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Optimal lay-up of hybrid composite beams, plates and shells using cellular genetic algorithm

S. Rajasekaran†, K. Nalinaa‡, S. Greeshma‡†,
N.S. Poornima‡† and V. Vinoop Kumar‡†

Department of Civil Engineering, PSG College of Technology, Coimbatore - 641004, Tamilnadu, India

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Abstract. Laminated composite structures find wide range of applications in many branches of technology. They are much suited for weight sensitive structures (like aircraft) where thinner and lighter members made of advanced fiber reinforced composite materials are used. The orientations of fiber direction in layers and number of layers and the thickness of the layers as well as material of composites play a major role in determining the strength and stiffness. Thus the basic design problem is to determine the optimum stacking sequence in terms of laminate thickness, material and fiber orientation. In this paper, a new optimization technique called Cellular Automata (CA) has been combined with Genetic Algorithm (GA) to develop a different search and optimization algorithm, known as Cellular Genetic Algorithm (CGA), which considers the laminate thickness, angle of fiber orientation and the fiber material as discrete variables. This CGA has been successfully applied to obtain the optimal fiber orientation, thickness and material lay-up for multi-layered composite hybrid beams plates and shells subjected to static buckling and dynamic constraints.

Key words: cellular automata; composites; genetic algorithm; optimisation; buckling load; frequency.

1. Introduction

Most of the methods used for design optimization assume that the design variables are continuous. In Structural optimization, almost all design variables are discrete. A simple Cellular Genetic Algorithm is used to obtain the optimal laminate thickness, fiber orientation and material of multi-layered composite plates. Cellular Automata combined with simple Genetic Algorithm is based on two operators namely cross over and mutation.

Cellular Automata(CA) are mathematical idealization of physical systems originally introduced by Von Newmann and Ulam (1974) to biological systems to model self reproduction in which space and time are discrete and physical quantities take on finite set of discrete values. CA evolves in discrete time steps, with the value of the variable at one set being affected by values of variables at sites in its "neighbourhood" in the previous time step. The variables at each site are updated simultaneously ("Synchronously") based on the values of neighbourhood and according to definite

† Professor of Infrastructural Engineering
‡ Senior Lecturer in Civil Engineering
‡† Graduate Students

set of local rules involving Darwinian theory of survival of the fittest and applying genetic operators such as crossover and Mutation. The working of Simple Genetic Algorithm is explained by Goldberg (1989). Rajeev and Krishnamoorthy (1992) have applied simple Genetic Algorithm to the optimization of two and three-dimensional pin-jointed members subjected to stress and deflection.

2. Cellular Genetic Algorithm (CGA) for the optimal lay-up of structures made of composite laminates

A beam, plate or shell is optimized for its weight considering laminate thickness, fiber orientation and material adopting Cellular Genetic Algorithm. The multi-layered structure composed of two, four, six… to sixteen layered plates of different thickness are considered separately and analysed for free vibration. With reference to the middle plane symmetrical and anti-symmetrical fiber orientations are adopted. The discrete variables are laminate thickness, fiber orientation and the type of material.

Binary coding system is used to represent the variable and a sub string of 4-bit length is used for representing laminate thickness, fiber orientation and material. A total length of $n \times 12/2$ bit represents one solution for even layers both for symmetric and anti-symmetric orientations. For symmetric orientation, the layers at center may be combined to make it as one layer. If the layers are even and are symmetric lay-up the same orientation is repeated for the other half and in case of anti-symmetric lay-up the angles are repeated with change in sign for the other half. For 4-bit string we can represent minimum and maximum values of any variable as 0000 and 1111 and the real coding being 0 to 15. If the minimum and maximum values for any variable are given one can find the incremental value as

$$X_{inc} = (X_{\max} - X_{\min})/15 \tag{1}$$

The decoded value of binary number 1101 is shown in Fig. 1 and the corresponding X value is given by

$$X = X_{\min} + 13 \times X_{inc} \tag{2}$$

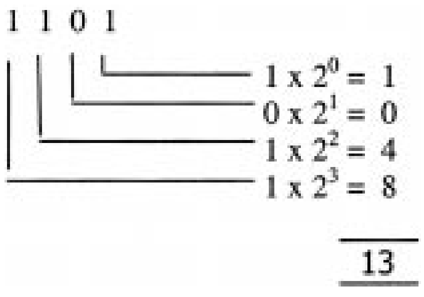

Fig. 1 Decoded value of 1101

The maximum and minimum values for the laminate thickness are given depending on the problem and the maximum and minimum values of the angles of fiber orientations are given as 90 deg and 0 deg and for the material 4.75 and 1 respectively. Herein we consider only four materials and the integer value is taken as the material value.

The composite structure is analysed and the objective function for each population in a cell of cellular automata is computed using the closed form solution as indicated below. The objective is to get the laminate thickness, optimal fiber orientation and material to give the least weight design subjected to deflection, buckling and frequency constraints. The population is matted with the best population in its neighbourhood and crossed between the cross-sites along the random lengths of the full string. The process is repeated till to get the minimum weight or minimum total thickness.

Composite materials are used for various applications and are ideal for structural applications where high strength-to-weight ratio is required. Aircraft and other vehicles are typical weight sensitive structures in which composite materials such as Boron/epoxy, Carbon/epoxy and Graphite/epoxy and Glass/epoxy have resulted in an increase in the use of laminated fiber reinforced plates.

Composite materials are nothing but the combination of two or more materials on a microscopic scale to form a macroscopic high strength and lightweight material. Usually the composite shows the best qualities of its ingredients and sometime desirable qualities that cannot be obtained by the use of parent material alone can also be achieved. Moreover the properties, which can be improved by combining proper materials for a composite are strength, stiffness, corrosion resistance, wear resistance, attractiveness, weight, fatigue life, temperature dependent behaviour, thermal insulation, thermal conductivity and acoustical insulation.

Composite structures which are made up of more than one material have found widespread applications in various fields of engineering such as aerospace, marine, automobile, electronic equipment, structures, etc. The term composite is used to denote layered laminates where each layer is made up of an orthotropic material, distinct from sandwich plate which is typically a plate having a core material that separates two relatively thin face sheets of higher modulus material. In the present work, the composite laminates composed of straight parallel fibres are used. Each layer is assumed to be a homogeneous orthotropic material having a value of Young's modulus considerably greater in longitudinal direction (E_L) than in transverse direction (E_T), but the longitudinal axes of adjacent laminae are generally not parallel. In composites the fiber bears the mechanical load while the matrix distributes the loads and holds the shape of the part. The main advantage of composites is the possibility of tailoring a laminate to suit the structural requirements. The properties of the materials are given in Table 1 as given by Kao (1997).

Table 1 Material properties

Composites	E_L KN/mm^2	E_T KN/mm^2	E_N KN/mm^2	γ_{LT}	G_{LT} KN/mm^2	Mass density
1.Graphite/Epoxy	181.0	10.30	10.30	0.28	7.17	0.022e-4
2. Boron/Epoxy	204.0	18.5	18.5	0.23	5.59	0.0208e-4
3. Kevlar/Epoxy	83.0	5.6	5.6	0.34	2.1	0.01e-4
4. Glass/Epoxy	38.6	8.27	8.27	0.26	4.14	0.025e-4

5	4	3
6	1	2
7	8	9

Fig. 2 Moore neighbourhood

2.2 Cellular genetic algorithm: An introduction

In the last six years Genetic Algorithms have emerged as a practical robust optimization and search method. First proposed by Holland (1975), GA's are attractive classes of computational models that mimic natural evaluation to solve the problems in wide variety of domains. Pioneering work by Holland, Goldberg (1989), Dejong Grefenstette Davis, Muhlenbein, Srinivas (1997) and others fueling the spectacular growth of GAs.

A genetic algorithm emulates biological evolutionary theory to solve optimization problems. Genetic Algorithm comprises a set of individual elements (the populations) and a set of biologically inspired operators defined out the population itself. According to evolutionary theory only the most suited element in a population is likely to survive and generate off spring, thus transmitting the biological heredity to new generation. In computing terms a GA maps a problem on to a set of (typically binary) strings each string representing a potential solution.

Cellular automata evolves in discrete time steps, with the value of the variable at one site being affected by the values of variables at sites in its neighbourhood on the previous time step. The neighbourhood of a site is typically taken to be the site itself and all immediately adjacent cells. The variables at each site are updated simultaneously i.e. synchronously, based on the values on the variables in their neighbourhood at the preceding time steps according to set of local rule. There are several possible lattices and neighbourhood structures for two-dimensional cellular automata. Four cells (5 including its own) in the neighbourhood is known as Von Neumann neighbourhood, and eight cells (9 including its own) known as Moore neighbourhood and 6 cells in a hexagonal pattern known as uniform neighbourhood respectively. In this paper Moore neighbourhood is considered as shown in Fig. 2.

2.2.1 Comparison between GA and other traditional methods

GAs differs from traditional optimization algorithm in many ways. A few are listed here.

- GA does not require a problem specific knowledge to carryout a search. For instance, Calculus-based search algorithms use derivative information to carryout a search.
- GA works on coded design variables, which are finite length strings. These strings represent artificial chromosomes. Every character in the string is an artificial gene. GA processes successive populations of these artificial chromosomes in successive generations.
- GA uses a population of points at a time in contrast to the single point approach by the traditional optimization methods. That means, at the same time GAs process a number of designs.
- GA uses randomized operators in place of the usual deterministic ones.

2.2.2 Comparison between the biological terms and the corresponding terms in GA

- "Chromosome", a small rod like body found in the living cells, which is responsible for the transmission of genetic information denotes coded design vector in GA.
- "Gene" which is a part of the chromosome carrying the hereditary information denotes each bit in the coded design vector in GA.
- "Population" denotes a number of coded design variables in a cell whereas "Generation" denotes the population of design vectors, which are obtained after one computation.

2.3 Power of genetic algorithm

Genetic Algorithm combines the Darwinian survival of the fittest procedure. Genetic algorithms are search procedures based on mechanics of natural genetics and natural selection. Genetic Algorithm derives its power from the following genetic operators.

1) Reproduction 2) Cross over 3) Mutation 4) Inversion 5) Dominance 6) Deletion 7) Intra-chromosomal duplication 8) Translocation 9) Segregation 10) Speciation 11) Migration 12) Sharing 13) Mating

In this paper a simple Genetic Algorithm combined with Cellular Automata uses Crossover and mutation operators to find the optimal lay-up of composite Laminates.

2.4 Working of Cellular Genetic Algorithms

The weight of a composite structure varies with the laminate thickness and the deflection, buckling load and frequency of the structure depends on fibre orientation, laminate thickness and the material in the laminates. The design problem of composite structure is to find the minimum weight or minimum thickness such that maximum deflection is less than allowable deflection and frequency is greater than the allowable maximum frequency or the buckling load is greater than the allowable load. To find the optimum parameters of fibres CGA is used. The orientation of fibres in layers is such that the laminates are either symmetric or anti-symmetric of layers in the structure and the scheme has been employed for the genetic algorithm. In case of even number of layers, the first half of the layers about the middle surface are taken as design variable for CGA and for second half layer, if symmetric orientation the same layers as the first half are used and if anti-symmetric the layers of first half with negative sign are used. Only one half of the layer orientations are used for all the operations of Reproduction and Crossover.

In this subsection, the working of CGA is explained with reference to a five-layered symmetric orientation of thin composite square plate composed of different subjected to dynamic loading. The assumed data for the plate is 40 × 40 mm and thickness 0.8 mm.

In this example, the design variable for five layered plates is three because only half of the layers above the middle layer and the middle layer itself are considered for the symmetric orientation of fibres. The orientation of fibres can be varied as discrete values and varies from 0 to 90 as minimum and maximum values and similarly the thickness of a lamina. For the material the minimum and maximum values are assumed to be 1 to 4.75 and hence the truncated value of the material is chosen.

A twelve bit binary string is used to code three variables in which case a variable can take 16 discrete values. Hence for five-layered symmetric orientation we require 36 bits as $(5 + 1) \times 12/2$. In the cellular automata, we consider cells consisting of four rows and four columns and each cell representing a population. The number of cells or the populations depends on the importance of the problem and the complexity involved.

2.4.1 Survival of the fittest

Each cell in the cellular automata represents a population containing the thickness, fiber orientation and material information for the square plate. Each population is analysed using closed form solutions for deflection, the minimum circular frequency and the buckling load and thus the objective function i.e. the weight for all the cells are determined. After the objective functions for all the populations are obtained, the next step is to generate the population for the next generation, which are the offspring for the current generation. Every cell is examined with the cells of its neighbourhood and the cell having minimum weight is determined.
The objective function is given by

$$\text{Min weight } W = \rho \sum_{k=1}^{n} t_k \tag{3}$$

Subjected to constraints as

$$\frac{\delta}{\delta_{all}} \le 1; \quad \frac{P}{P_{cr}} \ge 1; \quad \frac{\omega}{\omega_{all}} \ge 1 \tag{4}$$

The constraint equations may be written as

$$C_1 = \left(\frac{\delta}{\delta_{all}} - 1\right) \quad \text{if} \quad \delta > \delta_{all}$$
$$= 0 \text{ otherwise}$$

$$C_2 = \left(1 - \frac{P}{P_{cr}}\right) \quad \text{if} \quad P < P_{cr}$$
$$= 0 \text{ otherwise}$$

$$C_3 = \left(1 - \frac{\omega}{\omega_{all}}\right) \quad \text{if} \quad \omega < \omega_{all}$$
$$= 0 \text{ otherwise} \tag{5}$$

$$C = \sum_{i=1}^{3} C_i \tag{6}$$

where δ, P_{cr}, ω_{all} are the deflection buckling load and the natural maximum allowable frequency of the plate.

Using the method of Rajeev and Krishnamoorthy (1992) the constrained optimisation can be converted to unconstrained optimisation by modifying the objective function ϕ as

$$\phi = W(1 + kC) \tag{7}$$

For all practical purposes, k can be assumed as 10.

The cross over operator, which is responsible for the search in genetic space, is carried out now. The populations in the referred cell is matted with that of the cell of minimum value of objective function using the randomized cross-sites keeping the population of the cell having minimum objective function thus keeping the fittest individuals for the next population according to Darwinian Theory.

It can be observed that the minimum objective function in the second iteration is less than the previous generation. It clearly shows the improvement among the set of populations. As proceeds with more generations there may not be much improvement among the set of populations and the best individuals with only slight deviation from the fitness of the best individual may progress. The populations get filled by more fit individuals with only slight deviation from the fitness of the best individual so far found and the average fitness comes very close to the fitness of the best individual. Number of generations is left to the personal interest. If the satisfied result is obtained, iteration can be stopped when there is no significant improvement in the performance from generation to generation.

2.4.2 Cross over

Crossover operator is applied to the cells with a hope that it would create a better string. In this paper, strings are selected from the mating pool at random and some portions of the strings are exchanged between strings. The following types of cross over operators are available in GA. 1) Single point cross over 2) Two point cross over 3) Multi-point cross over 4) Uniform cross over and 5) Two dimensional cross over. In this paper two point cross over is applied.

In the first generation as shown in Fig. 3, assume each cell contains 12 bit strings and with reference to the first cell, the mating takes place between first and second cell according to the Darwinian theory of survival of the fittest for minimum objective function by considering itself (1 celll) and its neighbor cells 2,8 & 9. Individuals are taking place in mating cells with cross-site 1 as 1 and cross-site 2 as 10 and the cross over takes place between the cells first and second. Similarly the crossover operator was applied for all the 16 cells by considering itself and its neighbors assuming the probability of cross over is one. The populations obtained after cross over will form new population set for the next generation as shown in Fig. 4. The process is repeated until to get optimal design without the violation of constraints or with a little violation.

Generation 1.

101010011010 **2.30754** 1	111000000100 3.1301 2	000110110101 2.8731 3	000011000111 3.1179 4
001001000101 3.1280 5	011000101111 2.8326 6	000100010000 2.9953 7	100011000011 3.1784 8
100100111100 3.1423 9	010101101111 2.8551 10	100011011011 2.9971 11	110001110110 **1.9642** 12
110001101110 2.4532 13	010011111011 2.9866 14	000010101100 3.3802 15	101011001010 2.7242 16

Fig. 3 4 × 4 Cells representing Moore neighbourhood for 1st iteration

Generation 2.

111000000110 2.30754 1	111000000100 3.1301 2	000011000001 3.3521 3	000011000011 3.2876 4
000100111101 3.3255 8	000100111111 3.2897 7	000011000000 3.3354 6	100011000011 3.1784 5
100100111100 3.1423 9	000010101111 3.3131 10	1100011101101 1.9642 11	100010101110 3.4024 12
100100111110 2.7800 13	000010101111 3.4117 14	000010101100 3.3802 15	100010101110 1.8573 16

Fig. 4 4 × 4 Cells representing Moore neighbourhood for 2nd iteration

2.4.3 Mutation

After Crossover strings are subjected to mutation. Mutation of a bit involves flipping it, changing 0 to 1 and vice versa. Just as PC (probability of the crossover rate assumed as 1) controls the cross over, another parameter PM (probability of the mutation rate assumed as 0.03), gives the probability that a bit will be flipped. The bits of strings are independently mutated, i.e., the mutation of a bit does not affect the probability of mutation of other bits. The Simple GA treats the mutation as a genetic operator with the role of restoring lost genetic material. For example suppose all the strings in a population has converged to 0 at a given position and optimal solution has a 1 at that position, the cross over cannot regenerate 1 at that position while a mutation could do. Thus mutation is simply an insurance policy against irreversible loss of genetic material.

The mutation operator introduces new genetic structure in the population by randomly modifying some of its building blocks, helping the search algorithm escape from local minima's traps. Since the modification is not really to the previous genetic structure representing other sections of the search space.

3. Formulation

Stacking different composite materials and / or fiber orientations forms composite laminates. Composite laminates are used in applications that require axial and bending strengths. Therefore, composite laminates are treated as plate elements. Even though there are many theories such as 1) Equivalent single layer theory a) Classical laminate theory b) shear deformation laminate theory 2) three dimensional elasticity theory 3) multiple model methods in this formulation we use classical laminate theory as given by Reddy (2001).

Consider a plate shown in Fig. 5 of total thickness '*h*' composed of '*n*' orthotropic layers with the principal natural coordinates *L*, *T*, *Z* directions with *Z* axis is taken positive upward at middle plane. The following assumptions are made.

1) The layers are perfectly bonded together
2) The material of each layer is linearly elastic and has two planes of natural symmetry
3) Each layer is of uniform thickness
4) The strains and displacements are small

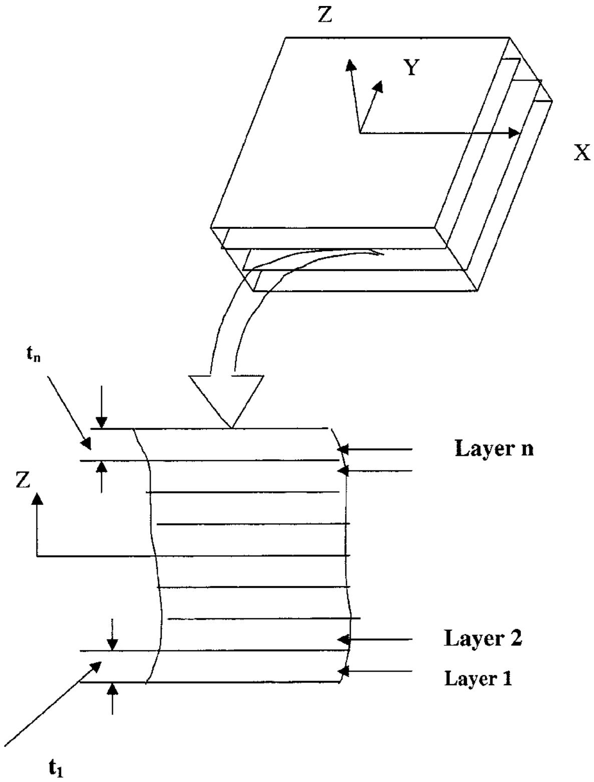

Fig. 5 Coordinate system and layer numbering

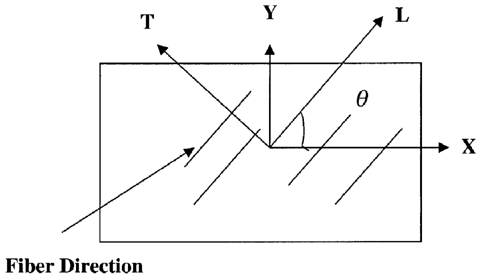

Fig. 6 *X, Y* and *L, T* system

5) The transverse shear stresses on the top and bottom surfaces of the laminates are zero.
6) Kirchhoff's assumption holds good.
7) The transverse normal does not suffer any elongation
8) The transverse normal rotates such that they remain perpendicular to the mid surface after deformation.

Taking a laminate shown in Fig. 6 and using the notations commonly adopted in composite literature as given by Kao (1997), one can give stresses in *XY* coordinate directions in terms of stresses in principal material coordinates namely *LT* as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = [T]\begin{Bmatrix} \sigma_{LL} \\ \sigma_{TT} \\ \sigma_{LT} \end{Bmatrix} \tag{8}$$

where $[T]$ is the stress transformation matrix.

Using the constitutive law, the stress - strain relationship in LT system can be given as

$$\begin{Bmatrix} \sigma_{LL} \\ \sigma_{TT} \\ \sigma_{LT} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{L} \\ \varepsilon_{T} \\ \gamma_{LT} \end{Bmatrix} \tag{9}$$

where

$$[Q] = \begin{bmatrix} \dfrac{E_L}{(1-\nu_{LT}\nu_{TL})} & \dfrac{\nu_{LT}E_T}{(1-\nu_{LT}\nu_{TL})} & 0 \\ \dfrac{\nu_{TL}E_L}{(1-\nu_{LT}\nu_{TL})} & \dfrac{E_T}{(1-\nu_{LT}\nu_{TL})} & 0 \\ 0 & 0 & G_{LT} \end{bmatrix} \tag{10}$$

Using the transformation law the constitutive matrix in XY system is obtained as

$$\begin{Bmatrix} \sigma_{XX} \\ \sigma_{YY} \\ \sigma_{XY} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{16} \\ S_{21} & S_{22} & S_{26} \\ S_{61} & S_{62} & S_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{X} \\ \varepsilon_{Y} \\ \gamma_{XY} \end{Bmatrix} \tag{11}$$

where

$$[S] = [T][Q][T]^T \tag{12}$$

and the transformation matrix is given by

$$[T] = \begin{bmatrix} \mathrm{Cos}^2\theta & \mathrm{Sin}^2\theta & -\mathrm{Sin}2\theta \\ \mathrm{Sin}^2\theta & \mathrm{Cos}^2\theta & \mathrm{Sin}2\theta \\ \dfrac{\mathrm{Sin}2\theta}{2} & -\dfrac{\mathrm{Sin}2\theta}{2} & \mathrm{Cos}2\theta \end{bmatrix} \tag{13}$$

where θ is the fiber orientation with respect to X axis.

Referring to Fig. 5 the displacements of any point in a laminate is given by

$$u(X, Y, Z) = u_0(X, Y) - Z\frac{\partial w_0}{\partial X}$$

$$v(X, Y, Z) = v_0(X, Y) - Z\frac{\partial w_0}{\partial y}$$

$$w(X, Y, Z) = w_0(X, Y) \tag{14}$$

where u_0, v_0 and w_0 are the displacements along the coordinate lines of a material point on XY plane where $Z = 0$ (mid plane)
For small displacement problem, the strain displacement relation is written as

$$\begin{Bmatrix} \varepsilon_X \\ \varepsilon_Y \\ \gamma_{XY} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial X} \\ \dfrac{\partial v}{\partial Y} \\ \dfrac{\partial u}{\partial Y} + \dfrac{\partial v}{\partial X} \end{Bmatrix} \tag{15}$$

Substituting for u, v we get

$$\begin{Bmatrix} \varepsilon_X \\ \varepsilon_Y \\ \gamma_{XY} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial X} \\ \dfrac{\partial v_0}{\partial Y} \\ \dfrac{\partial u_0}{\partial Y} + \dfrac{\partial v_0}{\partial X} \end{Bmatrix} - Z \begin{Bmatrix} \dfrac{\partial^2 w_0}{\partial X^2} \\ \dfrac{\partial^2 w_0}{\partial Y^2} \\ 2\dfrac{\partial^2 w_0}{\partial X \partial Y} \end{Bmatrix} \tag{16}$$

or

$$\{\varepsilon\} = \{\varepsilon\}^0 + Z\{\kappa\} \tag{17}$$

The above equation shows the linear relationship of the strain in a laminate to the curvature of the laminates. Now the stresses can be written as

$$\{\sigma\} = [S]\{\varepsilon\}^0 + Z[S]\{\kappa\} \tag{18}$$

From Eq. (18) it is clear that the stresses vary linearly through the thickness of each lamina as shown in Fig. 7. Even though strain is linear, the stresses however jump from lamina to lamina

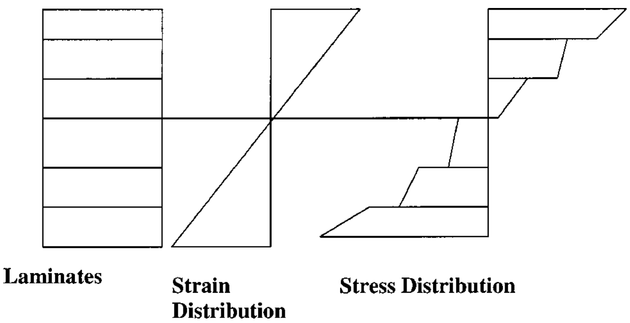

Fig. 7 Stress strain variation across laminates

since the transformed reduced stiffness matrix [S] changes from ply to ply as [S] depends on material orientation of the ply. Consider a laminate made up of 'n' plies shown in Fig. 5 each ply having the thickness of t_k. The thickness of the lamina is given by

$$h = \sum_{k=1}^{n} t_k \tag{19}$$

and

$$\begin{aligned} h_1 &= -h/2 \quad \text{(bottom surface)} \\ h_2 &= h/2 \quad \text{(top surface)} \end{aligned} \tag{20}$$

The forces and moments related to the mid plane are given in terms of strains and curvatures as

$$\begin{Bmatrix} N_{XX} \\ N_{YY} \\ N_{XY} \end{Bmatrix} = \sum_{k=1}^{n} \int_{Z_k}^{Z_{k+1}} \begin{Bmatrix} \sigma_{XX} \\ \sigma_{YY} \\ \sigma_{XY} \end{Bmatrix} dZ \tag{21a}$$

$$= \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{61} & A_{62} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_X^0 \\ \varepsilon_Y^0 \\ \gamma_{XY}^0 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{21} & B_{22} & B_{26} \\ B_{61} & B_{62} & B_{66} \end{bmatrix} \begin{Bmatrix} \kappa_{XX} \\ \kappa_{YY} \\ \kappa_{XY} \end{Bmatrix} \tag{21b}$$

$$\begin{Bmatrix} M_X \\ M_Y \\ M_{XY} \end{Bmatrix} = \sum_{k=1}^{n} \int_{Z_k}^{Z_{k+1}} \begin{Bmatrix} \sigma_{XX} \\ \sigma_{YY} \\ \sigma_{XY} \end{Bmatrix} Z \; dZ \tag{22a}$$

$$= \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{21} & B_{22} & B_{26} \\ B_{61} & B_{62} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_X^0 \\ \varepsilon_Y^0 \\ \gamma_{XY}^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{21} & D_{22} & D_{26} \\ D_{61} & D_{62} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_{XX} \\ \kappa_{YY} \\ \kappa_{XY} \end{Bmatrix} \tag{22b}$$

where A_{ij}, B_{ij} and D_{ij} are extensional, bending and extensional coupled and bending matrices respectively given by

$$(A_{ij}, B_{ij}, D_{ij}) = \sum_{k=1}^{n} \int_{Z_k}^{Z_{k+1}} S_{ij}^{(k)} (1, Z, Z^2) \; dZ \tag{23}$$

The coefficients of the above three matrices explicitly given by

$$A_{ij} = \sum_{k=1}^{n} S_{ij}^{(k)} (Z_{k+1} - Z_k) \tag{24a}$$

$$B_{ij} = \sum_{k=1}^{n} S_{ij}^{(k)}(Z_{k+1}^2 - Z_k^2) \tag{24b}$$

$$C_{ij} = \sum_{k=1}^{n} S_{ij}^{(k)}(Z_{k+1}^3 - Z_k^3) \tag{24c}$$

In the following problems, the objective function is the total thickness, which has to be minimized subjected to deflection, buckling and dynamic constraints.

4. Numerical results and discussion

Example 1. Flexural - torsional buckling of thin-walled symmetric I beam made of composite material (Lee and Kim 2001)

Consider a simply supported composite beam of span 'L' as shown in Fig. 8 subjected to uniformly distributed load 'q'. The object is to find the optimal lay up of the composite beam. It is assumed that stacking sequence is symmetric and the thin-walled beam is also symmetric with respect to Z axis. Assume A_{16}, D_{16} do not contribute much, the differential equations are uncoupled and one can calculate deflection and buckling loads very easily.

The deflection is given by (Lee and Kim 2001)

$$\delta = \frac{5qL^4}{384E_{33}} \tag{25}$$

where

$$E_{33} = EI_{ZZ} = b_1D_{11}^1 + 2y_1b_1B_{11}^1 + y_1^2b_1A_{11}^1 + b_2D_{11}^2 + 2y_2b_2B_{11}^2 + y_2^2b_2A_{11}^2 + b_3^3A_{11}^3/12 \tag{26}$$

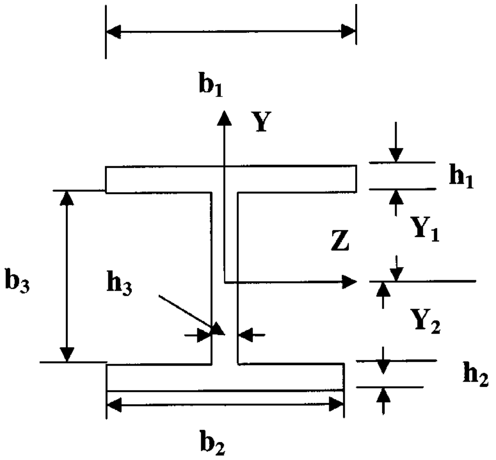

Fig. 8 Composite I beam

The various buckling loads in Euler and Torsional buckling modes are given by

$$P_{Y-cr} = \frac{\pi^2 E_{22}}{L^2} \tag{27a}$$

$$P_{Z-cr} = \frac{\pi^2 E_{33}}{L^2} \tag{27b}$$

$$P_{\phi} = \frac{A}{I_0}(\pi^2 E_{44}/L^2 + 4E_{55}) \tag{27c}$$

where

$$E_{22} = \frac{b_1^3}{12}A_{11}^1 + \frac{b_2^3}{12}A_{11}^2 + b_3 D_{11}^3 \tag{28a}$$

$$E_{44} = \frac{b_1^3 y_1^2 A_{11}^1}{12} + \frac{b_2^3 y_2^2 A_{11}^2}{12} \tag{28b}$$

$$E_{55} = b_1 D_{66}^1 + b_2 D_{22}^6 \tag{28c}$$

$$I_0 = I_Y + I_Z \tag{28d}$$

I_{YY}, I_{ZZ}, A are the moments of Inertia of I section about Y and Z axes and area of section respectively. The buckling load is the minimum of values of Euler Buckling about Y and Z axes as well as torsional buckling loads.

Numerical Example. A hybrid Composite Beam of symmetric I section is simply supported over a span of 8 m and subjected to uniformly distributed load of 1 N/m acting through shear centre. The widths of top and bottom flanges are 100 mm and the depth of the web is 200 mm. It is required to find the optimal lay-up of the hybrid beam such that the deflection should not exceed 5 mm and the buckling load should not be less than 7 kN. Cellular Genetic Algorithm (CGA) is applied to find the optimal lay-up of the beam. For each layer 12 bit binary string for each layer representing a population in which first four represents the thickness and the second four the angle of fibres and the last four representing the material. Varying the ratio of thickness of web to the flange (r), analysis is carried out and the objective function is arrived at for each population. Table 2 represents the optimal lay-up of laminates giving the thickness, fibre angle and the material. Fig. 9 shows the variation of thickness with respect to number of layers both for symmetric and anti-symmetric orientation of fibres. The minimum weight is obtained for $r = 1$ for the composite I beam with anti-symmetric laminates with 4 layers consisting of top layer of Graphite of 2.3 mm (18 deg) thickness and the next layer of Boron of 1.7 mm (18 deg) thickness. Similarly for $r = 0.5$ minimum weight is obtained with anti-symmetric laminate of top layer of Graphite of 3.2 mm (12 deg) and the next layer of Boron 0.8 mm (6 deg). This is obvious from Fig. 9. For any other number of layers also it is possible to get the design parameters.

Table 2 Optimal lay-up for simply supported composite thin-walled I beam

Symmetry/ Antisymmetry	r	Optimum Layer	Optimum Thickness in mm (total)	Unknowns		
				Thickness	Angle	Material
Symmetry	1	4	8.2	2.4	18	1
				1.7	18	2
Antisymmetry	1	4	8	2.3	18	1
				1.7	18	2
Symmetry	0.5	2	8.8	4.4	60	1
Antisymmetry	0.5	4	8	3.2	12	1
				0.8	6	1

(Note:- r = thickness of web/thickness of flange)

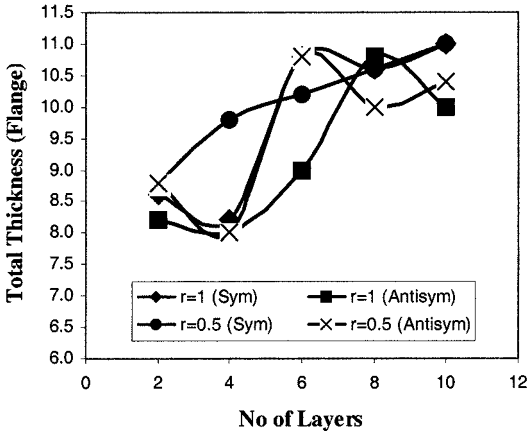

Fig. 9 Optimal lay-up for I section (thickness in mm)

Example 2. Thin walled composite beam with channel section (Lee and Kim 2002)

Consider a channel section simply supported with a span of L and subjected to uniformly distributed load 'q' as shown in Fig. 10. Assuming the stacking sequence is symmetric, and the thin-walled composite-beam is symmetric with respect to Z axis and assuming A_{16}, D_{16} do not contribute much we get uncoupled differential equations and the deflection and the frequency can be calculated very easily.

$$\delta = \frac{5qL^4}{384E_{33}} \tag{29a}$$

$$E_{33} = [A^1_{11}y^2_1 - 2B^1_{11}y_1 + D^1_{11}]b_1 + [A^2_{11}y^2_2 - 2B^2_{11}y_2 + D^2_{11}]b_2 + \frac{1}{12}A^3_{11}b^3_3 \tag{29b}$$

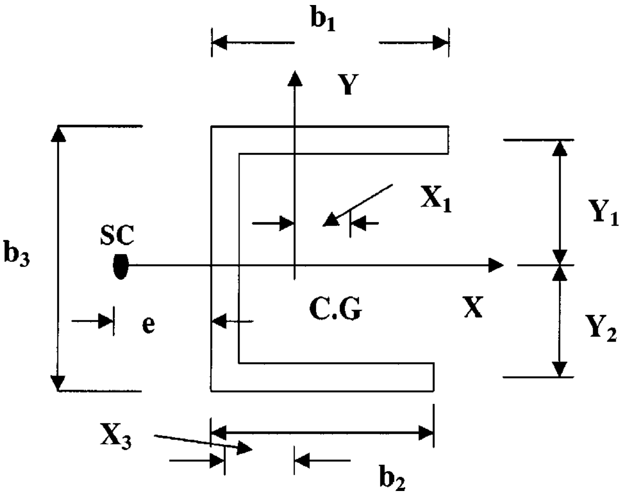

Fig. 10 Composite channel

and the natural frequency is given by

$$\omega = \frac{\pi^2}{L^2}\sqrt{\frac{E_{22}}{m_0}} \tag{30}$$

where

$$E_{22} = \frac{b_1^3}{12}A_{11}^1 + \frac{b_2^3}{12}A_{11}^2 + A_{11}^1 b_1 X_1^2 + A_{11}^2 b_2 X_2^2 + A_{11}^3 b_3 X_3^2 + [B_{11}^3 X_3 + D_{11}^3] b_3 \tag{31}$$

$$m_0 = I_0^1 b_1 + I_0^2 b_2 + I_0^3 b_3 \tag{32}$$

where

$$I_0 = \rho t \tag{33}$$

Numerical Example. A hybrid Composite Beam of Channel section is simply supported over a span of 8 m and subjected to uniformly distributed load of 1 N/m acting through shear centre. The widths of top and bottom flanges are 200 mm and the depth of the web is 400 mm. It is required to find the optimal lay-up of the hybrid beam such that the deflection should not exceed 3 mm and the fundamental frequency should not be less than 1 rad/sec. Cellular Genetic Algorithm (CGA) is applied to find the optimal lay-up of the beam. Table 3 represents the optimal lay-up of laminates giving the thickness, fibre angle and the material. Fig. 11 shows the variation of thickness with respect to number of layers both for symmetric and anti-symmetric orientation of fibres. The minimum weight is obtained for the channel section (r = 1) with a symmetric laminate of 4 layers with thickness of 0.8 mm (24 deg) of top layer of Boron and the next layer of 0.5 mm thickness (60 deg) of Graphite. The variation of thickness with respect to the number of layers is drawn in Fig. 11.

Table 3 Optimal lay-up for simply supported composite thin-walled channel (r = 1)

Symmetry/ Antisymmetry	Optimum Layer	Optimum Thickness	Unknowns		
			Thickness	Angle	Material
Symmetry	4	2.6	0.8	24	2
			0.5	60	1
Antisymmetry	6	3.0	0.5	12	2
			0.5	12	2
			0.5	18	1

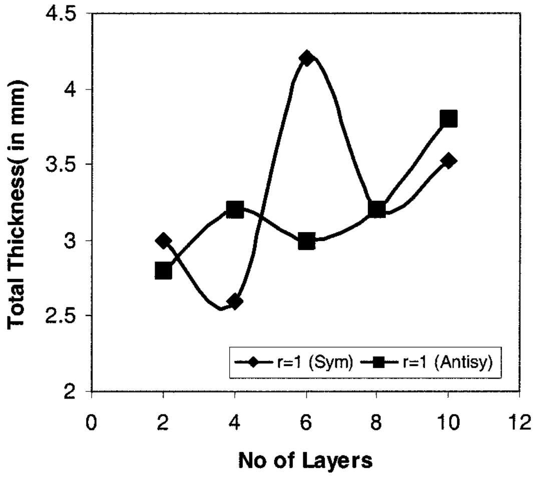

Fig. 11 Optimal lay-up for channel section

Example 3. Simply supported rectangular composite Plate (Reddy 2001)

We use either symmetric or anti-symmetric fiber orientation with respect to the middle layer in practice. It is now necessary to calculate the deflection due to uniformly distributed load, buckling load and the natural frequency for a rectangular plate shown in Fig. 12

$$k = \frac{N_y}{N_x} \tag{34a}$$

$$p = \frac{a}{b} \tag{34b}$$

The following constants are to be calculated

$$T_{11} = A_{11}m^2\pi^2 + A_{66}\pi^2 n^2 p^2 \tag{35a}$$

$$T_{12} = (A_{11} + A_{66})\pi^2 mnp \tag{35b}$$

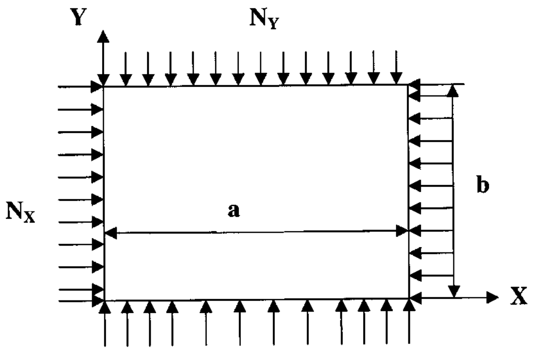

Fig. 12 Simply supported rectangular plate

$$T_{13} = -(3B_{16}\pi^2m^2 + B_{26}\pi^2n^2p^2)\pi np + B_{11}\pi^3m^3 + (B_{12} + 2B_{66})\pi^3n^2p^2 \tag{35c}$$

$$T_{22} = A_{66}\pi^2m^2 + A_{22}\pi^2n^2p^2 \tag{35d}$$

$$T_{23} = -(B_{16}\pi^2m^2 + 3B_{26}\pi^2n_2p^2)m\pi + B_{22}\pi^3n^3p^3 + (B_{12} + 2B_{66})\pi^3m^2np \tag{35e}$$

$$T_{33} = D_{11}\pi^4m^4 + 2(D_{12} + 2D_{66})\pi^4m^2n^2p^2 + D_{22}\pi^4n^4p^4 \tag{35f}$$

and the deflection due to uniformly distributed load of q_0 is given by

$$\delta = \frac{16q_0}{\pi^2mna^4\overline{D}}(T_{11}T_{22} - T_{12}^2) \tag{36}$$

where

$$\overline{D} = \frac{(T_{11}T_{22} - T_{12}^2)}{a^8}\left\{T_{33} + \frac{(2T_{12}T_{23}T_{13} - T_{22}T_{13}^2 - T_{11}T_{23}^2)}{(T_{11}T_{22} - T_{12}^2)}\right\} \tag{37}$$

Buckling load $N_x(cri)$ is given by

$$N_x(cri) = \frac{1}{\pi^2a^2(m^2 + kn^2p^2)}\left\{T_{33} + \frac{(2T_{12}T_{23}T_{13} - T_{23}T_{13}^2 - T_{11}T_{23}^2)}{(T_{11}T_{22} - T_{12}^2)}\right\}$$

The natural frequency is given by

$$\omega^2 = \frac{\pi^4}{\overline{I}_0^4b^4}\left[D_{11}m^4\left(\frac{b}{a}\right)^4 + 2(D_{12} + 2D_{66})m^2n^2\left(\frac{b}{a}\right)^2 + D_2n^4\right] \tag{39}$$

where

$$\overline{I}_0 = I_0 + I_2\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right] \tag{40}$$

where I_0, I_1, I_2 are mass moments and are given by

$$\begin{Bmatrix} I_0 \\ I_1 \\ I_2 \end{Bmatrix} = \int_{-h/2}^{h/2} \rho_0 \begin{Bmatrix} 1 \\ Z \\ Z^2 \end{Bmatrix} dZ \tag{41}$$

Numerical Example. A hybrid Composite plate of side 360 mm is simply supported on all sides and subjected to uniformly distributed load of 0.01 N/sq.mm. It is required to find the optimal lay-up of the hybrid plate such that the deflection should not exceed 2.75 mm and the buckling should not be less than 80 N and the fundamental frequency should not be less than 25 rad/sec. Cellular Genetic Algorithm (CGA) is applied to find the optimal lay-up of the plate. Analysis is carried out by varying the aspect ratio and the ratio of axial load in Y direction to axial load in X direction. Table 4 represents the optimal lay-up of laminates giving the thickness, fibre angle and the material.

Table 4 Optimal lay-up for simply supported rectangular composite plate

Aspect ratio *a/b*	*Ny/Nx*	Optimum Layer (material)	Optimum Thickness	Optimum Angle	Sym/Antisy
	0	4(2,1)	3.8(1.1,0.8)	12,48	
1	1	6(1,3,3)	4.8(0.5,1.4,0.5)	0,60,36	Symmetry
	4	6(2,2,4)	5.4(0.8,1.4,0.5)	24,84,72	
	0	8(1,1,3,1)	4(0.5,0.5,0.5,0.5)	0,12,0,48	
1	1	6(2,1,1)	4.2(0.5,0.8,0.8)	66,18,36	Antisym
	4	4(1,2)	4.4(1.7,0.5)	54,0	
	0	8(1,2,3,4)	4(0.5,0.5,0.5,0.5)	30,18,48,24	
1.5	1	4(1,2)	4.4(1.1,1.1)	72,24	Symmetry
	4	4(1,4)	5(0.8,1.7)	12,18	
	0	4(1,1)	4.4(1.7,0.5)	0,30	
1.5	1	4(1,3)	4.4(0.8,1.4)	36,48	Antisym
	4	6(2,2,3)	4.8(0.5,1.4,0.5)	6,48,36	

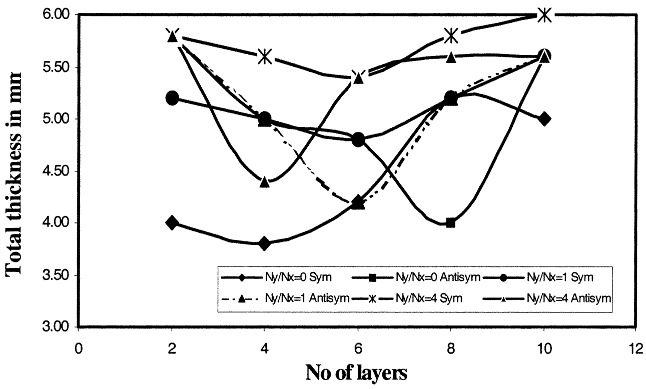

Fig. 13 Optimal lay-up for simply supported square plate (b/a = 1)

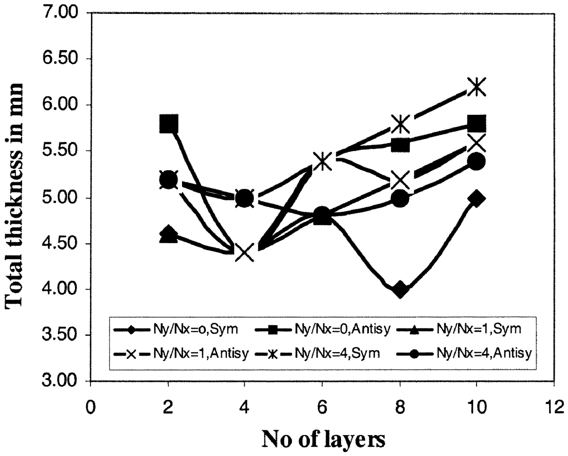

Fig. 14 Optimal lay-up for simply supported rectangular plate (b/a = 1.5)

Figs. 13 and 14 show the variation of thickness with respect to number of layers both for symmetric and anti-symmetric orientation of fibres for two aspect ratios of 1 and 1.5 respectively.

Example 4. Orthotropic circular cylindrical composite shell (Yao and Xiao 1987)

Optimal design for orthotropic circular cylindrical shell shown in Fig. 15 of hybrid laminate composites subjected to deflection and free vibration is carried out. From the equilibrium equation for the shell subjected to uniformly distributed load 'q' the maximum deflection and the natural frequency are obtained by Yao and Xiao (1987) as

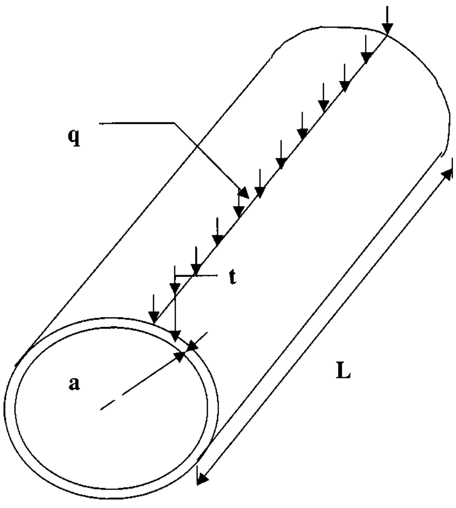

Fig. 15 Simply supported shell

$$\delta = \frac{q}{\beta} \tag{42}$$

where

$$\beta = \left[D_{11}\left(\frac{m\pi}{L}\right)^4 + 2(D_{12} + D_{66})\left(\frac{n}{a}\right)^2\left(\frac{m\pi}{L}\right)^2 + D_{22}\left\{\left(\frac{n}{a}\right)^4 + \frac{1}{a}\left(\frac{m\pi}{L}\right)^2\alpha\right.\right]$$

where

$$\alpha = \frac{f}{g} \tag{44}$$

f and g are given by

$$f = \frac{(A_{11}A_{22} - A_{12}^2)\left(\frac{m\pi}{L}\right)^2}{a} \tag{45a}$$

$$g = -\left[A_{11}\left(\frac{m\pi}{L}\right)^4 + A_{22}\left(\frac{n}{a}\right)^4 + \frac{(A_{11}A_{22} - A_{12}^2 - 2A_{12}A_{33})}{A_{33}}\left(\frac{n}{a}\right)^2\left(\frac{m\pi}{L}\right)^2\right] \tag{45b}$$

The natural frequency is given by

$$\omega^2 = \frac{1}{\rho h}\left(a_1 + \frac{b_1}{c_1}\right) \tag{46a}$$

$$a_1 = \left[D_{11}\left(\frac{m\pi}{L}\right)^4 + 2(D_{12} + D_{66})\left(\frac{n}{a}\right)^2\left(\frac{m\pi}{L}\right)^2 + D_{22}\left(\frac{n}{a}\right)^4\right] \tag{46b}$$

$$b_1 = (A_{11}A_{22} - A_{12}^2)\left(\frac{m\pi}{L}\right)^4 \tag{46c}$$

$$c_1 = a^2\left[A_{11}\left(\frac{m\pi}{L}\right)^4 + A_{22}\left(\frac{n}{a}\right)^4 + (A_{11}A_{22} - A_{12}^2 - 2A_{12}A_{66})\left(\frac{n}{a}\right)^2\left(\frac{m\pi}{L}\right)^2 / A_{66}\right] \tag{46d}$$

The above formulae are applicable only if the shell is simply supported and subjected to uniformly distributed load of 'q'. If the supports are changed different formulae must be used.

Numerical Example. A hybrid Composite simply supported tube of radius of 2000 mm and length of 8 m is simply supported and subjected to uniformly distributed load of 1 N/m. It is required to find the optimal lay-up of the hybrid composite tube such that the deflection should not exceed 0.095 mm and the fundamental frequency should not be less than 2 rad/sec. Cellular Genetic Algorithm (CGA) is applied to find the optimal lay-up of the tube. Analysis is carried out by varying the aspect ratio (radius/length). Table 5 represents the optimal lay-up of laminates giving the thickness, fibre angle and the material. Figs. 16 and 17 show the variation of thickness with respect to number of layers both for symmetric and anti-symmetric orientation of fibres for different aspect ratios.

Table 5 Optimal lay-up for simply supported composite tube

r/L	Optimum layer	Optimum thickness	Unknowns			Symmetry/ Antisymmetry
			Thickness	Angle	Material	
0.5	4	4.2	1.6 0.5	60 12	2 2	Antisymmetry
1	4	7.606	0.733 3.07	60 72	2 2	Antisymmetry
1.5	4	12.92	4.53 1.93	72 84	2 2	Antisymmetry
0.5	8	5.014	0.773 0.507 0.72 0.507	72 90 36 54	1 1 1 3	Symmetry
1	2	7.6	3.8	66	2	Symmetry
1.5	4	13.86	2 4.93	84 60	2 2	Symmetry

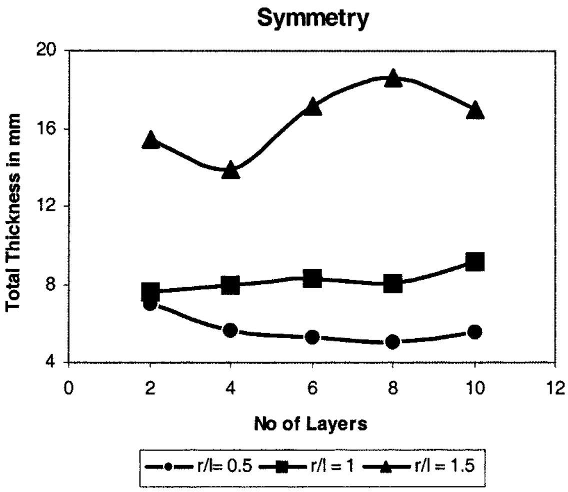

Fig. 16 Optimal symmetric lay-up for simply supported composite tube

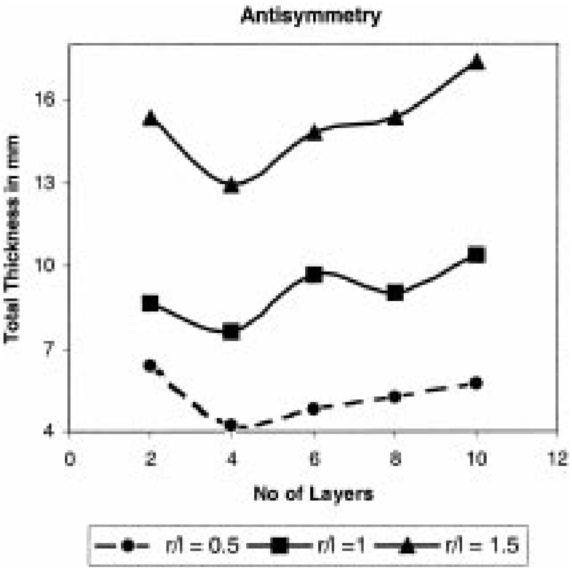

Fig. 17 Optimal anti-symmetric lay-up for simply supported composite tube

4. Conclusions

In this paper, the optimum fiber orientations, thickness and material and number of layers are obtained for multi-layered composite hybrid beams, plates and shells subjected to static and dynamic analysis. For composite I beam for $r = 1$, 4 layers of hybrid material of Graphite and Boron with anti-symmetric orientation give the minimum weight with thickness of 8 mm and for $r = 0.5$, 4 layers of Boron with anti-symmetric orientation give the minimum weight with thickness of 8 mm.

For hybrid composite channel, symmetric orientation with 4 layers of Graphite and Boron gives the minimum thickness of 2.6 mm.

For composite plates, the thickness of the plate vary from 3.8 to 4.8 mm depending on the ratio of axial loads in Y to X directions and the aspect ratio.

For the hybrid composite tube, 4 layers give the minimum weight for all ratios of r/L with thickness varying from 4.2 mm to 12.92 mm for anti-symmetric orientation and 5 mm to 13.86 for symmetric orientation. With the program developed, it is possible to get the minimum weight design of hybrid composite beams, plates and tube.

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Notation

The following symbols are used in the paper

a : Length in X direction of the plate

Aij : Extensional Constant

b : Length in Y direction of the plate

Bij	: Coupling constant
C	: Constraint
D	: Bending constant
E_L	: Modulus of elasticity in Longitudinal direction
E_T	: Modulus of elasticity in Transverse direction
E_N	: Modulus of elasticity in Normal direction
G_{LT}	: Modulus of rigidity in L-T direction
h	: Thickness of the laminate
I	: Moment of Inertia
M	: Moments
mo	: Mass
p	: *a/b* aspect ratio
q	: Lateral Load on the plate
Q	: Constitutive constants
u	: Displacement in *X* direction
v	: Displacement in *Y* direction
w	: Displacement in *Z* direction
X	: Variable
Z	: Distance of the lamina from centre
δ	: Deflection
ε	: Normal Strains
ϕ	: Objective function
γ	: Shear Strain
γ_{LT}	: Poison's ratio in L-T direction
ρ	: Density
σ	: Stress