

# Seismic resistance and mechanical behaviour of exterior beam-column joints with crossed inclined bars

P. G. Bakir†

*Faculty of Civil Engineering, Istanbul Technical University, Maslak 80626, Istanbul, Turkey*

*(Received January 23, 2003, Accepted July 3, 2003)*

**Abstract.** Attempts at improving beam-column joint performance has resulted in non-conventional ways of reinforcement such as the use of the crossed inclined bars in the joint area. Despite the wide accumulation of test data, the influence of the crossed inclined bars on the shear strength of the cyclically loaded exterior beam-column joints has not yet been quantified and incorporated into code recommendations. In this study, the investigation of joints has been pursued on two different fronts. In the first approach, the parameters that influence the behaviour of the cyclically loaded beam-column joints are investigated. Several parametric studies are carried out to explore the shear resisting mechanisms of cyclically loaded beam-column joints using an experimental database consisting of a large number of joint tests. In the second approach, the mechanical behaviour of joints is investigated and the equations for the principal tensile strain and the average shear stress are derived from joint mechanics. It is apparent that the predictions of these two approaches agree well with each other. A design equation that predicts the shear strength of the cyclically loaded exterior beam-column joints is proposed. The design equation proposed has three major differences from the previously suggested design equations. First, the influence of the bond conditions on the joint shear strength is considered. Second, the equation takes the influence of the shear transfer mechanisms of the crossed inclined bars into account and, third, the equation is applicable on joints with high concrete cylinder strength. The proposed equation is compared with the predictions of the other design equations. It is apparent that the proposed design equation predicts the joint shear strength accurately and is an improvement on the existing code recommendations.

**Key words:** mechanical behaviour; deformation; earthquake resistant structures; crossed inclined bars; cyclic loads; joints; shear properties; strut and tie models; anchorage; bond (concrete to reinforcement); beams; columns; reinforced concrete; connections; structural analysis; shear strength.

---

## 1. Introduction

The Kocaeli and Duzce earthquakes in Turkey showed that, even when the beams and columns in reinforced concrete multi-storey residences are only slightly damaged after the main shock or aftershocks, the integrity of a building was threatened if the joint, where these members connected failed, as mentioned in previous papers of the author (Bakir 2003a, 2003b, Bakir and Boduroglu 2002a, 2002b, 2002c). The author has investigated the shear resisting mechanisms and the factors influencing the failure modes of monotonically loaded beam-column joints in companion papers (Bakir and Boduroglu 2002d, 2002e). The design of multi-storey structures for gravity loads causes

---

† Lecturer

no serious problems. Nevertheless, due to the unexpected nature of earthquakes, many aspects of the seismic design of structures still need to be investigated. The aim of this paper is to investigate the shear resisting mechanisms of cyclically loaded exterior beam-column joints and the influence of the crossed inclined bars on the shear strength and the shear resisting mechanisms.

There are still differences in codes regarding the design of beam-column joints. The New Zealand Design Code (1995) is based on the assumption that there are two types of shear resisting mechanisms in beam-column joints as shown in Figs. 1(a) and 1(b) and as first suggested by Paulay (1975). These are the diagonal strut mechanism and the truss mechanism. The strut mechanism transfers shear forces via a diagonal concrete strut which sustains compression only and is assumed to be inclined at an angle close to that of the potential corner-to-corner failure plane. The truss mechanism consists of the contribution of the horizontal reinforcement inside the joint core. The New Zealand Code, which takes into account the influence of both the strut mechanism and the

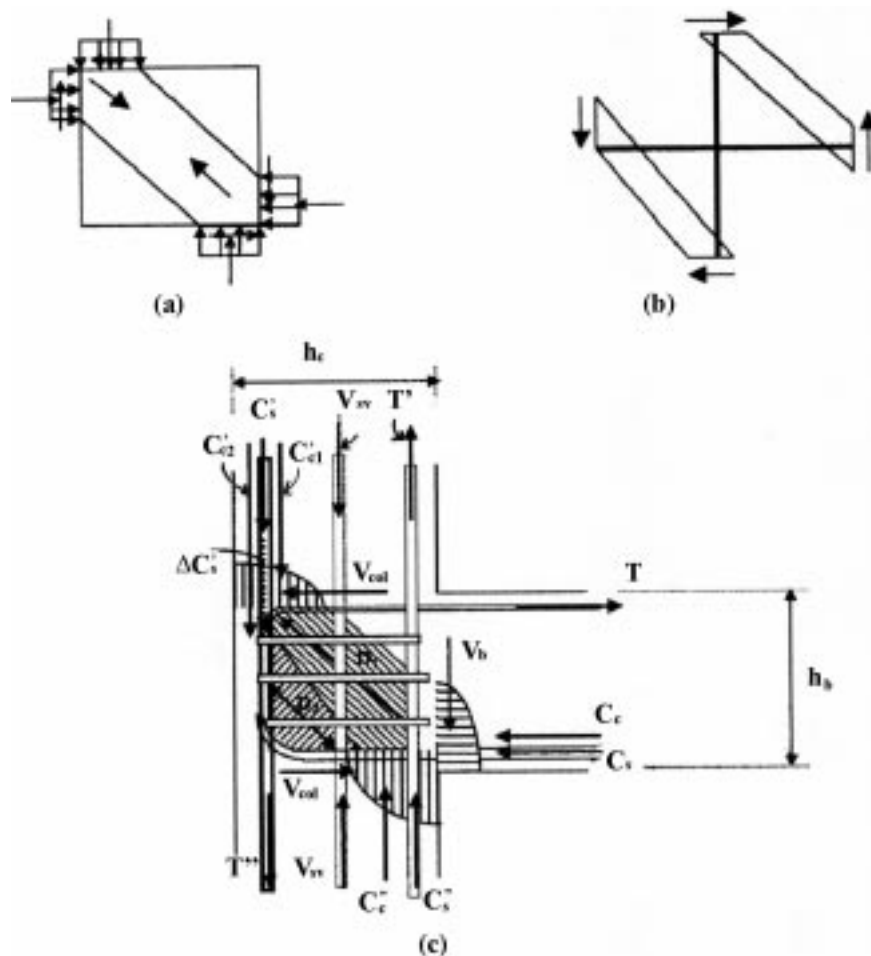


Fig. 1 The diagonal strut mechanism; (a) and truss mechanism, (b) in interior beam-column joints, (c) idealised stress paths in exterior joints

truss mechanism, recommends that the beam bars should be properly anchored within the joint area in order to have a workable truss mechanism. This is because the truss mechanism can only exist when there is good bond transfer in the beam bars. Thus, the bar size is strictly limited in the New Zealand Code relative to the joint dimensions. The New Zealand Code advocates that the bond deterioration of beam bars within a joint is undesirable because pinching in the hysteresis curves increases after bond deterioration, the compressive stresses in the diagonal strut increase and the beam deformations increase due to the loss of bond. In addition to this requirement, the New Zealand Code necessitates large amounts of vertical and horizontal shear reinforcement to be used in the joint area to equilibrate the truss mechanism, as it is based on the assumption that joint shear strength is considerably increased by the provision of vertical and horizontal shear reinforcement. The United States (1995, 1998) and the Japanese Design Codes (1990), on the other hand, are based on the assumption that joint shear is resisted entirely by the direct strut mechanism and the stirrups are only necessary to confine the joint core. Further complicating the problem, all the design recommendations in codes today, are based on tests of joints of normal strength concrete with concrete compressive strengths between 20 and 50 MPa. However, in recent years, high strength concrete is frequently used in the construction industry. Thus, it is more appropriate to alter the design recommendations so as to also cover high strength reinforced concrete structures using the available recent tests on joints with high strength concrete specimens.

## 2. Previously suggested models and code recommendations for cyclically loaded exterior beam column joints

There is already a large amount of experimental data available, related to the behaviour of joints. Nevertheless, seismic design provisions for beam-column joints are still controversial, despite the great deal of research that has been conducted throughout the years. In the following sections, previously suggested models and design recommendations for cyclically loaded beam-column joints will be reviewed.

### 2.1 Model of Paulay

The shear resisting mechanisms of interior beam-column joints as shown in Figs. 1(a) and 1(b) are different from the shear resisting mechanisms of exterior beam-column joints as shown in Fig. 1(c) according to Paulay (Paulay and Priestley 1992). Because in an exterior joint only one beam frames into a column, the shear load input into a joint will generally be less than that encountered with interior joints. As in the case of interior joints, shear forces, both in horizontal and vertical directions, can be sustained by a diagonal concrete compression field together with horizontal and vertical joint shear reinforcement. A major diagonal strut, sustaining a compression force  $D_1$  can develop at the bend of the top beam bars. The horizontal component of this strut is the tension force  $T$ , assumed to be developed at the beginning of the hook, less the column shear force  $V_{col}$ . The vertical component of the strut consists of the concrete force  $C_{c1}'$ , a part of the compression force on the column reinforcement  $\Delta C_s'$  which is transmitted by bond near the bend of the beam bar, and the compression force originating from the anchorage of the intermediate column bar, acting as vertical shear reinforcement. At the lower and inner abutment of the strut, the horizontal component necessary to support the diagonal force  $D_1$ , will consist of part of the beam concrete compression

force  $C_{c1}$ , reduced by the shear force  $V_{col}$ . The remainder of the horizontal force,  $V_{sh}$ , must be supplied by horizontal ties. To support the diagonal strut  $D_1$  near the beam, horizontal ties are required. To absorb the tension forces in these ties at the outer face of the joint, another diagonal compression field  $D_2$ , needs to be developed. The associated horizontal forces are the bond forces transmitted from the bottom beam bars,  $C_s$ , and the remainder of the beam flexural compression force,  $C_c - C_{c1}$ . The vertical components at the upper end of the strut  $D_2$ , originate from bond forces in the outer column bars,  $T''$  and  $C_s' - \Delta C_s'$ , and from some column compression force  $C_{c2}'$  entering the joint core via the cover concrete (Paulay and Park 1984).

It should be noted however that, due to the interchange of forces between concrete and steel, load transfer within the joint is inseparable from the mechanisms of bond. When a plastic hinge develops adjacent to the joint, with the beam bars entering also the strain hardening range, yield penetration into the joint core and consequent drastic bond deterioration is unavoidable. As a result, after a few cycles of inelastic loading, significant anchorage can be provided only by the hook. Serious bond deterioration in interior joints results in significant loss of stiffness and energy dissipation. Anchorage failure of beam bars in exterior joints, on the other hand, results in complete failure (Paulay and Priestley 1992, Paulay and Park 1984, Paulay *et al.* 1978).

## 2.2 Model of Tsonos

Tsonos (1997, 1999, 2000, 2001a, 2001b) has carried out extensive experimental and theoretical work on beam-column joints and has suggested a model which is based on the assumption that both the strut and the truss mechanisms depend on the core concrete strength. Thus, the ultimate concrete strength of the joint core under compression/tension also gives the ultimate strength of the connection. From the vertical and horizontal equilibrium, Eqs. (1) and (2) are obtained in the model of Tsonos as shown in Fig. 2.

$$D_{cy} + (T_1 + \dots + T_4 + D_{vy}) = D_{cy} + D_{sy} = V_{jv} \quad (1)$$

$$D_{cx} + (D_{1x} + \dots + D_{vx}) = V_{jh} \quad (2)$$

The vertical normal compressive stress  $\sigma$  and the shear stress  $\tau$  uniformly distributed over the whole section are given by the Eqs. (3), (4).

$$\sigma = \frac{D_{cy} + D_{sy}}{h'_c \times b'_c} = \frac{V_{jv}}{h'_c \times b'_c} \quad (3)$$

$$\tau = \frac{V_{jh}}{h'_c \times b'_c} \quad (4)$$

The relationship between the average normal compressive stress  $\sigma$  and the average shear stress  $\tau$  are shown in Eq. (5).

$$\sigma = \frac{V_{jv}}{V_{jh}} \tau \quad (5)$$

where

$$\frac{V_{jv}}{V_{jh}} = \frac{h_b}{h_c} = \alpha \quad (6)$$



provided as the joint stirrups are assumed to increase the joint ductility by confining the cracked core concrete and because they increase the bond conditions for the column bars. The AIJ Guidelines require that the minimum amount of stirrup ratio in joints should be 0.002, and the following criterion should be satisfied:

$$\rho_{stir} > 0.003 \frac{V_j}{V_{ju}} \quad (12)$$

$$\rho_{stir} = \frac{A_s}{(7db_c/8)} \quad (13)$$

## 2.4 The ACI-ASCE Committee 352 Recommendations

The recommendations of the ACI-ASCE Committee 352 are based on the assumption that the joints should be stronger than the incoming beams, and failure should occur by hinging in the incoming beams rather than in the joint itself. The cyclically loaded beam-column joints are classified as Type 2 joints in the ACI-ASCE Committee 352 Recommendations, and the design shear force is calculated based on the yield capacity of the beam longitudinal reinforcement as shown in Eq. (14).

$$V_j = A_s f_y - V_{col} \quad (14)$$

The horizontal joint shear force  $V_{jhor}$  should not exceed a maximum value, taken as:

$$V_{jhor} = V_u = \phi \gamma \sqrt{f_c} b_j h_c \quad (15)$$

$\sqrt{f_c}$  is in units of psi.

$\phi$  is taken as 0.85.

$\gamma$  is 15 for Type 2 exterior beam-column joints.

If  $f_c$  is defined in units of MPa, the values of  $\gamma$  should be multiplied by 0.083.

## 3. Method

In this paper, the parameters that influence the behaviour of the cyclically loaded beam-column joints are explored. An experimental database consisting of a large number of cyclically loaded exterior beam-column joints are used to investigate the shear resisting mechanisms of joints, as shown in Table 1. The tests in Table 1 are all exterior beam-column joint tests by different researchers (Megget and Park 1974, Paulay and Scarpas 1981, Ehsani and Wight 1985, Alameddine 1990, Kaku and Akasuka 1991, Fuji and Morita 1991, Tsonos *et al.* 1992, Tsonos 1997). The specimens are chosen according to the following criteria:

1. Specimens with slabs, transverse beams, beam bars with plate anchorage, beam bottom bars bent downward into the lower column, or specimens that have eccentricity between column and beam axis are omitted.
2. Only specimens failing in a joint or a beam adjacent to a column are considered; specimens with a relocated beam hinge or those that exhibited column or anchorage failures are omitted.

Table 1(a) The experimental database

| No | Researcher            | Specimen | $f_c$ (MPa) | $h_b$  | $h_c$  | $b_b$  | $b_c$  | $V_{cal}/V_{exp}$ | $V_{aci}/V_{exp}$ |
|----|-----------------------|----------|-------------|--------|--------|--------|--------|-------------------|-------------------|
| 1  | Kaku&Asakusa (1991)   | 2        | 41.7        | 220    | 220    | 160    | 220    | 0.74              | 0.85              |
| 2  | "                     | 3        | 41.7        | 220    | 220    | 160    | 220    | 0.85              | 0.97              |
| 3  | "                     | 5        | 36.7        | 220    | 220    | 160    | 220    | 0.72              | 0.94              |
| 4  | "                     | 6        | 40.4        | 220    | 220    | 160    | 220    | 0.82              | 0.99              |
| 5  | "                     | 8        | 41.2        | 220    | 220    | 160    | 220    | 0.74              | 0.85              |
| 6  | "                     | 9        | 40.6        | 220    | 220    | 160    | 220    | 0.76              | 0.88              |
| 7  | "                     | 11       | 41.9        | 220    | 220    | 160    | 220    | 0.76              | 0.90              |
| 8  | "                     | 12       | 35.1        | 220    | 220    | 160    | 220    | 0.75              | 1.00              |
| 9  | "                     | 13       | 46.4        | 220    | 220    | 160    | 220    | 0.94              | 0.99              |
| 10 | "                     | 14       | 41          | 220    | 220    | 160    | 220    | 0.77              | 0.92              |
| 11 | "                     | 15       | 39.7        | 220    | 220    | 160    | 220    | 0.73              | 0.90              |
| 12 | Fuji&Morita (1991)    | B2       | 30.6        | 250    | 220    | 220    | 220    | 0.82              | 1.74              |
| 13 | "                     | B3       | 30.6        | 250    | 220    | 220    | 220    | 0.65              | 3.72              |
| 14 | "                     | B4       | 30.6        | 250    | 220    | 220    | 220    | 0.67              | 3.54              |
| 15 | Ehsani (1985)         | 1B       | 33.6        | 480.06 | 299.72 | 259.08 | 300    | 0.64              | 1.35              |
| 16 | "                     | 3B       | 40.9        | 480    | 300    | 259    | 300    | 0.73              | 1.26              |
| 17 | "                     | 4B       | 44.6        | 439    | 300    | 259    | 300    | 0.72              | 1.21              |
| 18 | "                     | 5B       | 24.3        | 480.06 | 340.36 | 299.72 | 340.36 | 0.66              | 1.53              |
| 19 | Scarpas&Paulay (1981) | 1        | 22.6        | 610    | 457    | 356    | 457    | 0.81              | 0.76              |
| 20 | "                     | 2        | 22.5        | 610    | 457    | 356    | 457    | 0.58              | 0.85              |
| 21 | "                     | 3        | 26.9        | 610    | 457    | 356    | 457    | 0.84              | 0.75              |
| 22 | Alameddine (1990)     | LL8      | 55.84       | 508    | 356    | 317.5  | 356    | 0.83              | 2.55              |
| 23 | "                     | LH8      | 55.84       | 508    | 356    | 317.5  | 356    | 0.92              | 2.61              |
| 24 | "                     | HH8      | 55.84       | 508    | 356    | 317.5  | 356    | 0.78              | 2.74              |
| 25 | "                     | LL11     | 73.77       | 508    | 356    | 317.5  | 356    | 1.12              | 2.77              |
| 26 | "                     | LH11     | 73.77       | 508    | 356    | 317.5  | 356    | 0.99              | 2.31              |
| 27 | "                     | HH11     | 73.77       | 508    | 356    | 317.5  | 356    | 0.90              | 2.59              |
| 28 | "                     | HH14     | 93.77       | 508    | 356    | 317.5  | 356    | 1.05              | 2.60              |
| 29 | Tsonos (1992)         | S3       | 18.96       | 300    | 200    | 200    | 200    | 0.91              | 1.19              |
| 30 | "                     | X3       | 26.98       | 300    | 200    | 200    | 200    | 1.26              | 1.05              |
| 31 | "                     | S4       | 20.96       | 300    | 200    | 200    | 200    | 0.83              | 1.90              |
| 32 | "                     | X4       | 16.97       | 300    | 200    | 200    | 200    | 0.99              | 1.78              |
| 33 | "                     | S5       | 24.96       | 300    | 200    | 200    | 200    | 0.77              | 1.70              |
| 34 | "                     | X5       | 22.01       | 300    | 200    | 200    | 200    | 0.82              | 1.45              |
| 35 | "                     | S6       | 32.96       | 300    | 200    | 200    | 200    | 0.86              | 1.63              |
| 36 | "                     | X6       | 26.98       | 300    | 200    | 200    | 200    | 0.78              | 1.27              |
| 37 | "                     | S61      | 28.96       | 300    | 200    | 200    | 200    | 0.75              | 1.52              |
| 38 | "                     | X7       | 18.01       | 300    | 200    | 200    | 200    | 0.71              | 1.46              |
| 39 | "                     | X8       | 18.98       | 300    | 200    | 200    | 200    | 0.78              | 1.67              |
| 40 | "                     | P1       | 16          | 300    | 200    | 200    | 200    | 0.71              | 2.50              |
| 41 | "                     | Y1       | 22.96       | 300    | 200    | 200    | 200    | 0.76              | 2.18              |
| 42 | "                     | O1       | 19.99       | 300    | 200    | 200    | 200    | 0.87              | 2.14              |
| 43 | "                     | F2       | 23.99       | 300    | 200    | 200    | 200    | 0.88              | 1.98              |

Table 1(a) Continued

| No | Researcher    | Specimen | $f_c$ (MPa) | $h_b$ | $h_c$ | $b_b$ | $b_c$ | $V_{cal}/V_{exp}$ | $V_{aci}/V_{exp}$ |
|----|---------------|----------|-------------|-------|-------|-------|-------|-------------------|-------------------|
| 44 | Tsonos (1997) | NC1      | 18.96       | 300   | 200   | 200   | 200   | 0.90              | 1.49              |
| 45 | "             | NCZ1     | 21.99       | 300   | 200   | 200   | 200   | 0.96              | 1.46              |
| 46 | "             | N1       | 20.96       | 300   | 200   | 200   | 200   | 0.82              | 1.20              |
| 47 | "             | NZ1      | 19.99       | 300   | 200   | 200   | 200   | 0.85              | 1.27              |
| 48 | "             | N2       | 32.96       | 300   | 200   | 200   | 200   | 0.79              | 1.47              |
| 49 | "             | NZ2      | 19.99       | 300   | 200   | 200   | 200   | 0.87              | 2.14              |
| 50 | "             | NZO2     | 15.99       | 300   | 200   | 200   | 200   | 0.72              | 2.49              |
| 51 | "             | NZM2     | 28.96       | 300   | 200   | 200   | 200   | 0.76              | 1.52              |
| 52 | "             | N3       | 24.96       | 300   | 200   | 200   | 200   | 0.76              | 1.66              |
| 53 | "             | NZ3      | 23.99       | 300   | 200   | 200   | 200   | 0.78              | 1.72              |
| 54 | "             | A2       | 31.03       | 300   | 200   | 200   | 200   | 1.10              | 1.35              |
| 55 | "             | A3       | 25.99       | 300   | 200   | 200   | 200   | 1.01              | 1.41              |

Table 1(b) The average and standard deviation values for the proposed equation and the ACI Code Recommendations

|                                    | Proposed equation | ACI-ASCE Committee 352 Recommendations |
|------------------------------------|-------------------|--|
| Average $V_{predicted}/V_{actual}$ | 0.82              | 1.63                                   |
| Standard deviation                 | 0.13              | 0.7                                    |

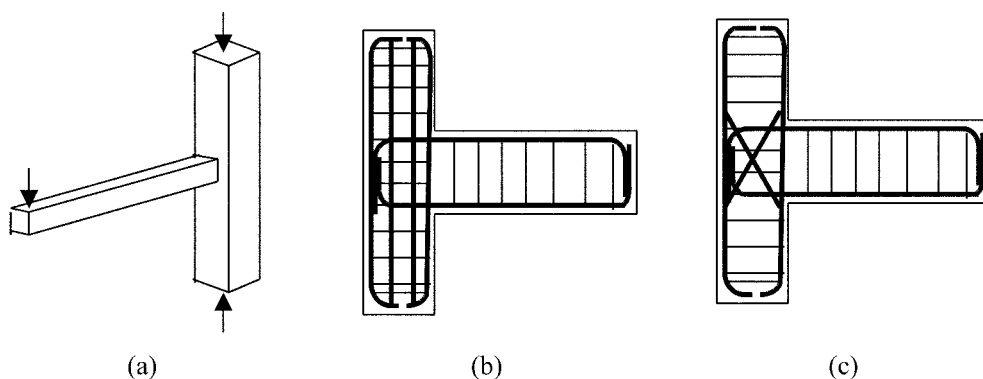


Fig. 3(a) A typical specimen in the experimental database, (b) Joints without crossed inclined bars, (c) Joints with crossed inclined bars

3. Specimens with flexural over-strength ratios higher than 3 and less than 1 are not included in the experimental database.

The typical specimen type used in the experimental database is shown in Fig. 3(a). The database contains both the joints with and without the crossed inclined bars, as shown in Figs. 3(b) and 3(c).

In analysing the tests, multiple linear regression analysis is used. The possibility of non-linearity is also investigated for each regressor by controlling the residual plots, as will be explained later. For  $p$  independent variables, the model for multiple linear regression is:



$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_p X_p + \varepsilon \quad (16)$$

which can be written in matrix notation as;

$$[Y] = [X][\beta] + [\varepsilon] \quad (17)$$

where

$$[Y] = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ \vdots \\ \vdots \\ Y_n \end{bmatrix}, \quad [X] = \begin{bmatrix} 1 & X_{11} & X_{12} & \cdots & X_{1p} \\ 1 & X_{21} & X_{22} & \cdots & X_{2p} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & X_{n1} & X_{n2} & \cdots & X_{np} \end{bmatrix}, \quad \beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \vdots \\ \beta_p \end{bmatrix} \quad (18)$$

The use of more than one variable, however, can result in many difficulties and, as stated by Snedecor and Cochran (1980): ‘Multiple linear regression is a complex subject’. The complexity arises due to the following facts:

1. It is particularly difficult to select variables and decide which methods to use to obtain the best subset of variables.
2. It is very difficult to interpret the results, more specifically, the regression coefficients.
3. It is difficult to decide whether to use least squares or robust regression methods as there may be outliers or leverage points in the data.

Naturally, hand calculations are impractical when  $p$  is greater than 2 and the use of computers is necessary. The objective in multiple linear regression analysis is to minimise

$$\sum_{i=1}^n \varepsilon_i^2 = \sum (Y_i - \beta_0 - \beta_1 X_{i1} - \beta_2 X_{i2} - \dots - \beta_p X_{ip})^2 \quad (19)$$

The above equation is differentiated with respect to  $\beta_0, \beta_1, \beta_2, \beta_3, \dots, \beta_p$  to produce  $(p+1)$  equations. The coefficients can be expressed in matrix form as shown below:

$$[\beta] = (X^T X)^{-1} X^T Y \quad (20)$$

The general form of the ANOVA tables are given in Table 2. A small  $P$  value indicates that the

Table 2 The explanation of the ANOVA Table

| Source     | Degrees of freedom | Sum of squares SS               | Mean square MS                                  | $F$         |
|------------|--------------------|---------------------------------|---|-------------|
| Regression | $p$                | $[\beta]^T [X]^T [Y]$           | $(1/p)[\beta]^T [X]^T [Y]$                      | Msreg/MSres |
| Residual   | $n - p - 1$        | $[Y]^T Y - [\beta]^T [X]^T [Y]$ | $(1/(n - p - 1)) [Y]^T Y - [\beta]^T [X]^T [Y]$ |             |
| Total      | $n - 1$            | $[Y]^T [Y]$                     |   |             |

regression equation is of high value in predicting  $Y$ . The  $R^2$  value is given as;

$$R^2 = \frac{SS_{reg}}{SS_{total}} \quad (21)$$

The closer the  $R^2$  value to 1, the better the variability in  $Y$  is explained by the regression equation. In multiple linear regression however,  $R^2$  will always increase when variables are added to the model. A better measure is  $R_{adj}^2$  which is expressed as shown in Eq. (22).

$$R_{adj}^2 = 1 - (1 - R^2) \left( \frac{n-1}{n-p} \right) \quad (22)$$

A confidence interval will be constructed for each parameter which has a form as Eq. (23):

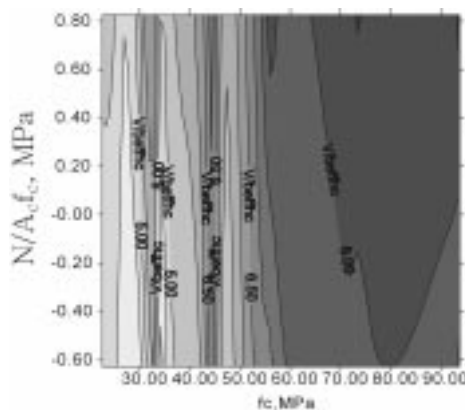
$$\hat{\beta}_i \pm t_{\alpha/2, n-p-1} \sqrt{MS_{res} (c_{ii})^{1/2}} \quad (23)$$

The next step is to determine the most appropriate subset. Various methods have been used for many years for this purpose. Forward selection is one of them. With this method, a subset is obtained by sequentially adding variables one at a time until the marginal contribution of a regressor is estimated as insignificant by the  $F$  test. The opposite of forward selection is backward elimination. With this method, the starting point is the full equation with all of the available variables. A variable is then deleted if the  $t$  test indicates that its marginal contribution is not significant. Stepwise regression is basically a combination of these two methods in that it allows for both the addition and deletion of variables. After an  $F$  test indicates that a variable should be added to the equation, subsequent  $F$  tests are carried out to determine whether any of the other variables in the equation have become unnecessary and should thus be eliminated. Stepwise multiple regression analysis is used in this study to obtain the best subset of regressors. A large number of subsets are tried for the analysis. In choosing the best subset, the following criteria is accepted:

1.  $R^2$  value should be very close to 1.
2. If the above criterion is true, then  $R_{adj}^2$  should be checked as given by Eq. (22).  $R_{adj}^2$  is a better measure because, as variables are added to a model,  $R^2$  will certainly increase. However,  $R_{adj}^2$  is not affected by this, because as apparent from Eq. (22),  $n-p$  becomes smaller as  $p$  increases and thus  $1-R^2$  must decrease at a faster rate so that  $R_{adj}^2$  increases.
3. A significance level of 0.05 is used, and it is checked whether the hypothesis that  $\beta = 0$  will be rejected. The  $F$  value should be considerably greater than the minimum value needed to reject  $H_0: \beta = 0$ . In this study, it is accepted that the  $F$  value should be at least four times the minimum value needed to reject  $H_0: \beta = 0$ . Any subset of regressors which have a ratio less than 4 are not accepted.
4. If  $F$  is greater than the minimum value needed to reject  $H_0$ , the  $t$  tests of the coefficients are checked. The tests can indicate whether any of the regressors are irrelevant.
5. The confidence intervals are built with the assumption that error terms have a normal distribution and are independent. The assumptions of independent errors and constant error variance are checked by plotting the errors against the particular regressor. The spread of residuals should be reasonably constant over  $X$ , and the residuals should not illustrate an obvious pattern. Plots illustrating non-random residuals can imply that regression is inappropriately used for time series data. Plots illustrating a horse shoe shaped, non-constant residual variance, can indicate that a non-linear relationship exists between the regressor and  $Y$ .

#### 4. The analysis of the tests

The experimental database is analysed in order to investigate the influence of several different parameters on the joint shear strength. As apparent from Fig. 4(a), for a constant concrete cylinder strength, the joint shear strength is independent of the column axial stress. However, for a constant column axial stress, as the concrete cylinder strength increases, the joint shear strength increases substantially and the highest joint shear strength is obtained when the joints have concrete cylinder strengths higher than 55 MPa. The parametric study in Fig. 4(b) also shows that the joint shear strength is independent of the column axial stress. Other researchers have also suggested that the joint shear strength in exterior beam-column joints is independent of the column axial stress (Vollum 1998, Pantazopoulou 1992, Uzumeri 1975, Paulay 1985, Kitayama 1991, Kurose 1993). Uzumeri (1977) comments that the presence of a large axial compressive force is of help at the early stages of loading and whether a large axial force continues to be of help once joint deterioration starts is debatable. During the latter stages of loading, anchorage of the beam steel is provided at the bend of the beam steel. At this stage, the concrete in the core acts as a series of struts anchored at their ends by the joint steel. Uzumeri concludes that a large axial compressive force applied to these struts may be detrimental rather than helpful. Vollum (1998) states that the joint shear strength is totally independent of the column axial stress. Pantazopoulou (1992) states that the shear strength of a joint depends on the usable compressive strength of concrete, which decreases with increasing principal tensile strain. The principal tensile strain, on the other hand, increases with increasing column axial stress. Consequently, the joint shear strength decreases with the increase in principal tensile strain due to the increase in column axial load. Paulay and Park (1984) state that the beneficial effect of axial compression on the shear strength of exterior beam-column joints depends on the aspect ratio of the joint and is less significant than in the case of interior joints. Paulay further comments that the axial load on the column is not likely to significantly reduce yield penetration and; for this reason, the benefit of axial compression in 'inelastic joints' is likely to be less than in 'elastic joints' (Paulay *et al.* 1978). Kitayama *et al.* (1991) suggests that the column axial load does not seem to influence the joint shear strength even



in interior beam-column joints. Kurose (1993) also states that the axial load does not influence the joint shear strength.

In this study, the influence of all possible variables is investigated by using a large number of subsets of regressors as explained in section 3. The tried regressors are the joint aspect ratio  $h_b/h_c$ , the stirrup index defined as  $A_{sje} * f_{yw} / (b_{eff} h_c)$ , where  $b_{eff}$  is the average width of the beam and the columns, the column longitudinal reinforcement ratio, the beam longitudinal reinforcement ratio, the ratio of the cross-sectional height of the column to the diameter of the beam bars, the ratio of the cross-sectional height of the beam to the diameter of the column bars, the ratio of the crossed inclined bars and the column axial stress. The regression analysis is carried out on 32 specimens with flexural over-strength values between 1 and 1.5 out of the 59 specimens. The best regression statistics are obtained by using the product of the stirrup ratio and the stirrup yield strength, the concrete cylinder strength, the ratio of the height of the column to the diameter of the beam bars and ratio of the crossed inclined bars in the joint area. Thus the equation takes the following form:

$$V = \left( \frac{b_c + b_b}{2} \right) * h_c * \lambda * \left( 0.092 * f_c + 0.34 * \frac{A_{inc} * f_{inc} * \cos \theta}{\left( \frac{b_c + b_b}{2} \right) h_c} + 0.55 * \ln \left( \frac{h_c}{d_b} \right) + 0.23 * \frac{A_{sje} f_{yw}}{\left( \frac{b_b + b_c}{2} \right) h_c} \right) \quad (24a)$$

where  $\lambda$  is a capacity reduction factor of 0.78 and  $\cos \theta = h_c / \sqrt{(h_c^2 + h_b^2)}$ .

Table 3 shows the regression statistics, Table 4 shows the ANOVA Table and Table 5 shows the confidence intervals. The proposed equation shows that stirrups increase the joint shear strength; however, the stirrups' contribution to the joint shear strength is much less than their yield capacity. The joint shear strength is considerably increased by increasing the concrete cylinder strength and the ratio of the crossed inclined bars in the joint area. It is apparent that the crossed inclined bars contribute to the joint shear strength by a mechanism explained in Fig. 21 and section 7 as suggested by Tsonos *et al.* Tsonos suggests that the inclined bars contribute to the joint shear strength by their yield capacity. However, in this study, it is found that a capacity reduction factor of  $\beta$  should be used, as shown in Eqs. (24b) and (24c), to account for the fact that the crossed inclined bars' contribution to the joint shear strength is much less than their yield capacity (34%).

$$V_{sx} = \beta * A_{inc} * f_{inc} * \cos \theta \quad (24b)$$

$$\tan \theta = \frac{h_b}{h_c} \quad (24c)$$

The model of Paulay for exterior beam-column joints has shown that load transfer within the joint is inseparable from the mechanisms of bond. Thus, it is very important to prevent the yield penetration from beam bars to the joint area and prevent the formation of plastic hinges at the face of the column so that the joint can remain elastic. To reduce bond stresses, it is necessary to use the smallest bar diameter that is compatible with practicality. The equation proposed is applied on the experimental database. As apparent from Table 1, it gives very accurate predictions of the joint shear strength. Figs. 5 to 12 show the residual and trend-line plots of the step-wise regression analysis.

Table 3 Regression statistics

| Regression statistics |      |
|-----------------------|------|
| Multiple $R$          | 0.82 |
| $R$ Square            | 0.67 |
| Adjusted $R$ Square   | 0.59 |
| Standard deviation    | 0.64 |
| Observation           | 32   |

Table 4 ANOVA tables

|            | degrees of freedom | Sum of squares | Mean Square | $F$      | $p$      |
|------------|--------------------|----------------|-------------|----------|----------|
| Regression | 4                  | 22.86314       | 5.715785    | 13.88183 | 2.84E-06 |
| Residual   | 28                 | 11.52888       | 0.411746    |          |          |
| Total      | 32                 | 34.39202       |             |          |          |

Table 5 Confidence intervals, coefficients, standard deviations and  $t$  statistics

|                                   | Coefficients | Standard deviation | $t$ Statistics | $p$ -value | Low %95  | High %95 |
|-----------------------------------|--------------|--------------------|----------------|------------|----------|----------|
| Intersection                      | 0            | -                  | -              | -          | -        | -        |
| $f_c$ (MPa)                       | 0.092343     | 0.01296            | 7.12504        | 9.41E-08   | 0.065795 | 0.118891 |
| $\ln(h_c/d_b)$                    | 0.551015     | 0.160776           | 3.427212       | 0.001904   | 0.221679 | 0.880351 |
| $A_{inc}f_{inc}\cos\theta/b_ch_c$ | 0.343445     | 0.127032           | 2.703618       | 0.011529   | 0.083232 | 0.603658 |
| $A_{sje}f_{yw}/b_{eff}h_c$        | 0.228464     | 0.060018           | 3.806616       | 0.000704   | 0.105523 | 0.351405 |

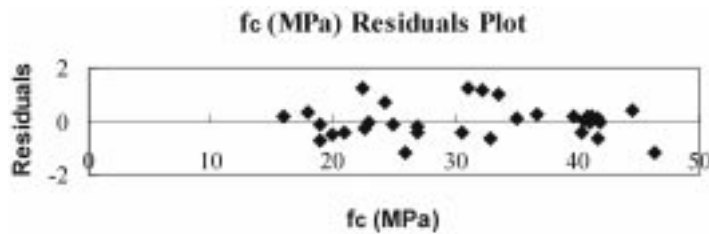


Fig. 5 The residuals plot for the concrete cylinder strength

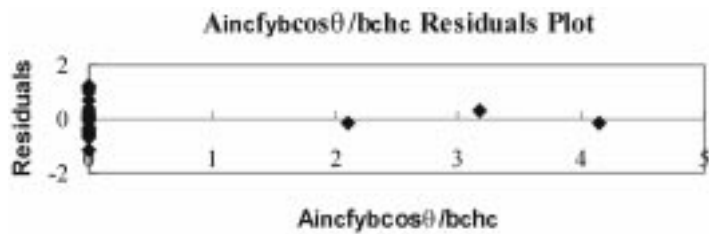


Fig. 6 The residuals plot for the crossed inclined bar ratio

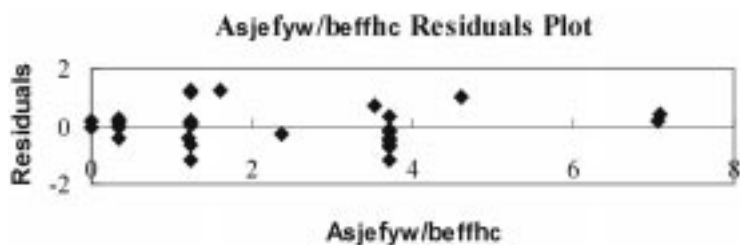


Fig. 7 The residuals plot for the stirrup index

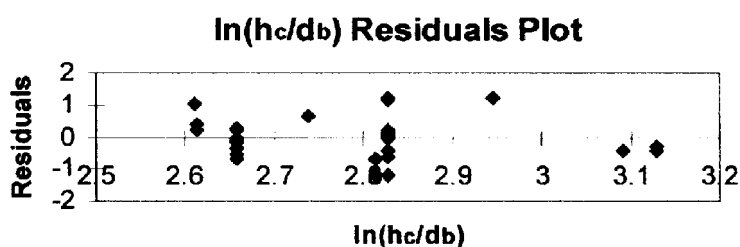


Fig. 8 The residuals plot for the ratio of the cross sectional height of the column to the diameter of the beam longitudinal reinforcement ratio

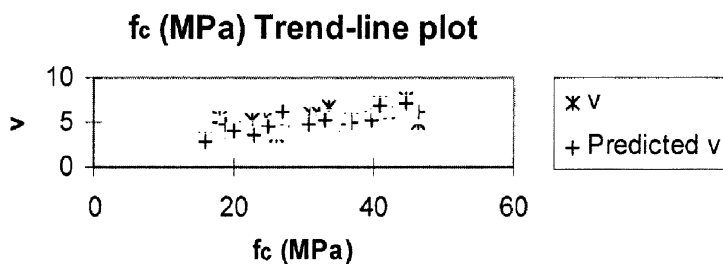
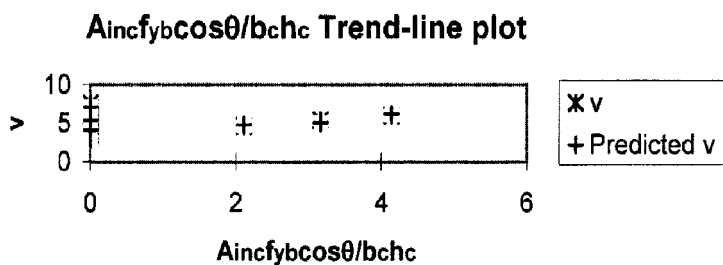


Fig. 9 The trend-line plot for the concrete cylinder strength

Fig. 10 The trend-line plot for  $A_{inc}f_{inc}\cos\theta/b_{eff}h_c$

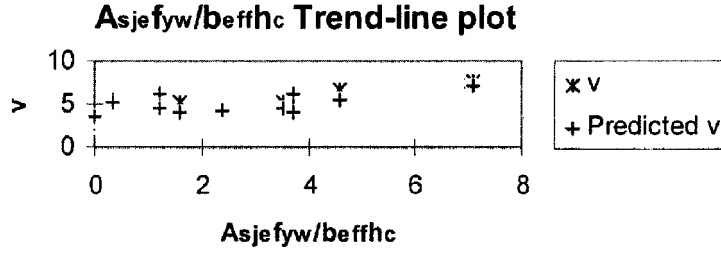


Fig. 11 The trend-line plot for the product of the stirrup ratio and the stirrup yield strength

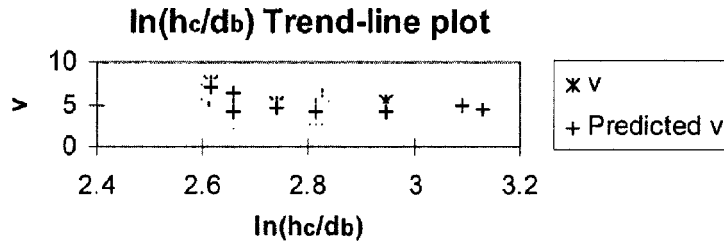


Fig. 12 The trend-line plot for the  $h_c / d_b$

## 5. Comparison of the parametric studies with the established principles of joint mechanics

In order to investigate the reliability of Eq. (24), the equation is compared with the established equations on the basic mechanics of reinforced concrete beam-column joints. This has been also previously discussed by Paulay (1986) and by Bonacci & Pantazopoulou (1992) in detail for interior joints, who have also taken into account the joint deformations. Bonacci & Pantazopoulou, as well as Paulay, use the average stresses for equilibrium as shown in Fig. 13, and the typical loading system considered in the analysis of the exterior beam-column joints is shown in Fig. 14. This study is complementary to the model of Bonacci and Pantazopoulou in that, the inclined bars are also incorporated into their model.

The equilibrium of forces in the horizontal direction require the average transverse compressive stress in the joint  $\sigma_x$  when inclined bars are used in the joint area defined as:

$$\sigma_x = -\frac{A_{sb}}{d_y d_z} f_s - \frac{A_{sje}}{d_y d_z} f_w - \frac{A_{inc}}{d_y d_z} f_{inc} \cos \theta \quad (25)$$

Consequently, the average normal concrete stress in the y direction  $\sigma_y$  can be expressed as:

$$\sigma_y = -\frac{A_{scol}}{d_x d_z} f_{scol} - \frac{N}{d_x d_z} - \frac{A_{inc}}{d_x d_z} f_{inc} \sin \theta \quad (26)$$

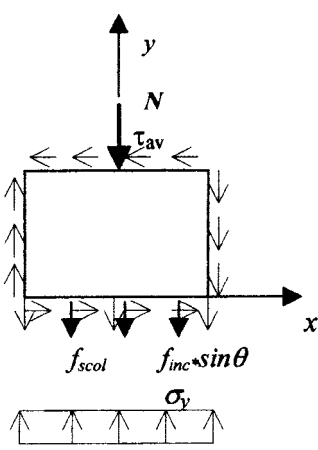


Fig. 13 Stress equilibrium in joints

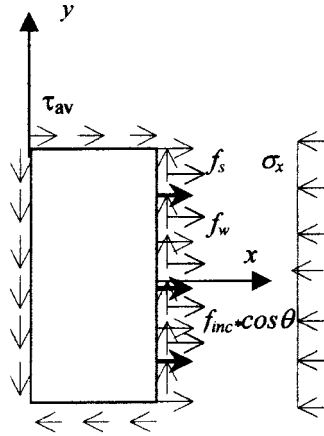


Fig. 14 Joint dimensions

In this study, the last two terms in Eqs. (25) and (26) have been added to account for the shear transfer mechanisms of the crossed inclined bars. Defining the average joint shear stress in the joint as  $\tau_{av}$ , the maximum principal stress associated with the stress tensor is given as;

$$\sigma^3 - I_1 \sigma^2 + I_2 \sigma - I_3 = 0 \quad (27)$$

where  $\sigma_z$  is the confining stress provided by stirrups in the  $z$  direction.

$$\sigma = \begin{bmatrix} \sigma_x & \tau_{av} & 0 \\ \tau_{av} & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{bmatrix} \quad (28a)$$

In order to determine the principal stresses, the Eq. (28a) has to be solved;

where

$$I_1 = \sigma_x + \sigma_y + \sigma_z \quad (28b)$$

$$I_2 = \sigma_x \sigma_y + \sigma_y \sigma_z + \sigma_x \sigma_z - \tau_{av}^2 \quad (28c)$$

$$I_3 = \sigma_x \sigma_y \sigma_z - \sigma_z \tau_{av}^2 \quad (28d)$$

The tensile stress in the concrete is negligible and therefore  $\sigma_1 = 0$ , which consequently gives;

$$\sigma_y = \frac{\tau_{av}^2}{\sigma_x} \quad (29)$$

From the Mohr's circle,

$$\tan 2\theta = \frac{2\tau_{av}}{\sigma_x - \sigma_y} \quad (30)$$

If Eq. (29) is substituted into Eq. (30), the following quadratic equation ensues;



$$\tau_{av}^2 + \left( \tan \theta + \frac{1}{\tan \theta} \right) \sigma_x \tau_{av} - \sigma_x^2 = 0 \quad (31)$$

which gives;

$$\sigma_y = -\frac{\tau_{av}}{\tan \theta} \quad (32)$$

Using Eq. (29), we have;

$$\tau_{av} = -\frac{\sigma_x}{\tan \theta} \quad (33)$$

Collins and Mitchell (1991) suggest the following equation for the maximum stress in concrete panels;

$$f_{2\max} = \frac{f_c}{0.8 + 170\varepsilon_1} < f_c \quad (34)$$

The principal compressive stress is given by;

$$\sigma_2 = \left( 2 \left( \frac{\varepsilon_2}{-0.002} \right) - \left( \frac{\varepsilon_2}{-0.002} \right)^2 \right) f_{2\max} \quad (35)$$

$\sigma_2$  is also given from Mohr's circle as;

$$\sigma_2 = \sigma_x + \sigma_y = -\tau_{av} \left( \tan \theta + \frac{1}{\tan \theta} \right) \quad (36)$$

Thus, the average joint shear stress can be expressed as;

$$\tau_{av} = -\frac{\sigma_2}{\left( \tan \theta + \frac{1}{\tan \theta} \right)} \quad (37)$$

Eqs. (34) to (36) show very clearly that, as the principal tensile strain increases, the average joint shear stress decreases. Thus, it is necessary to express the principal tensile strain in terms of the strains in the  $x$  and  $y$  directions in order to investigate the factors that influence the joint shear strength. From Mohr's circle, it is known that;

$$\tan 2\theta = \frac{\gamma}{\varepsilon_x - \varepsilon_y} \quad (38)$$

From Mohr's circle, the principal tensile strain will be;

$$\varepsilon_1 = \frac{(\varepsilon_x + \varepsilon_y)}{2} + \sqrt{\left( \frac{\varepsilon_x - \varepsilon_y}{2} \right)^2 + \left( \frac{\gamma}{2} \right)^2} \quad (39)$$

If Eq. (38) is substituted into Eq. (39) and appropriate trigonometric transformations are carried out, Eq. (40) given by Bonacci and Pantazopoulou is obtained.

$$\varepsilon_1 = \left( \frac{\varepsilon_x - \varepsilon_y \tan^2 \theta}{1 - \tan^2 \theta} \right) \quad (40)$$

The next step will be to express the strains in the  $x$  and  $y$  directions in terms of the stresses.

$$\begin{aligned} \sigma_x = -\tau_{av} \tan \theta &= -\frac{A_{sb} f_s}{b_{eff} h_b} - \frac{A_{sje} f_w}{b_{eff} h_b} - \frac{A_{inc} f_{inc} \cos \theta}{b_{eff} h_b} \\ &= -\left( \frac{A_{sb}}{b_{eff} h_b} \mu + \frac{A_{sje}}{b_{eff} h_b} + \frac{A_{inc} \cos \theta}{b_{eff} h_b} \beta \right) f_w \end{aligned} \quad (41)$$

$$\begin{aligned} \sigma_y = -\frac{\tau_{av}}{\tan \theta} &= -\frac{A_{scol} f_{scol}}{d_x d_z} - \frac{N}{d_x d_z} - \frac{A_{inc} f_{inc} \sin \theta}{d_x d_z} \\ &= f_{scol} \left( -\frac{A_{scol}}{d_x d_z} - \frac{\gamma A_{inc} \sin \theta}{d_x d_z} \right) - \frac{N}{d_x d_z} \end{aligned} \quad (42)$$

where  $\mu = f_s/f_w$  which is a null value for full bond,  $\beta = f_{inc}/f_w$  and  $\gamma = f_{inc}/f_{scol}$

The strain in the  $x$  direction can therefore be expressed as;

$$\varepsilon_x = \frac{f_w}{E_s} = \frac{\tau_{av} \tan \theta}{E_s \left( \frac{A_{sb} \mu}{d_y d_z} + \frac{A_{sje}}{d_y d_z} + \frac{A_{inc} \beta \cos \theta}{d_y d_z} \right)} \quad (43)$$

The strain in the  $y$  direction can similarly be expressed as;

$$\varepsilon_y = \frac{f_{scol}}{E_s} = \frac{1}{E_s} \left( \frac{\left( \frac{\tau_{av}}{\tan \theta} - \frac{N}{d_x d_z} \right)}{\left( \frac{A_{scol}}{d_x d_z} + \frac{\gamma A_{inc} \sin \theta}{d_x d_z} \right)} \right) \quad (44)$$

If Eqs. (43) and (44) are substituted into Eq. (40);

$$\begin{aligned} \varepsilon_1 &= \frac{1}{E_s (1 - \tan^2 \theta)} \left( \tau_{av} \tan \theta \left( \frac{1}{\frac{A_{sb}}{d_y d_z} \mu + \frac{A_{sje}}{d_y d_z} + \frac{A_{inc} \beta \cos \theta}{d_y d_z}} \right) \right. \\ &\quad \left. - d_x d_z \tan^2 \theta \left( \frac{\frac{\tau_{av}}{\tan \theta} - \frac{N}{d_x d_z}}{A_{scol} + \gamma A_{inc} \sin \theta} \right) \right) \end{aligned} \quad (45)$$

where the angle of inclination can be expressed as equal to the corner-to-corner potential failure plain as shown in Eq. (46).

$$\tan \theta = \frac{h_b}{h_c} \quad (46)$$

The above equation shows that the principal tensile strain is increased by increasing the column longitudinal reinforcement ratio and the axial load on the column, whereas it is decreased by increasing the stirrup ratio. The shear stress in the joint is dependent on the principal tensile strain as evident from Eqs. (34), (35) and (36). It is therefore evident from Eqs. (25) and (45) that the joint shear strength increases as the transverse reinforcement ratio increases. Eq. (26) shows that the joint shear strength increases as the column load and the column longitudinal reinforcement increase but Eq. (45) shows that, as the longitudinal column reinforcement and the column load increase, the principal tensile strain increases, which consequently decreases the normalised joint shear strength. Therefore, the increase in the joint shear strength due to Eq. (26) is offset by the increase in the principal tensile strain. As apparent from Eq. (25), crossed inclined bars are only effective in the horizontal direction. Thus, the steeper the angle between the horizontal direction and the crossed inclined bars, the less effective the crossed inclined bars will be in increasing the joint shear strength. Due to geometrical constraints, this angle is dependent on the joint aspect ratio defined as  $h_b/h_c$  and the smaller the joint aspect ratio is, the more the crossed inclined bars will contribute to the joint shear strength. The above conclusions are totally in accordance with the predictions of the author's equation. Figs. 15 and 16 are the 3D plots for the exterior beam-column joints. As apparent from Fig. 15, the joint shear strength increases when the  $h_c/d_b$  ratio increases and the ratio of the crossed inclined bars is kept constant and vice versa. Fig. 16 shows that for a constant concrete cylinder strength, the joint shear strength increases by increasing the product of the stirrup ratio and the yield strength of stirrups, and likewise, for a constant stirrup ratio, the joint shear strength increases with increasing concrete cylinder strength.

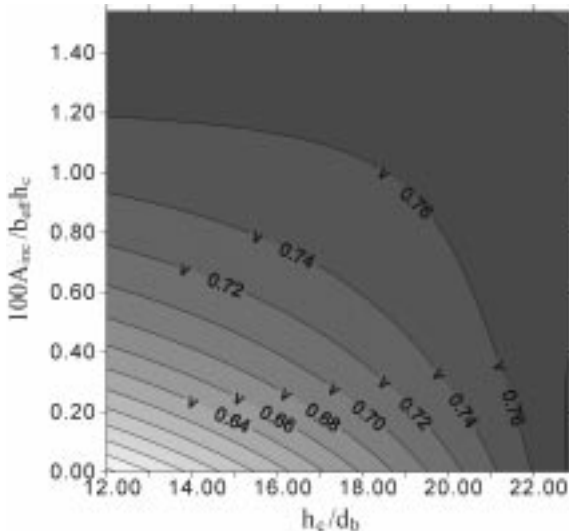


Fig. 15 The influence of the  $h_c/d_b$  ratio and the crossed inclined bars on the joint shear strength

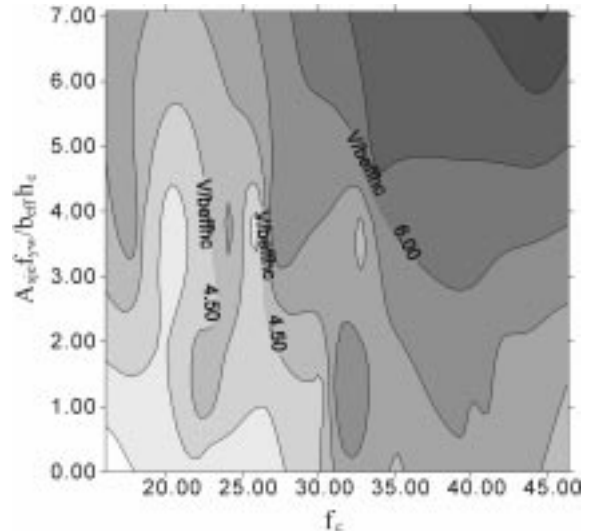


Fig. 16 The influence of the  $A_{sje}f_{yw}/b_{eff}h_c$  and the concrete cylinder strength on the joint shear strength

## 6. Present design guidelines

Both the ACI-ASCE Committee 352, and AIJ Guidelines calculate the joint shear strength on the assumption that the stirrups do not contribute to the joint shear strength. The above methods are considered to be inadequate by the author because they neglect the influence of the stirrups and the influence of the bond conditions on the joint shear strength. The ACI-ASCE Committee 352 Recommendations are compared with the equation of the author in Figs. 17 to 20.

Eq. (24) proposed by the author and the ACI-ASCE Committee 352 Recommendations are applied on the experimental database in Table 1(a). It is apparent from Table 1(b) that the average  $V_{predicted}/V_{actual}$  is 0.82 for the proposed equation and 1.63 for the ACI Code Recommendations. The standard deviations for the proposed equation is 0.13 for the proposed equation and 0.70 for the

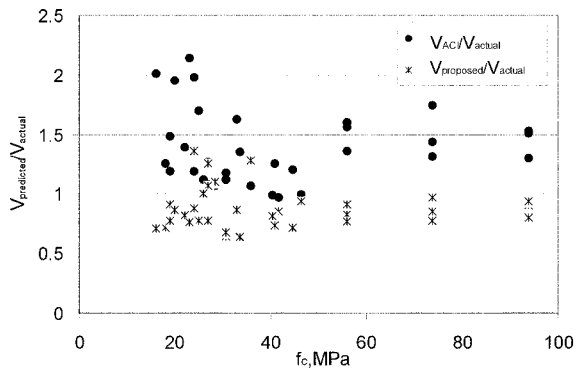


Fig. 17 The influence of the concrete cylinder strength on the predicted joint shear strength of the proposed equation and the equation of ACI

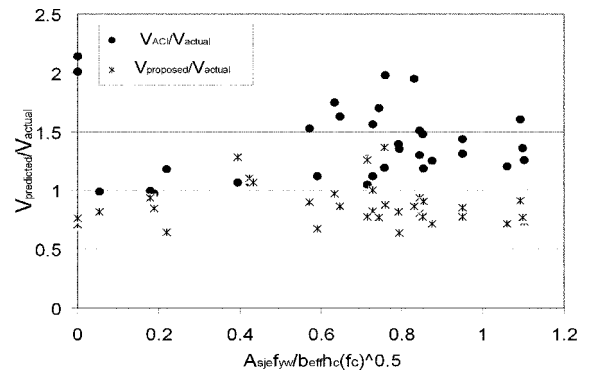


Fig. 18 The influence of the stirrup index on the predicted joint shear strength of the proposed equation and the equation of ACI

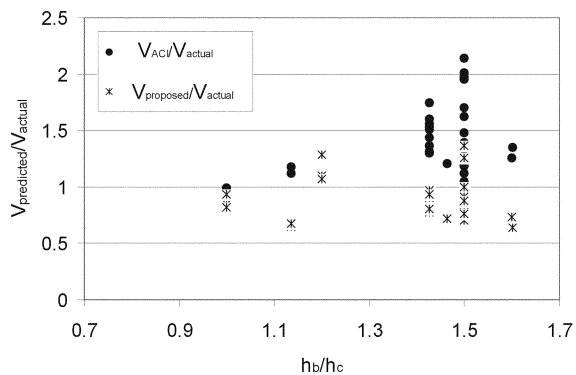


Fig. 19 The influence of the joint aspect ratio on the predicted joint shear strength of the proposed equation and the equation of ACI

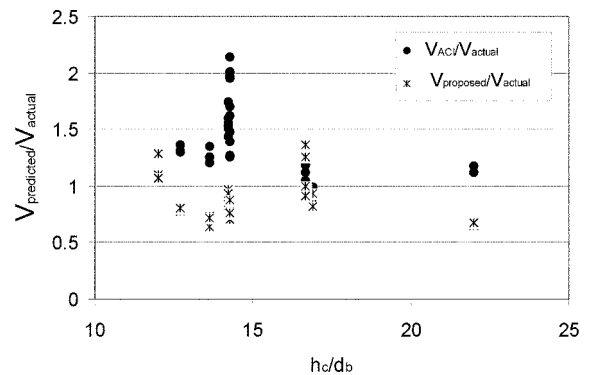


Fig. 20 The influence of the  $h_c/d_b$  ratio on the predicted joint shear strength of the proposed equation and the equation of ACI

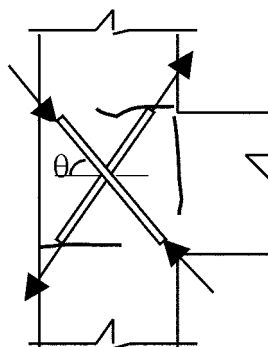


Fig. 21 The shear transfer mechanism of the crossed inclined bars (Tsonos *et al.* 1992)

recommendations of the ACI. It is apparent that the ACI-ASCE Committee 352 Recommendations substantially overestimate the shear strength of the cyclically loaded exterior beam-column joints. The standard deviation of the ACI-ASCE Committee 352 is very high compared to the equation of the author. Thus, the American Code is non-conservative. The proposed equation, on the other hand, gives quite accurate and conservative predictions of the joint shear strength with minimal standard deviation and is an improvement on the existing design codes for joints. It is also evident from Figs. 18 and 19 that the recommendations of ACI overestimates the shear strength at high joint aspect ratio values and high stirrup ratios. It is also evident from Fig. 20 that the ACI code can give very non-conservative results at low  $h_c/d_b$  ratios.

## 7. The shear resisting mechanisms of the cyclically loaded exterior beam-column joints with crossed inclined bars

As mentioned before, cyclically loaded beam-column joints resist the joint shear via two mechanisms. These are the strut mechanism and the truss mechanism, as shown by the model of Paulay in Figs. 1(b) and 1(c). The strut mechanism represents the contribution of concrete to the joint shear strength, whereas the truss mechanism is used to account for the contribution of stirrups. The presence of the inclined bars introduces an additional new mechanism of shear transfer, as shown in Fig. 21, as first suggested by Tsonos *et al.* (1992). This is the truss mechanism of the inclined bars. However, in this study, it is shown that the contribution of the inclined bars to the joint shear strength is much less than their yield capacity (34%).

## 8. Conclusions

This study aims at understanding the influence of different parameters on the shear strength of cyclically loaded exterior beam-column joints. Particular emphasis is given to codify the influence of the crossed inclined bars on the joint shear strength. A step-wise multiple regression analysis is carried out and the predictions of this analysis are compared with the joint mechanics; the stress equilibrium and the strain compatibility as well as the 3D interaction plots. The conclusions of the present study are as follows:

1. Crossed inclined bars are a feasible solution for increasing the shear strength of the cyclically loaded beam-column joints. But due to the geometrical constraints, the increase in the joint shear strength due to the crossed inclined bars is dependent on the joint aspect ratio. The greater the joint aspect ratio ( $h_b/h_c$ ) is, the less the contribution of the crossed inclined bars will be to the joint shear strength.
2. The contribution of the crossed inclined bars is much less than their yield capacity (34%).
3. The stepwise regression analysis shows that the joint shear strength increases as the concrete cylinder strength and the  $h_c/d_b$  ratio increases, and the joint shear strength is independent of the column axial stress or the column longitudinal reinforcement ratio. These findings are also confirmed by the joint mechanics as apparent from Eq. (45), which shows that the principal tensile strain is increased by the column longitudinal reinforcement ratio and the axial load on the column, whereas it is decreased by increasing the stirrup ratio. The shear stress in the joint is dependent on the principal tensile strain, as evident from Eqs. (34), (35) and (36). It is clear from Eqs. (25) and (45) that the joint shear strength increases as the transverse reinforcement ratio increases. Eq. (26) shows that the joint shear strength increases as the column load and the column longitudinal reinforcement increase, but Eq. (45) shows that as the longitudinal column reinforcement and the column load increases, the principal tensile stresses increase, which consequently decrease the normalised joint shear strength. Therefore, the increase in the joint shear strength due to Eq. (26) is offset by the increase in the principal tensile strain.
4. The 3D interaction plots also confirm the findings of the joint mechanics and the proposed equation.
5. A large number of subsets of regressors are tried for the step-wise multiple regression analysis and any subset of regressors which have a ratio less than 4 are not accepted. Throughout the step-wise regression analysis, a significance level of 0.05 is used, and it is checked whether the hypothesis that  $\beta=0$  will be rejected. The  $F$  value should be considerably greater than the minimum value needed to reject  $H_0: \beta=0$ . In the present analysis the  $F$  value is five times the minimum value needed to reject  $H_0: \beta=0$ . The residual plots are checked for the possibility of non-linearity, however, the spread of the residuals are fairly constant over  $x$  and there are no plots illustrating a horse-shoe shaped non-constant residual variance.
6. The existing code recommendations are thought to be inadequate by the author because they do not take into account the beneficial influence of the crossed inclined bars and the ratio of the height of the column to the diameter of the beam longitudinal reinforcement. In addition to this, both the ACI-ASCE Recommendations and the Japanese Code are based on the assumption that the joint shear strength is not significantly influenced by stirrups. The present study has shown, however, that the joint shear strength is increased by increasing the ratio of stirrups, but this increase is much less than their yield capacity (23% of their yield capacity) as opposed to that recommended by the New Zealand design philosophy for joints. The proposed equation has been compared with the equation of ACI in Figs. 17 to 20. It is apparent that the proposed equation is an improvement on the equation of ACI. The equation of ACI is non-conservative and substantially overestimates the shear strength of cyclically loaded exterior beam-column joints. The standard deviation of the equation of the ACI-ASCE Code is also substantially high. The proposed equation, on the other hand, predicts the shear strength of the cyclically loaded beam-column joints quite accurately and with minimal standard deviation.
7. The proposed equation takes into account the bond conditions of the beam bars. It is apparent from this study that the shear strength of beam-column joints is closely related to the bond

conditions of the beam bars. The equation requires that the diameter of the beam bars relative to the column cross-sectional height should be kept as small as possible to increase the joint shear strength.

8. The equation is also applicable on joints with high strength concrete because it is derived from a database consisting of a large number of high strength concrete specimens.

## References

- ACI-ASCE Committee 352R-91 (1998), "Recommendations for the design of beam-column joints in monolithic, reinforced concrete structures", ACI Manual of Concrete Practice: Part 3, Detroit, American Concrete Institute, 1998.
- ACI Committee 318 (1995), "Building code requirements for structural concrete (ACI 318-95) and commentary (318R-95)", American Concrete Institute, Farmington Hills, Michigan, 369pp.
- Alameddine, F.F. (1990), "Seismic design recommendation for high strength concrete beam-to-column connections", PhD Thesis, University of Arizona, 257pp.
- Architectural Institute of Japan (1990), "Design guidelines for earthquake resistant reinforced concrete buildings based on ultimate strength concept and commentary", 340pp.
- Bakir, P.G. (2003a), "A proposal for national mitigation strategy in Turkey", Submitted to *Natural Hazards Journal* for Possible Publication.
- Bakir, P.G. (2003b), "Shear transfer mechanisms in exterior beam-column joints", *Journal of Advanced Materials*, Accepted for Publication in 2003.
- Bakir, P.G. and Boduroglu, M.H. (2002a), "Earthquake risk and hazard mitigation in Turkey", *Earthquake Spectra*, **18**(3), 427-447, August, EERI.
- Bakir, P.G. and Boduroglu, M.H. (2002b), "Mitigating against earthquakes in Turkey", (special paper), *Proc. of the 12<sup>th</sup> European Conf. on Earthq. Eng.*, London, UK.
- Bakir, P.G. and Boduroglu, M.H. (2002d), "A new design equation for predicting the shear strength of the monotonically loaded exterior-beam column joints", *Eng. Struct. J.*, **24**(8), 1105-1117, August, Elsevier Science.
- Bakir, P.G. and Boduroglu, M.H. (2002e), "Predicting the failure modes of monotonically loaded reinforced concrete exterior beam-column joints", *Struct. Eng. Mech., An Int. J.*, **14**(3), 307-330, September, Techno-Press.
- Boduroglu, M.H. and Bakir, P.G. (2002c), "Proposals for earthquake planning in Turkey", *Proc. of the 12<sup>th</sup> European Conf. on Earthq. Eng.*, London, UK.
- Collins, M.P. and Mitchell, D. (1991), *Prestressed Concrete Structures*, Prentice Hall, Englewood Cliffs, New Jersey, 07362.
- Ehsani, M.R. and Wight, J.K. (1985), "Exterior reinforced concrete beam-to-column connections subjected to earthquake type loading", *ACI Struct. J.*, **82**(3), 343-349.
- Fuji, S. and Morita, S. (1991), "Comparison between interior and exterior reinforced concrete beam-column joint behaviour", *Design of Beam-Column Joints for Seismic Resistance*, SP-123, J.O. Jirsa ed., American Concrete Institute, Farmington Hills, Michigan, 167-185.
- Kaku, T. and Akasuka, H. (1991), "Ductility estimation of exterior beam-column subassemblies in reinforced concrete frames", *Design of Beam-Column Joints for Seismic Resistance*, SP-123, J.O. Jirsa ed., American Concrete Institute, Farmington Hills, Michigan, 167-185.
- Kitayama, K., Otani, S. and Aoyama, H. (1991), "Development of design criteria for RC interior beam-column joints", *Design of Beam-Column Joints for Seismic Resistance*, SP-123-4, J.O. Jirsa ed., American Concrete Institute, Farmington Hills, Michigan, 167-185.
- Kurose, Y. (1993), "Design of beam-column joints for shear", A Volume Honouring Hiroyuki Aoyama, University of Tokyo publications, 488.
- Megget, L.M. (1974), "Cyclic behaviour of exterior reinforced concrete beam-column joints", *Bulletin of the New Zealand National Society for Earthquake Engineering*, **7**(1), 27-47.

- Pantazopoulou, S. and Bonacci, J. (1992), "Consideration of questions about beam-column joints", *ACI Struct. J.*, **89**(1), 27-39.
- Park, R. and Paulay, T. (1975), "Reinforced concrete structures", John Wiley and Sons, New York, p.769.
- Paulay, T. and Park, R. (1984), "Joints in reinforced concrete frames designed for earthquake resistance", *Research Report 84-9*, Department of Civil Engineering, University of Canterbury, Christchurch, New Zealand.
- Paulay, T., Park, R. and Priestley, M.J.N. (1978), "Reinforced concrete beam-column joints under seismic actions", *Journal of the ACI, Proceedings*, **75**(11), 585-593.
- Paulay, T. and Priestly, M.J.N. (1992), "Seismic design of reinforced concrete and masonry buildings", John Wiley & Sons, New York, 744.
- Paulay, T. and Scarpas, A. (1981), "Behaviour of exterior beam-column joints", *Bulletin of the New Zealand National Society for Earthquake Engineering*, **14**, 131-144.
- Paulay, T. (1986), "Critique of the special provisions for seismic design of the building code requirements for reinforced concrete (ACI 318-83)", *ACI Struct. J.*, **83**(2), 274-283.
- Snedecor, G.W. and Cochran, W.G. (1980), *Statistical Methods*, 7<sup>th</sup> edition. Iowa State University Press, Ames.
- The Design of Concrete Structures, NZS 3101:1995 (1995), Standards 3101:1995, Standards New Zealand, Wellington.
- Tegos, I.A. (1984), "Contribution to the study and improvement of earthquake-resistant mechanical properties of low slenderness structural elements", PhD Thesis, Aristotle University of Thessaloniki, 185pp. (in Greek).
- Tsonos, A.G., Tegos, I.A. and Penelis, G. Gr. (1992), "Seismic resistance of type 2 exterior beam-column joints reinforced with inclined bars", *ACI Struct. J.*, **89**(1), 307-330.
- Tsonos, A.G. (1997), "Shear strength of ductile reinforced concrete beam-to-column connections, for seismic resistant structures", *J. European Earthq. Eng.*, **2**, 54-64.
- Tsonos, A.G. (1999), "Lateral load response of strengthened reinforced concrete beam-to-column joints", *ACI Struct. J.*, **96**(1), 46-56.
- Tsonos, A.G. (2000), "Effect of vertical hoops on the behaviour of reinforced concrete beam-to-column connections", *J. European Earthq. Eng.*, **2**, 13-26.
- Tsonos, A.G. (2001a) "Seismic retrofit of R/C beam-to-column joints using local three-sided jackets", *J. European Earthq. Eng.*, **1**, 48-64.
- Tsonos, A.G. (2001b), "Seismic rehabilitation of reinforced concrete joints by the removal and replacement technique", *J. European Earthq. Eng.*, **3**, 29-43.
- Uzumeri, S.M. (1977), "Strength and ductility of cast-in-place beam-column joints", *Reinforced Concrete in Seismic Zones*, Publication SP-53, American Concrete Institute, Detroit, 1977, 293-350.
- Vollum, R.L. (1998), "Design and analysis of exterior beam-column connections", PhD thesis, Imperial College of Science Technology and Medicine-University of London.

## Notation

|                  |   |
|------------------|---|
| $A_{inc}$        | : cross-sectional area of the crossed inclined bars   |
| $A_s$            | : cross-sectional area of the beam longitudinal top reinforcement   |
| $A_{scol}$       | : cross-sectional area of the total column reinforcement  |
| $A_{sje}$        | : area of joint stirrups  |
| $b_{a1}, b_{a2}$ | : smaller of one quarter of the column depth and one half the distance between the beam and column faces  |
| $b_b$            | : beam width  |
| $b_c$            | : width of the column   |
| $b_j$            | : effective joint width which is taken as $b_j = (b_b + b_c)/2$ but not greater than the beam width plus one half the column depth on each side of the beam |
| $b_{eff}$        | : average of the beam and the column width  |
| $c_{ii}$         | : $i$ th diagonal element of $(X^T X)^{-1}$   |
| $d$              | : effective depth of the beam   |



|                                 |   |
|---------------------------------|---|
| $d'$                            | : cover   |
| $d_b$                           | : diameter of the beam longitudinal reinforcement   |
| $C_c$                           | : concrete compressive force in the flexural compression zone   |
| $C_s$                           | : compression force in the compression reinforcement  |
| $D_1$                           | : diagonal concrete compressive force originating at the beam bar anchorage hook and subjected to a clamping force from the intermediate column bars          |
| $D_2$                           | : diagonal concrete compressive force maintaining equilibrium of the joint ties   |
| $D_j$                           | : taken as equal to column depth for interior joints and as equal to the horizontal projected length of the beam reinforcement in exterior beam-column joints |
| $E_s$                           | : modulus of elasticity of steel  |
| $f_2$                           | : principal compressive stress in concrete  |
| $f_{2\max}$                     | : maximum stress in concrete panels   |
| $f_c$                           | : concrete cylinder strength  |
| $f_{inc}$                       | : average stress in the crossed inclined bars   |
| $f_s$                           | : average stress in the beam reinforcement  |
| $f_{scol}$                      | : average stress in the column reinforcement  |
| $f_y$                           | : yield strength of the beam longitudinal reinforcement   |
| $f_{yw}$                        | : yield strength of the stirrups  |
| $f_w$                           | : average stress in the transverse reinforcement  |
| $H$                             | : total height of the column  |
| $h_c$                           | : column depth in the direction of joint shear  |
| $h_b$                           | : beam depth  |
| $k$                             | : factor that is dependent on the type of beam-column joint which is equal to 0.3 for interior beam-column joints and 0.18 for exterior beam-column joints    |
| $N$                             | : column axial load   |
| $p$                             | : number of independent variables   |
| $SS$                            | : sum of squares  |
| $SS_{reg}$ and the $SS_{total}$ | : sum of squares for the regression and total analysis as explained in Table 2  |
| $T_1, \dots, T_4$               | : forces acting in the longitudinal column bars between the corner bars in side faces of the column   |
| $V_{col}$                       | : shear force in the upper column   |
| $V_{cv}$                        | : ideal vertical joint shear strength provided by concrete shear resisting mechanism  |
| $V_j$                           | : applied joint shear   |
| $V_{jh}$                        | : total horizontal shear force across a joint   |
| $V_{jv}$                        | : total vertical shear force across a joint   |
| $V_{ju}$                        | : joint shear strength  |
| $V_{sv}$                        | : ideal vertical joint shear strength provided by vertical joint shear reinforcement  |
| $\alpha$                        | : joint aspect ratio defined as $h_b/h_c$   |
| $\Delta T$                      | : bond force  |
| $\epsilon_1$                    | : principal tensile strain  |
| $\epsilon_2$                    | : compressive strain  |
| $\epsilon_c$                    | : compressive strain at failure (−0.002)  |
| $\epsilon_x$                    | : tensile strain in the $x$ direction   |
| $\epsilon_y$                    | : tensile strain in the $y$ direction   |
| $\theta$                        | : angle between the direction of the principal compressive stress and the transverse tensile strain $\epsilon_t$  |
| $\gamma$                        | : joint shear stress expressed as a multiple of $\sqrt{f_c}$  |
| $\rho_{stir}$                   | : lateral reinforcement ratio   |
| $\sigma_x$                      | : average normal concrete stress in the $x$ direction   |
| $\sigma_y$                      | : average normal concrete stress in the $y$ direction   |
| $\sigma_z$                      | : confining stress provided by stirrups in the $z$ direction  |