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Analytical and experimental study on the behavior of elastically supported reinforced concrete decks

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Abstract. Current design specifications prescribe that the upper and lower reinforcement mat is required in the same amount to resist negative and positive moment in bridge decks. This design concept is primarily based on the unrealistic assumption that the girder plays a role of rigid support against deck deflection. In reality, however, girders are flexible and the deflection of girders affect the behavior of deck slabs. In the present study, an analytical method was developed to take the effect of the girder flexibility on the deck behavior into account. The method was formulated based on the slope-deflection equations of plates and harmonic analysis. Unlike the conventional finite element analysis, the input and output schemes are simple and convenient. The validity of the presented study was verified by a series of comparative studies with finite element analyses and experimental tests. It was shown from the analyses that the negative transverse moments of decks were significantly reduced in many cases when the girder flexibility were appropriately taken into consideration whereas the positive moments tend to increase. This poses a strong need to improve the conventional design concept of decks on rigid girders to those on flexible girders.

Key words: bridge decks; girder deflection; negative bending moment; harmonic analysis; elastic support.

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1. Introduction

In a slab-on-girder bridge system, most of bridge decks have been designed according to the American Association of State Highway and Transportation Officials (AASHTO) specifications of live load bending moment formula based on the study by Westergaard (1930), which is associated with positive bending moment in simply supported plates. The transverse bending moment formula for bridge decks are based on the assumptions that the girders are rigid. Based on this bending moment, the lower and upper reinforcement mats in a deck shall be provided in the same reinforcement proportions to resist positive and negative bending moments respectively. Also, the provisions prescribe that in slabs continuous over three or more supports a continuity factor of 0.8 and in a slab over simple support a factor of 1.0, shall be applied to both positive and negative bending moment. However, the aforementioned design moment provided in the specifications does not accurately represent the actual behavior of the deck slab due to the limitations in defining merely them as a function of girder spacing and magnitude of loads. In other words, in an actual bridge system, the transverse bending moments should be expressed by the sum of one due to elastic support as well as one due to rigid support. It has been observed that shrinkage cracks often occur over the upper transverse bars, inducing increased exposure to deleterious substances such as deicing chemicals (Cao *et al.* 1996). Once the deleterious substances attack the upper reinforcement, the reinforcement is susceptible to corrosion, which results in the premature deck failure. As a consequence, it can be said that the unnecessary excessive upper reinforcement may rather accelerate the deck deterioration.

There have been many research activities to clarify the behavior of bridge decks and to propose an adequate design moment criteria. The effect of girder deflections was discussed by Newmark (1949), Cusens and Pama (1975), Bakht and Jeager (1985), Allen (1991), and Cao *et al.* (1996). During revision of the Ontario Highway Bridge Design Code (OHBDC), many experimental investigations on the behavior of bridge decks were performed. The OHBDC specifications were originally based on the compression membrane force in deck slabs, which increases the flexural capacity (Arching Effect). Slabs usually fail by punching shear due to static or fatigue loads. In fact, the deck slabs subjected to service load have been known to show no serious cracks in the elastic range.

The actual behavior of bridge decks subjected to service loads can be divided into two parts (Cao *et al.* 1996). One part is the primary bending due to local deflection that is developed in the continuous slabs under the assumption that the supporting girders are rigid. The other is the secondary bending moment that is caused by deflection of the supporting girder. The derivation of these bending moments presented by Cao *et al.* is known as a simplified method based on the force method using the isotropic and orthotropic plate theory and the applications of this derivation are quite limited to the various cases such as multiple girder and continuous spans bridges. It should be noted that there are numerous parameters affecting the secondary bending moment such as the width to span ratio, relative stiffness of girder to deck, number of girders, and number of cross beams, etc.

In the present study, a new analytical method was developed based on the slope-deflection equations of plates and harmonic analysis and can be applied in handling the various slabs-on-elastic-girders with different aforementioned parameters. This method was encoded into a computer program (ASGB, Analysis of Slab on Girder Bridge). Meanwhile, the developed method is verified through an intensive comparative study by conventional finite element analyses (LUSAS) and a

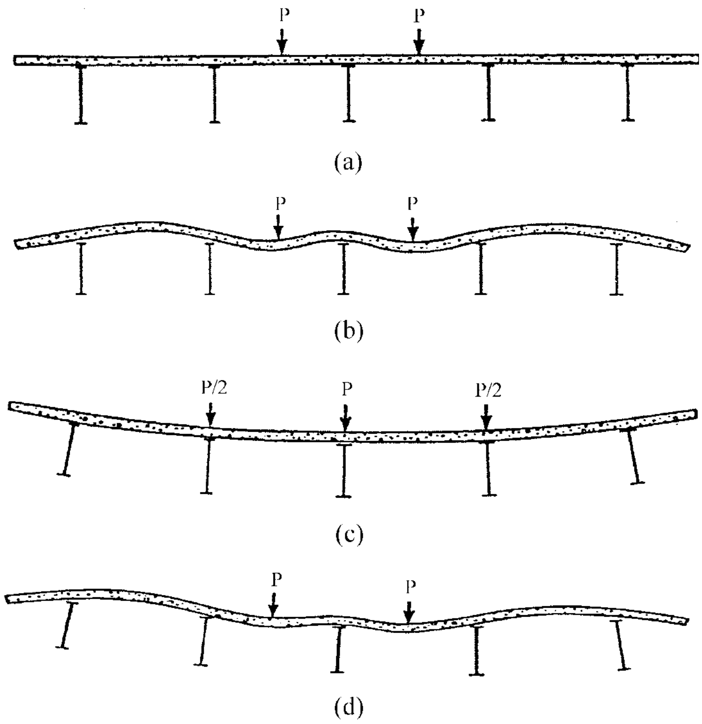


Fig. 1 Cross-section deformations due to transverse moments (Bakht and Jaeger 1985): (a) Cross-section before deformations; (b) Deformations due to local deflection; (c) Deformations due to global deflection; (d) Deformations due to actual transverse deflection

series of experimental results. It is shown that in many cases the performance of the slab decks can be considerably enhanced by taking into account the effect of girder flexibility.

2. Bridge deck analysis

Analytical methods of plate theory were introduced by Kirchhoff-Love and Reissner-Mindlin (Timoshenko *et al.* 1984 and Szilard 1974). Kirchhoff-Love's plate theory assumes that deformation of the plate is mainly caused by bending effects without shearing effects. While, Reissner-Mindlin's plate theory considers both the bending and shearing effects. In this study, an analytical solution of thin plate bridge-deck behavior derived from the former theory has been adopted. Bending moments in the direction perpendicular to the flow of traffic are referred to as transverse moments. To determine the transverse moments in slab-on-girder bridges, it is common to divide the response into two parts, i.e., global moments and local moments. Local moments are obtained from the local deflections, as shown in Fig. 1(b), of deck slabs between adjacent girders under the assumption of rigid-girder supports and non-yielding girders whereas global moments are obtained through the overall deflections of the deck slab, as shown in Fig. 1(c). Henceforth, the actual bending moments of slab-on-girder bridges consist of local and global moments as illustrated in Fig. 1(d).

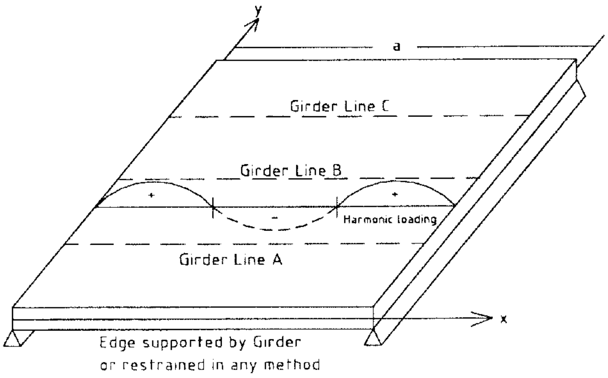


Fig. 2 Bridge deck continuously supported by girders

2.1 Harmonic analysis of bridge deck

Harmonic analysis is a mathematical method to express the behavior of plates. Navier's solution and Levy's method, which are the basic methods of plate analysis, use harmonic functions (Timoshenko *et al*. 1984). Fig. 2 prescribes bridge deck with simply supported of both-end and girder supported of both-end, which is applied the sinusoidal harmonic loading. When harmonic loading is applied parallel to the x-axis of a rectangular plate that is supported by girders, and is applied perpendicular to a y-axis that is simply supported, the deflection of the plate can be interpreted with harmonic terms. That is, the deflection is

$$w = Y_n\left(\frac{n\pi y}{a}\right)\sin\left(\frac{n\pi x}{a}\right) \tag{1}$$

where the symbols a and Y_n denote span length of the slab and amplitude factor of deflection relating a function of y, respectively. Solving the differential equation of the plate by Eq. (1), the amplitude factor Y_n, slope-deflection $\partial w/\partial y$, transverse bending moment M_y, shear force V_y, and reaction R_y can be obtained by

$$Y_n = c_1\sinh\eta + c_2\cosh\eta + c_3\eta\sinh\eta + c_4\eta\cosh\eta \tag{2}$$

$$\frac{\partial w}{\partial y} = \frac{\pi}{s}Y_n'\sin\xi\ ; \qquad M_y = D\frac{\pi^2}{s^2}(-Y_n'' + \upsilon Y_n)\sin\xi \tag{3,4}$$

$$V_y = D\frac{\pi^3}{s^3}(Y_n' - Y_n''')\sin\xi \tag{5}$$

$$R_y = D\frac{\pi^3}{s^3}[(2-\upsilon)Y_n' - Y_n''']\sin\xi \tag{6}$$

where $s = a/n$, n = harmonic number, c_1 - c_4 = coefficient of amplitude factor, D = flexural rigidity of plate, $\xi = \pi x/s$, $\eta = \pi y/s$, and υ = Poisson's ratio. As shown from Eq. (1) to Eq. (6), because the deflection shape of the plate is assumed to be a sinusoidal function, the slope-deflection and the bending moment as well as the shearing force are expressed with the same function.

2.2 Responses in bridge deck by slope-deflection method

The slope-deflection method is a displacement-based method that relates internal moments at the ends of a member to slope and deflection of the members. This method is generally used in the analysis of indeterminate beam and frame structures, and in the analysis of bridge decks.

As shown in Fig. 2, when girder line A and B are rigidly supported, a rigid moment induced by harmonic loading can be obtained. The moments at the slab on each girder line caused by deflection and slope can be expressed with multiple stiffness factor and deflection term. Therefore, the bending moment and reaction derived from the slope, deflection, and fixed support are divided into three parts, which are derived from one rectangular plate between girder line A and B.

The bending moment and reaction for the first part are calculated under following conditions; girder line A (y = 0) is rigidly supported, and slope occurs at girder line B ($y = b$), $\Phi = \Phi_0 \sin\xi$. In this case, boundary conditions are $\Phi = \partial w/\partial y = 0$, $\Delta = w = 0$ at $y = 0$ and $\Phi = \Phi_0 \sin\xi$, $\Delta = w = 0$ at $y = b$. The solutions of plate or slab for bending moment and reaction as well as deflection under the boundary are

at $y = 0$; $M = M_y = -kK\Phi_0 \sin\xi$ at $y = b$; $M = M_y = K\Phi_0 \sin\xi$

$$k = \frac{\beta\cosh\beta - \sinh\beta}{\sinh\beta\cosh\beta - \beta} \qquad K = 2\beta\frac{\sinh\beta\cosh\beta - \beta}{\sinh^2\beta - \beta^2}\frac{D}{b}$$

$$R = R_y = -qQ\Phi_0\sin\xi \qquad R = R_y = Q\Phi_0\sin\xi$$

$$q = \frac{2\beta\sinh\beta}{\sinh^2\beta + \beta^2 + \upsilon(\sinh^2\beta - \beta^2)} \qquad Q = \beta^2\left[\frac{\sinh^2\beta + \beta^2}{\sinh^2\beta - \beta^2} + \upsilon\right]\frac{D}{b^2} \tag{7,8}$$

$$w = \frac{1}{\sinh^2\beta - \beta^2}\frac{a}{n\pi}\Phi_0\{\beta\sinh\beta\sinh\eta$$
$$+(\beta\cosh\beta - \sinh\beta)\eta\sinh\eta - \beta\eta\sinh\beta\cosh\eta\}\sin\xi \tag{9}$$

where the coefficient $\beta = \pi b/s$, K, Q = stiffness factor due to the shape of the deck and the flexural rigidity of the plate, and k, q = carry-over factor due to the shape of the deck. When one edge of the deck is rigidly supported and the other edge has a defined slope, the bending moment and reaction are graphically presented in Fig. 3(a).

The bending moment and reaction for the second part are also derived from the following conditions; girder line A is rigidly supported and girder line B occurs deflection, $\Delta = \Delta_0 \sin\xi$. In this case, boundary conditions are $\Phi = \partial w/\partial y = 0$, $\Delta = w = 0$ at $y = 0$ and $\Phi = 0$, $\Delta = w = \Delta_0 \sin\xi$ at $y = b$. The solutions of plate or slab for bending moment and reaction as well as deflection under the boundary conditions are

at $y = 0$; $M = M_y = -qQ\Delta_0\sin\xi$ at $y = b$; $M = M_y = Q\Delta_0\sin\xi$

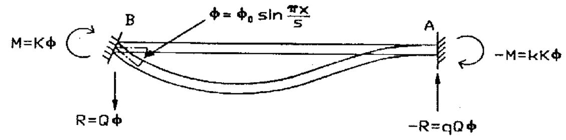
$$R = -R_y = -tT\Delta_0\sin\xi \qquad R = R_y = T\Delta_0\sin\xi$$

$$t = \frac{\beta\cosh\beta + \sinh\beta}{\sinh\beta\cosh\beta + \beta} \qquad T = 2\beta^3\frac{\sinh\beta\cosh\beta + \beta}{\sinh^2\beta - \beta^2}\frac{D}{b^3} \tag{10, 11}$$

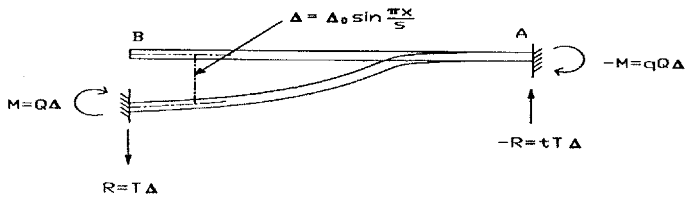
$$w = \frac{\Delta_0}{\sinh^2\beta - \beta^2}\{(\beta\cosh\beta + \sinh\beta)\sinh\eta + \beta\eta\sinh\beta\sinh\eta$$
$$-(\beta\cosh\beta + \sinh\beta)\eta\cosh\eta\}\sin\xi \tag{12}$$

where T = stiffness factor due to the shape of the deck and the flexural rigidity of the plate, and t = carry-over factor due to the shape of the deck. The bending moment and reaction is shown graphically in Fig. 3(b). Similarly, using the Maxwell's law of reciprocal deflection at girder line A, the moment and reaction due to slope and deflection are also obtained.

For the third part, the fixed end moment and reaction are derived from the followings; a lane load applied in the longitudinal direction on a two-edge-simply supported and two-edge-fixed plate as shown in Fig. 4. In this case, edge A and B are fixed supports and the other two edges are simple supports. When lane load $F = F_{on}\sin\xi$ is applied to any lane ($y = y_1$), the shape of deflection can be assessed from the Maxwell's law of reciprocal deflection. For example, when lane load is applied at



(a)



(b)

Fig. 3 Bending moments and reactions of bridge deck: (a) Bending moments and reactions due to slope; (b) Bending moments and reactions due to deflection

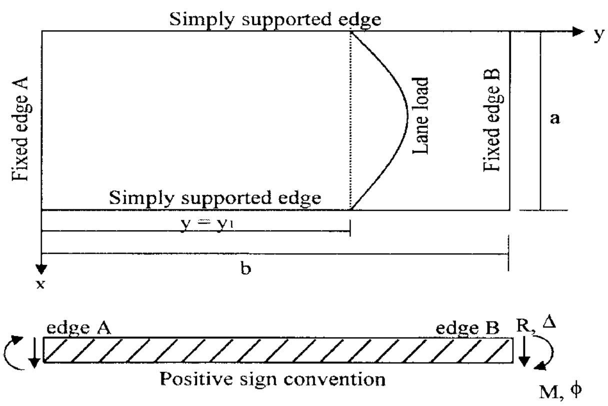


Fig. 4 Bridge deck applied lane load

$y = y_1$, the influence line of the fixed end moment of edge B is equal to the deflection under a unit slope, $\Phi = \Phi_0 \sin\xi$, at edge B. The fixed end reaction can be obtained by the same procedure as described for the fixed end moment. Thereby, the fixed end moment and reactions at $y = 0$ and $y = b$ are

$$\text{at } y = 0; \quad M^F = -\frac{F_0 s}{\pi} \frac{\sinh\beta(\eta_1 \sinh\alpha) - \beta\alpha \sinh\eta_1}{\sinh^2\beta - \beta^2} \sin\xi$$

$$R^F = -\frac{F_0}{\sinh^2\beta - \beta^2}[\sinh\beta(\sinh\alpha + \eta_1 \cosh\alpha) - \beta(\sinh\eta_1 + \alpha\cosh\eta_1)]\sin\xi \tag{13}$$

$$\text{at } y = b; \quad M^F = -\frac{F_0 s}{\pi} \frac{\alpha \sinh\beta \sinh\eta_1 - \beta\eta_1 \sinh\alpha}{\sinh^2\beta - \beta^2} \sin\xi$$

$$R^F = -\frac{F_0}{\sinh^2\beta - \beta^2}[\sinh\beta(\sinh\eta_1 + \alpha\cosh\eta_1) - \beta(\sinh\alpha + \eta_1 \cosh\alpha)]\sin\xi \tag{14}$$

where $\alpha = \beta - \eta_1 = (n\pi/a)(b - y_1)$, y_1 = location of lane load, and F_0 = magnitude of lane load.

2.3 Bending moments and reactions in continuously supported slab on rigid girders

In sequel to the previous solutions, moment and reaction at each girder support of a continuously supported slab are also derived in this study. With further consideration of the effect of local deflection on a slab, the fixed end moment and reaction in each slab subjected to a harmonic lane load are obtained according to the slope-deflection method. The bending moment and reaction due to the slope are calculated in terms of the corresponding stiffness factor. The moment equilibrium equation is always valid in each girder line that is restrained against vertical deflections in a continuously supported slab, and is used in Eqs. (7), (8), (13), and (14).

The definition of notations for bending moment, reaction, and slope of continuously supported slabs by rigid girders are as follows.

$M^{Slab\ No.}_{(Girder\ No.)} = M^j_{(i)}$ = Bending moment applied to the slab on the i^{th} girder in the j^{th} slab.

$R^{Slab\ No.}_{(Girder\ No.)} = R^j_{(i)}$ = Reaction applied to the slab on the i^{th} girder in the j^{th} slab.

$R^r_{(Girder\ No.)} = R^r_{(i)}$ = Reaction applied to the i^{th} girder.

$\Phi^{Slab\ No.}_{(Girder\ No.)} = \Phi^j_{(i)}$ = Slope applied to the slab on the i^{th} girder in the j^{th} slab

The graphical representation of a continuously supported bridge deck is depicted in Fig. 5. Using these notations, the bending moment, $M^1_{(1)}$, applied to the slab on the first girder of the first slab is obtained by the sum of the fixed end moment of the first support, $M^{F1}_{(1)}$, the moment due to the slope of the first support, $K_1\Phi_{(1)}$, and the moment due to the slope of the second support, $k_1 K_1 \Phi_{(2)}$.

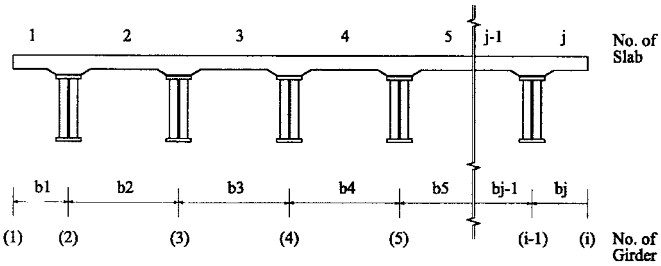


Fig. 5 Notations used for bridge deck

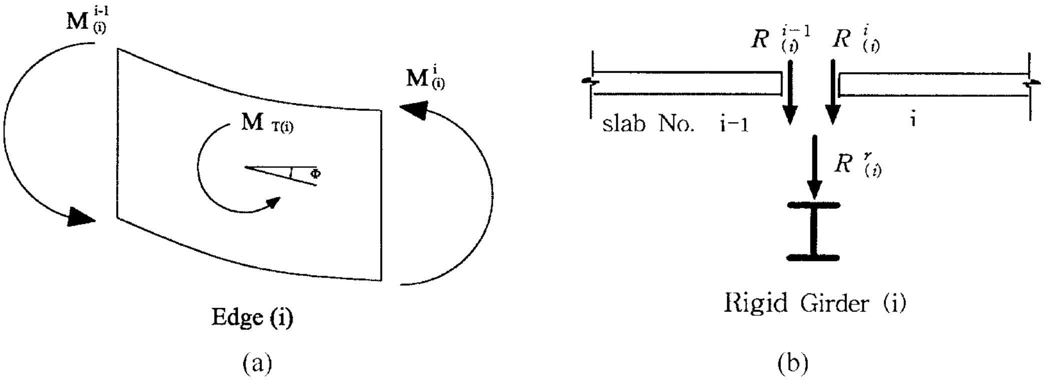


Fig. 6 Equilibrium condition of moments and reactions at a support due to rigid girders: (a) Equilibrium condition of moments (b) Equilibrium condition of reactions

The reactions at this support are obtained by the same procedure. Equations for the $i-1^{\text{th}}$ and i^{th} slab on the i^{th} rigid girder for the bending moment and reaction involving a stiffness factor and carry-over factor are as follows.

$$
\begin{aligned}
M_{(i)}^{i-1} &= [M_{(i)0}^{Fi-1} + K_{i-1}\Phi_{i(0)} + k_{i-1}K_{i-1}\Phi_{(i-1)0}]\sin\xi \\
M_{(i)}^{i} &= [M_{(i)0}^{Fi} + K_{i}\Phi_{(i)0} + k_{i}K_{i}\Phi_{(i+1)0}]\sin\xi \\
R_{(i)}^{i-1} &= [R_{(i)0}^{Fi-1} - Q_{i-1}\Phi_{(i)0} - q_{i-1}Q_{i-1}\Phi_{(i-1)0}]\sin\xi \\
R_{(i)}^{i} &= [R_{(i)0}^{Fi} + Q_{i}\Phi_{(i)0} + q_{i}Q_{i}\Phi_{(i+1)0}]\sin\xi
\end{aligned}
\tag{15}
$$

As shown in Fig. 6, the bending moment and reaction at any support in a continuously supported slab on rigid girders are determined by using the moment and reaction equilibrium equation. Whereby the moment equilibrium equation, $M_{(i)}^{i-1} + M_{(i)}^{i} + M_{T(i)} = 0$, where $M_{T(i)}$ is the torsional moment resisting slope (Φ) of a support. The relationship between the fixed end moment and slope is obtained in terms of the matrix, $[K]\{\Phi_{(i)}\}=\{-M_{(i)}^{F}\}$. Thus, substituting the slope matrix, $\{\Phi_{(i)}\}$, into the second equation of (15), the bending moment due to rigid girders is determined. Similarly, in the reaction equilibrium equation, $R_{(i)}^{r} = R_{(i)}^{i-1} + R_{(i)}^{i}$, the reactions due to rigid girders are determined by the following equations.

$$M_{(i)}^{i\ rigid} = [M_{(i)0}^{Fi} + K_i\Phi_{(i)0} + k_iK_i\Phi_{(i+1)0}]\sin\xi$$

$$R_{(i)}^{rigid} = [(R_{(i)0}^{Fi-1} + R_{(i)0}^{Fi}) - q_{i-1}Q_{i-1}\Phi_{(i-1)0} + (Q_i - Q_{i-1})\Phi_{(i)0} + q_iQ_i\Phi_{(i+1)0}]\sin\xi \qquad (16)$$

2.4 Bending moments and reactions in continuously supported slab on flexible girders

The effect of girder deflection is considered by the slope-deflection method. The end moment and reaction due to girder deflection in each slab are expressed in terms of a stiffness factor, a carry-over factor, and deformation. The equilibrium equations of moment and vertical reaction are applied to each girder line in continuously supported bridge decks. Notations of bending moment, reaction, and slope in continuously supported slabs due to girder deflection are as follows. The superscript Δ denotes the effect of the differential deflection of girders.

$M_{(Girder\ No.)}^{\Delta\ Slab\ No.} = M_{(i)}^{\Delta j}$ = Bending moment applied to the slab on the i^{th} girder in the j^{th} slab due to girder deflection.
$R_{(Girder\ No.)}^{\Delta\ Slab\ No.} = R_{(i)}^{\Delta j}$ = Reaction applied to the slab on the i^{th} girder in the j^{th} slab due to girder deflection.
$\Phi_{(Girder\ No.)}^{\Delta} = \Phi_{(i)}^{\Delta}$ = Slope applied to the slab on the i^{th} girder due to girder deflection.

When deflection $\Delta_{(i)} = \Delta_{(i)0}\sin\xi$ occurs at each girder due to harmonic loading $F = F_{0n}\sin\xi$, the bending moment and vertical reaction equations for a continuously supported slab are expressed by Eqs. (7), (8), (10), and (11). Hence, the bending moment and vertical reaction at both ends of the i^{th} support are expressed by

$$M_{(i)}^{\Delta i-1} = [Q_{i-1}\Delta_{(i)0} - q_{i-1}Q_{i-1}\Delta_{(i-1)0} + K_{i-1}\Phi_{(i)0}^{\Delta} + k_{i-1}K_{i-1}\Phi_{(i-1)0}^{\Delta}]\sin\xi$$

$$M_{(i)}^{\Delta i} = [Q_i\Delta_{(i)0} - q_iQ_i\Delta_{(i+1)0} + K_i\Phi_{(i)0}^{\Delta} + k_iK_i\Phi_{(i+1)0}^{\Delta}]\sin\xi$$

$$R_{(i)}^{\Delta i-1} = [T_{i-1}\Delta_{(i)0} - t_{i-1}T_{i-1}\Delta_{(i-1)0} - Q_{i-1}\Phi_{(i)0}^{\Delta} - q_{i-1}Q_{i-1}\Phi_{(i-1)0}^{\Delta}]\sin\xi$$

$$R_{(i)}^{\Delta i} = [T_i\Delta_{(i)0} - t_iT_i\Delta_{(i+1)0} + Q_i\Phi_{(i)0}^{\Delta} + q_iQ_i\Phi_{(i+1)0}^{\Delta}]\sin\xi \qquad (17)$$

As shown in Fig. 7, using the moment and reaction equilibrium equations at each support in a continuously supported slab, the bending moment and reaction due to the girder deflection at the support are determined. Applying Eq. (17) to the moment equilibrium equation, $M_{(i)}^{\Delta i-1} + M_{(i)}^{\Delta i} + M_{T(i)}^{\Delta} = 0$, where $M_{T(i)}^{\Delta}$ is the torsional moment that is the resisting moment due to the effect of differential deflection of girders, the bending moment is represented in the first equation of (18). Likewise, considering the reaction equilibrium equation, $R_{(i)}^{r} + R_{(i)}^{\Delta i-1} + R_{(i)}^{\Delta i} + F_{(i)} = 0$, the reaction matrix with the effect of deflection of girders is derived by substituting the slope and deflection calculated from the second equation of (18) into the fourth equation of (17).

$$M_{(i)}^{\Delta i} = [Q_i\Delta_{(i)0} - q_iQ_i\Delta_{(i+1)0} + K_i\Phi_{(i)0}^{\Delta} + k_iK_i\Phi_{(i+1)0}^{\Delta}]\sin\xi$$

$$\begin{pmatrix} \{0\} \\ \{-R_{(i)}^{r}\} \end{pmatrix} = \begin{bmatrix} [Q^{\Delta}] & [K] \\ [T] & [Q^{\Phi}] \end{bmatrix} \begin{pmatrix} \{\Delta_{(i)}\} \\ \{\Phi_{(i)}^{\Delta}\} \end{pmatrix} \qquad (18)$$

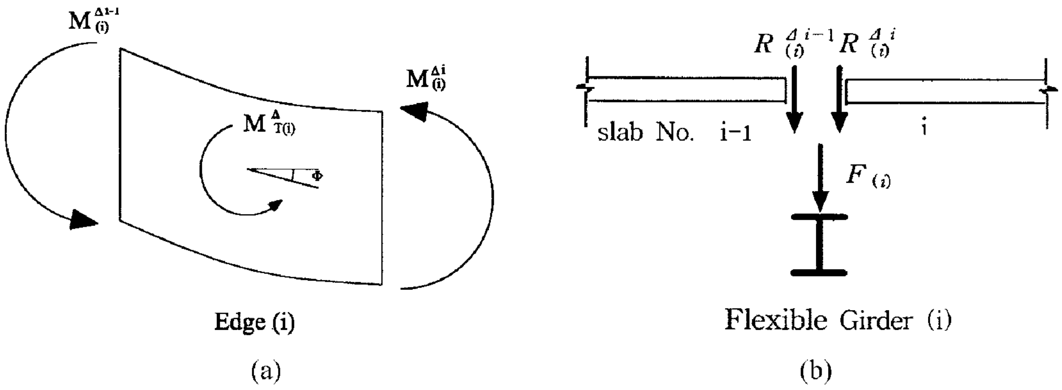


Fig. 7 Equilibrium condition of moments and reactions at each support due to flexible girders: (a) Equilibrium condition of moments (b) Equilibrium condition of reactions

2.5 Total negative bending moment in bridge decks

The total bending moment occurring at each support of a deck slab is obtained by adding the bending moments due to rigid girders and flexible girders. Whereby, the first equation of (16) denotes the bending moment due to a rigid girder at a support and the second equation of (17) represents the bending moment due to girder deflection at each support. These bending moments indicate an application in the transverse direction perpendicular to the traffic axis, i.e., the y direction shown in Fig. 2. Finally, the total negative bending moment in deck slabs is calculated by

$$M^{total}_{(i)} = \sum_{i=1,2,\ldots}^{\infty} [M^{i}_{(i)} + M^{\Delta i}_{(i)}]$$

$$= \sum_{i=1,2,\ldots}^{\infty} [M^{Fi}_{(i)0} + K_i\Phi_{(i)0} + k_iK_i\Phi_{(i+1)0}]\sin\xi + \sum_{i=1,2,\ldots}^{\infty} [Q_i\Delta_{(i)0} - q_iQ_i\Delta_{(i+1)0} + K_i\Phi^{\Delta}_{(i)0} + k_iK_i\Phi^{\Delta}_{(i+1)0}]\sin\xi$$

$$= \sum_{i=1,2,\ldots}^{\infty} [M^{Fi}_{(i)0} + K_i(\Phi_{(i)0} + \Phi^{\Delta}_{(i)0}) + k_iK_i(\Phi_{(i+1)0} + \Phi^{\Delta}_{(i)0}) + Q_i\Delta_{(i)0} - q_iQ_i\Delta_{(i+1)0}]\sin\xi \qquad (19)$$

2.6 Positive bending moment in bridge decks

In general, a point load which induces a positive bending moment is considered as a distributed load with a constant width, and as such, the bending moment presented by Westergarrd (1930) and AASHTO (1996) were assumed to be an equivalent load with constant width. In this study, it is assumed that the distributed width of a traffic load is the tire contact area as indicated in AASHTO specifications (1996). The dimensions pertaining to this specification are a rectangular area of 0.01 P in square inches and length in the direction of traffic/width with a ratio of 1/2.5. The positive bending moment in a deck slab is obtained by the sum of moments both due to distributed load as

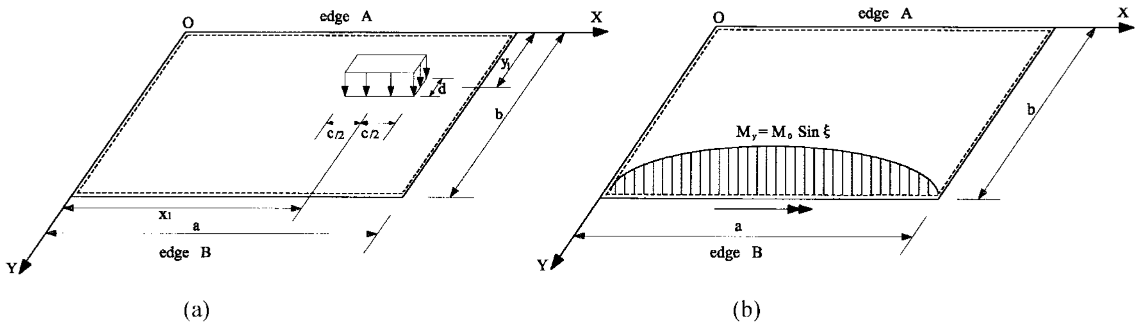


Fig. 8 Deck slabs applied distributed load and edge moment: (a) Deck slab applied distributed load (b) Deck slab applied edge moment due to girder deflection

shown in Fig. 8(a) and due to the effect of girder deflection as shown in Fig. 8(b). The deflection and positive bending moment according to this procedure are expressed as

$$w = M_0 \frac{b^2}{D} \frac{1}{2\beta^2} \frac{\sinh\eta}{\sinh\beta}(\beta\coth\beta - \eta\coth\eta)\sin\xi$$

$$M_y = (m_y + \upsilon m_x) M_0 \sin\xi$$

$$m_x = \frac{1}{2}\frac{\sinh\eta}{\sinh\beta}(\beta\coth\beta - \eta\coth\eta)$$

$$m_y = \frac{\sinh\eta}{\sinh\beta} - m_x \tag{20}$$

The detailed derivation for Eq. (1) to Eq. (20) was presented by Kang *et al.* (1999). The positive bending moment may increase within a deck due to the effect of differential deflection of girders. However, it is recommended that the continuity factor be eliminated for the positive bending moment. This leads to a small increase in the reinforcement of the lower slab (Cao 1999).

2.7 Effect of cross beam and continuous span bridge

To consider the transverse negative bending moment in a deck slab with a non-composite cross beam, the cross beam is modeled as a one-dimensional-beam element, which has two degrees of freedom at each node, i.e., a vertical reaction and rotation at the nodal point, as shown in Fig. 9. The stiffness matrix for the beam element is expressed in the first equation of Eq. (21), $\{f\} = [K]\{\delta\}$, and the relation between the deflection and vertical reaction for n cross beams is represented in the second equation of (21), $[K]_i\{\delta\}_i = \{R\}_i$. The reaction vector is obtained from Eq. (21) using flexibility matrix, $[W]$, and displacement vectors, $[W]\{R_c\} = \{D\}$ and $\{\delta\} = \{dp\} + \{D\}$. Whereby, $\{D\}$ denotes the displacement vector due to the reaction of a non-composite cross beam, $\{dp\}$ is the displacement vector at each node due to external forces in deck slabs without non-composite cross beams, and $\{\delta\}$ is the displacement vector at each node due to external forces in deck slabs with non-composite cross beams.

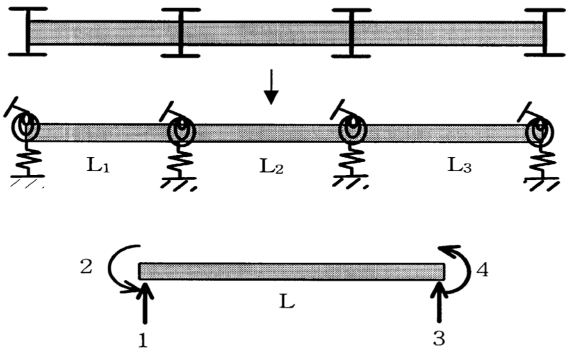


Fig. 9 Modeling of a cross beam section

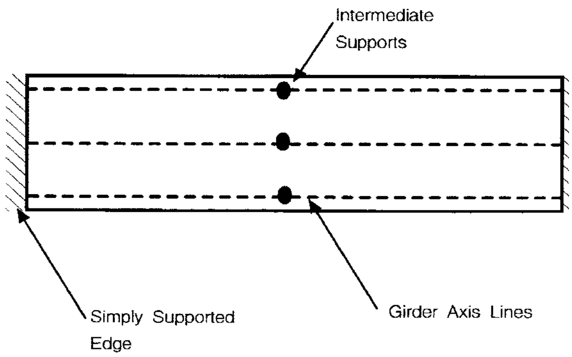


Fig. 10 Typical section of a continuous span bridge

$$\begin{Bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{Bmatrix} = \frac{EI_C}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{Bmatrix}$$

$$\begin{bmatrix} [K]_{(1)} & [0] & [0] & [0] \\ [0] & [K]_{(i-1)} & [0] & [0] \\ [0] & [0] & [K]_{(i)} & [0] \\ [0] & [0] & [0] & [K]_{(n)} \end{bmatrix}\begin{Bmatrix} [\delta]_{(1)} \\ [\delta]_{(i-1)} \\ [\delta]_{(i)} \\ [\delta]_{(n)} \end{Bmatrix} = \begin{Bmatrix} [R_C]_{(1)} \\ [R_C]_{(i-1)} \\ [R_C]_{(i)} \\ [R_C]_{(n)} \end{Bmatrix} \tag{21}$$

The transverse negative bending moment in a deck slab with a non-composite cross beam is obtained by the sum of the bending moment in a deck without a cross beam expressed in Eq. (19) and the bending moment due to the reaction of a cross beam expressed in Eq. (22).

$$\{R_C\} = -[[I] + [[K]_{(i)}] \cdot [W]]^{-1} \cdot [[K]_{(i)}]\{dp\} \tag{22}$$

where $[I]$ and $[W]$ indicate identity and flexibility matrix, respectively.

In the continuous span bridge shown in Fig. 10, intermediate supports are restrained to vertical deflection, while rotation is allowed. For the purpose of simplicity, the intermediate supports of a continuous span bridge are modeled as a cross beam with infinitely large stiffness providing restraint for the vertical deflection. Thus, the procedure of analysis of transverse negative bending moment in a continuous span bridge may be the same as that of a non-composite cross beam considered to be the vertical restraint of the intermediate supports. The detailed derivations for (21) and (22) were also presented by Kang *et al.* (1999).

3. Analysis program for bridge deck (ASGB)

In this study, an analytical program for moment and deflection in a bridge deck is referred to as

ASGB, and coded by FORTRAN language. The ASGB program is considered the effect of deflection of girders, based on the displacement method not force method, which can be applied in complex and various cases such as the multiple girder system, placement of cross beam, and continuous span bridge. Results of the program show positive and negative bending moments in slabs, bending moment and deflection in girders, and shear force in girders. A flowchart of this program using harmonic function and the slope-deflection method based on plate theory is illustrated in Fig. 11.

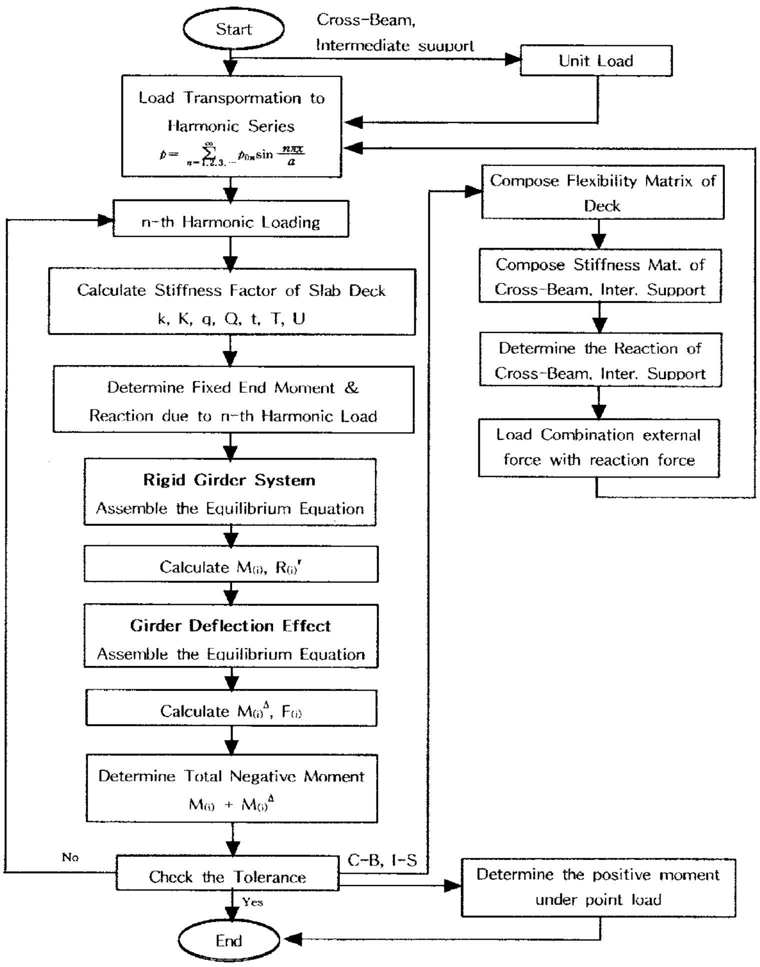


Fig. 11 Flowchart of the analytical program for a bridge deck (ASGB)

Verification of analytical method

In this study, a comparison of the negative bending moment of a finite-element model is performed. The finite-element model consists of plates and beams. A plate-element contains eight nodes and a beam-element has three nodes. A rigid beam is used to connect the plate and beam elements. The maximum negative bending moments calculated from both the finite-element model and the ASGB analysis are presented with respect to span length for a simply supported bridge with a three-girder deck, as depicted in Fig. 13. The dimensions of a simply supported bridge, as shown in Fig. 12(a), are 10 m (32.8 ft) long, 6 m (19.68 ft) wide, and 2 m (6.56 ft) girder spacing. Values of 2.083×10^{-2} m^4 (2.4136 ft^4) and 1.0417×10^{-2} m^4 (1.2068 ft^4) are calculated for the inertia moment corresponding to the major axis of each girder and cross beam, and the torsional constant of each girder is 4.4314×10^{-3} m^4 (0.5134 ft^4). The section properties of each girder and cross beam are almost comparable to those of W36 × 650 and W36 × 359 steel. Two concentrated loads of 196.14 kN (44.08 kips) are applied at mid-span. As illustrated in Fig. 13, the maximum negative bending moments in the transverse direction attained from the finite-element model and ASGB analysis have more or less identical results.

Fig. 14 illustrates a comparison of the maximum negative bending moment attained from the finite-element model and the ASGB analysis with respect to span length of a continuous span bridge with five girders. The dimensions selected for this two-span continuous bridge, as shown in Fig. 12(b), are 3 m (9.84 ft) for girder spacing, 30 m (98.4 ft) for length and 15 m (49.2 ft) for width. Values of 3.0×10^{-3} m^4 (0.3475 ft^4) and 1.5×10^{-3} m^4 (0.1738 ft^4) are calculated for the

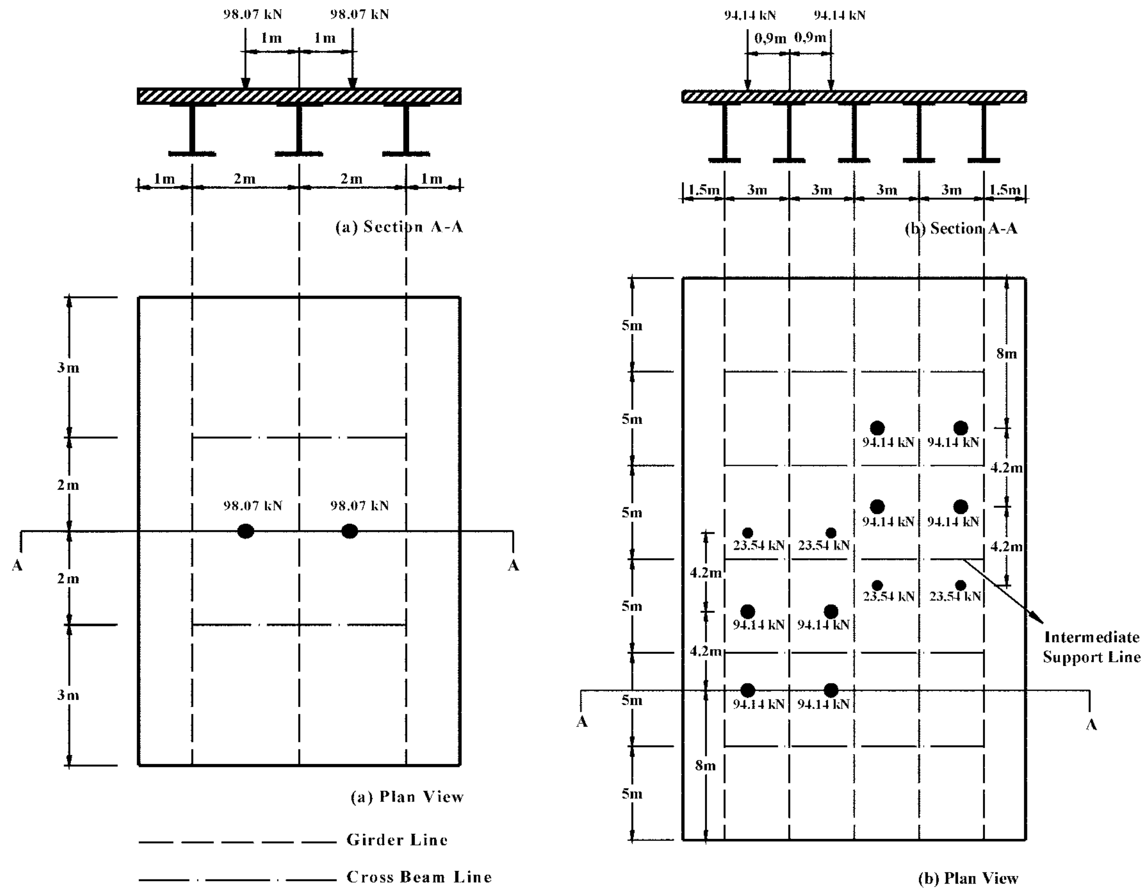


Fig. 12 Loading and geometric details: (a) simply supported bridge (b) continuous span bridge

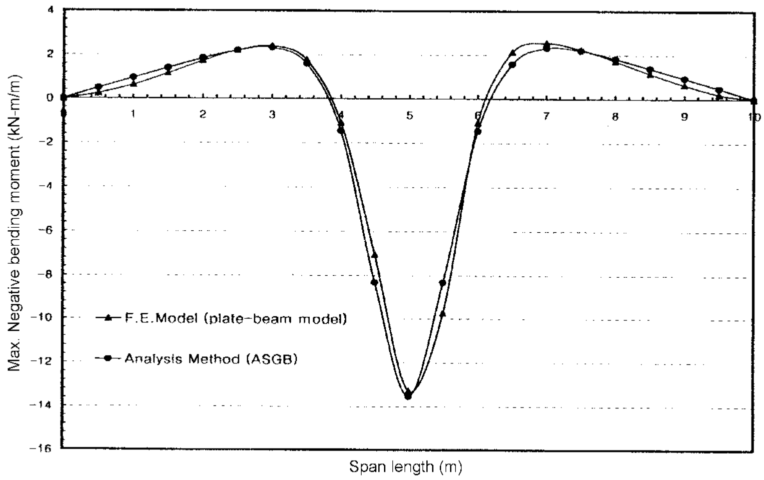


Fig. 13 Maximum negative bending moment with respect to span length in a simply supported bridge [Three-girder deck, S = 2 m (6.56 ft), 1 kN-m/m = 0.225 kips-ft/ft, 1 m = 3.28 ft]

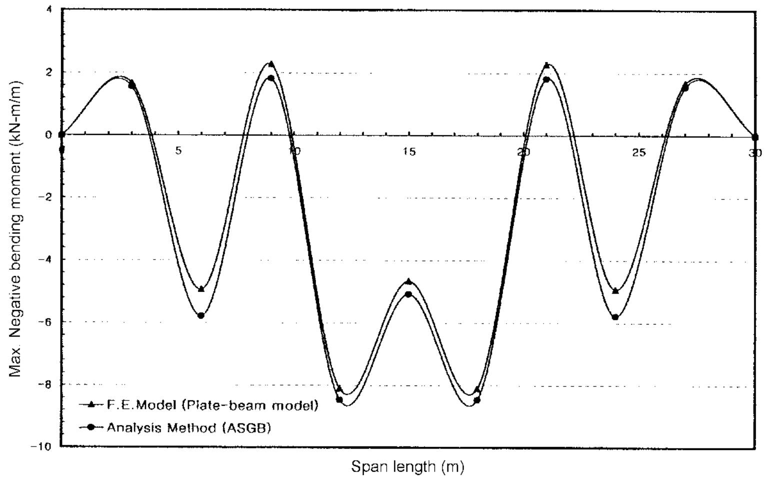


Fig. 14 Maximum negative bending moment with respect to span length in a continuous span bridge [Five-girder deck, S = 3 m (9.84 ft), 1 kN-m/m = 0.225 kips-ft/ft, 1 m = 3.28 ft]

inertia moment corresponding to the major axis of each girder and cross beam, and the torsional constant for each girder is 1.5×10^{-4} m^4 (0.0174 ft^4). The section properties of each girder and cross beam are almost comparable to those of W33 × 141 and W30 × 90 steel. Based on the live load specifications of the Korean Ministry of Construction and Transportation (1996), the load of a truck is prescribed as 423.66 kN (95.21 kips), which is 32% over that of the AASHTO standard HS20-44 truck. Herein, two trucks as shown in Fig. 12(b) are applied to the slab. The maximum negative bending moments in the transverse direction according to the finite-element model and the analytical method have the same results. The finite-element method program used for verification of the ASGB analysis is LUSAS (FEA Ltd. United Kingdom 1970).

4. Experimental behavior of bridge deck

4.1 Configurations of experimental decks

Two experimental plate-girder-bridge decks with three girders are constructed and static loading tests are performed to verify the effect of the deflection of girders. Owing to laboratory configuration, experimental decks are fabricated as bridge decks of quarter-scale based on the rule of similitude ratio. The experimental decks consist of two types according to girder spacing. One type has girder spacing of 0.5 m (1.64 ft) and the other has 0.6 m (1.97 ft). As shown in Fig. 15, that is girder spacing of 0.5 m (1.64 ft), geometric properties of the experimental decks are calculated to length of 3.75 m (12.3 ft), depth of 0.065 m (0.213 ft), cantilever arms of 0.25 m (0.82 ft), transverse and longitudinal wire-mesh mats of 0.004 m and 0.003 m (0.16 in and 0.12 in) diameter. And inertia moment of 2.1×10^{-5} m^4 (2.4×10^{-3} ft^4) corresponding to the major axis of the steel girder is almost the same as that of W8 × 15 steel.

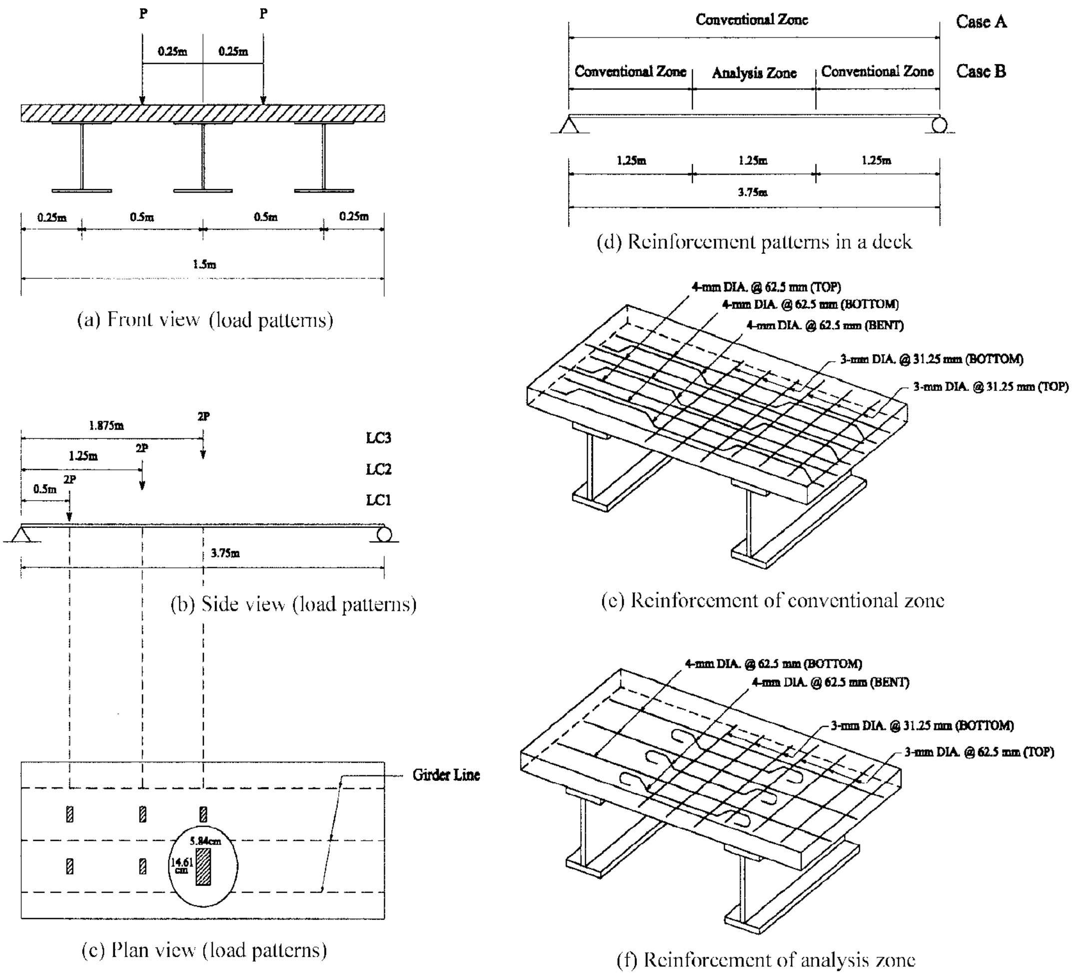


Fig. 15 Configurations of experimental bridge deck [Girder spacing, S = 0.5 m (1.64 ft), 1 m =3.28 ft]

The tire contact area is assumed to be 0.008 m^2 (12.43 sq. in) according to the AASHTO standard HS20 truck (0.01P in^2). The wheel load (2P), calculated to 25.3 kN (5.68 kips), is applied on the bridge deck parallel to the transverse direction of the deck at the same distance as the girder spacing.

Material properties of the experimental decks are based on the specifications of the Korean Ministry of Construction and Transportation (1996). The material properties are 26.5 MPa (867.8 psi) compressive strength of deck concrete, 3.24 MPa (106.1 psi) flexural strength of deck concrete, 22,606 MPa (740.3 ksi) elastic modulus of deck concrete, 200,062 MPa (6,551 ksi) elastic modulus of girder steel-plate, 314.2 MPa (10.3 ksi) yield stress of girder steel-plate and 393 MPa (12.9 ksi) yield stress of wire-mesh mat. It is assumed that Poisson's ratio of concrete deck and steel-plate girder are 0.17 and 0.3.

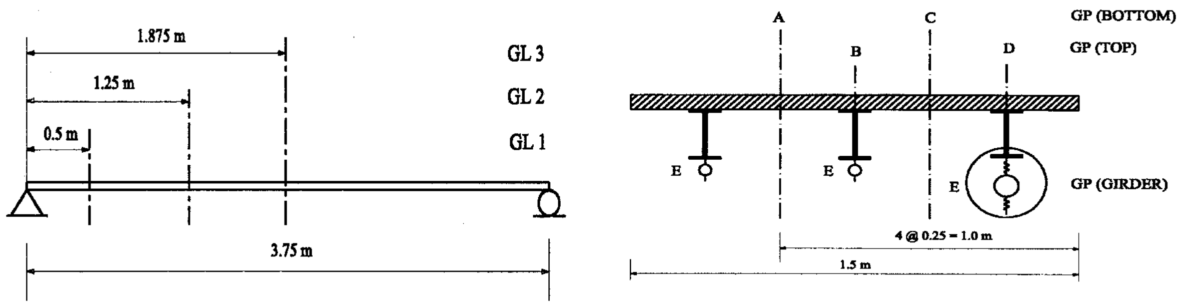
Fig. 15(d) shows two patterns of reinforcements in a slab with respect to the rigid support and flexural support system. One patterns follows the arrangement of conventional reinforcing mats throughout the deck (Case A), and the other removes the upper transverse reinforcing mat according to the analytical method in the region between 1.25 m (4.1 ft) and 2.5 m (8.2 ft) of the longitudinal direction (Case B). In this experimental work, the Case B pattern is fabricated and is performed the static load test. The analysis zone of Case B is based on results of the analytical method, which the transverse tensile stress in slab-on-girder due to the effect of deflection of girders is smaller than the modulus of rupture of concrete in a deck. According to the analytical method, it is possible to remove the transverse reinforcing bars in the upper slab from a distance like girder spacing. However, in this experimental study, a starting line for the analysis zone of 1.25 m (4.1 ft) is adopted due to the nonlinear material properties of concrete, the ignorance of the impact effects of the truck load and the disregard for sidewalks. The reinforcement arrangements for the conventional zone and the analysis zone are shown in Fig. 15(e) and (f). It is seen that the latter method leads to a simpler reinforcement arrangement.

Loading is applied in three separate cases. One loading case is applied at one girder length from the abutment, another is applied at $L/3$ (where L equals the span length), and the last is applied at $L/2$. For a girder spacing of 0.5 m (1.64 ft), gauge lines are located at 0.5 m (1.64 ft), 1.25 m (4.1 ft), and 1.875 m (6.15 ft) from the abutment, as shown in Fig. 16(a). Each gauge line corresponds to each loading case and has eleven gauge points as shown in Fig. 16(b). They are comprised of eight gauge points for measurement of the strain in the longitudinal and transverse directions of the concrete-deck surface (Gauge Points A to D), and three gauge points for measurement of the deflection of the girders (Gauge Point E).

In the case of the experimental deck with girder spacing of 0.6 m (1.97 ft), all geometric and material properties are identical to decks with girder spacing of 0.5 m (1.64 ft), except for the width of the deck (1.7 m, 5.58 ft) and the location of load case 1 (0.6 m, 1.97 ft).

4.2 Comparison of analytical and experimental results

To evaluate the bending moment due to the deflection of girders, static loading tests are performed in the laboratory. Static forces are applied through the loading actuator at intervals of 4.9 kN (1.10 kips) to a maximum of 25.3 kN (5.68 kips), and longitudinal and transverse strains at each force-step are measured. As shown in Fig 16(b) and Table 1, because the maximum negative tensile stress for each load case developed at gauge point B, strains with respect to each force-step are summarized for gauge point B. Converting strains from the stress-strain relationship, the maximum



(a) (b)

Fig. 16 Gauge locations of experimental bridge deck [Girder spacing, S = 0.5 m (1.64 ft), 1 m = 3.28 ft]: (a) Gauge lines (side view); (b) Gauge points (front view)

negative tensile stresses in the transverse direction are obtained.

As illustrated in Fig. 17, the maximum negative tensile stresses measured in the experimental three-girder bridge deck with a girder spacing of 0.5 m (1.64 ft) for load case 1 to 3 are 37%, 18%, and 17% smaller than that of the analytical results. The much difference of both stresses for load case 1 is regarded to the effect of the boundary condition. And, for each force-step, these stresses of analysis and experiment represent that the farther from the abutment, the smaller the negative tensile stresses. It is noted that the farther from the abutment, the larger the effect of the deflection of girders. Also, all maximum negative tensile stresses developed in both the experimental measurements and analytical results are smaller than the modulus of rupture of concrete.

The experimental and analytical results for girder spacing of 0.6 m (1.97 ft) are compared in Fig. 18. Comparing the tensile stresses for elastically supported system shown in Fig. 18, the maximum transverse tensile stresses with respect to the span length are larger when closer to the abutment, which is regarded as the effect of the boundary condition. And, the experimental tensile stress of 1.28 MPa (0.186 ksi) at mid-span is 21% smaller than the analytical tensile stress of 1.62 MPa (0.235 ksi) for elastically supported system. Also, comparing the tensile stresses for elastically supported and rigidly supported system, the analytical tensile stress for rigidly supported system is

Table 1 Maximum strain with respect to force-steps at gauge point B [S = 0.5 m (1.64 ft)]

Load (kN)	Load Case 1	Load Case 2	Load Case 3
	Strain ($\times 10^{-6}$)	Strain ($\times 10^{-6}$)	Strain ($\times 10^{-6}$)
4.9	–7 / +8	–15 / +6	–17 / +6
9.8	–21 / +16	–30 / +15	–32 / +15
14.7	–30 / +27	–46 / +23	–50 / +23
19.6	–40 / +37	–61 / +33	–65 / +32
24.5	–51 / +48	–77 / +43	–84 / +39
25.3	–53 / +50	–80 / +45	–87 / +41

Note : 1 kN = 0.225 kips, longitudinal strain/transverse strain

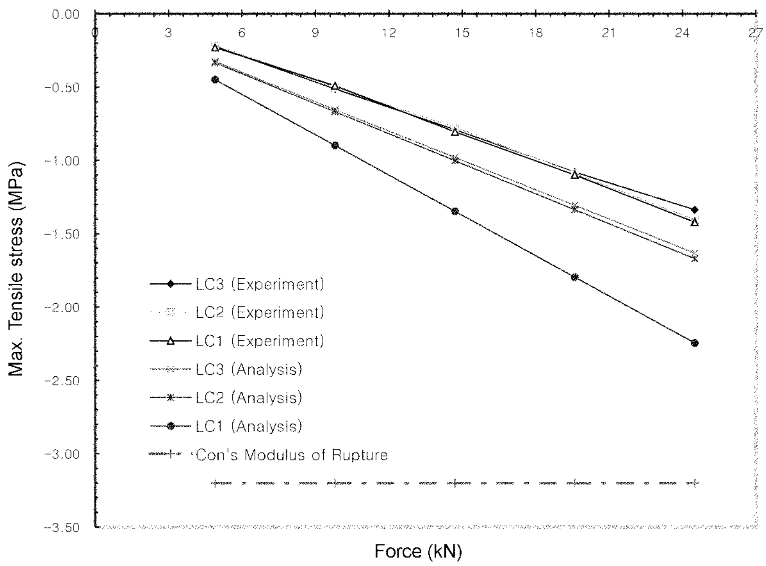


Fig. 17 Maximum negative tensile stress along force-steps [S = 0.5 m (1.64 ft), 1 MPa = 0.145 ksi, 1 kN = 0.225 kips]

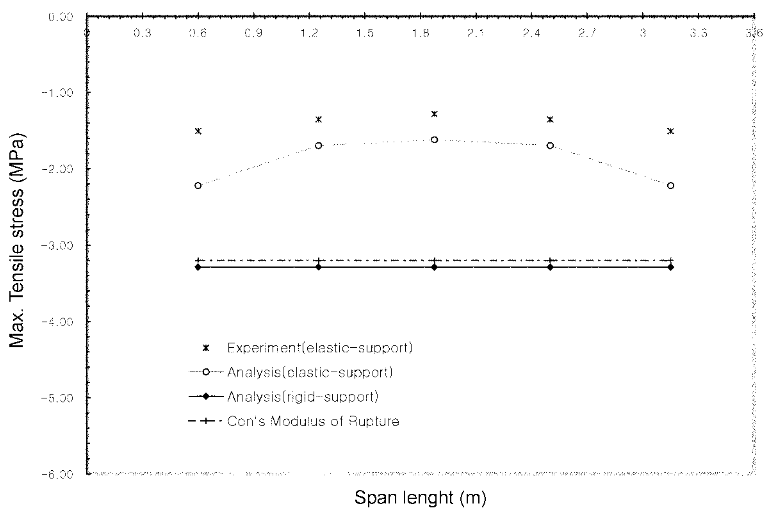


Fig. 18 Maximum negative tensile stress along span length [S = 0.6 m (1.97 ft), 1 MPa = 0.145 ksi, 1 m = 3.28 ft]

much larger than the experimental and analytical tensile stresses for elastically supported system. It is noted that considering the effect of the deflection of girders in a slab design, the tensile stresses can be considerable reduced.

Maximum tensile stresses at gauge line 3, i.e., the mid-span of the bridge deck, for a girder spacing of 0.5 m (1.64 ft), are compared in Table 2 and Fig. 19. Comparing the analytical tensile

Table 2 Maximum tensile stress along transverse distance at gauge line 3 [S = 0.5 m (1.64 ft)]

Transverse Distance (m)	Analytical stress for elastically supported (MPa)	Analytical stress for rigidly supported (MPa)	Experimental stress for elastically supported (MPa)	Modulus of rupture of concrete (MPa)
0.25	1.63	2.02	0.98	3.20
0.50	3.19	2.07	2.49	3.20
0.75	1.68	3.31	1.34	3.20
1.00	3.19	2.07	2.43	3.20
1.25	1.63	2.02	0.98	3.20

Note : 1 MPa = 0.145 ksi, 1 m = 3.28 ft

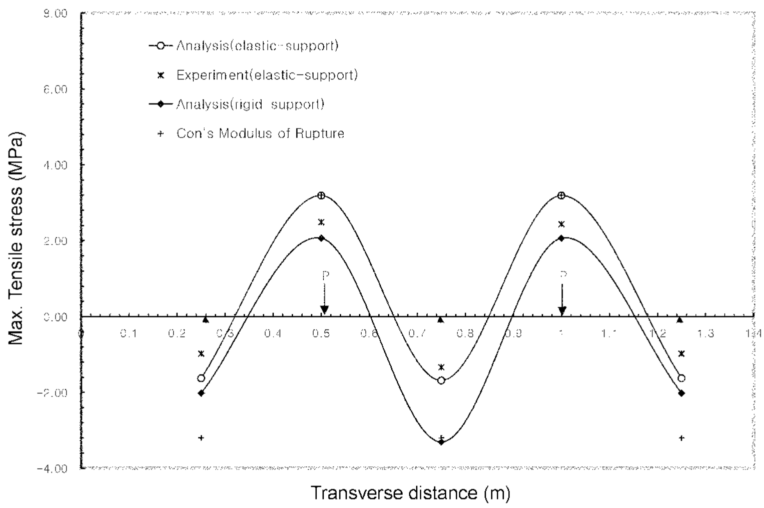


Fig. 19 Maximum tensile stress along transverse distance at gauge line 3 [S = 0.5 m (1.64 ft), 1 MPa = 0.145 ksi, 1 m = 3.28 ft]

stress of the elastically supported and rigidly supported system at gauge point B, the former of 1.68 MPa (0.24 ksi) is 49% smaller than the latter of 3.31 MPa (0.48 ksi). And, comparing the analytical and experimental tensile stress for elastically supported system at the same point, the former of 1.68 MPa (0.24 ksi) is 25% larger than the latter of 1.34 MPa (0.19 ksi). Although the analytical and experimental results for elastically supported system differ a little, the both are smaller than the modulus of rupture of concrete as well as the analytical tensile stress of the rigidly supported system. It is indicated that the tensile stress in the top fibers of the bridge decks can be reduced to a considerable extent. This casts a strong feasibility of improving the deck slab reinforcement if the girder flexibility appropriately taken into account.

While, comparing the analytical tensile stress of the elastically supported and rigidly supported system at gauge point A or C, the former of 3.19 MPa (0.46 ksi) is about 50% larger than the latter of 2.07 MPa (0.30 ksi). The increment of this analytical tensile stress is related to the additional tensile stress due to the effect of girder deflection. Although the increment of the tensile stress occurs owing to the effect of girder deflection, the analytical tensile stress is close to the modulus of rupture of concrete.

From these comparisons of the tensile stress, the analytical and experimental behaviors of slab-on-girder bridge involving the effect of girder deflection are investigated. Summarizing the behaviors of elastically supported system corresponding to the rigidly supported system in a slab, there are two major trends. The first trend is that the farther from the abutment, the smaller the negative tensile stresses for elastically supported system, as shown in Fig. 17 to 18. The second trend is that comparing the negative tensile stresses of elastically supported and rigidly supported system, the former is quite a little smaller than the latter due to the effect of girder deflection, as illustrated in Fig. 18 to 19. From the second trend, the upper reinforcements to resist the negative bending moment will be considerably reduced.

The modulus of rupture of concrete chosen in this study has been calculated according to formula 318 of the American Concrete Institute (ACI 1996).

5. Comparison with Cao's method

In bridge decks, the reduction of negative bending moment due to differential deflection of girders has been reported by Cao (1996). He has termed the closed-form solution based on the force method as the "simplified method". This simplified method, which was derived to superimpose two bending moments that is the bending moments due to rigid girder and flexural girder, applies to slab-on-girder bridges with three girders only. Which use the isotropic plate theory in transverse direction of deck and the orthotropic plate theory in longitudinal direction of deck.

To compare the simplified method and the analytical method presented in this study, an example bridge deck with three girders has been adopted as shown in Fig. 20. The properties of this slab-on-girder bridge are 14.94 m (49 ft) long, 2.13 m (7 ft) girder spacing, 89 kN (20 kips) wheel load of a HS20-44 truck (AASHTO) including the effect of impact, and a W36 × 150 steel girder. The wheel load of a HS20-44 truck is assumed to apply at mid-span causing maximum girder deflection. At mid-span according to the simplified method, the bending moment due to the rigid girder M_1 is −19.082 kN-m/m (−4.2880 kips-ft/ft), and the reduction factor K_d is equal to 0.3509. The additional bending moment due to flexible girders M_0 is 24.33 kN-m/m (5.4665 kips-ft/ft), and the total negative bending moment, $M_t = M_1 + K_d \times M_0$, in the transverse direction is −10.55 kN-m/m (−2.3698 kips-ft/ft). According to the analytical method presented (ASGB), the negative bending moment in the transverse direction equals −10.69 kN-m/m (−2.4031 kips-ft/ft), which is 1.3% larger than in the simplified method. The results are practically identical.

The simplified method by Cao can express the transverse bending moment in a closed-form solution for simple cases such as a bridge deck with three girders, but for relatively complex cases such as multiple girder systems, placement of cross beams, or continuous span bridges, the derivation of a closed-form solution may become extremely difficult. However, the analytical method presented can be applied to complex cases such as multiple girder system and continuous span bridges.

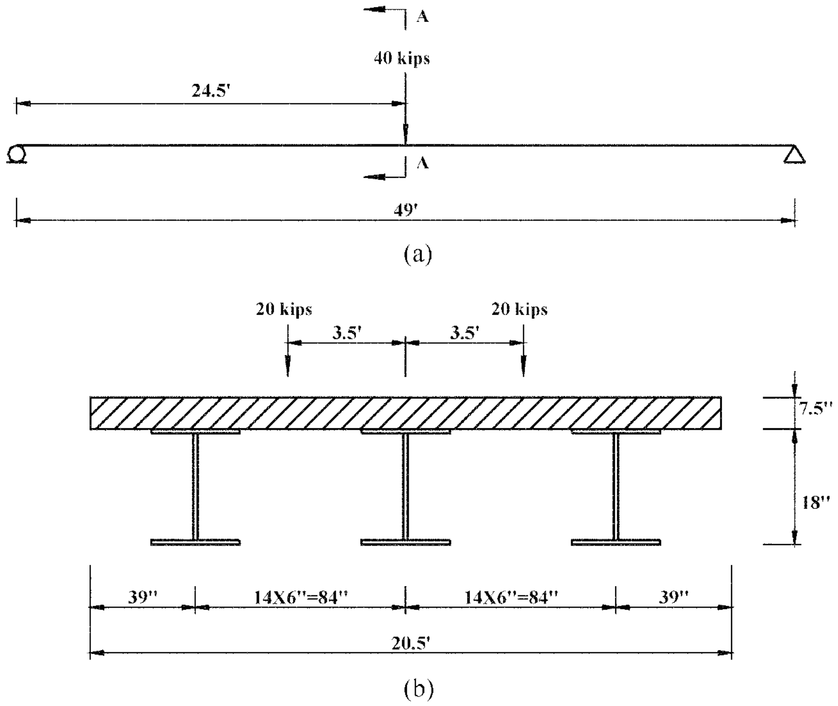


Fig. 20 Loading and geometric details (Cao 1996): (a) Elevation; (b) Cross section A-A

6. Conclusions

In the design of bridge decks, AASHTO specifications prescribe that the evaluation of bending moment only considers rigidly supported decks. However, the effects of deflection of girders, i.e., the effect of elastically supported decks, must be included for accurate representation of the actual behavior of bridge decks. Considering the stress distribution of a continuously supported bridge deck in the transverse direction, the positive bending moment arises in deck between girders and the negative bending moment arises in slab-on-girder.

In this study, the analytical method, which was derived using the harmonic analysis and the slope-deflection method, was developed to describe the actual behavior of slab-on-girder bridge involving the effect of girder deflection and was verified as plate-beam model of commercial finite element program. Because the major effect of girder deflection or flexural girder was related to the transverse bending moment in slab-on-girder, in this work this bending moment was illustrated in detail. Meanwhile, the other resultants such as shear force, displacement, and slope also were calculated in this work.

In addition, to measure the effect of girder deflection, experimental decks in this study were fabricated based on design according to the analytical method, and static loads were applied in compliance with the tire contact area of an AASHTO standard HS20-44 truck (0.01 P in^2). Comparing the negative tensile stress of both the elastically supported system and the rigidly supported system in a slab at mid-span, the former was about 50% smaller than the latter due to the effect of girder deflection. It is noted that if the effect of girder deflection is considered in a bridge-deck design, the upper reinforcements to resist negative bending moment in transverse direction can be quite a little reduced.

Comparing this analytical method with the force method, the analytical method has a wider range of application in various and complex cases such as the multiple girder system, placement of cross beams, and continuous span bridges. Also, in the bridge-deck design, this method and program will be applied to ease because requires small input and output tasks.

Acknowledgements

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References

ACI Manual of Concrete Practice. (1996), Part 4, ACI 318, American Concrete Institute (ACI), Detroit.

Allen, J.H. (1991), "Cracking serviceability and strength of concrete bridge decks", *Transp. Res. Rec. 1290*, Transportation Research Board, National Research Council, Washington, D.C., 152-171.

Bakht, B. and Jaeger, L.G. (1985), *Bridge Analysis Simplified*, McGraw-Hill, New York.

Bakht, B. and Jaeger, L.G. (1989), *Bridge Analysis by Microcomputer*, McGraw-Hill, New York.

Cao, L., Allen, J.H., Shing, P.B. and Woodham, D. (1996), "Behavior of RC bridge decks on flexible girders", *J. Struct. Engrg.*, ASCE, **122**(1), 11-19.

Cao, L. and Shing, P.B. (1999), "Simplified analysis method for slab-on-girder highway bridge decks", *J. Struct. Engrg.*, ASCE, **125**(1), 49-59.

Cusens, A.R. and Pama, R.P. (1975), *Bridge Deck Analysis*, Wiley, New York.

Kang, Y.J., Yu, C.S., Choi, J.Y. and Park, N.H. (1999), "New concept for top reinforcement design of decks on steel i-girder bridges", *R&D 96-0122*, Ministry of Construction and Transportation, Korea.

Lusas User Manual, (1970), Fea. Ltd., Forge House. UK.

Newmark, N.M. (1949), "Design of I-beam bridges", *Symp. of Hwy. Bridge Floors, ASCE Trans.*, 114, 979-1072.

Ontario Highway Bridge Design Code (OHBDC), (1983), Ministry of Transportation and Communications, Ontario, Canada.

Szilard, R. (1974), *Theory and Analysis of Plates*, Prentice Hall, New Jersey.

Standard Specifications for Highway Bridges. (1992), 15^{th} Ed., Am. Assoc. of State Hwy. and Transp. Officials (AASHTO), Washington, D.C.

Standard Specifications for Highway Bridges. (1996), Korean Ministry of Construction and Transportation, Korea.

Timoshenko, S.P. *et al*. (1984), 2^{nd} Ed., *Theory of Plates and Shells*, McGraw-Hill, New York.

Westergaard, H.M. (1930), "Computations of stresses in bridge slabs due to wheel loads", *Public Roads*, (Mar.), 1-23.