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Modelling of timber joints made with steel dowels and locally reinforced by DVW discs

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Abstract. Local reinforcement in dowel type timber joints is essential to improve ductility, to increase load carrying capacity and to reduce the risk of brittle failure, especially in the case of using solid dowel. In many types of reinforcing materials available today, DVW (densified veneer wood) has been demonstrated to be the most advantages in terms of compatibility, embedding performance and ductility. Preliminary studies show that using appropriately sized DVW discs bonded into the timber interfaces may be an effective way to reinforce the connection. In this paper, non-linear 3-dimensional finite element models, incorporating orthotropic and non-linear material behaviour, have been developed to simulate structural performance of the timber joints locally reinforced by DVW discs. Different contact algorithms were applied to simulate contact conditions in the joints. The models were validated by the corresponding structural tests. Correlation between the experimental results and the finite element simulations is reasonably good. Using validated finite element models, parametric studies were undertaken to investigate effects of the DVW disc sizes and the end distances on shear stresses and normal stresses in a possible failure plane in the joint.

Key words: DVW; local reinforcement; timber joint; finite element; parametric study; end distance.

1. Introduction

Local reinforcement in dowel type timber joints has been shown to improve ductility, to increase load carrying capacity and to reduce the risk of brittle failure. There are many types of reinforcing materials available today, which are not only suitable for strengthening timber connections, but also for general timber members. Researchers initially used steel as a local reinforcing material (Leijten 1988, Clarke *et al.* 1993), then resin composites (Gardner *et al.* 1994) and plywood (Rodd 1996). More recently, Glass Fibre Reinforced Plastics (GFRP) have been extensively studied for the same purpose (Chen and Haller 1994, Larsen 1996, Claisse and Davis 1998, Gilfillan *et al*. 2001, Gentile *et al.* 2002), so have Carbon Fibre Reinforced Plastics (CFRP) (Triantafillou and Deskvic 1992, Plevris and Triantafillou 1995, Johns and Lacroix 2000). Densified Veneer Wood (DVW) has been also studied as a reinforcing material for timber connections in the last a few years (Leijten *et al*. 1994), which has been shown to be the most advantages in terms of compatibility, embedding performance and ductility (Guan and Rodd 2000, Guan and Rodd 2001).

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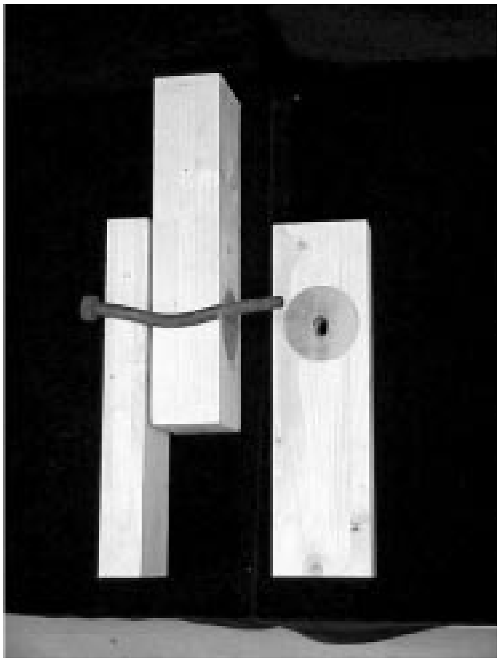

Fig. 1 A timber joint locally reinforced by DVW discs

Traditionally, DVW has mostly been used in electrical engineering as an insulating material, but in recent years it has been used as a local reinforcement in timber connections in Europe due to better availability and reduction of the price.

In current reinforcing practice DVW sheets are placed over the whole interface areas of the joints. Although very effective this inevitably increases material and handling costs. An alternative way to reinforce the connection is to strengthen the timber using appropriately sized DVW discs bonded into the timber interfaces, as shown in Fig. 1. Preliminary results (Rodd and Pope 2001) show that this may be an effective way forward.

In the new connector, a disk of DVW is bonded into a circular recess that is cut into each shear interface of a jointed member. This is a very localised form of timber reinforcement. In a timber-timber joint there will be at least two disks per shear plane (one in each member). In a three-member timber joint there will be a minimum of four disks (one in each side member and two in the middle member). The depth of the recess requires that the outer surface of the disk is flush with the surface of the timber member. It is essential that there is a good bond between the internal surfaces of the recess and the mating surfaces of the disk in order to achieve necessary rigidity and reasonable load carrying capacity.

Owing to complexity introduced by interaction between the timber and the DVW, the timber and the steel dowel, the DVW and the steel dowel, also 3-dimensional and non-linear nature of the problem, the most promising way to undertake the theoretical analysis is to develop suitable finite element (FE) models. Then the models need to be validated against the related experimental work. Using the validated models, systematically designed parametric studies can be carried out to optimise the joint.

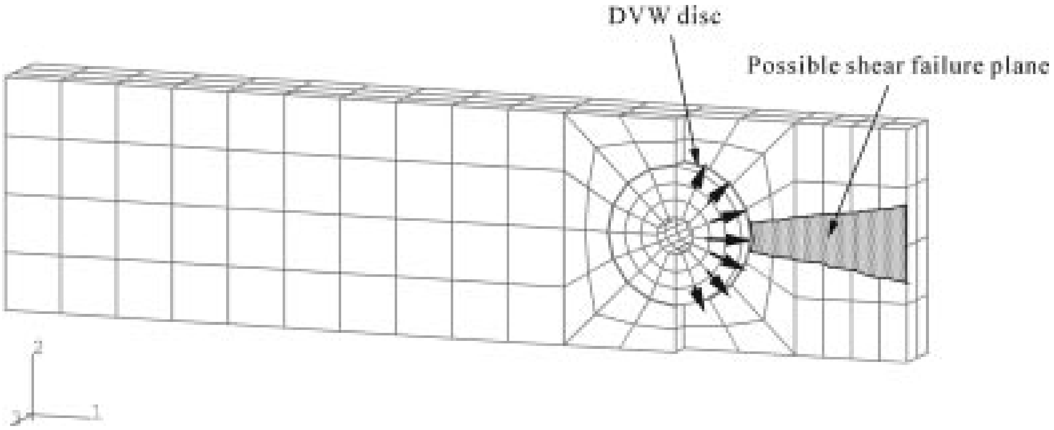

Fig. 2 Possible shear failure plane in the timber member

In this newly developed joint, one of the potential problems to be addressed is the possible shearing failure in the plane within the timber components when a joint is loaded in tension, as shown in Fig. 2. This failure is primarily dependent upon the size of the disc and the end distance. Therefore, it is necessary to undertake parametric studies of various disc sizes and end distances with validated FE models and to optimise such dimensions. In addition, normal stress distributions between the disc and the end of the timber member are of important effect on the punch through failure. Thus, such normal stress distributions were linked to shear stress distributions in the possible failure plane.

In this paper, non-linear 3-dimensional finite element models, incorporating orthotropic elastic and elasto-plastic material behaviour, have been developed to simulate structural performance of the timber joints with various dowel diameters locally reinforced by DVW discs. Both finite slide and small slide algorithms were applied to simulate variable interactions between different interfaces in the joints. The model was validated against the related structural tests, which were carried out with almost “defect” free samples. Good correlation has been obtained between the experimental results and the finite element simulations. With verified FE models, parametric studies were carried out to determine effects of the disc size and the end distance on shear and normal stress distributions in the possible critical plane. Optimum size of the DVW disc and the end distance were discussed.

2. Finite element simulation

Since different materials are used in the joint appropriate constitutive models need to be employed to simulate variable performances of constituent materials, especially for dominant materials. For a joint locally reinforced by DVW disc, the steel dowel and the DVW are such dominant materials and elasto-plasticity is such an appropriate constitutive model. In other words, timber members have secondary influence on overall structural behaviour for these locally reinforced joints. In addition, various contact conditions between different joint members need to be modelled adequately in order to simulate the overall structural behaviour of the joint reasonably well.

2.1 Elasto-plasticity

The basic finite element equations in matrix form for elasto-plastic problems in engineering have been well developed (Yamada and Yoshimura 1968, Zienkiewicz *et al*. 1969, Owen and Hinton 1980, Chen and Han 1988). The incremental stress-strain relationship can be expressed as

$$\boldsymbol{d}\{\sigma\} = [\boldsymbol{D}]_{ep}\boldsymbol{d}\{\varepsilon\} \tag{1}$$

where $[\boldsymbol{D}]_{ep}$ is elasto-plastic matrix, which is dependent upon the elastic matrix $[\boldsymbol{D}]$, the yield function and the hardening function H. The later refers to the change of the equivalent stress against the change of plastic strain (Zienkiewicz *et al*. 1969). Clearly, in the case of perfect plasticity the hardening function is zero.

After initial yielding, total deformation will be partly elastic and partly plastic. Therefore, the strain increment in Eq. (1) is composed of two parts,

$$\boldsymbol{d}\{\varepsilon\} = \boldsymbol{d}\{\varepsilon\}_{el} + \boldsymbol{d}\{\varepsilon\}_{pl} \tag{2}$$

In large non-linear geometric situations, such as the steel dowel experiencing large plastic flow, all strain measurements are based on logarithmic strain definition, which is

$$\varepsilon_{ij} = \ln(\lambda_{ii}) \tag{3}$$

where λ_{ii} are the 'principal stretches' which are defined as the ratio of current and initial lengths between any two points within a structure.

For modelling of the steel dowel and the DVW plate, elasto-plasticity with non-linear strain hardening was used. Therefore, the strain hardening function H depends upon variable plastic strains. Experimental work indicated that the timber used did not show significant strain hardening under embedding conditions and, therefore, for this situation perfect plasticity was utilised. As a result of this, the strain function H is simply zero for timber under embedding conditions.

Timber members, with the exception of the embedding areas beneath the dowel, were taken to behave as a linear orthotropic elastic material. Timber can only sustain relatively small deformations prior to brittle failure when it is subject to tension. Thus, reasonably good simulation of timber members was obtained using the above constitutive models.

In linear orthotropic elastic case, stress-strain relationship can be expressed as

$$d\{\sigma_{ij}\} = [D]_{orth}d\{\varepsilon_{ij}\} \tag{4}$$

where $[D]_{orth}$ is an orthotropic elastic matrix (6 × 6), e.g., $D_{LL} = E_L(1 - \upsilon_{TR}\upsilon_{RT})\gamma$, $D_{LT} = E_R(\upsilon_{LT} - - \upsilon_{RT}\upsilon_{LR})\gamma$, $D_{LR} = E_T(\upsilon_{LR} - \upsilon_{LT}\upsilon_{TR})\gamma$, and $\gamma = 1/(1 - \upsilon_{LT}\upsilon_{TL} - \upsilon_{TR}\upsilon_{RT} - \upsilon_{RL}\upsilon_{LR} - 2\upsilon_{TL}\upsilon_{RT}\upsilon_{LR})$ which is an effective factor based on Poisson's ratios.

2.2 Modelling of contact conditions

Dowel type joint with local reinforcement is usually of different interfaces between the steel dowel and the DVW/the timber, the DVW and the timber, the timber and the timber. There would be different interactions on those interfaces, some with finite slippage, some with small slide and some without separation. In the joint studied, finite slide can only occur between the side member and the middle member. However, such finite slide occurs initially in DVW-DVW interface and later in DVW-timber interface, since each of these members has a DVW disc encased in the timber in the interface area. Thus, interface elements with 3-D slide lines for generalised contact conditions between two deforming bodies were used (Guan and Rodd 2000). This contact algorithm allows interface elements and 3-D slide lines to be generated on both the timber and the DVW. Contact surface can only be prescribed in a single material. Moreover, sliding directions in the DVW-DVW interface can be specified to follow mesh patterns, which give great flexibility to simulate the contact conditions on this specific interface.

Contacts between the dowel and the timber, the dowel and the DVW plates, the washer and the timber are subject to small slide. Contact between the timber and the DVW discs is fixed due to gluing assuming that there is no breakdown of the bond so as to allow separation during test. Therefore, these contact conditions can be modelled by the use of the 3-D contact surface algorithm (Guan and Rodd 2000).

In all contact pairs, a slave surface (relatively soft) and a master surface (relatively hard) needs to be selected. The slave surface should be numerically smaller than the master surface to avoid numerical problems.

2.3 Implementation of the FE modelling

Finite element code Abaqus (Hibbitt *et al.* 1998a, 1998b), in conjunction with all essential features, was utilised to obtain the theoretical simulations. Fisrt order isoparametric brick elements and the related interface elements were used for fully 3-D problems. Comparing with the second order elements, such elements give greater efficiency or a better convergence rate for modelling of the complex contact problems that exist in a joint of this type. Within the element the incremental displacement at any point can be expressed based on the nodal degree of freedom as

$$[\Delta\{\boldsymbol{u}\}] = [\boldsymbol{N}]\Delta\{\boldsymbol{q}_i\} \tag{5}$$

where $[N]$ is the shape function matrix and $\Delta\{q_i\}$ is the differential increment in nodal displacements.

The equations of equilibrium for a discretised structure can be then expressed in terms of the unknown nodal displacement $\delta\{q_i\}$ as

$$[\boldsymbol{K}]_T^{(k-1)}\delta\{\boldsymbol{q}_i\}^{(k)} - \delta\{\boldsymbol{P}_i\}^{(k-1)} = 0 \tag{6}$$

where $[K]_T$ is the tangential stiffness matrix in a load step and $\delta\{P_i\}$ are the incremental loads dependent on convergence criteria. The full Newton-Raphson scheme was used to solve the above equations as this is a powerful method for extremely non-linear geometric problems such as the steel dowel under large local plastic deformations. Under the scheme $[K]_T$ is evaluated and factorised for each iteration.

In order to develop the corresponding numerical models the joint needs to be descretized. Fig. 3 shows mesh generations of timber members, DVW discs and the steel dowel, including geometry, boundary and loading conditions. Only half of the joint is created due to symmetrical nature of the problem. 8-node brick elements were used in the modelling.

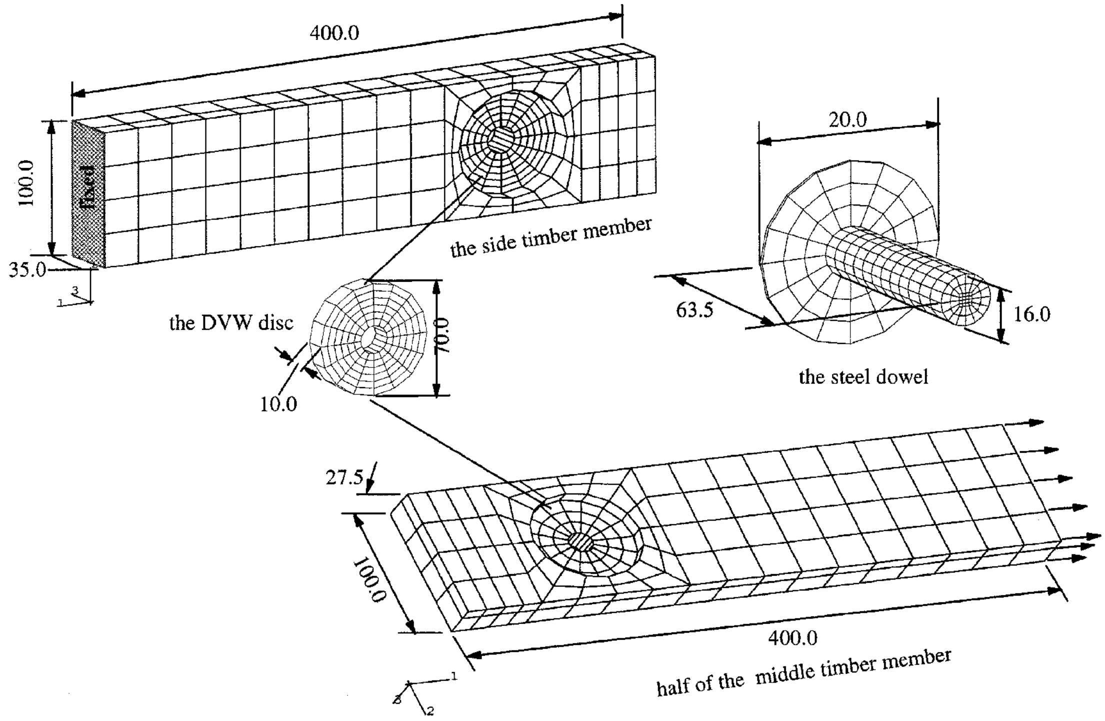

Fig. 3 Mesh generation, loading and boundary conditions for a timber joint locally reinforced by DVW discs

3. Results and discussion

Material properties used in the FE modelling are listed as follows.

Timber (European Whitewood)(Hearmon 1948):

$E_L = 10700$ N/mm^2, $E_R = 471$ N/mm^2, $E_T = 430$ N/mm^2, $G_{LT} = 620$ N/mm^2,
$G_{LR} = 500$ N/mm^2, $G_{TR} = 23$ N/mm^2, $\upsilon_{LT} = 0.025$, $\upsilon_{LR} = 0.030$, $\upsilon_{TR} = 0.510$
$\sigma_y = 35$ N/mm^2

DVW:

$E = 19000$ N/mm^2, $\upsilon = 0.14$
$\sigma_{y1} = 55$ N/mm^2

Steel:

$E = 200000$ N/mm^2, $\upsilon = 0.30$
$\sigma_{y1} = 300$ N/mm^2

Plastic strains are calculated using following equation when consider strain hardening.

$$(\varepsilon_L)_{pl} = (\varepsilon_L)_{total} - \frac{\sigma_{true}}{E} \text{ for } \sigma_{true} \geq \sigma_y \tag{7}$$

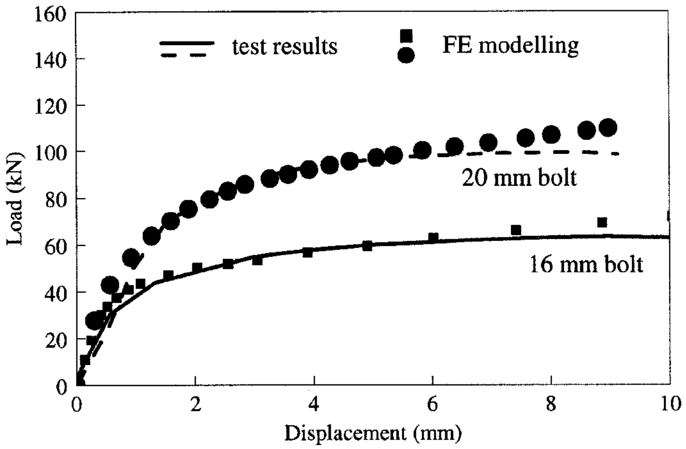

Fig. 4 Load-displacement curves of joints with 16 & 20 mm diameter solid dowels

3.1 Simulations of experimental results

Fig. 4 shows load-displacement relationships obtained from the FE analysis. The corresponding experimental results are also shown in the same diagram in order to make easy comparison. Clearly, correlation is reasonably good, which indicates that essential features of the joint have been simulated appropriately. As mentioned before, properly modelling of predominant materials (steel and DVW) and contact conditions are crucial. One more factor needs to be pointed out is that DVW, as a local reinforcing material, has very good embedding performance, i.e. in a ductile manner. This offers some convenience to simulate this type of the joint. If materials with poorer embedding behaviour, such as plywood, are used as local reinforcing materials, correct modelling of the embedding behaviour, accompanied by a large local crush, will be essential to obtain good correlation between the experimental and numerical results. It needs to point out that there is mild discrepancy in the later stage of the load-displacement curves. Practically, severe local crush in both the timber and the DVW is expected prior to joint failure. Therefore, the curve tends to flat until the failure. At present, modelling of such crush has not been included in the FE simulation. This is a main reason for the discrepancy mentioned above.

Displaced meshes of the joint, the DVW disc and the dowel are displayed in Fig. 5, in which only joint with 16 mm diameter bolt is shown. Clearly, overall displacements are from excessive bending of the dowel, which contributes large embedding deformations in the DVW discs as shown in Fig. 5, i.e. the shape of its inner hole was changed from the circular one to the oval one. There is a clear opening between the middle timber member and the side timber member, which is caused by different embedding performances of the DVW and the timber. Such opening can be significantly reduced by replacing the solid dowel with a hollow dowel. This has been verified in a joint made with a hollow dowel locally reinforced by DVW plates (Guan and Rodd 2001). Moreover, a plastic hinge can be easily seen in the dowel where there is an interface between the side member and the middle member. Solid dowel has a different deformation mode than that of the hollow dowel (Guan and Rodd 2000). The former is bending dominant and the latter is local shearing dominant. Simulations of the embedding behaviour in the DVW disc and strain hardening in the dowel are essential to obtain the reasonable modelling.

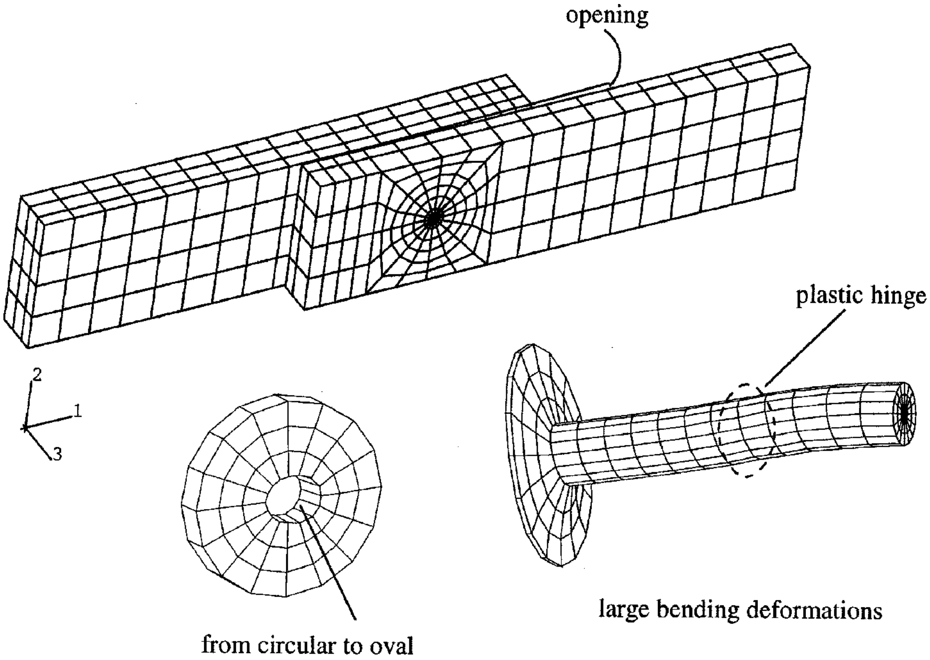

Fig. 5 Displaced meshes of a joint with separate diagrams of the dowel and a DVW disc

3.2 Parametric studies of the disc size

The load applied to the DVW disc from the dowel needs to be transferred to the timber through contact normal and shear stresses between the disc and the timber. The contact normal stresses are relied on shear resistance of the timber in the critical plane. Therefore, it is necessary to investigate variation of both stresses with changing diameters of the disc. Variation of the disc thickness only has effect on embedding behaviour since the total area where shear stresses act is not dependent upon the disc thickness.

Using validated FE models, parametric studies were undertaken to evaluate the disc size effect on shear stresses and normal stresses in the critical plane (see Fig. 2), whilst make no dimensional changes to other joint members. There were five disc diameters investigated, i.e. 30, 45, 60, 75 and 90 mm. A load of 60 kN was applied to all cases.

Fig. 6 shows longitudinal normal stress distributions for two typical disc diameters of 30 and 90 mm, respectively. The critical stresses in the disc-timber interface areas vary from −5.8 to −40.4 N/mm^2, corresponding to the dramatic increasing such stresses generated by the normal contact between the disc and the timber. Clearly, with decreasing the disc size the high stress areas are expanded well into the timber, which would contribute to the failure of the timber. Relationship between the disc size and the normal stress is displayed in Fig. 7, which may be expressed as a logarithmic function as follows.

$$\sigma_{11} = -145.34 + 31.26 \ln(D) \tag{8}$$

where D is the diameter of the DVW disc.

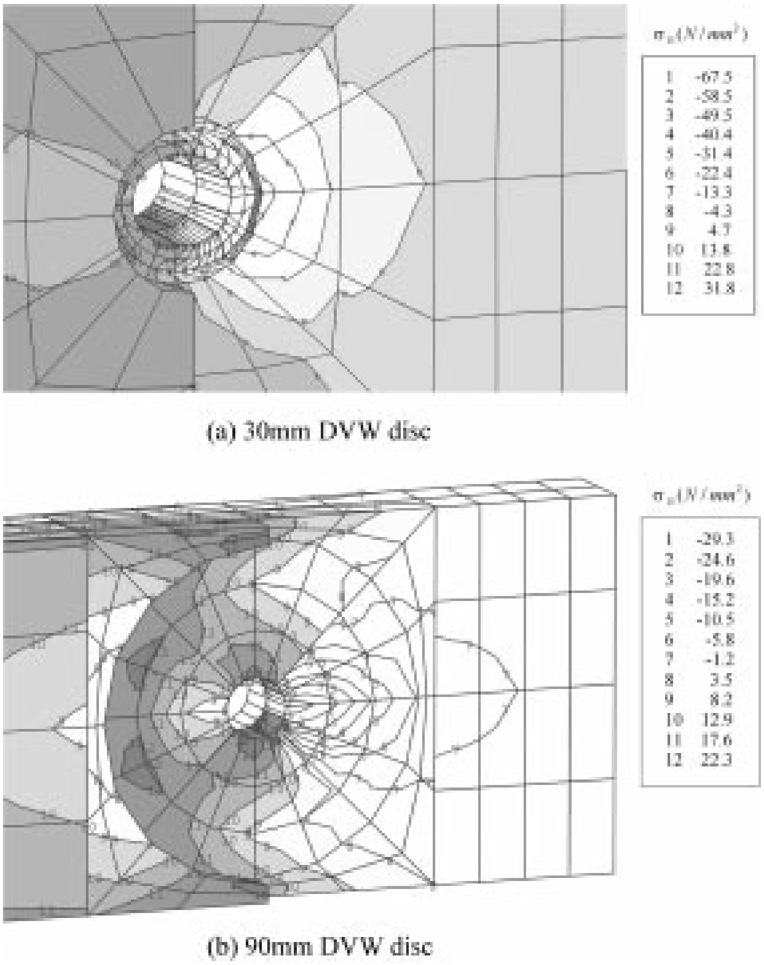

Fig. 6 Typical longitudinal normal stress distributions in the critical areas on the timber member, subjected to changing DVW disc sizes

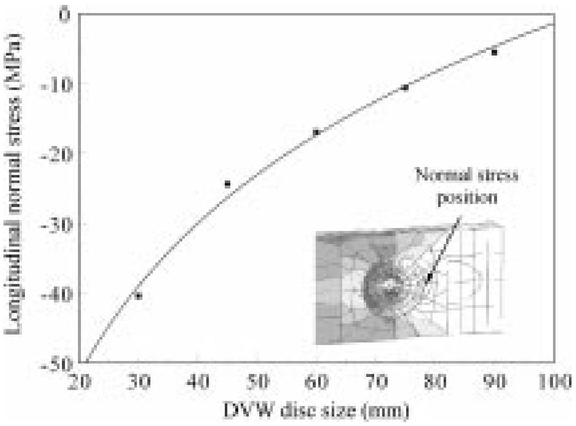

Fig. 7 Variation of the longitudinal normal stresses against DVW disc sizes

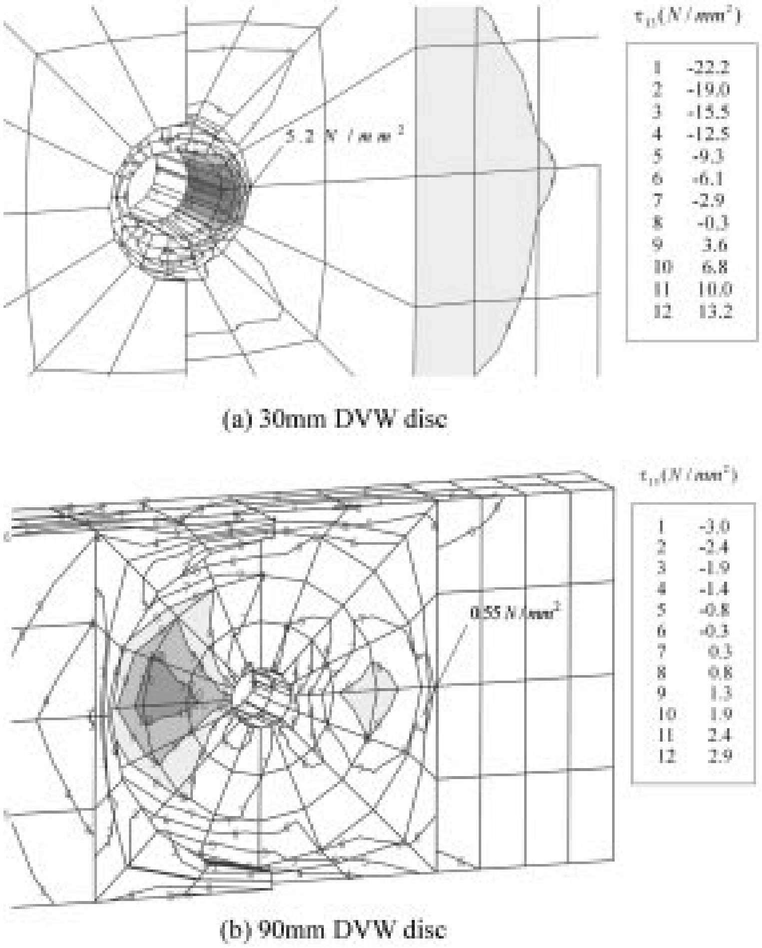

Fig. 8 Typical longitudinal shear stress distributions in the possible failure plane on the timber member, subjected to changing DVW disc sizes

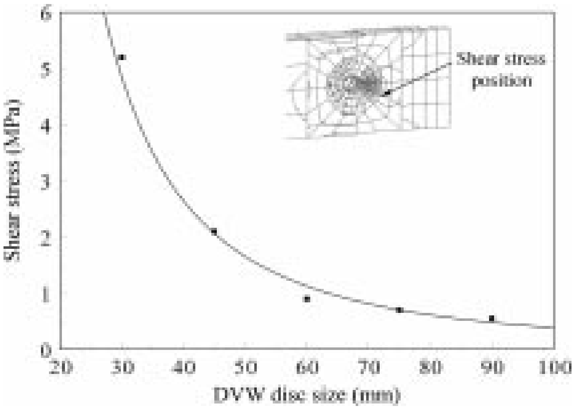

Fig. 9 Variation of the critical longitudinal shear stresses against DVW disc sizes

Fig. 8 shows longitudinal shear stress distributions for the two typical disc sizes described above in the possible failure plane. It can be seen that the shear stress in the timber adjacent to the disc (see Fig. 8) varies from 5.2 to 0.55 N/mm^2. In other words, with decreasing the disc size there are significant increases in the shear stress, which could reach a critical level to cause shear failure. It is worth pointing out that much higher shear stresses are developed in the disc-timber shear interface areas which is close to the critical disc edge-timber contact region. Once the bonding strength is reached in those areas, debonding may occur. Thus higher shear stresses are expected in the critical shear plane in the timber by transferring load from the dowel through the disc to the timber. Fig. 9 gives relationship between the disc size and the shear stress, which may be presented following a power function.

$$\tau_{13} = 6233.72D^{-2.1053} \tag{9}$$

Based on the above studies, a disc size of about 60 mm in diameter may be a proper one for a joint with dimensions studied to satisfy both embedding strength and shear strength.

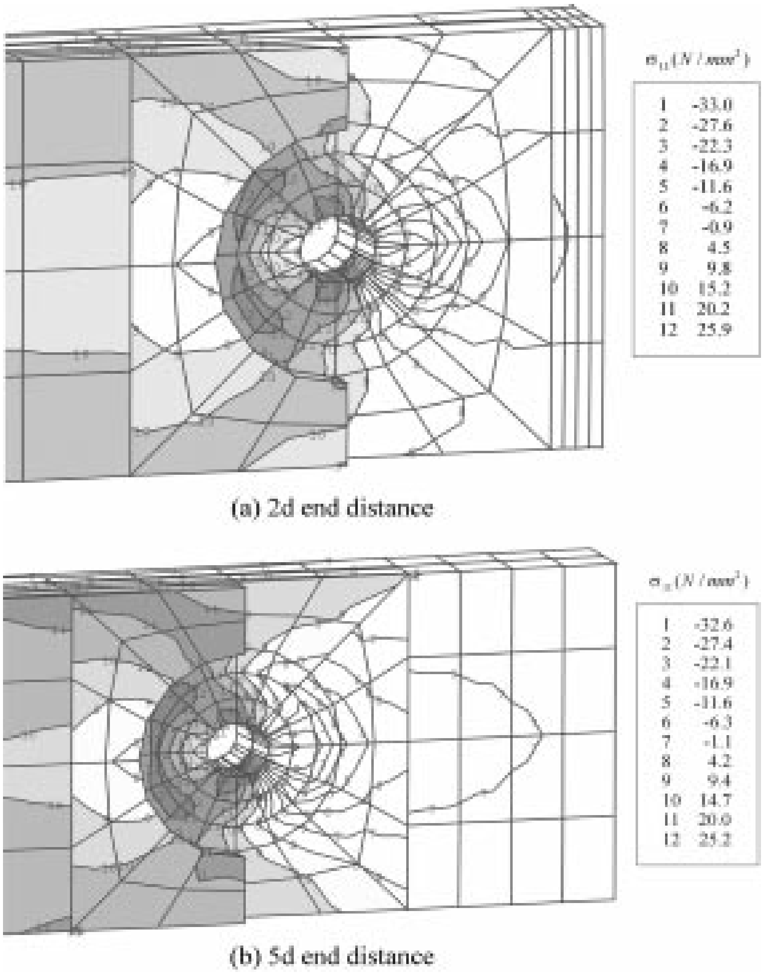

Fig. 10 Typical longitudinal normal stress distributions in the critical areas on the timber member, subjected to changing the end distances

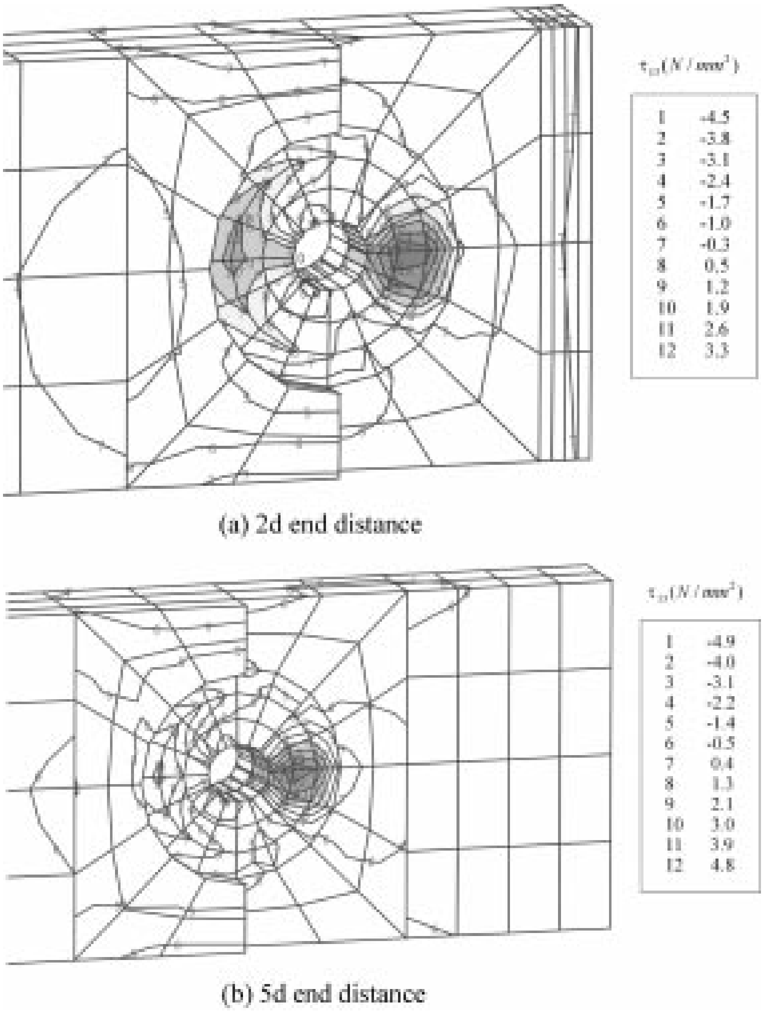

Fig. 11 Typical longitudinal shear stress distributions in the possible failure plane on the timber member, subjected to changing the end distances

3.3 Parametric studies of the end distance

It is also necessary to study the end distance from underneath the dowel to the end of the timber member along the loading direction since it also affects load carrying capacities and failure modes of the joint. Paramertic studies, which cover 2*d*, 3*d*, 4*d*, 5*d* and 6*d* end distances, were undertaken, where *d* is the external diameter of the dowel. As the end distance is also dependent upon the diameter of the DVW disc, a diameter of 60 mm was selected in order to minimise such variable. The reason to choose this disc diameter is to coincide the recommended disc size and to avoid the extreme cases, such as 90 mm and 30 mm.

Fig. 10 shows longitudinal normal stress distributions from two typical end distances of 2*d* and 5*d*. Clearly, high stress regions are gradually migrated away from the end with increasing the end distance. For the end distance of around 5*d*, the critical stress region is well inside of the timber underneath the disc. Therefore, it can be taken as a safe end distance. The corresponding longitudinal shear stress distributions in a possible failure plane underneath the dowel are displayed in Fig. 11. Similar situations occur here, i.e. the critical shear stress region is well away from the end when the end distances are increased from 2*d* to 5*d*. Moreover, the size of the critical shear stress area in the joint with 5*d* end distance is significantly smaller than that for the joint with 2*d* end distance. This is understandable since the critical area has to be bigger for the latter joint in order to balance the similar resultant shear force for both joints. Again, 5*d* end distance may be regarded as a safe one when considering the shear stress distributions.

More work has been proposed to obtain structural behaviour of the joint made with hollow steel dowels of different diameters and wall thicknesses. Also modelling of local crush will be included in future FE simulations.

4. Conclusions

Non-linear 3-D finite element models have been developed to simulate structural behaviour of the timber connections locally reinforced by DVW discs. Linear orthotropic elasticity and elasto-perfect plasticity are adequate for modelling timber members in tension and in compression, respectively, whilst elasto-plasticity with strain hardening is suitable for simulating the steel dowel and the DVW disc. Both small slide and finite slide contact algorithms have been used to deal with different contact conditions within the joint. All essential features of the joint have been modelled reasonably well. Good correlation between the test results and the FE modelling has been obtained. With validated models parametric studies of variation of the disc size and the end distance have been undertaken to recommend reasonable disc sizes and end distances.

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