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Ambient vibration measurements and finite element modelling for the Hong Kong Ting Kau Bridge

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Abstract. The Ting Kau Bridge in Hong Kong is a cable-stayed bridge comprising two main spans and two side spans. The bridge deck is supported by three towers, an end pier and an abutment. Each of the three towers consists of a single reinforced concrete mast which reduces its section in steps, and it is strengthened by transverse cables and struts in the transverse vertical plane. The bridge deck is supported by four inclined planes of cables emanating from anchorages at the tower tops. In view of the threat from typhoons, the dynamic behaviour of long-span cable-supported bridges in the region is always an important consideration in their design. This paper is devoted to the ambient vibration measurements of the bridge for evaluation of dynamic characteristics including the natural frequencies and mode shapes. It also describes the modelling of the bridge. A few finite element models are developed and calibrated to match with the field data and the results of subsequent structural health monitoring of the bridge.

Key words: bridges; cables & tendons; composite structures; concrete structures; dynamics; steel structures.

1. Introduction

Hong Kong is located to the east of the Pearl River estuary on the south coast of China. It consists of Hong Kong Island, the Kowloon Peninsula, and the New Territories, which include the

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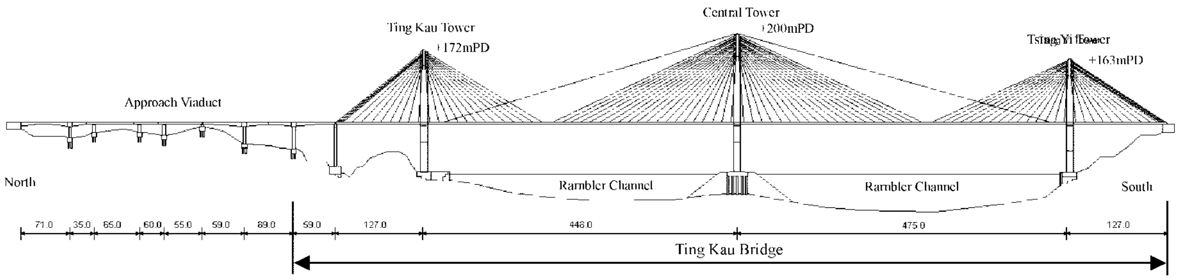

Fig. 1 Elevation of Ting Kau Bridge and approach viaduct (dimensions in m)

mainland area lying largely to the north, together with around 230 large and small offshore islands. Hong Kong lies at the northern fringe of the tropical zone. Apart from the north-easterly monsoon in winter and the south-easterly monsoon in summer, Hong Kong is also affected by tropical cyclones, or typhoons, generally occurring between June and October. The strong winds from the typhoons with the torrential downpours are a constant threat to life and property in Hong Kong.

Therefore the dynamic behaviour of long-span cable-supported bridges in the region is always an important consideration in their design. The 1,177 m long Ting Kau Bridge in Hong Kong, as shown in Fig. 1, is a cable-stayed bridge carrying a dual three-lane carriageway over Rambler Channel providing important access to Chek Lap Kok Airport from the North part of New Territories. The bridge consists of two main spans and two side spans. The bridge deck is supported by three towers, namely the Ting Kau Tower, the Main Tower on a man-made Central Island and the Tsing Yi Tower, the Ting Kau End Pier and the Tsing Yi Abutment. The three towers are similar in construction. They rise to levels ranging from 163 m above the Principal Datum (P.D.) to 200 m P.D. Each tower consists of a single reinforced concrete mast which reduces its section in steps, and it is stabilised by transverse cables and struts in the transverse vertical plane. In addition, the top of the Main Tower is stiffened by longitudinal stabilising cables. The concrete mast of each tower is essentially made up of three sections of hollow members with transverse diaphragms provided at the levels at which changes of sections take place. The tower is supported on a piled foundation. The bridge deck of Ting Kau Bridge actually comprises two composite beam-and-slab decks separated by an air gap but connected by cross girders. Each deck carries three traffic lanes and a hard shoulder, and it consists of precast concrete slab panels supported on a grid of steel beams. The bridge deck is supported by four inclined planes of cables emanating from anchorages at the tower tops. The bridge deck is constrained laterally at the three towers. It is fixed to the Main Tower longitudinally while the other two towers allow longitudinal movements to take place. The Ting Kau End Pier and the Tsing Yi Abutment also provide vertical support to the bridge deck with transverse restraints.

In the design of a complex structure, the adoption of certain simplifying assumptions is inevitable. Where such assumptions are important, validation at a later stage is desirable. Ting Kau Bridge is a rather complex structure, and it is worthwhile to conduct tests on its major components as well as the complete structure. The bridge towers are important components which would affect the stability and serviceability of the entire bridge. A series of tests were carried out to determine their vibration characteristics under ambient conditions. The behaviour of the finished cable-stayed bridge depends on the construction sequence and adjustments to cable forces, among other factors. It is therefore useful to measure the vibrations of the complete bridge between the Ting Kau End Pier and the

Tsing Yi Abutment. Sections of the bridge deck and the towers are chosen at which accelerometers were used to pick up the translational and rotational vibrations.

From the results, the natural frequencies and vibration modes can then be derived. These parameters are important for the assessment of the structural behaviour of the complete bridge. They are also very useful for the formulation of the mathematical model for the monitoring system, which was subsequently developed for the bridge. This paper will focus on the ambient vibration measurements and finite element modelling of the bridge.

2. Vibration measurements and data analysis

The use of vibration measurements has long been regarded as important tool in establishing the dynamic characteristics of long-span bridges (Brownjohn *et al.* 1987, 1989, 1997, Macdonald *et al.* 1997, 1998, Casas and Aparicio 1998, Fujino *et al.* 2000, Paultre *et al.* 2000). They are normally classified under ambient vibrations caused by the environment, and the forced vibrations which are induced artificially by a shaker for the purpose of the measurement. The measurement of ambient vibrations is often regarded as more convenient as no heavy equipment is necessary. Fairly comprehensive methodologies have also been developed by updating using the finite element method (Brownjohn *et al.* 2001). According to the original plan, vibration measurements were to be carried out in two stages in parallel with the construction of the bridge. In the first stage, vibration measurements were carried out on the free-standing concrete masts of Ting Kau Tower and the Main Tower. The second stage was supposed to cover the same two towers while cantilever erection of deck segments was going on. Because of the tight construction schedule, the vibration monitoring team was allowed very stringent periods for measurements. The weather conditions also placed additional constraints on the implementation. Subsequently the second stage was postponed and amended to cover the bridge upon substantial completion.

Fig. 2 shows an elevation of the concrete mast of a typical tower without the top cable anchorages, the struts and the transverse stabilising cables in the free-standing state just after the completion of the concrete works. Vibration measurements were conducted at Ting Kau Tower (The University of Hong Kong 2000a) and the Main Tower (The University of Hong Kong 2000b) under ambient conditions. As the bridge towers are tall slender structures, measurements were carried out in sections at three different levels which are accessible. They include the section at the lower cable anchorage, the section at deck level and the section at the underside of the upper cable anchorage close to the top. The pile cap is taken to be substantially fixed. The layout of accelerations at various levels is also shown in Fig. 2. The top level where vibration is expected to be the maximum has been taken as the reference section. Simultaneous measurements at two different levels comprising the reference section and another section were carried out for correlation.

To formulate a strategy for the vibration measurement of the complete bridge (The University of Hong Kong 2000c), a preliminary finite element analysis was carried out to get the vibration mode shapes. A total of 11 servo-accelerometers have been used for the vibration measurements. The locations of accelerometers are determined with reference to the above information and with consideration of the actual site and time constraints. To achieve the objectives of the field measurement, the accelerometers are located such that the vertical bending, lateral bending and torsional modes of Ting Kau Bridge can be identified.

The three towers, which include the Tsing Yi Tower, the Main Tower and the Ting Kau Tower, as

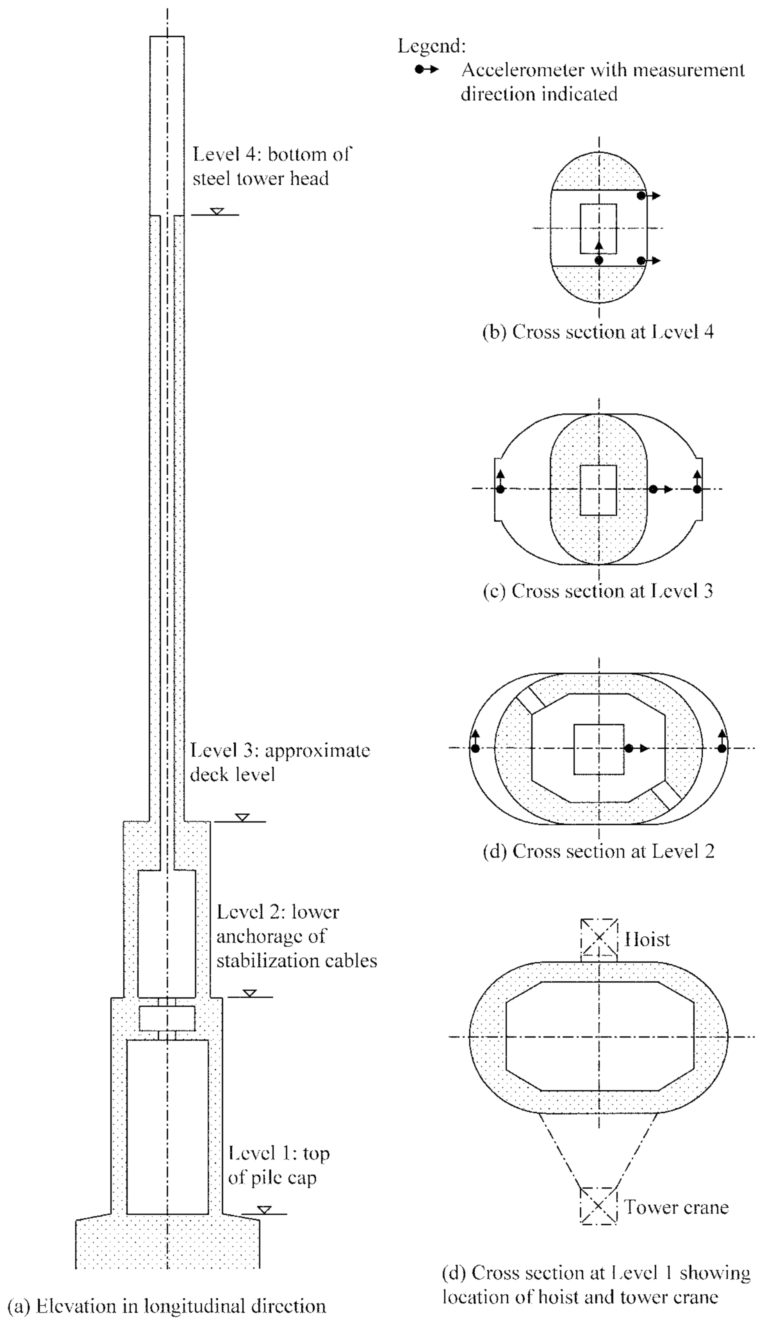

Fig. 2 Locations of accelerometers for vibration measurements of the concrete mast of free-standing tower

well as a total of 13 cross sections of the whole bridge deck between the Ting Kau End Pier and the Tsing Yi Abutment are chosen at which accelerometers were used to pick up the vibration signals, as shown in Fig. 3. Such cross sections of the deck are so chosen that they can be easily identified on site by, for example, cable anchorages or towers. At each cross section of the bridge deck,

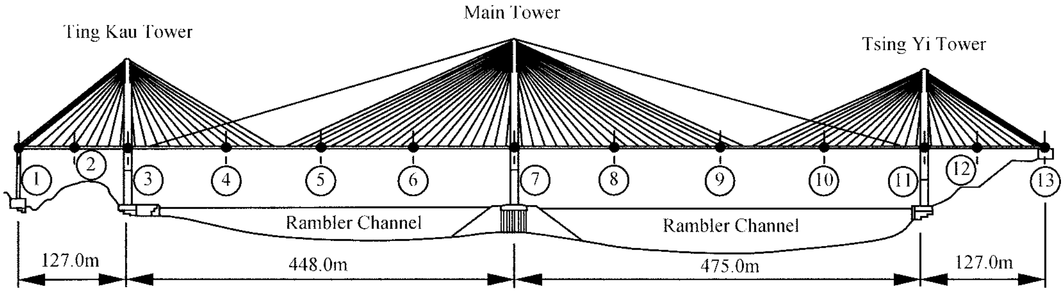

Fig. 3 Elevation of Ting Kau Bridge showing measurement positions on deck

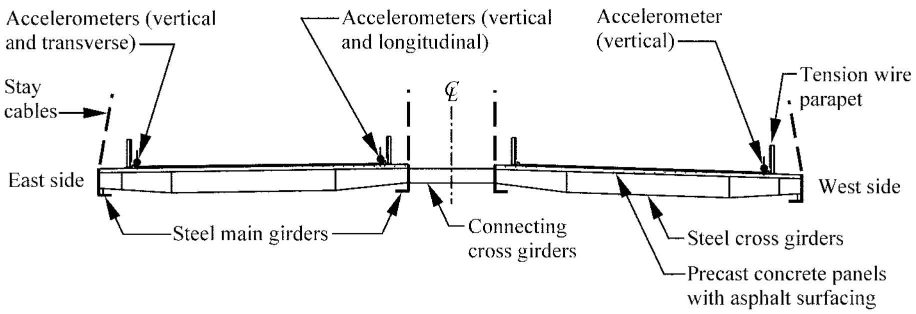

Fig. 4 Typical cross section of deck showing arrangement of accelerometers

normally 5 accelerometers were placed at the left kerb, right kerb, and either the left or right of the median gap, as shown in Fig. 4. There were three channels of vertical vibrations, i.e., at the two kerbs and median, to pick up the torsional vibration. There was one channel for transverse vibration, and the one channel for longitudinal vibration was only used at the reference section.

For each tower, the accelerometers were mounted at three levels: the lower anchorage of the transverse stabilising cables, the strut level and tower top, as shown in Fig. 5. It is relatively difficult to measure the torsional vibrations at the lower anchorage and tower top in view of the size, and hence only two channels were measured at each of these two levels. At the strut level, measurements were carried out at the tips of the two struts as well as at the mast. At the tip of strut, the longitudinal and vertical vibrations were measured; while at the mast, the lateral and longitudinal vibrations were measured. Since the number of transducers and signal conditioners were limited, the orientations of the accelerometers had to be changed during the measurement process.

During the monitoring process, simultaneous measurements at two cross sections were carried out to establish their relationship. They normally include a ‘reference’ section and a ‘moving’ section. Sections 5 and 9, which are roughly at the middle of the northern and southern main spans respectively, were chosen as reference deck sections from which the vibration at other measured locations can be related. Inspection of the theoretical mode shapes obtained from the preliminary finite element analysis indicated these sections to be suitable, as the modal amplitudes are expected to be relatively large for many of the modes. Section 5 was chosen as the reference section for roughly the northern half of the bridge, namely Sections 1, 2, 3, 4, 6 & 7 of the deck, sections of the Ting Kau Tower and the Main Tower. Section 9 was selected as the reference section for the

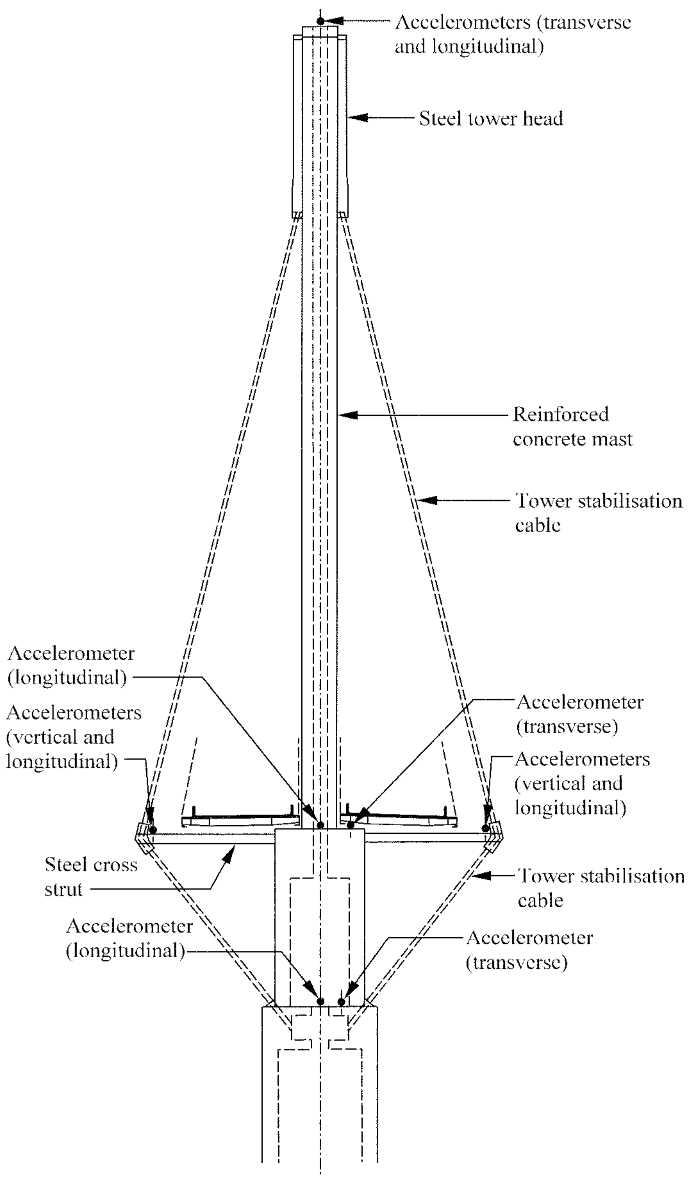

Fig. 5 The main tower showing locations of accelerometers

southern half of the bridge, namely Sections 7, 8, 10, 11, 12 & 13 of the deck, sections of the Main Tower and the Tsing Yi Tower. Simultaneous measurements at the two reference sections then provided the necessary basis for establishing the mode shape information. For each configuration of transducers, measurements were taken for 20 to 30 minutes and measurements were repeated to provide a bigger bank of data for subsequent analysis. Where results were considered doubtful, further repetitions of measurements were carried out.

The signals from the accelerometers were acquired by a personal computer with an analogue-to-digital card as well as recorded on an analogue tape recorder and subsequently analysed. Using a data acquisition and analysis package, the accelerometers were scanned simultaneously and readings were taken and displayed automatically. These measurements were used to derive the natural frequencies of Ting Kau Bridge. According to the preliminary finite element analysis, the first twenty natural modes of Ting Kau Bridge are within the frequency range 0-0.8 Hz. It was therefore decided to filter out all frequencies higher than 10 Hz using a low-pass filter with a cut-off frequency of 10 Hz to prevent aliasing. The sampling frequency for data conversion was taken as 25 Hz. As the vibration measurements were carried out during the construction stage, normal construction activities giving rise to minor vibrations were therefore expected. In the planning of the vibration measurements, the measurement team had tried to avoid construction activities such as laying and compaction of bituminous materials which may give rise to strong persistent vibrations. The captured data were also carefully checked for any out-of-range measurements. Wind speeds and air temperatures were also measured at half-hourly intervals. The ambient vibration measurements were taken in spring during which time the site was not particularly windy. For most of the time, the maximum wind speed recorded at the top of the Main Tower was below 5 m/s although it occasionally reached 8 m/s. Such wind speeds are far below those that may give rise to vortex shedding. The air temperature measured at deck level ranged from 19^{o}C to 35^{o}C during the period.

The Hanning Window was used for the Fast Fourier Transform to minimize spectral leakage (Lynn and Fuerst 1998). With a sampling frequency of 25 Hz and a data block size of 4096, the frequency resolution of the spectral analysis (Bendat and Piersol 1993) was about 0.0061 Hz. This frequency resolution was fine enough to identify the lower modes within the frequency range of 0-0.8 Hz.

Ting Kau Bridge is subjected to environmental dynamic excitation mainly including the wind load, ground micro-tremors, vehicular traffic and other mechanical disturbances. These sources of ambient vibration are random in character, and cannot be accurately or conveniently simulated. If the environmental excitations can be assumed to be a stationary stochastic process, reasonable estimates of the fundamental dynamic properties of Ting Kau Bridge can be deduced by measuring and analysing the structural responses. When a lightly damped structure, such as a long-span bridge, is subjected to such a random excitation, the output auto-spectrum at any response point will reach a maximum at frequencies of the excitation or the natural frequencies of the structure. In the absence of strong prevalent wind or mechanical excitations, one may assume that there are no peaks in the excitation spectrum. Therefore, peaks in the response spectra can generally be assumed to represent the natural modes of the structure. To identify these output spectral peaks, one can make use of the fact that all points on a structure responding in a lightly damped natural mode of vibration will be either in phase or 180^{o} out of phase with one another, depending only on the shape of the natural mode. Furthermore, the oscillation of the structure at its natural frequencies produces a very coherent motion of the structure at all points. The examination of coherence obtained in the cross-spectral analysis is therefore useful for verifying that the peaks in the response spectra are due to the motion in the particular natural modes.

Having identified the natural frequencies, the order and shape of each natural vibration mode can be determined if sufficient responses are obtained over the full length and height of the structure. For the kth natural mode, the response $\Phi_k(y_i)$ at the ith location of the bridge is approximately related to that at the reference section $\Phi_k(y_R)$ by

$$\frac{\Phi_k(y_i)}{\Phi_k(y_R)} = \sqrt{\frac{G_{ii}(f_k)}{G_{RR}(f_k)}} \tag{1}$$

where $G_{ii}(f_k)$ and $G_{RR}(f_k)$ are the auto-spectral density functions, respectively, at the ith location and the reference section of the bridge at the kth natural frequency. The modal vector of the kth natural mode can therefore be gradually built up.

The spectral peaks in the output auto-spectral measurements actually occur at resonance frequencies f_r but not at the undamped natural frequencies f_n required in normal-mode determination. These frequencies are related approximately by

$$f_r = f_n\sqrt{1-2\zeta^2} \quad \text{for} \quad \zeta^2 \le 0.5 \tag{2}$$

and they are in close agreement only for very small values of the modal damping ratio ζ. Hence the application of Eq. (1) should be limited to cases where the damping ratio is small ($\zeta < 0.05$) and the coupling among the natural modes can be ignored. In addition, the auto-spectral peaks can provide reasonably accurate estimation of the relative modal shape only when they are not or slightly contaminated by extraneous noise. Problems arising from the extraneous noise in the response measurements, coupling among the natural modes, and other sources such as non-linearity will be revealed by the coherence and phase results among the response signals. Extraneous noise, coupling in the natural modes or non-linearity will cause the coherence functions between the measurement at one location and measurement at other locations to be less than unity. Also the phase data among at least some of the measurements will be other than zero or 180°. As a general rule, the auto-spectrum at a specific location can be used to define a natural mode shape if the response measurement produces near-unity coherence and near-zero or 180° phase difference with other response measurements and if the contributions of other modes at that frequency are small.

The phase information is obtained by calculating the cross-spectral density function $G_{ij}(f)$ between two response measurements at the ith and the jth locations, and the corresponding coherence function $r_{ij}^2(f)$ is defined as (Bendat and Piersol 1993)

$$r_{ij}^2(f) = \frac{|G_{ij}(f)|^2}{G_{ii}(f)\cdot G_{jj}(f)}; \qquad 0 \le r_{ij}^2(f) \le 1 \tag{3}$$

As each tower of Ting Kau Bridge is in the form of a single concrete mast with the four planes of stay cables converging to its top anchorage, the vibration mode shape is largely characterised by that of the bridge deck. On the other hand, the deck vibration is characterised by movements in the vertical and lateral directions while the longitudinal movement can be regarded as essentially uniform. The response signals at the measuring points for vertical and lateral directions were analysed to identify the natural frequencies of Ting Kau Bridge for vertical, torsional and lateral vibration modes. Their respective auto-spectra, cross-spectra (for magnitude and phase) and coherence functions were calculated. The vertical, torsional and lateral vibration modes were then identified by examining the information on phase and coherence. Subtracting the signals of vertical vibration at different points of the same cross section, the torsional modes were identified. However, the natural frequencies of bending and torsional vibration modes are closely spaced and their respective resonance peaks tend to merge. As a consequence, special attention must be paid to the vibration measurement programme to obtain a clear picture of the vibration modes.

For the concrete mast of Ting Kau Tower and the Main Tower, the lowest six natural frequencies each for the transverse and longitudinal modes were worked out from the field results, and they are respectively shown in Tables 1 and 2. The first forty measured natural frequencies of the complete

Ting Kau Bridge and their corresponding forms of vibration modes are listed in Table 3 in ascending order. The auto-spectra, cross-spectra and coherence functions for the measuring cross sections were studied for confirmation. The auto-spectra exhibit narrow-band peaks at all the bending and torsional natural frequencies. The corresponding coherence values of the first eight natural modes are mostly more than 0.90, and phase differences are mostly between 0-15^o and 165-180^o. However, for higher natural frequencies, the coherence values and phase differences may not be as good.

3. Computer modelling

As Ting Kau Bridge is a fairly complicated structure, the finite element method has been chosen for the analysis. Several finite element models have been developed for different purposes. A comprehensive model employing beam and shell elements has been developed for rigorous calibration against the field vibration measurements. On the other hand, a simplified model for analysis of wind-induced vibration has also been developed. The results from the vibration measurements also serve as useful references for its verification. In both cases, only Ting Kau Bridge itself is modelled without taking into account the approach viaduct. The bridge substructures are assumed to be fixed at the top levels of pile caps.

3.1 Comprehensive structural model for free vibration analysis

The comprehensive structural model employing beam and shell elements was adopted to enable accurate modelling and calibration using the field vibration measurements. The concrete mast of the tower was modelled as a vertical cantilever Euler beam using space beam elements with six degrees of freedom at each node. Each section of hollow member was regarded as a thin-walled tube in the calculation of sectional properties. The struts were also modelled as space beam elements. The transverse cables were modelled as truss elements using the Ernst modulus (Xanthakos 1994) to account for the sag effect. The bridge deck essentially consists of precast concrete slab panels supported on a grid of steel beams. The deck slabs were modelled as shell elements so that the shear lag effect could be taken into account. The steel beams were modelled as space beam elements with eccentricities. The stay cables were modelled as space truss elements using the Ernst modulus to account for the sag effect. The prevalent cable forces on completion of the bridge were used to compute the Ernst moduli of the cables. All together 3255 elements were used for modelling the complete bridge.

3.2 Simplified structural model for wind-induced vibration

The modelling of the towers and cables does not pose any special difficulties. However to model every beam and slab of the bridge deck by the comprehensive structural model is not too practical for the analysis of wind-induced vibrations. Therefore a few commonly used simplified models have been investigated. They include the single-girder model, the double-girder model and the triple-girder model as shown in Fig. 6.

In the single-girder model or spine girder model shown in Fig. 6(a), all stiffness and mass properties of the entire bridge deck are assumed to be modelled by the fictitious central girder. The

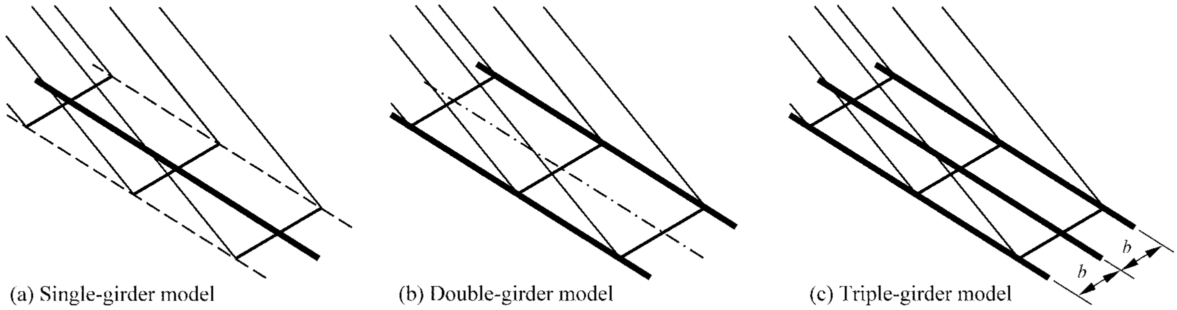

Fig. 6 Simplified structural models of bridge deck for analysis of wind-induced vibration

stiffness characteristics modelled include the vertical and lateral bending stiffnesses as well as the torsional stiffness. The translational mass and the torsional moment of mass inertia of the bridge deck are also represented by the central girder. The stay cables are connected to the central girder through rigid outriggers. Its major drawback is that the warping of the bridge deck section due to constrained torsion cannot be properly described. Therefore the model is mostly suitable for box girder bridges with relatively large free torsional stiffness. This is obviously not a suitable choice for Ting Kau Bridge.

The free torsional stiffness of a beam-and-slab deck is relatively small, and the stiffness for constrained torsion must be carefully considered. The double-girder model consists of two fictitious side girders connected by rigid cross beams as shown in Fig. 6(b). There is significant interaction among the vertical bending, lateral bending and torsion. However there are only two fictitious girders which the designer can adjust. Improper choice of the girder properties may give rise to misleading results such as the lower torsional frequencies. Therefore a more refined model, which can correctly simulate various stiffness characteristics, is required for analysis of the Ting Kau Bridge.

The triple-girder model (Xiang *et al.* 1996, Fan 1997) as shown in Fig. 6(c) consists of three fictitious girders arranged in a symmetrical pattern, comprising a central girder and two side girders, which are connected together by rigid cross beams. The properties of the central girder and the side girders used in the modelling of the bridge deck are, respectively, the areas A_c and A_s, the vertical moments of inertia I_{vc} and I_{vs}, the lateral moments of inertia I_{hc} and I_{hs}, and the pure torsional constants J_c and J_s. Based on various criteria of equivalence, the stiffness and mass properties of the bridge deck can be properly apportioned to the fictitious girders.

The cross-sectional area A, the lateral moment of inertia I_h and the pure torsional constant J of the original bridge deck are entirely modelled by the central girder only, i.e.,

$$A_c = A;\ I_{hc} = I_h;\ J_c = J;\ A_s = 0;\ I_{hs} = 0;\ J_s = 0 \tag{4}$$

The equivalence of vertical stiffness between the original bridge deck and the simplified model gives

$$I_{vc} + 2I_{vs} = I_v \tag{5}$$

Assuming that the torsional warping of the original bridge deck is modelled by the overall twisting of the triple-girder grillage, the torsional warping constant J_ω (Nakai and Yoo 1988) of the original bridge deck is modelled by the vertical bending stiffnesses of the two side girders, i.e.,

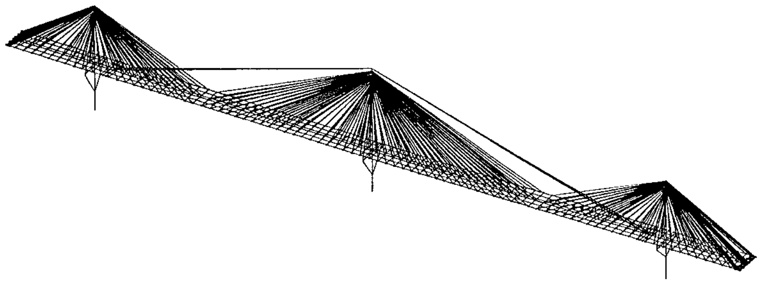

Fig. 7 Simplified finite element model of Ting Kau Bridge using twin triple-girder modelling

$$2I_{vs}b^2 = J_\omega \tag{6}$$

where b is the distance between the central girder and the side girder. Solving Eqs. (5) and (6), the vertical bending stiffnesses of the fictitious girders can be obtained as

$$I_{vc} = I_v - \frac{J_\omega}{b^2}; \quad I_{vs} = \frac{J_\omega}{2b^2} \tag{7}$$

Similarly, the mass of the original bridge deck M is assumed to be distributed over the three fictitious girders, while its moment of mass inertia I_M is provided by the mass distributed over the two side girders. Therefore one has

$$M_c + 2M_s = M \tag{8}$$

$$2M_s b^2 = I_M \tag{9}$$

where M_c and M_s are the mass per unit length in the central girder and the side girders, respectively. Solving Eqs. (8) and (9) leads to

$$M_c = M - \frac{I_M}{b^2}; \quad M_s = \frac{I_M}{2b^2}; \tag{10}$$

It can be seen that the actual stiffness and mass properties of the original bridge deck are well modelled in the triple-girder model through certain criteria of equivalence. The numerical results also indicate that the model can yield more accurate torsional frequencies, which are important parameters as far as the aerodynamic behaviour of long-span bridges is concerned.

As the deck of Ting Kau Bridge actually comprises two composite beam-and-slab decks separated by an air gap but connected by cross girders, each carriageway is modelled by a triple-girder model as described above. This simplified finite element model as shown in Fig. 7 consists of 1120 space beam elements for the bridge decks, 101 space beam elements for the towers and 404 space truss elements for the stay cables. There are a total of 740 nodes.

4. Comparison of results and calibration of numerical models

4.1 Free-standing concrete masts of bridge towers

The specified characteristic cube strength of the concrete is 60 MPa but the actual strength achieved was higher than this value. In the analysis, the mass density of reinforced concrete was taken to be 2500 kg/m^3 while the modulus of elasticity was taken as 36×10^9 N/m^2 in the light of the local design code (Highways Department 1997). It is noted that the torsional and axial vibration modes are of much higher frequencies. They are therefore outside the range of frequencies being considered. The lowest six frequencies each of longitudinal and transverse vibrations are obtained from the finite element analysis. Such natural frequencies obtained for the concrete mast only of Ting Kau Tower are shown in Table 1 and compared with results from the field measurements. Reasonable agreement is observed although it tends to deteriorate from the fourth mode upwards. Sensitivity studies were carried out to study the effects of variations of material properties such as the modulus of elasticity of concrete, aggregate density of concrete, etc with a view to possible

Table 1 Free-standing concrete mast of Ting Kau Tower: natural frequencies

Mode no.		Field results (Hz)	Numerical results (Hz)	
			Without crane & hoist	With crane & hoist
Transverse	1	0.263	0.258	0.259
	2	0.807	1.06	1.06
	3	2.31	2.57	2.56
	4	5.84	5.44	5.55
	5	7.89	8.31	8.38
	6	10.7	12.2	12.3
Longitudinal	1	0.317	0.288	0.305
	2	1.26	1.08	1.22
	3	2.66	2.51	2.87
	4	4.37	4.50	4.89
	5	7.92	7.57	7.99
	6	10.9	11.9	12.5

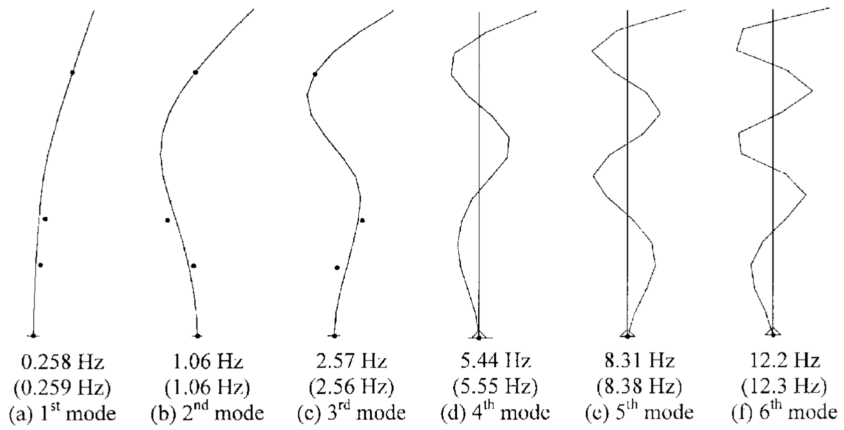

Fig. 8 Free-standing concrete mast of Ting Kau Tower: computed mode shapes and natural frequencies of transverse vibration (Notes: 1. Frequencies shown in brackets have been computed taking into account the crane and hoist. 2. The round dots represent estimated mode shape based on field measurements.)

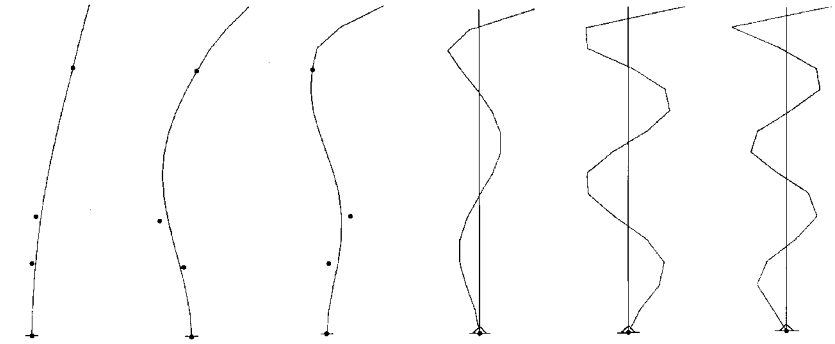

Fig. 9 Free-standing concrete mast of Ting Kau Tower: computed mode shapes and natural frequencies of longitudinal vibration. Notes for Fig. 8 also apply.

manual tuning of the model. However such discrepancies could not be accounted for by such variations. In the construction stage, a tower crane and a hoist were attached to the tower. They were aligned longitudinally, and a typical cross section of the assembly is shown in Fig. 2(d). Their effects on the vibration frequencies were also studied. Another numerical model of the tower taking into account the tower crane and hoist was therefore formulated and analysed. The results are also shown in Table 1, and their effects are seen mainly in the natural frequencies for longitudinal vibrations. The lower frequencies for longitudinal vibrations are indeed closer to the measured values when the crane and hoist are taken into account. The mode shapes for transverse and longitudinal vibrations for the concrete mast only of Ting Kau Tower are shown in Figs. 8 and 9 respectively.

Similar analyses were carried out for the Main Tower and the results are shown in Table 2. The

Table 2 Free-standing concrete mast of the Main Tower: natural frequencies

Mode no.		Field results (Hz)	Numerical results (Hz)	
			Without crane & hoist	With crane & hoist
Transverse	1	0.269	0.246	0.248
	2	0.807	0.951	0.958
	3	2.06	2.10	2.19
	4	4.33	3.93	4.08
	5	6.77	6.50	6.56
	6	9.82	10.1	10.3
Longitudinal	1	0.245	0.211	0.232
	2	1.07	0.985	1.13
	3	1.96	1.84	2.22
	4	3.87	3.86	3.93
	5	6.60	6.88	7.34
	6	8.36	8.99	8.73

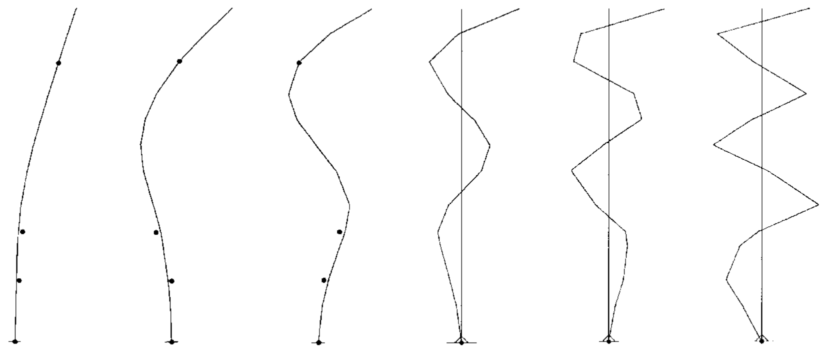

Fig. 10 Free-standing concrete mast of the Main Tower: computed mode shapes and natural frequencies of transverse vibration. Notes for Fig. 8 also apply.

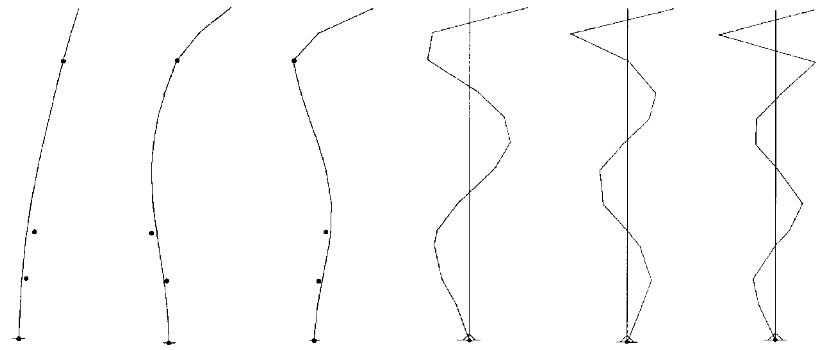

Fig. 11 Free-standing concrete mast of the Main Tower: computed mode shapes and natural frequencies of longitudinal vibration. Notes for Fig. 8 also apply.

effects of the tower crane and hoist are observed to be similar to Ting Kau Tower. The mode shapes for transverse and longitudinal vibrations for the concrete mast only of the Main Tower are shown in Figs. 10 and 11 respectively.

4.2 Complete bridge

In the present analysis, the mass density of reinforced concrete was taken to be 2500 kg/m^3 while the modulus of elasticity was taken as 36×10^9 N/m^2 (Highways Department 1997). The mass density of steel was taken to be 7800 kg/m^3 while the modulus of elasticity was taken as 200×10^9 N/m^2. Sensitivity studies were then carried out to study the effects of variation of these material properties on the lower frequencies, but their effects were comparatively small. However it is discovered that the natural frequencies for the lowest vertical bending modes are rather sensitive to the forces in the longitudinal stabilising cables, which in turn determine their Ernst moduli.

Table 3 Theoretical and measured natural frequencies (Hz) of completed Ting Kau Bridge

Mode	Field results	Comprehensive model	Simplified model	Remarks
1	0.165	0.171	0.166	Vertical Bending
2	0.220	0.202	0.219	Lateral Bending
3	0.244	0.246	0.245	Lateral Bending
4	0.262	0.263	0.282	Lateral Bending
5	0.293	0.291	0.305	Lateral Bending
6	0.311	0.337	0.331	Vertical Bending
7	0.366	0.354	0.353	Torsion
8	0.378	0.373	0.357	Vertical Bending
9	0.409	0.409	0.390	Vertical Bending
10	0.485	0.485	0.484	Vertical Bending
11	0.491	0.486	0.485	Torsion
12	0.507	0.503	0.501	Torsion
13	0.549	0.549	0.564	Torsion
14	0.579	0.576	0.577	Vertical Bending
15	0.586	0.579	0.593	Torsion
16	0.610	0.604	0.607	Vertical Bending
17	0.623	0.648	0.666	Torsion
18	0.671	0.677	---	Torsion
19	0.690	0.689	0.695	Vertical Bending
20	0.696	0.696	0.700	Torsion
21	0.714	0.713	0.723	Vertical Bending
22	0.726	0.723	0.758	Lateral Bending + Torsion
23	0.763	0.769	0.776	Vertical Bending
24	0.775	0.774	0.793	Lateral Bending + Torsion
25	0.787	0.779	0.798	Vertical Bending
26	0.806	0.814	0.845	Lateral Bending + Torsion
27	0.830	0.823	0.860	Lateral Bending + Torsion
28	0.848	0.848	0.846	Vertical Bending
29	0.854	0.860	0.906	Lateral Bending + Torsion
30	0.867	0.863	0.893	Vertical Bending
31	0.873	0.897	---	Torsion
32	0.909	0.907	---	Torsion
33	0.916	0.914	---	Lateral Bending + Torsion
34	0.922	0.917	---	Vertical Bending
35	0.928	0.927	---	Torsion
36	0.939	0.939	---	Vertical + Lateral Bending
37	0.946	0.948	---	Vertical + Lateral Bending
38	0.958	0.955	---	Vertical Bending
39	0.964	0.959	---	Torsion
40	0.970	0.968	---	Lateral Bending + Torsion

The first 40 natural frequencies and mode shape classifications obtained from the comprehensive finite element model are listed in Table 3 and compared with the field results. The comparison is also graphically shown in Fig. 12. The first 20 natural frequencies obtained from the simplified

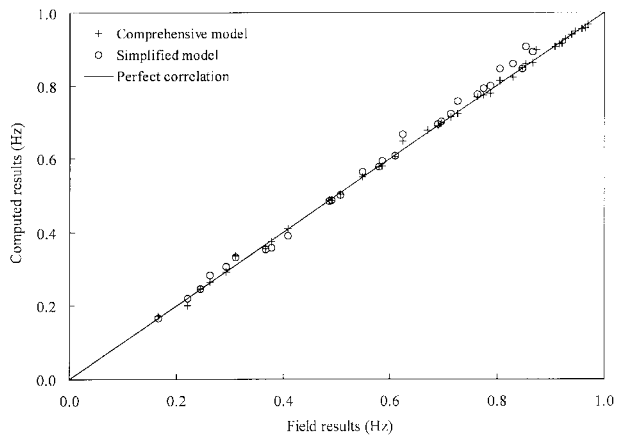

Fig. 12 Comparision between measured and computed frequencies

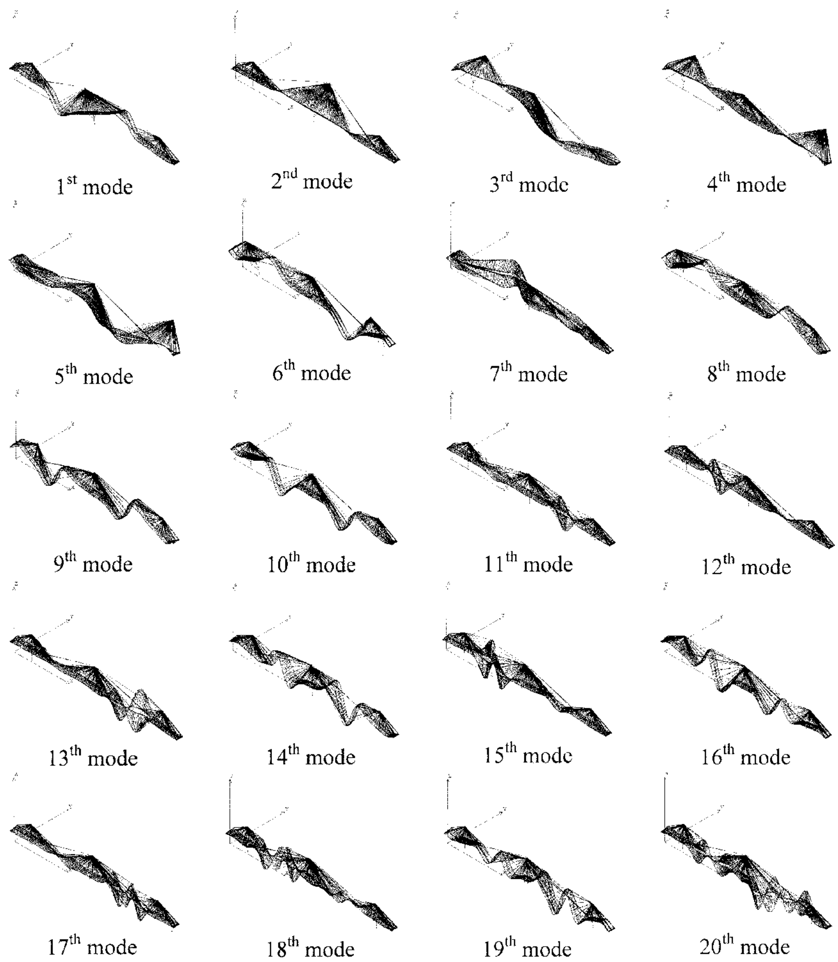

Fig. 13 Complete Ting Kau Bridge: first 20 mode shapes obtained from comprehensive model

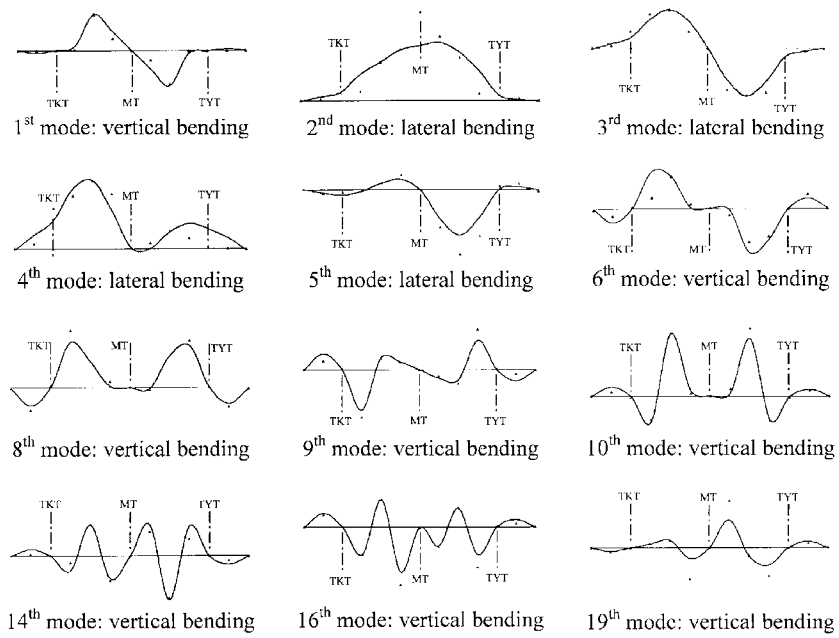

Fig. 14 Complete Ting Kau Bridge: mode shapes with significant bending components. Notes: 1. The triangular markers represent estimated mode shape based on field measurements. 2. TKT: Ting Kau Tower; MT: Main Tower; TYT: Tsing Yi Tower

model as shown in Table 3 also agree well with the others. The first 20 mode shapes obtained from the comprehensive model are shown in Fig. 13. Those with significant bending components are further plotted in Fig. 14 and compared with the field measurements. A permanent structural health monitoring system was installed on Ting Kau Bridge (Lau *et al.* 2000), and the system is being operated by the Tsing Ma Control Area (TMCA) Division of the Highways Department, Hong Kong. For comparison purpose, the vibration modes have been classified as vertical, lateral and torsional and tabulated in Table 4 together with the long term monitoring results and computed results from the Highways Department (Wong *et al.* 2000), as well as the computed results by Flint & Neill Partnership (FNP). In general, good agreement is observed. However no one single numerical model can pick up all natural frequencies. It is also observed that the simplified model using triple-girder modelling of each carriageway is reasonably accurate in predicting the lower natural frequencies, which are essential for the subsequent analysis of wind-induced vibrations.

It should be noted that the vibration measurements described in this paper were carried out when the bridge was substantially complete with only a few outstanding items including the top friction course and some of the tensioned safety fences. As the weight of these item compared to the total weight of bridge deck is minimal, the effects of their omission is considered insignificant. The vibration measurements cover more positions of the bridge but the length of records could not be too long because of various constraints. On the other hand, the TMCA measurements were obtained through the permanent structural health monitoring system and records up to 10 hours have been used. The bridge carried normal highway traffic during the TMCA measurements but their effects on the measured dynamic properties are considered insignificant, as the weight of the vehicles is small compared with the total weight of the bridge deck.

Table 4 Comparison of natural frequencies (Hz) of completed Ting Kau Bridge with other results

Mode	Computed FNP (Checker)	Computed TMCA	Computed Comprehensive	Computed Simplified	Measured TMCA (Operation)	Measured (Present)
Vertical						
1	0.176	0.176	0.171	0.166	0.1618	0.165
2	0.301	0.328	0.337	0.331	0.3145	0.311
3	0.351	0.338	---	---	0.3527	---
4	0.377	---	0.373	0.357	0.3727	0.378
5	---	0.411	0.409	0.390	0.4091	0.409
6	0.461	0.485	0.485	0.484	0.4764	0.485
7	0.564	0.568	0.576	0.577	0.5655	0.579
8	---	0.592	0.604	0.607	0.6091	0.610
9	0.634	0.649	---	---	0.6364	---
10	0.658	0.669	0.689	0.695	0.6582	0.690
11	0.713	0.712	0.713	0.723	0.7073	0.714
12	0.729	---	---	---	0.7624	---
13	0.768	0.763	0.769	0.776	0.7618	0.763
Lateral						
1	0.213	0.217	0.202	0.219	0.2264	0.220
2	---	0.239	0.246	0.245	0.2518	0.244
3	0.252	0.276	0.263	0.282	0.2591	0.262
4	0.282	0.305	0.291	0.305	0.2855	0.293
Torsional						
1	---	0.359	0.354	0.353	0.3591	0.366
2	0.455	0.432	---	---	0.4427	---
3	0.471	0.478	0.486	0.485	0.4809	0.491
4	0.506	0.499	0.503	0.501	0.5155	0.507
5	0.534	0.544	0.549	0.564	0.5345	0.549
6	0.579	0.573	0.579	0.593	0.5718	0.586
7	0.624	0.625	0.648	0.666	0.6264	0.623
8	0.684	0.679	0.677	---	0.6764	0.671
9	0.725	0.728	0.696	0.700	0.7255	0.696
10	0.758	0.728	0.723	0.758	0.7609	0.726
11	0.787	0.758	0.774	0.793	0.7891	0.775

5. Conclusions

The measurement of structural response to ambient excitation from wind load, ground micro-tremors and other disturbances has proved to be an effective means for identification of the dynamic properties of a full-scale flexible structure with fairly low natural frequencies. The dynamic properties identified include the natural frequencies and vibration mode shapes. The ratio among the lowest frequencies of the more important categories is approximately: 1.0 (vertical bending): 1.33 (lateral bending): 2.21 (torsional twist). The relative relationship among the lateral bending stiffness, vertical bending stiffness and torsional stiffness of Ting Kau Bridge can therefore be obtained taking into account the generalised masses for these mode shapes. The measured modal properties are in good agreement with the numerical results by various parties as well as the results from long term monitoring. It is also observed that the simplified model using triple-girder modelling of each carriageway is reasonably accurate in predicting the lower natural frequencies, which are essential for the subsequent analysis of wind-induced vibrations.

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