Structural Engineering and Mechanics, Vol. 14, No. 4 (2002) 491-504
DOI: http://dx.doi.org/10.12989/sem.2002.14.4.491

A state space method for coupled flutter analysis of long-span bridges

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(Received July 17, 2000, Accepted October 29, 2000)

Abstract. A state-space method is proposed to analyze the aerodynamically coupled flutter problems of long-span bridges based on the modal coordinates of structure. The theory about complex modes is applied in this paper. The general governing equation of the system is converted into a complex standard characteristic equation in a state space format, which contains only two variables. The proposed method is a single-parameter searching method about reduced velocity, and it need not choose the participating modes beforehand and has no requirement for the form of structure damping matrix. The information about variations of system characteristics with reduced velocity and wind velocity can be provided. The method is able to find automatically the lowest critical flutter velocity and give relative amplitudes, phases and energy ratios of the participating modes in the flutter motion. Moreover, the flutter analysis of Jiangyin Yangtse suspension bridge with 1385 m main span is performed. The proposed method has proved reliable in its methodology and efficient in its use.

Key words: long-span bridges; coupled flutter; state space method; multimode; complex mode; Jiangyin suspension bridge.

1. Introduction

One important aspect in the design of a cable-stayed or suspension bridge is the analysis of their aerodynamic stability because these structures are prone to the well-known flutter instability phenomenon. Since the infamous Tacoma Narrows Bridge failure of 1940, the bridge engineering community has been faced with the design consideration of aerodynamic flutter. Wind-tunnel tests have been widely used for that purpose and considerable understanding has been obtained using this tool. Furthermore, with progress in computer technology, analytical prediction methods in combination with wind-tunnel testing are now more commonly applied. The analytical methods provide a great deal of information about the aeroelastic phenomena and have made it possible to design much longer bridges. Controlling the flutter instability is a key factor in the ability to realize long-span bridges.

Flutter may occur in both smooth and turbulent flows. Natural winds are turbulent in nature. It is thought that the effects of turbulence on flutter are included in the flutter derivatives of bridge deck

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measured in turbulent flow (Scanlan 1987, Scanlan and Jones 1990). So the flutter analysis method in turbulence flow is the same as that in smooth flow.

The problem of flutter is closely associated with structural natural modes so the analysis in the frequency domain is direct and convenient. Because of the large computational efforts, time-domain methods are seldom used. A great deal of analytical investigations related to the flutter problems of long-span bridges have been made (e.g., Bleich 1948, Scanlan 1978, Xie and Xiang 1985, Miyata and Yamada 1988, Agar 1989, Namini 1992, Chen 1994, Jain *et al*. 1996, Dung *et al*. 1998, Chen and Matsumoto *et al*. 2000). Scanlan (1978) proposed a basic theory for multimode flutter analysis. He also suggested a mode-by-mode approach based on the fact that practical flutter problems of long-span bridges are most likely damping-driven flutter and are dominated by the action of a single mode. Xie and Xiang (1985) employed a planar model of unsteady aerodynamic forces and presented a state-space method for multimode flutter problems. Agar (1989) converted the flutter motion equation into the eigenvalue problem of a real unsymmetric matrix, but the flutter analysis requires a two-parameter searching process. Namini (1992) proposed the pK-F method for multimode flutter problems. The method is to solve iteratively nonlinear equations and can provide the information about variations of structural dynamic behaviors with the wind speed. Chen (1994) transferred the flutter problem into a complex generalized eigenvalue problem and proposed the M-S method that did not require iteration in non-damping situations. Jain (1996) also presented a complex method for the coupled flutter problem, but he solved the real and imaginary parts of the characteristic polynomial. Dung *et al*. (1998) suggested a direct flutter analysis and solved the characteristic equation by the mode tracing method. Chen *et al*. (2000) expressed the aerodynamic forces of a bridge deck by rational functions whose coefficients were derived from flutter derivatives. He analyzed the flutter problem by the state-space method. Those methods are mainly based on the modal coordinates of the structure, so the computations are efficient and each method has its own advantages. However, the participating modes in the flutter motion must be chosen beforehand and much personal participation is required in the flutter analysis in those methods.

In this paper, a new state-space method is proposed to analyze the aerodynamically coupled flutter problems based on the modal coordinates of structures, and a simple and practical process of automatic searching is introduced to find the critical flutter velocity. The model of self-excited forces that contains 18 flutter derivatives is employed and is expressed as a complex form. The general governing equation of the system is converted into a complex standard eigenvalue problem, which contains only two variables, under the condition of reasonable approximation. It is noted that the approximation does not affect the analytical results in the critical flutter state. The flutter analysis is a single-parameter searching process so the proposed method has many advantages. Finally, the flutter analysis of Jiangyin Yangtse suspension bridge with 1385 m main span is performed as an example.

2. General formulation

It is assumed that the buffeting forces have no influence on aerodynamic stability and are excluded in the flutter analysis. Thus the governing equation of motion of a bridge structure excited by aerodynamic forces is given in a matrix form by

$$\boldsymbol{M}\ddot{\boldsymbol{X}} + \boldsymbol{C}\dot{\boldsymbol{X}} + \boldsymbol{K}\boldsymbol{X} = \boldsymbol{F}_{se} \tag{1}$$

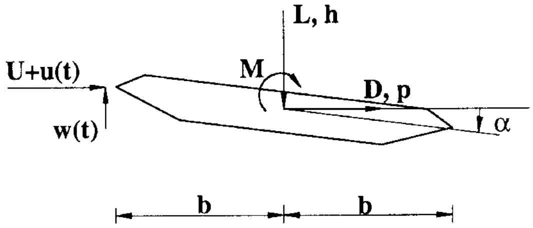


Fig. 1 Aerodynamic forces of bridge deck

where $\boldsymbol{M}$, $\boldsymbol{C}$, and $\boldsymbol{K}$ = mass, damping, stiffness matrices, respectively; $\boldsymbol{X}$ = nodal displacement vector; each dot denotes the partial differentiation with respect to time t; $\boldsymbol{F}_{se}$ = nodal vector of the self-excited force.

The self-excited forces per unit span are expressed in Scanlan's extended format below (Scanlan 1978, 1993):

$$L_{se}(t) = \frac{1}{2}\rho U^2(2B)\left(KH_1^*\frac{\dot{h}}{U} + KH_2^*\frac{B\dot{\alpha}}{U} + K^2H_3^*\alpha + K^2H_4^*\frac{h}{B} + KH_5^*\frac{\dot{p}}{U} + K^2H_6^*\frac{p}{B}\right) \tag{2a}$$

$$D_{se}(t) = \frac{1}{2}\rho U^2(2B)\left(KP_1^*\frac{\dot{p}}{U} + KP_2^*\frac{B\dot{\alpha}}{U} + K^2P_3^*\alpha + K^2P_4^*\frac{p}{B} + KP_5^*\frac{\dot{h}}{U} + K^2P_6^*\frac{h}{B}\right) \tag{2b}$$

$$M_{se}(t) = \frac{1}{2}\rho U^2(2B^2)\left(KA_1^*\frac{\dot{h}}{U} + KA_2^*\frac{B\dot{\alpha}}{U} + K^2A_3^*\alpha + K^2A_4^*\frac{h}{B} + KA_5^*\frac{\dot{p}}{U} + K^2A_6^*\frac{p}{B}\right) \tag{2c}$$

where ρ = air density; U = mean wind velocity; $B = 2b$ = bridge deck width; $K = \omega B/U$ = reduced frequency; ω = circular frequency of vibration; h, p, and α = vertical, lateral, and torsional displacements, respectively; the over-dot denotes the partial differentiation with respect to time t; and H_i^*, P_i^*, A_i^* (i = 1~6) = non-dimensional flutter derivatives, which are functions of the reduced frequency and depend on the geometrical configuration of the bridge section and the approach flow. The aerodynamic forces and the displacements are shown in Fig. 1.

For the bluff bridge deck sections, the flutter derivatives can be determined experimentally. This is actually done through a system identification method in the frequency or time domain using free vibration or forced vibration testing in a wind tunnel. The measurement of the free vibration is simple and is often applied. At present a two-degree-of-freedom section model of the bridge deck is widely used to identify the flutter derivatives H_i^* and A_i^* (i = 1~4). The drag and components associated with lateral motion are generally negligible, but may become important for certain bridge deck configurations (Miyata *et al.* 1994). The quasi-steady theory is invoked to consider the effects in the absence of the measured results in the wind tunnel.

$$\begin{aligned}
&P_1^* = -\frac{1}{K}C_D, \quad P_2^* = \frac{1}{2K}C_D', \quad P_3^* = \frac{1}{2K^2}C_D' \\
&P_5^* = \frac{1}{2K}C_D', \quad H_5^* = \frac{1}{K}C_L, \quad A_5^* = -\frac{1}{K}C_M \\
&P_4^* = P_6^* = H_6^* = A_6^* = 0
\end{aligned} \tag{3}$$

where C_L, C_D, and C_M = static lift, drag, moment coefficients (referred to deck width B), respectively; $C_D' = dC_D/d\alpha$ and (Scanlan 1987).

The expressions (2) are the real-number form of self-excited forces. In complex notation, the corresponding expressions of aerodynamic force read (Starossek 1998)

$$L_{se}(t) = \omega^2 \rho B^2 (C_{Lh} h + C_{Lp} p + BC_{L\alpha} \alpha) \tag{4a}$$

$$D_{se}(t) = \omega^2 \rho B^2 (C_{Dh} h + C_{Dp} p + BC_{D\alpha} \alpha) \tag{4b}$$

$$M_{se}(t) = \omega^2 \rho B^2 (BC_{Mh} h + BC_{Mp} p + B^2 C_{M\alpha} \alpha) \tag{4c}$$

where C_{rs} ($r = D, L, M$; $s = h, p, \alpha$) are the complex coefficients of self-excited forces.

The relationships between real and complex aerodynamic coefficients can be established by comparing the corresponding aerodynamic force expressions. The following relations are found:

$$C_{Lh} = H_4^* + iH_1^*, \quad C_{Lp} = H_6^* + iH_5^*, \quad C_{L\alpha} = H_3^* + iH_2^* \tag{5a}$$

$$C_{Dh} = P_6^* + iP_5^*, \quad C_{Dp} = P_4^* + iP_1^*, \quad C_{D\alpha} = P_3^* + iP_2^* \tag{5b}$$

$$C_{Mh} = A_4^* + iA_1^*, \quad C_{Mp} = A_6^* + iA_5^*, \quad C_{M\alpha} = A_3^* + iA_2^* \tag{5c}$$

When comparing the real force expressions (2) with the equivalent complex expressions (4), the complex equations are more compact. The reason is that the complex coefficients naturally represent the phasing between displacements and displacement-induced aerodynamic forces. In the real notation, the velocity terms are included in order to account for phasing properly. Although the two forms of self-excited forces are equivalent in essence, the presented method will exhibit that the complex expressions (4) are more beneficial. Moreover, the phasing between displacements and aerodynamic forces becomes quite obvious in the complex form.

In the FEM analysis, the distributed forces of a bridge deck are converted into equivalent nodal loads at member ends as follows:

$$\boldsymbol{F}_{se}^e = \omega^2 \boldsymbol{A}_{se}^e \boldsymbol{X}^e \tag{6}$$

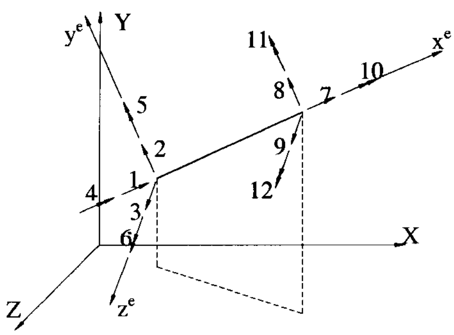


Fig. 2 Positive directions of 12-degree-of-freedom space frame member

where the subscript e represents the local coordinates of the member (see Fig. 2). $\boldsymbol{A}_{se}^{e}$ is a 12 by 12 aerodynamic matrix of the member, and has lumped and consistent forms as mass matrix. The lumped aerodynamic matrix of a bridge deck member with L length is

$$\boldsymbol{A}_{se}^{e} = \begin{bmatrix} \boldsymbol{A}_1 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{A}_1 \end{bmatrix} \tag{7}$$

where

$$\boldsymbol{A}_1 = \frac{1}{2}\rho B^2 L \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & C_{Lh} & C_{Lp} & BC_{L\alpha} & 0 & 0 \\ 0 & C_{Dh} & C_{Dp} & BC_{D\alpha} & 0 & 0 \\ 0 & BC_{Mh} & BC_{Mp} & B^2 C_{M\alpha} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{8}$$

Since aerodynamic forces are non-conservative, the aerodynamic matrix of the member is generally unsymmetrical and is a function of reduced frequency. When the aerodynamic matrices of the members are transformed into the structural global coordinates and are assembled, then

$$\boldsymbol{F}_{se} = \omega^2 \boldsymbol{A}_{se} \boldsymbol{X} \tag{9}$$

where $\boldsymbol{A}_{se}$ = structural aerodynamic matrix. Obviously, $\boldsymbol{A}_{se}$ is a complex matrix.

3. Multimode flutter analysis

Based on the preceding part, the governing motion equations are expressed in complex form as

$$\boldsymbol{M}\ddot{\boldsymbol{X}} + \boldsymbol{C}\dot{\boldsymbol{X}} + \boldsymbol{K}\boldsymbol{X} = \omega^2 \boldsymbol{A}_{se} \boldsymbol{X} \tag{10}$$

Let $\boldsymbol{X} = \boldsymbol{R}e^{st}$, where $\boldsymbol{R}$ is the complex mode response of the system including the structure and airflow; its corresponding complex frequency $s = (-\xi + i)\omega$ (where ξ and ω are the damping ratio and circular frequency of the complex mode, respectively, and $i^2 = -1$), substituting this into Eq. (10) yields

$$(s^2\boldsymbol{M} + s\boldsymbol{C} + \boldsymbol{K} - \omega^2 \boldsymbol{A}_{se})\boldsymbol{R}e^{st} = 0 \tag{11}$$

The complex mode response of the system can be expressed approximately by m structural natural modes as

$$\boldsymbol{R} = \boldsymbol{\Phi}\boldsymbol{q} \tag{12}$$

where $\boldsymbol{\Phi}$ = an n-row by m-column matrix of natural modes, can be given by the dynamic characteristic analysis in the loaded state; $\boldsymbol{q}$ = an m-row vector of generalized coordinates; and n = the total number of degrees of freedom. Inserting Eq. (12) into Eq. (11) and multiplying with $\boldsymbol{\Phi}^T$ by the left yields

$$[s^2\boldsymbol{I} - \omega^2\overline{\boldsymbol{A}}_{se} + s\overline{\boldsymbol{C}} + \boldsymbol{\Lambda}]\boldsymbol{q}e^{st} = 0 \tag{13}$$

where $\boldsymbol{\Lambda}$ = the diagonal eigenvalue matrix from the dynamic characteristic analysis; $\boldsymbol{I}$ = unit matrix; and the matrix $\overline{\boldsymbol{A}}_{se} = \boldsymbol{\Phi}^T\boldsymbol{A}_{se}\boldsymbol{\Phi}$ and $\overline{\boldsymbol{C}} = \boldsymbol{\Phi}^T\boldsymbol{C}\boldsymbol{\Phi}$.

Considering the fact that the damping ratios of the system (positive or negative) are small, the circular frequency of a complex mode is approximately given by $\omega = -si$ (where $i^2 = -1$), inserting this into Eq. (13) yields

$$[s^2(\boldsymbol{I} + \overline{\boldsymbol{A}}_{se}) + s\overline{\boldsymbol{C}} + \boldsymbol{\Lambda}]\boldsymbol{q}e^{st} = 0 \tag{14}$$

The corresponding vibration of self-excited force expressions (4) is harmonic and constant-amplitude. The effect of the damping ratio on self-excited forces is still an open problem for further discussion. Note that the damping ratio of the system is equal to zero in the critical flutter state, i.e., $\omega = -si$, so the above approximation has no influence on the critical flutter state and flutter velocity.

The above equation can be expressed in the following state-space format:

$$(\boldsymbol{A} - s\boldsymbol{I})\boldsymbol{Y}e^{st} = \boldsymbol{0} \tag{15}$$

where

$$\boldsymbol{Y} = \begin{Bmatrix} \boldsymbol{q} \\ s\boldsymbol{q} \end{Bmatrix}, \quad \boldsymbol{A} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\overline{\boldsymbol{M}}\boldsymbol{\Lambda} & -\overline{\boldsymbol{M}\boldsymbol{C}} \end{bmatrix} \tag{16a}$$

$$\overline{\boldsymbol{M}} = (\boldsymbol{I} + \overline{\boldsymbol{A}}_{se})^{-1} \tag{16b}$$

For a nontrivial solution to exist and the exponential is never zero, the analysis of complex modes of the system is converted into the following standard eigenvalue problem:

$$\boldsymbol{A}\boldsymbol{Y} = s\boldsymbol{Y} \tag{17}$$

where the characteristic matrix of the system $\boldsymbol{A}$ is a 2 m by 2 m complex matrix, and has a single variable, i.e., reduced frequency (or reduced velocity). Because the employed self-excited force expressions (4) do not contain any term in U, neither does Eq. (17), and thus it is to be solved for only two variables, s and K. Then a process of two-parameter searching is avoided for the flutter analysis. After fixing K, standard linear eigensolvers are available to compute the 2 m sets of eigenvalues s and corresponding eigenvectors $\boldsymbol{Y}$ from Eq. (17), and

$$s = (-\xi + i)\omega, \quad \boldsymbol{q} = \boldsymbol{a} + \boldsymbol{b}i \tag{18}$$

The m eigenvalues with positive imaginary part are the complex frequencies of the system and the upper half vectors $\boldsymbol{q}$ in the corresponding eigenvectors $\boldsymbol{Y}$ are the complex mode shapes of the system, and the other m ones with negative imaginary part are meaningless. Because of aerodynamic coupling, the mode shapes are coupled. In a prescribed complex mode shape, the magnitude and phase of the kth natural mode are given as

$$|q_k| = \sqrt{a_k^2 + b_k^2}, \quad \varphi_K = \tan^{-1}(b_k / a_k) \tag{19}$$

If the damping ratios of all complex modes are positive the system is stable; if at least one damping ratio is equal to zero the system is neutrally stable; if at least one damping ratio is negative the system is unstable. Therefore, the flutter analysis is to find the critical state that at least one damping ratio of the system is zero through searching reduced velocity V^r. The corresponding circular frequency is the flutter circular frequency ω_f and the critical flutter velocity can be computed by $U_{cr} = B\omega_f / K$. Several critical flutter states may occur in the range of reduced velocity. It is not sure that the flutter velocity corresponding to the lowest V^r of flutter onset is actually the lowest critical flutter velocity. An automatic searching procedure is therefore employed to find the lowest critical flutter velocity in the following part.

At the critical flutter velocity, the generalized modal coordinate $\boldsymbol{q}(t)$ and nodal displacement are expressed by the complex flutter mode as

$$\boldsymbol{q}(t) = \{|q_i| \sin(\omega_f t + \varphi_i)\} \tag{20}$$

$$\boldsymbol{X}(t) = \sum_{i=1}^{m} \phi_i |q_i| \sin(\omega_f t + \varphi_i) = \boldsymbol{X}_0 \sin(\omega_f t + \bar{\varphi}) \tag{21}$$

where ϕ_i = the ith natural mode shape; ω_f = flutter circular frequency; $\boldsymbol{X}_0$ and $\bar{\varphi}$ = amplitude and phase of $\boldsymbol{X}(t)$; m = the number of participating modes. It is clear that the coupled flutter motion is three-dimensional and that the phase shift exists among mode components.

The total motion energy of the characteristic motion in the critical flutter state is

$$E = \frac{1}{2}\{\dot{\boldsymbol{X}}_{\max}\}^T \boldsymbol{M} \{\dot{\boldsymbol{X}}_{\max}\} = \frac{1}{2}\omega_f^2 \sum_{i=1}^{m} |q_i|^2 \tag{22}$$

and the energy of the ith natural mode is

$$E_i = \frac{1}{2}\omega_f^2 |q_i|^2 \tag{23}$$

So the ratio of modal energy e_i is equal to E_i / E. The ratios of modal energy also provide a uniform measurement of the contribution of natural modes to flutter motion.

4. Procedure of automatic searching

First, one must choose a range of reduced velocity, from V^r_{low} to V^r_{high} in order to perform searching. Generally V^r_{low} can be set at zero. Because the searching method presented will automatically stop if the lowest critical flutter velocity is found, the upper bound V^r_{high} is only required to be high enough (proper between 20 to 30). Although the choice of the increment of reduced velocity V^r_{inc} has no restriction and has no influence on the final results, use of a small

V_{inc}^r can improve the stability of searching process. The procedures of automatic searching are as follows:

1. Compute the first m natural modes of the structure that are employed as the participating modes in flutter analysis.
2. Compute the current reduced velocity V_i^r, from within the range of reduced velocities V_{low}^r to V_{high}^r, and incrementing V_{inc}^r.
3. Compute the reduced frequency $K_i = 2\pi/V_i^r$; determine the corresponding complex matrix $\overline{\boldsymbol{M}}$, interpolate or extrapolate if necessary.
4. Solve the standard eigenvalue problem of Eq. (17); compute the damping ratios and circular frequencies of all complex modes of the system.
5. Loop over the kth complex modes, $k = 1, 2, \ldots, m$, when the damping ratio of the kth complex mode is less than zero, execute steps 6 and 7. After the loop, find out the minimum circular frequency $\omega_{\min}$ in the stable complex modes and go to step 8.
6. Using a linear prediction scheme (see Fig. 3), choose the next value of V^r as

$$V^r = \frac{V_2^r \xi_k^1 - V_1^r \xi_k^2}{\xi_k^1 - \xi_k^2} \tag{24}$$

   In the first computation, V_1^r and V_2^r are equal to V_{i-1}^r and V_i^r, respectively; ξ_k^1 and ξ_k^2 are the corresponding damping ratios of the kth complex mode.
7. Execute steps 3 and 4; when the absolute value of the damping ratio ξ_k of the kth complex mode is below tolerance, return to step 5. Otherwise, if $\xi_k > 0$, $V_1^r = V^r$ and $\xi_k^1 = \xi_k$, else $V_2^r = V^r$ and $\xi_k^2 = \xi_k$, repeat steps 6 and 7.
8. Compute the lowest critical flutter velocity $U_{\min}^{cr}$ below the reduced velocity V_i^r if possible, and compute $U_1 = \frac{1}{2\pi} B V_i^r \omega_{\min}$. When $U_{\min}^{cr} > U_1$, continue step 2, otherwise skip the loop of reduced velocity and stop.

For the undamped structures, the check value of damping ratio in step 5 should be a very small number (e.g., 0.0001). The reason is that the damping ratio of some participating modes (e.g., modes of cable vibration) which have no relation with the aerodynamic forces is not just equal to zero, but maybe positive or negative due to the error of numerical computation.

U_1 is the lowest wind velocity that has been searched for the stable complex modes in the ith loop

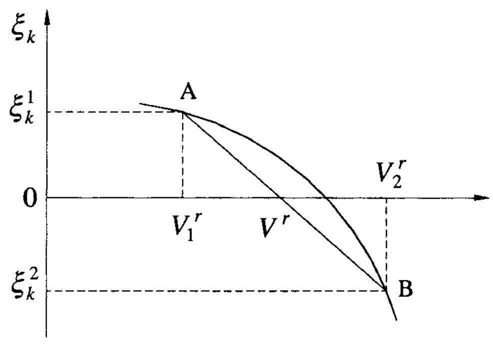


Fig. 3 Linear root predictor of reduced velocity

of reduced velocity. If the lowest flutter velocity found is less than U_1, it indicates that the critical flutter of structure has occurred below wind velocity U_1. Thus it is unnecessary to continue the process of searching.

5. Jiangyin suspension bridge

The proposed method has been coded into an analyzing software, which is able to analyze the spatial flutter problems of long-span bridges and take into consideration the nonlinear deformation induced by static wind loads. The multimode flutter problem of Jiangyin suspension bridge is analyzed with this software. Jiangyin Yangtse suspension bridge with 1385 m-long main span is the longest-span bridge that has been constructed in China, as shown in Fig. 4. The bridge section is a streamlined box one with 36.9 m width and 3.0 m height, see Fig. 5.

Structural properties are as follows. Center cables: area = 0.4825 m^2, mass = 3974.3 kg/m; beside cables: area = 0.5053 m^2, mass = 4081.8 kg/m, $E = 2.0 \times 10^8$ kN/m^2. Hangers: area = 0.0064 m^2, mass = 50 kg/m, $E = 1.4 \times 10^8$ kN/m^2. A single column of Towers: area = 33.389~43.148 m^2, I (longitudinal) = 371.17~847.55 m^4, I (lateral) = 137.15~183.52 m^4, J (torsional) = 299.56~479.23 m^4, ρ = 2550 kg/m^3, $E = 3.5 \times 10^7$ kN/m^2. Deck: area = 1.1 m^2, I (vertical) = 1.844 m^4, I (lateral) = 93.32 m^4, J (torsional) = 4.82 m^4, mass = 18000 kg/m, moment of inertia = 1.426×10^6 kgm^2/m, overall deck width = 36.9 m.

A model of the bridge deck with two degrees of freedom is used to measure the flutter derivatives H_i^* and A_i^* (i=1~4) in the wind tunnel, the results are shown in Fig. 6. The static lift, drag, and moment coefficients of the model at different angles of incidence are measured in laminar flow. The static coefficients of the deck section at 0 degree of incidence are C_D = 0.0697, C_L = –0.128, C_M = –0.0074, $dC_D/d\alpha$ = 0.0. Since there is no measured result of the flutter derivatives related to the lateral motion, these flutter derivatives are calculated by Eq. (3) based on the quasi-steady theory. The structural damping ratio for each natural mode is assumed to 0.005. To simplify discussion of the fundamental characteristics of the flutter problem, only the aerodynamic forces acting on bridge

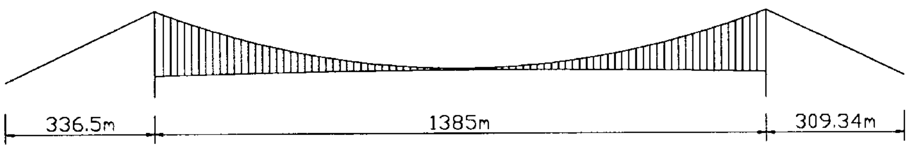


Fig. 4 Jiangyin Yangtse suspension bridge

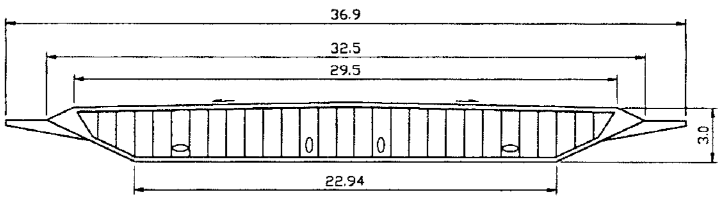
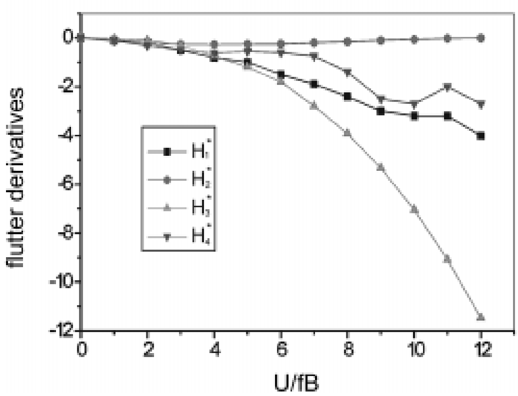


Fig. 5 Cross section of the deck of Jiangyin bridge



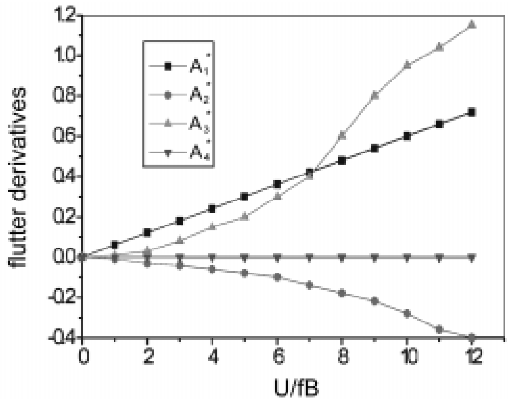


Fig. 6 Flutter derivatives H_i^* and A_i^* of bridge deck

Table 1 Main modes of bridge deck

No. Mode	Frequency (Hz)	Mode shape	No. Mode	Frequency (Hz)	Mode shape
1	0.0516	S-L	15	0.2730	S-T
2	0.0891	A-V	16	0.3107	A-V
3	0.1237	A-L	27	0.3707	S-V
4	0.1316	A-V	30	0.4132	S-T
5	0.1338	S-V	31	0.4322	A-V
6	0.1883	S-V	36	0.4990	S-V
7	0.2005	A-V	38	0.5304	A-T
12	0.2468	S-L	41	0.5690	A-V
13	0.2583	S-V	44	0.6444	S-V
14	0.2677	A-T	45	0.6640	S-T

Note: *S* - Symmetric; *A* - Antisymmetric; *V* - Vertical; *L* - Lateral; *T* - Torsional

Table 2 Flutter results of different methods

The analyzing method	Critical wind velocity (m/s)	Flutter frequency (Hz)
The pK-F method	66.8	0.2410
The present method	66.95	0.2422

deck are involved. The aerodynamic parameters are assumed to be uniform along the bridge axis, and the deformation due to static wind is ignored.

The analytical model of the suspension bridge is set up based on the design data. The first 50 natural modes are computed by the Lanczos method, and the major modes of bridge deck are listed in Table 1. The Sturm check on the first 50 modes is applied to prevent the missing of modes, and no mode is found missing.

To confirm the reliability of the present method for analyzing flutter problems, both the present method and the pK-F method are employed simultaneously to analyze the same flutter case. The mode 5, 6, and 15 are chosen as the participating modes of flutter. The results in critical flutter state

Table 3 Results of all critical flutter states found

Flutter state	Reduced velocity (U/fB)	Critical wind velocity (m/s)	Flutter frequency (Hz)	Original natural mode
1	7.585	67.55	0.2413	First symmetric torsion
2	8.621	111.74	0.3513	Second symmetric torsion
3	8.757	183.48	0.5676	Third symmetric torsion
4	8.820	73.73	0.2265	First antisymmetric torsion
5	9.055	148.01	0.4430	Second antisymmetric torsion

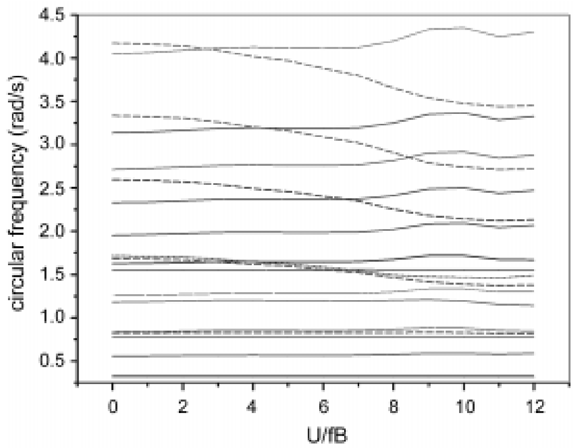


Fig. 7 Frequencies versus reduced velocity

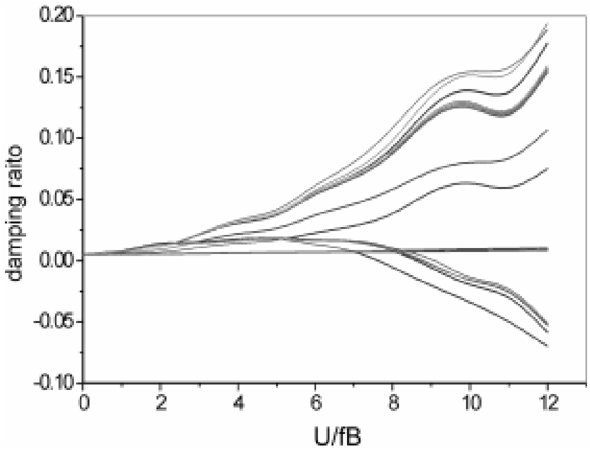


Fig. 8 Damping ratios versus reduced velocity

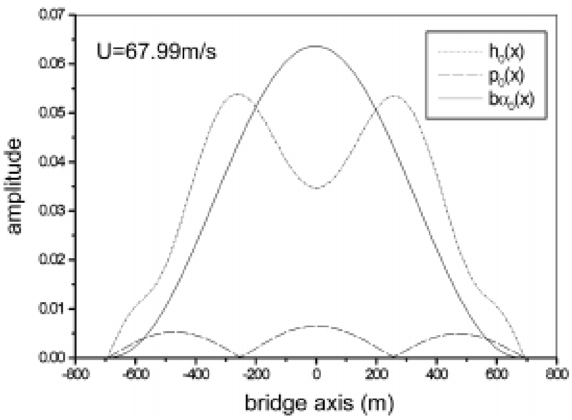


Fig. 9 Amplitude of coupled flutter motion along the bridge axis

are listed in Table 2. It is obvious that the two sets of results have good agreement, hence both the proposed method and the corresponding analyzing software are proved reliable.

Since the proposed method has no limit on the number of modes, all of the first 50 natural modes are employed as the participating modes of flutter. The automatic searching of flutter is performed within the range of measured reduced velocity. Five critical flutter states are found totally and are listed in Table 3. The lowest critical flutter velocity is 67.55 m/s, and the corresponding flutter frequency is 0.2413 Hz. From the experiment of the full-bridge model in a wind tunnel, flutter velocity is determined at 67 m/s (Xiang *et al.* 1996). Thus the analytical results have good agreement with the experimental ones. The flutter motion initiates from the first symmetric torsion and is obviously coupled. Figs. 7 and 8 show the variations of frequencies and damping ratios of main complex modes with reduced velocity. It is noted that the proposed method is not subject to the intersections of frequencies. And Fig. 9 shows the amplitude of the vertical, lateral and torsional displacement along the bridge axis in the flutter motion. Since only the amplitude is presented here, it should be noted that phase shifts exist among response components.

Figs. 10, 11, and 12 show the relative amplitudes, phases, and energy ratios of the 50 participating modes in the flutter motion, respectively. It is noted that modes 5, 6, and 15 are major participating modes of flutter and the phase shifts among those modes are quite obvious. Although the lateral-motion modes of bridge deck also participate in the flutter motion, the degree of participation is small.

The proposed method can also analyze the flutter problem in different mode combinations. The results in different mode combinations of the suspension bridge are listed in Table 4. It should be noted that the participation of high-frequency modes has positive or negative effects on the critical

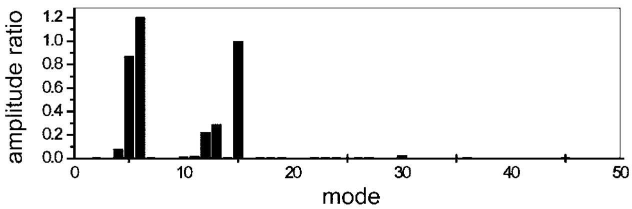


Fig. 10 Relative amplitudes of natural modes in flutter motion

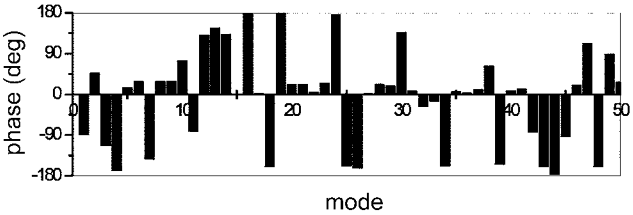


Fig. 11 Phases of natural modes in flutter motion

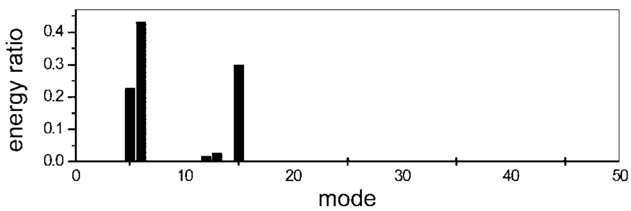


Fig. 12 Energy ratios of natural modes in flutter motion

Table 4 Flutter results of different combinations of natural modes

Mode combinations	Critical wind velocity (m/s)	Flutter frequency (Hz)
5,15	85.89	0.2200
5,6,15	66.95	0.2422
1-6,15	66.88	0.2423
1-50	67.55	0.2413

flutter velocity. Even then the effects are small. Further research is needed for qualitative conclusions.

6. Conclusions

This paper has proposed a state-space method for analyzing the aerodynamically coupled flutter problems of long-span bridges based on the modal coordinates of structures, and a simple and practical process of automatic searching is introduced to find the critical flutter velocity. The theory about complex modes is applied in this paper. The model of self-excited forces that contains 18 flutter derivatives is employed and is expressed as a complex form. The general governing equation of the system is converted into a complex standard eigenvalue equation in a state space format, which contains only two variables. This format is used for flutter analysis by solving the complex standard eigenvalue problem.

The proposed method can be applied to analyze the aerodynamically coupled flutter of long-span bridges in completed or construction stages. Based on the method, the effects of each flutter derivative on the flutter of various bridges can be studied. The advantages of the proposed method are:

1. It is a single-parameter searching method about the reduced velocity, simple and practical, which proceeds automatically to a fair degree.
2. It is not necessary to choose the participating modes beforehand and there is no requirement for the form of the structural damping matrix.
3. Information about variations of the main complex modes of the system with reduced velocity and wind velocity can be provided.
4. The process can automatically locate the lowest critical flutter velocity and give relative amplitudes, phases and energy ratios of the participating modes in the flutter motion.

The proposed method has been coded into an analyzing software, which is able to analyze the spatial flutter problems of long-span bridges. The multimode flutter problem of Jiangyin suspension bridge is analyzed with the software as an example. It has been seen that the results of flutter analysis have good agreement with the results of the previous method in the same flutter case and with the experimental ones of full-bridge model in the wind tunnel.

For the suspension bridge, flutter occurs in the first symmetric torsion mode and is obviously coupled. Although the lateral-motion modes of bridge deck also participate in the flutter motion, the degree of participation is small. From the flutter analysis of this bridge, it has been noted that the participation of multimodes may have positive or negative effects on the critical flutter velocity. These effects will be addressed in a future publication.

Acknowledgements

This paper is part of a large research project financially supported by the National Natural Science Foundation of China (Grants 59895410), with Prof. Haifan Xiang as its program director. The first writer wishes to express thanks to Prof. Xiang (his advisor) and to the National Natural Science Foundation of China.

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