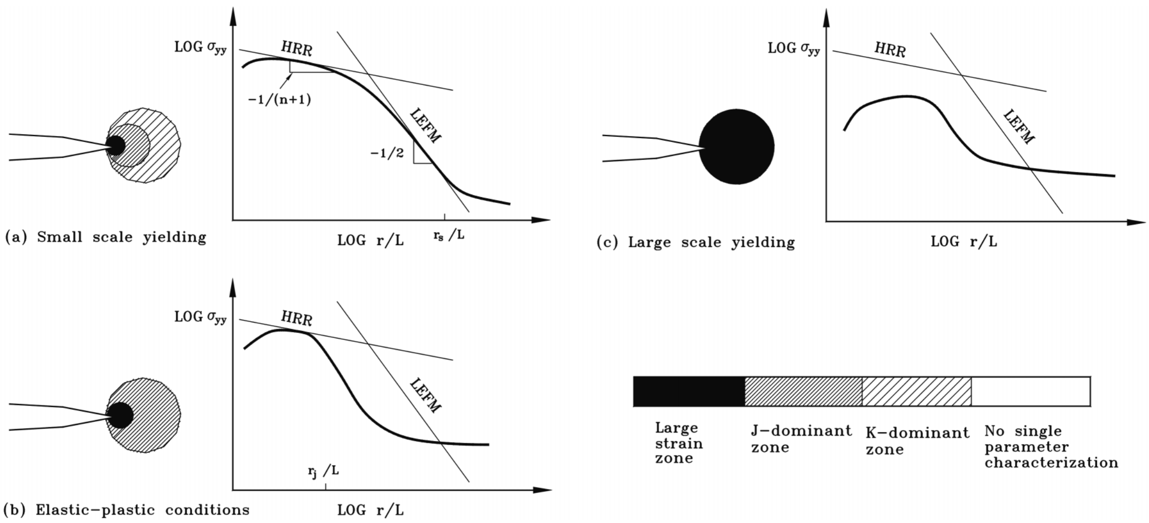


Fig. 1 Stress field in the vicinity of a crack tip

3. The shape of the plastic zone

The estimation of size and shape of plastic zone is very important to characterize the elastic-plastic behavior near a crack tip. The hypothetical solution for a small scale yielding was proposed by Irwin (1971). The boundary for the plastic zone is a function of θ that is defined by the stress intensity factor K instead of J. In this hypothesis, the stress dose not increase much after yielding. For a plane strain case, it can be obtained by von-Mises yield criteria;

$$r(\theta) = \frac{1}{4\pi}\left(\frac{K}{\sigma_{ys}}\right)^2\left[(1-2\nu)^2(1+\cos\theta) + \frac{3}{2}\sin^2\theta\right] \tag{7}$$

where σ_{ys} is the unaxial yield stress. On the other hand, the shape of the plastic zone for a plane stress case is derived by;

$$r(\theta) = \frac{1}{4\pi}\left(\frac{K}{\sigma_{ys}}\right)^2\left[1 + \frac{3}{2}\sin^2\theta + \cos\theta\right] \tag{8}$$

A different approach to finding the extent of the plastic zone was followed by Dugdale (1960). He considered an effective crack that is longer than the physical crack. The crack edges in front of the physical crack carry the yield stress σ_{ys} tending to close crack. In this approach, the stress intensity K under the uniform tension has to be compensated by the stress intensity K_p due to the wedge forces σ_{ys}. However, the proposed hypotheses are doubtful because of the limited validity of the expressions for K which are based on elastic solutions if the plastic zone is larger with respect to the crack. Several investigators (Miller and Kfouri 1974, Gdoutos and Papakaliatakis 1986) showed the plastic zone for a large scale yielding by numerical method as well as experimental method.

4. *J*-Integral by finite element method

The integral type parameter, *J*-integral, is defined for two-dimensional problems as integrals over an arbitrary closed contour Γ_c around the crack tip. The coordinate system x_i is on the basis of crack line direction and perpendicular direction to crack line, respectively. For a cracked body with an arbitrary counter-clockwise path, Γ_c around the crack tip, a formal definition of *J*-integral under mode-I condition is expressed by (Gdoutos and Papakliatakis 1986);

$$J = \int_{\Gamma_c}[Wn_1 - T_i u_{i,1}]dS \tag{9}$$

where $W = \int\sigma_{ij}d\varepsilon_{ij}$ is the strain energy density with σ_{ij} and ε_{ij} representing components of stress and strain tensors, respectively, u_i and $T_i = \sigma_{ij}n_j$ are the *i*-th component of displacement and traction vectors, n_j is the *j*-th component of unit outward normal to integration path, dS is the differential length along the closed contour Γ_c, and $u_{i,1} = \partial u_i/\partial x_1$ is the differentiation of displacement with respect to x_1.

For elastic-plastic applications, the strain energy density can be separated into elastic and plastic components.

$$W = W_e + W_p \tag{10}$$

then W_e is given by;

$$W_e = \frac{1}{2}\sigma_{ij}(\varepsilon_{ij})_e \tag{11}$$

where $(\varepsilon_{ij})_e$ denotes the elastic components of strain. The plastic work contribution is given by;

$$W_p = \int_0^{\bar{\varepsilon}_p} \bar{\sigma}\, d\bar{\varepsilon}_p \tag{12}$$

in which $\bar{\sigma}$ and $\bar{\varepsilon}_p$ are the effective stress and effective plastic strain.

Subject to the definition of Eq. (8), the finite element process for J-integral can be established by the following equation that is based on the standard coordinate system denoted by ξ and η.

$$J = \int_{-1}^{+1} \left\{ \frac{1}{2}[\sigma_{11}(\varepsilon_{11})_e + \sigma_{12}(\varepsilon_{12})_e + \sigma_{22}(\varepsilon_{22})_e]\frac{\partial x_2}{\partial \eta} + W_p \frac{\partial x_2}{\partial \eta} - \right.$$
$$\left. \left[(\sigma_{11}n_1 + \sigma_{12}n_2)\frac{\partial u}{\partial x_1} + (\sigma_{12}n_1 + \sigma_{22}n_2)\frac{\partial v}{\partial x_1}\right]\sqrt{(\partial x_1/\partial \eta)^2 + (\partial x_2/\partial \eta)^2} \right\} d\eta \tag{13}$$

where n_1 and n_2 are unit normal vectors to the integration path at any point. The integration path is conveniently chosen to coincide with the line $\xi = \xi_p$ that passes through the certain Gauss points.

5. Fatigue life prediction by EPFM approach

In the linear portion of the crack growth curve (stage II), the crack growth rate (da/dN) can be related to stress intensity range (ΔK). So, a crack growth law has been suggested by Paris and Erdogan (1963) which is now well established and is of the form;

$$\frac{da}{dN} = C(\Delta K)^m \tag{14}$$

where a is crack length, N is number of cycles, ΔK is stress intensity range, and C, m are material constants. In the regime of LEFM, ΔK and J-integral range (ΔJ) are related by the following equation;

$$\Delta J = \frac{(\Delta K)^2}{E} \tag{15}$$

where E is the elastic modulus. ΔJ can therefore be used in place of ΔK as long as gross plasticity is not present. An attempt has been made by Dowling and Begley (1976) in relating fatigue crack growth rate and ΔJ on the pressure vessel steel by using Eq. (15). Also, Srivastava and Garg (1988) reported the fact that the fatigue crack propagation law can also be expressed as a function of ΔJ. The ΔJ value for a elastic-plastic material may be defined as (Srivastava and Garg 1988);

$$\Delta J = \Delta J_e + \Delta J_p \tag{16}$$

where ΔJ_e and ΔJ_p are elastic and plastic component of J-integral ranges. Thus, the *modified Paris equation* can be derived from Eq. (14) by substituting ΔJ for the range of stress intensity factor denoted by ΔK. It is noted that the material constants C and m in Eq. (17) are totally different from those in Eq. (14) which are determined by the fatigue test.

$$\frac{da}{dN} = C(\Delta J)^m \tag{17}$$

The estimation of fatigue life can be obtained by integrating Eq. (17);

$$N = \int_{a_i}^{a_f} \frac{da}{C(\Delta J)^m} \tag{18}$$

where a_i is an initial crack length and a_f is a critical crack length. In this study, a_f is assumed to be a critical crack length for centrally cracked panels when the crack ligament along the crack line causes full yielding like;

$$a_f = \frac{W}{2}\left(1 - \frac{q}{\sigma_{ys}}\right) \tag{19}$$

where W is half width of cracked panels, q is uniform tensile load, and σ_{ys} is uniaxial yield stress.

6. *p*-Version finite element method

In *p*-version formulations, it is assumed that as the degree p of the approximating functions approaches a large value, the solution converges to the true solution. Key to the formulation is the appropriate selection of the shape functions to approximate the system variables (Basu and Lamprecht 1979, Babuska and Szabo 1982). In order to achieve the advantages of ease of data preparation and model refinement, shape functions must be selected that provide the order to be increased by simply augmenting the lower order shape functions. In this study, integrals of Legendre polynomials are used as shape functions that are in hierarchical nature. There are several attractiveness of hierarchical element like high accuracy, robustness and computational efficiency, especially in fracture mechanics (Woo 1993, Woo and Jung 1994, Woo and Lee 1995, Woo *et al*. 1998). High accuracy is due to an exponential rate of convergence in the case of an analytical exact solution. This exponential rate can even be obtained for problems with singularities when an increase of the polynomial order is combined with local mesh refinement in an *hp*-version. The robustness of the *p*-version allows the use of strongly distorted elements and prevents from Poisson locking in cases of nearly incompressible materials and from shear locking in thin plate situations based on Reissner-Mindlin theory. Furthermore, it has been shown that the *p*-version is superior in parallel efficiency as compared to a classical *h*-version approach.

In this work, the incremental theory of plasticity has been adopted for strain hardening materials. The constitutive relationship is described by incorporating associated flow theory into the isotropic hardening model for elastic-plastic materials. The discrete incremental matrix is solved by the full Newton-Raphson method. The Gaussian quadrature is used for numerical integration where the number of Gauss points is extended to 10×10 integration points. The yielding criteria are applied at each Gauss point whether the plastic deformation may occur. The computer code has been developed for this purpose that is actually based on the *h*-version program written by Owen and Fawkes (1983). A lot of modifications have been carried out to calculate the *J*-integral and fatigue life N by the *p*-version of the finite element method.

7. Numerical results

7.1 A centrally cracked panel with elastic-perfectly plastic materials

The geometry of a centrally cracked panel (CCP) under simple tensile loading is shown in Fig. 2. Due to symmetry, a quarter of panel is modeled by the *p*-version and *h*-version of the finite element method. In case of the *p*-version finite element model in Fig. 2, a quarter of the structure with height $2H$, width $2W$, and crack length $2a$ is discretized into only four elements where H/W and a/W are fixed as 2.5 and 0.4, respectively. On the other hand, the *h*-version finite element model by ADINA consists of 9 three-node elements for the crack-tip region and 90 four-node elements for other regions. Also, 20 eight-node serendipity elements are modeled by Owen (1983) Those *h*-version models are determined from the convergence test. The convergence characteristics of *J*-integral by the *p*-version finite element model are plotted in Fig. 3 with respect to the inverse of degree of freedoms as the load factors are increased, where the load factor is denoted by $L.F. = (q/\sigma_{ys}) \times 10$. It is shown that *J*-integral values with $p = 5$ begin to converge to the reference values by ADINA.

The material properties assumed in this study are as follows: elastic modulus ($E = 2.1\times10^6$), Poisson's ratio ($\nu = 0.3$), uniaxial yield stress ($\sigma_{ys} = 2{,}400$), strain hardening parameter ($\hat{H} = 0$).

The *J*-integral values obtained by the *p*-version finite element model with $p = 5$ agree very well with those by the *h*-version finite element model within ±0.5% relative errors, which are shown in Fig. 4 with respect to the radial distance of the integral contour from the crack tip as the load factor is increased from $L.F. = 3.333$ to $L.F. = 5.0$. It is apparent that the *J*-integral is path-dependent when the selected integration contours pass within the plastic zone. However, for a path-independent *J*-integral, the selected integration contour should be greater than the plastic zone size. This result can be confirmed by McMeeking (1977), Stump (1993), Sivaneri (1991) and Kuang (1996). The path-dependent *J*-integral becomes more serious as the development of plastic zone is increased.

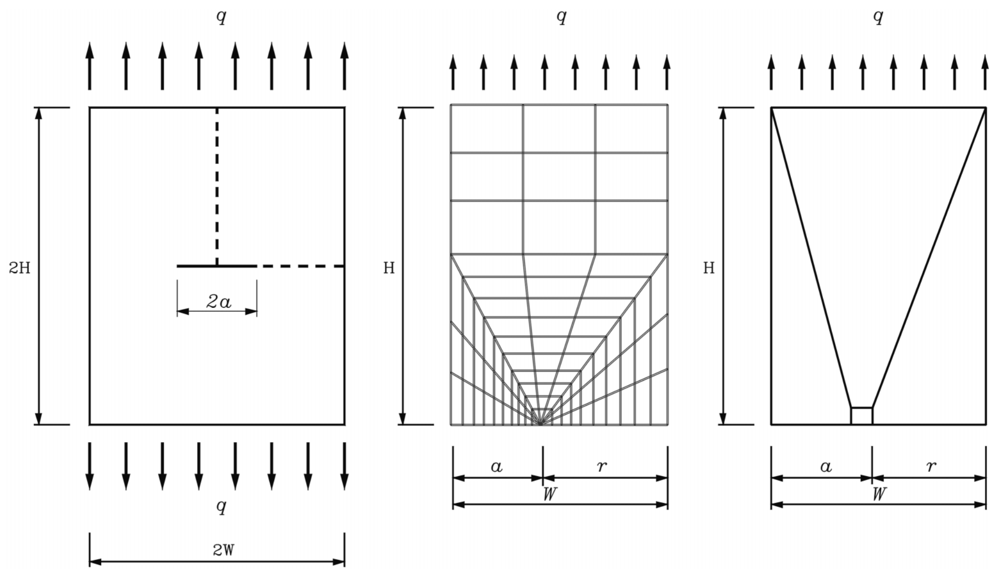


Fig. 2 Geometric configuration of a centrally cracked panel and finite element models

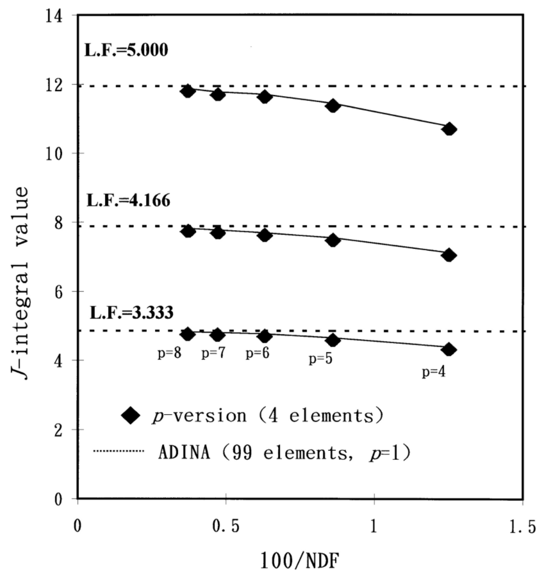


Fig. 3 Convergence characteristics of *J*-integral with respect to different load factors by the *p*-version finite element model

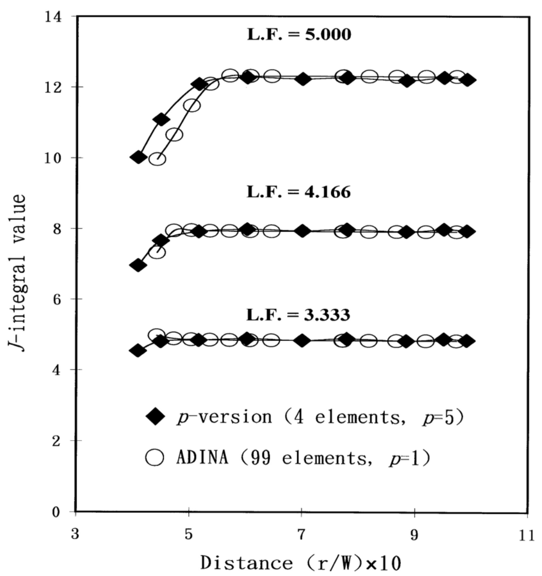


Fig. 4 The values of *J*-integral with different load factors according to integral paths

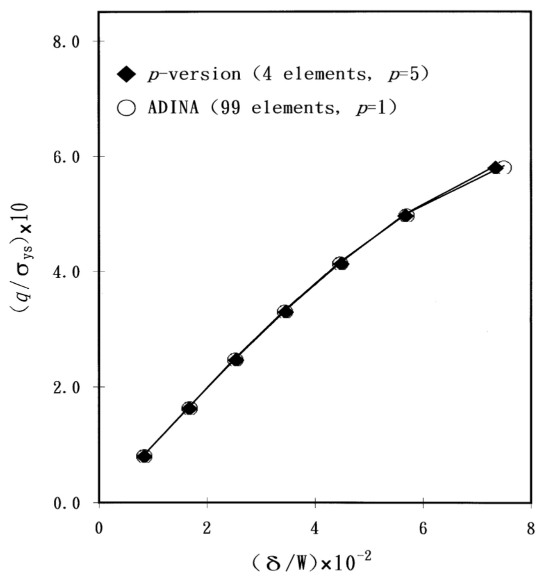


Fig. 5 The non-dimensional crack opening displacement with respect to load increment

The non-dimensional crack opening displacement (COD) by the proposed model is compared with that obtained by ADINA software at the center of crack surface, shown in Fig. 5, as the tensile load increases. The results by the proposed model are almost in line with those obtained by ADINA.

The numerical results for stress distribution of σ_{yy} near a crack tip, perpendicular to the crack line, by several *p*-version models are compared with the theoretical solution by Irwin (1971). As we expected, the stress distribution, especially near crack tip, by EPFM analysis showed large discrepancies compared with other results by LEFM analysis and theoretical estimation, which is plotted in Fig. 6 under plane stress condition with *L.F.* = 5.0. It may be noted that the stress level should be decreased near a crack tip because of stress redistribution as the plastic zone begins to be developed ahead of crack tip. To show the accuracy of the proposed model, the stress distribution of σ_{yy} is compared with the results by the *h*-version models in Fig. 7 under plane stress condition where the load factor is fixed as *L.F.* = 5.0. The results by the fifth order *p*-version four element model exhibit good comparisons with those by Owen and ADINA. It is also concluded that the degree of freedoms by the proposed model are much less than that by the *h*-version models.

The variation of COD from the center of crack surface to crack tip is shown in Fig. 8 as *L.F.* is increased from 4.0 to 6.0. It demonstrates that all results by three different approaches show good comparison each other, however, the numerical results by ADINA show some discrepance when *L.F.* = 6.0. The *J*-integral values are shown in Fig. 9 under the different load factors varying from

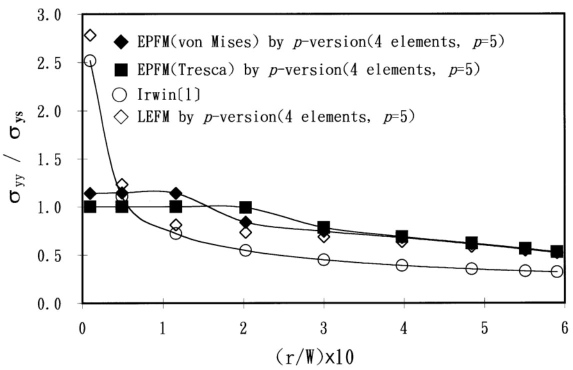


Fig. 6 Distribution of σ_{yy} near a crack tip under plane stress condition when *L.F.* = 5.0

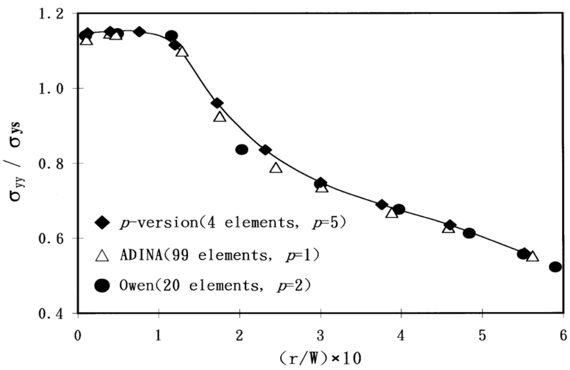


Fig. 7 Comparison of EPFM analyses of σ_{yy} near a crack tip under plane stress condition when *L.F.* = 5.0

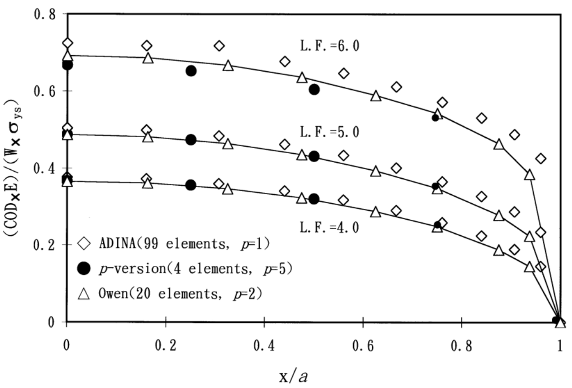


Fig. 8 Comparison of crack opening displacement with respect to different load factors

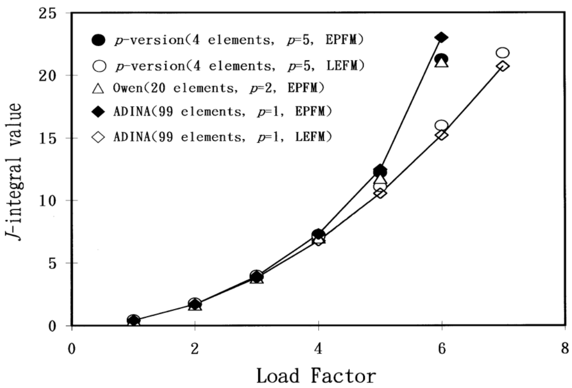
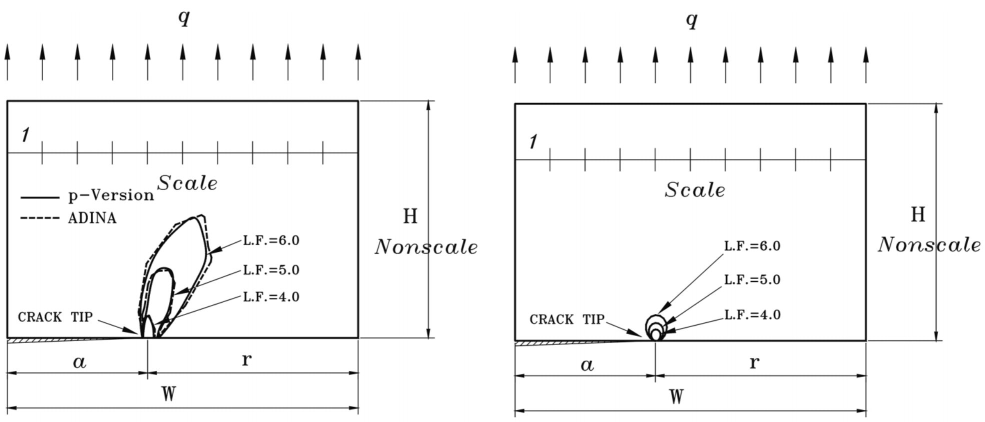


Fig. 9 Comparison of *J*-integral values with respect to different load factors

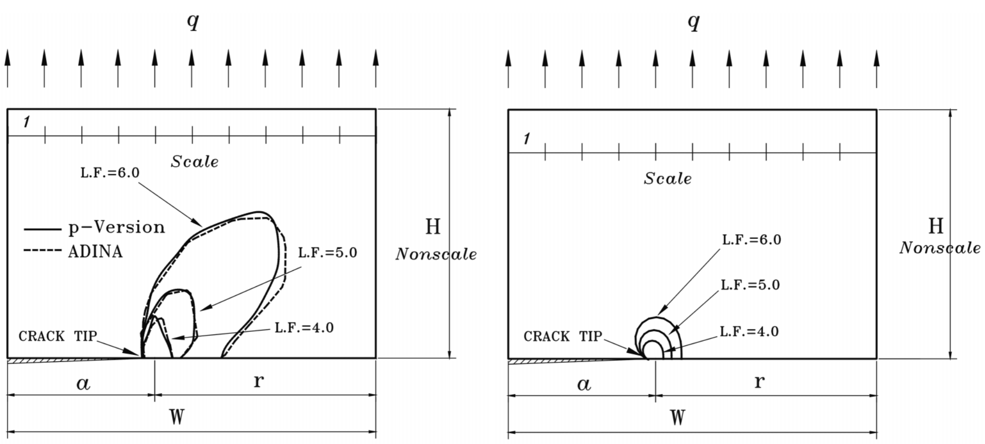
$L.F = 1.0$ to $L.F. = 7.0$. From $L.F. = 4.0$ that is equivalent tensile load to 40% of yield stress, the difference of J-integral appears larger between LEFM and EPFM analyses. Therefore, the EPFM approach is very essential tool for large-scale yielding problems.

The size and shape of plastic zone near a crack tip is investigated by the *p*-version finite element model in comparison with the hypothetical solution for a small scale yielding by Irwin (1971). The shape of plastic zone has been estimated in Figs. 10-11 under three applied loads factors and the conditions of plane stress/strain that is based on von-Mises yield criteria. The results by ADINA are compared to validate the numerical results by the *p*-version finite element model. As explained earlier, the proposed hypothesis by Irwin is doubtful because of the limited validity of the expressions for K which are based on elastic solutions if the plastic zone is larger with respect to the crack. It is noted, thereby, that the plastic zones are about twice as large as the analytical solution by Irwin when $L.F. = 4.0$. This tendency appears much more serious for plane stress condition as the load factor is increased.



(a) p-Version (b) Irwin

Fig. 10 Shape of plastic zone based on von-Mises yield criteria under plane strain condition

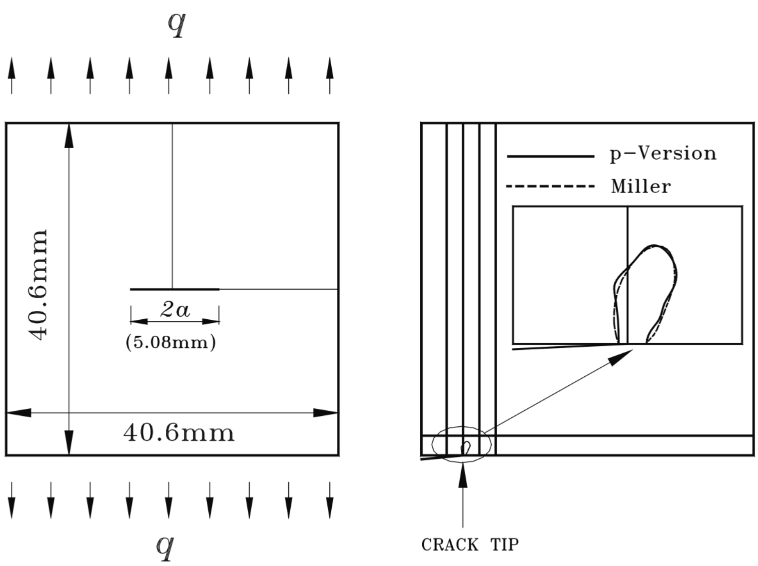


(a) p-Version (b) Irwin

Fig. 11 Shape of plastic zone based on von-Mises yield criteria under plane stress condition

7.2 A centrally cracked panel with bilinear elastic-plastic materials

A center-cracked specimen (40.6 × 40.6 × 1.0 mm) with a crack length of 5.08 mm is considered. This is assumed to be under the condition of plane strain and made of bilinear elastic-plastic materials (elastic modulus $E = 206.85$ kN/mm^2, tangent modulus of elasticity $E_T = 965.3$ N/mm^2, hardening parameter $\hat{H} = 969.8$ N/mm^2, Poisson's ratio $\nu = 0.3$, uniaxial yield stress $\sigma_{ys} = 310.26$ N/mm^2, and tensile load $q = 177$ N/mm^2). The geometry and material properties are in accordance with Miller's model (1974) to verify the shape of plastic zone. Since the system is symmetrical, it is only required to analyze quadrant 1 whose finite element mesh is idealized in Fig. 12. The mesh by Miller involves 99 linear elements and 121 nodes, on the other hand, the ten fifth order hierarchical elements are used in the *p*-version finite element model with 10 × 10 Gauss points, for the convenience of accurate plotting of plastic zone. The occurrence of plasticity near a crack tip has been checked at each Gauss point by von-Mises yield criteria. It may be noted that the length of the plastic zone radiating from the crack tip is approximately 1.15 mm and is inclined at 65^o to the crack plane when *L.F.* = 5.7. These facts are very similar to those by Miller shown in Fig. 12. In the *p*-version finite element model, the element shape seems to be very slender which gives a large aspect ratio. However, it was proven that the *p*-version finite element model tolerates the large aspect ratio up to 4000 if we use 5% accuracy in (Woo 1993).



(a) CCP specimen (b) p-version finite element model

Fig. 12 Shape of plastic zone of bilinear strain hardening materials under plane strain condition

7.3 A centrally cracked panel with nonlinear elastic-plastic materials

To estimate the elastic-plastic behaviors for nonlinear strain hardening materials, the uniaxial stress-strain curve is used for numerical analysis that comes from the experimental data given by Gdoutos (1986). Since the stress-strain relation is nonlinear as shown in Fig. 13, the nonlinear part of stress-strain curve after yielding is divided by sixteen segments for linearization. From this, the hardening parameter $\hat{H}$ can be defined at each separated region. Therefore, the current stress is calculated by $\hat{H}$ multiplied by the plastic strain increment $d\varepsilon_p$ in addition to the previous stress. The geometry of square CCP specimen is fixed as same model of Gdoutos with width of 25.4 cm, length of 25.4 cm shown in Fig. 14. In the case of p-version finite element model, a quarter of plate is discretized into ten hierarchical elements with p = 5 in order to draw the shape of plastic zone accurately. From the Fig. 14, it is known that the development of plastic zone by the proposed model shows an excellent agreement with the results by Gdoutos as the load factor is increased.

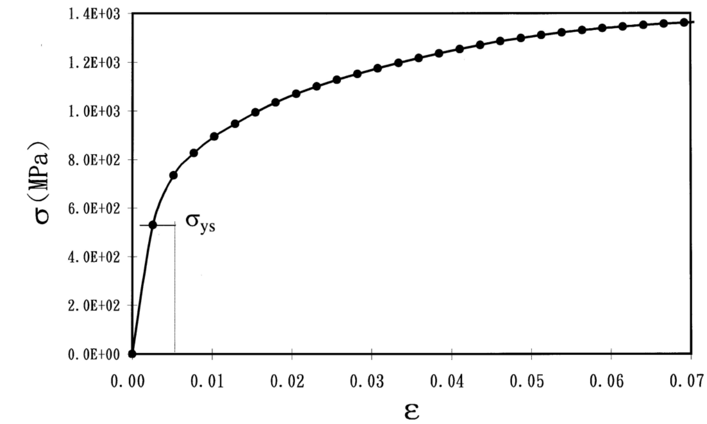


Fig. 13 Uniaxial stress-strain relation for nonlinear strain hardening materials given by Gdoutos (1986)

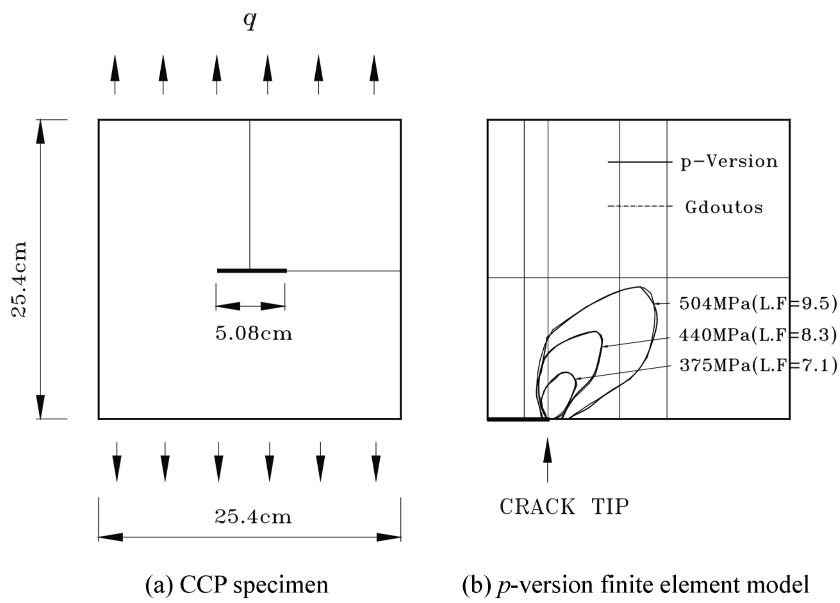


Fig. 14 Shape of plastic zones of nonlinear strain hardening materials with respect to different load factors under plane strain condition

7.4 Fatigue life prediction of CCP specimen by EPFM approach

To predict the fatigue life, the CCP specimen with elastic-perfectly plastic relation has been analyzed by EPFM approach as well as the conventional LEFM approach for the purpose of comparison. The experimental fatigue test has been performed on five same CCP specimens made of structural steel that is shown in Fig. 15 to validate the accuracy of both approaches by the *p*-version of finite element method. The mechanical properties of CCP specimen are summarized in Table 1. The fatigue test was carried out on MTS system with a load capacity of 100 kN operating under load control. Sinusoidal cyclic loadings of constant amplitude were applied at a frequency of 10 Hz. The maximum load for five tests was kept constant at 10 kN for the stress ratio R of 0.05. The initial crack length is measured by a_i = 3.3 mm, the critical crack length of a_f = 8.75 mm can be calculated by Eq. (19), and the thickness of panel is 3.2 mm denoted by t.

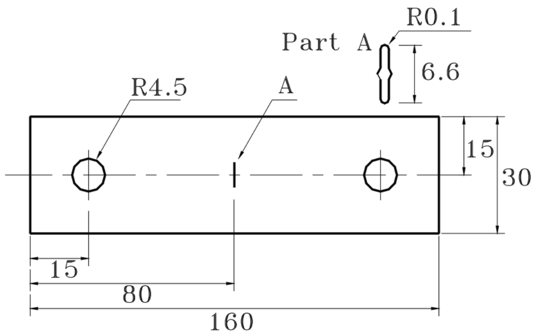


Fig. 15 Geometry of CCP specimen (unit: mm)

Table 1 Mechanical properties of CCP specimen

σ_{ys} (MPa)	σ_u (MPa)	E (MPa)	Elongation (%)
257.6	379.5	235,000	23.7

From the fatigue test, the number of cycles to failure are 422,000 from the initial crack length 3.3 mm to critical crack length. Also, the actual critical crack length was measured by 11.25 mm. As described earlier, the calculated critical crack length was 8.75 mm. This difference does not have effects on prediction of fatigue life due to the characteristics of crack-growth curve shown in Fig. 16. The material constants C and m for modified Paris equation are determined by 1.55×10^{-5}, and 1.79, respectively, from the relation of da/dN and ΔJ that is plotted in Fig. 17.

The *p*-version finite element model is shown in Fig. 18 where the *p*-order of shape function is fixed as eight. The range of *J*-integral, denoted by ΔJ, has been estimated with respect to the increment of crack size varying from a/t = 1.03(a_i = 3.3 mm) to a/t = 2.73(a_f = 8.75 mm) that is summarized in Table 2. Also, the relationship between ΔJ and a has been plotted in Fig. 19. The equation of the range of *J*-integral can be derived by the least square exponential fitting. It is noted that the relationship between the crack length a and ΔK in LEFM analysis shows a strong linearity, on the other hand, the nonlinear relationship between a and ΔJ is detected in EPFM analysis. Therefore, this trend will be depended especially in the case of large scale yielding.

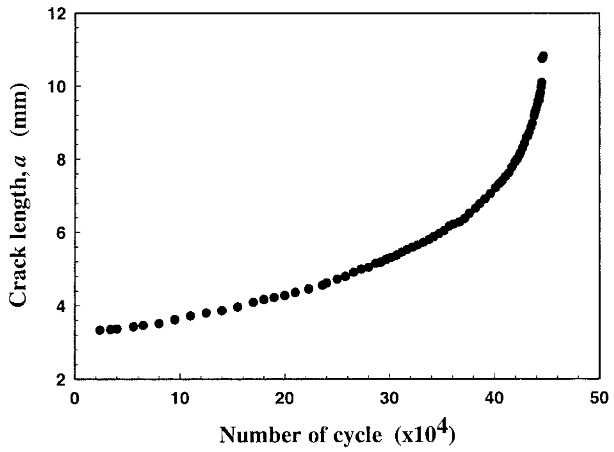


Fig. 16 Crack growth curve of CCP specimen

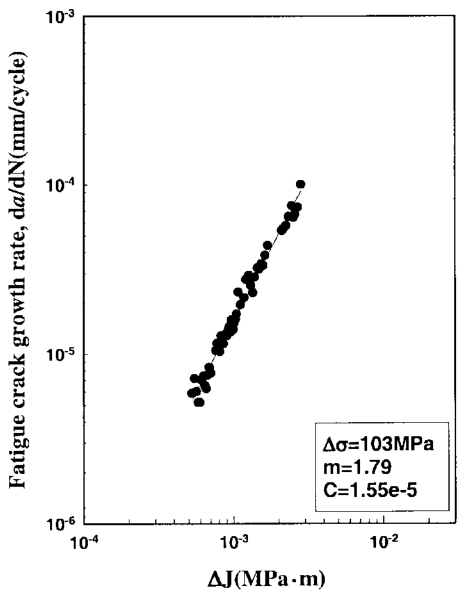


Fig. 17 Relation between da/dN and ΔJ

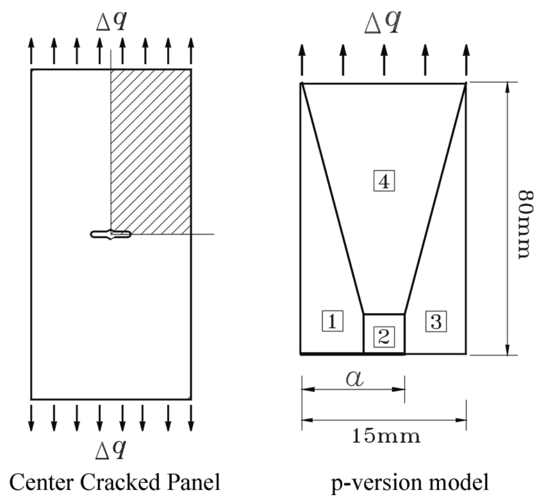


Fig. 18 The *p*-version four element model

Table 2 The range of *J*-integral with respect to increment of crack size by EPFM analysis

a	*a/t*	ΔJ (MPa · mm)
3.3	1.03	0.4910
5.0	1.56	0.8105
7.0	2.18	1.3573
8.75	2.73	2.4632

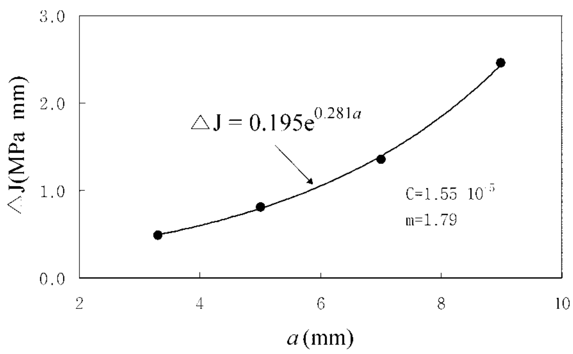


Fig. 19 Curve fitting for ΔJ and a by EPFM analysis

The final results for fatigue life of CCP specimen can be calculated by Eq. (18) and are summarized in Table 3. The result by EPFM analysis based on the p-version of the finite element method shows 429,000 cycles that is very close to the experimental result of 422,000 cycles. Also, these results are compared with that by the conventional LEFM analysis considering two different cases that are based on the same p-version finite element model. For Case I, the material constants C and m are assumed to be 2.18×10^{-13} and 3 because CCP specimen in this study can be classified as the ferrite-pearlite steel. For Case II, however, the values of C and m are determined as 6.30×10^{-10} and 3.74 from the relation of ΔK and a by experiments. It is concluded that the LEFM analysis is not suitable for the strain hardening materials to predict the fatigue life.

Table 3 Results of fatigue life of CCP specimen

Methodology	Fatigue Test	LEFM Analysis		EPFM Analysis
		Case I	Case II	
Fatigue Life N(cycle)	422,000	245,000	385,000	429,000

8. Conclusions

The new finite element model is proposed to characterize the elastic-plastic behaviors near a crack tip by using the fracture parameter J-value for strain hardening materials. Results in this paper can be summarized as:

(1) The J-integral values obtained by the p-version finite element model agree very well with those by the h-version finite element model within ±0.5% relative errors.

(2) The J-integral is path-dependent when the selected integration contours pass within the plastic zone. However, for a path-independent J-integral, the selected integration contour should be greater than the plastic zone size. The path-dependent J-integral becomes more serious as the development of plastic zone is increased.

(3) The hypothesis by Irwin is doubtful because of the limited validity of the expressions for K which are based on elastic solutions if the plastic zone is larger with respect to the crack. It is noted, thereby, that the plastic zones are about twice as large as the analytical solution by Irwin when $L.F. = 4.0$. This tendency appears much more serious for plane stress condition as

the load factor is increased.

(4) The accurate prediction of fatigue life is obtained for CCP specimen by EPFM anlysis. It is noted that relationship between a and ΔK in LEFM analysis shows a strong linearity, on the other hand, the nonlinear relationship between a and ΔJ is detected in EPFM analysis. Therefore, this trend will be depended, especially in the case of large scale yielding problem.

From these results, it may be concluded that the *p*-version finite element model is suitable for EPFM analysis as well as LEFM analysis to calculate the *J*-integral near a crack tip and fatigue life for strain hardening materials.

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