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Identification of modal damping ratios of structures with closely spaced modal frequencies

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Abstract. This paper explores the possibility of using a combination of the empirical mode decomposition (EMD) and the Hilbert transform (HT), termed the Hilbert-Huang transform (HHT) method, to identify the modal damping ratios of the structure with closely spaced modal frequencies. The principle of the HHT method and the procedure of using the HHT method for modal damping ratio identification are briefly introduced first. The dynamic response of a two-degrees-of-freedom (2DOF) system under an impact load is then computed for a wide range of dynamic properties from well-separated modal frequencies to very closely spaced modal frequencies. The natural frequencies and modal damping ratios identified by the HHT method are compared with the theoretical values and those identified using the fast Fourier transform (FFT) method. The results show that the HHT method is superior to the FFT method in the identification of modal damping ratios of the structure with closely spaced modes of vibration. Finally, a 36-storey shear building with a 4-storey light appendage, having closely spaced modal frequencies and subjected to an ambient ground motion, is analyzed. The modal damping ratios identified by the HHT method in conjunction with the random decrement technique (RDT) are much better than those obtained by the FFT method. The HHT method performing in the frequency-time domain seems to be a promising tool for system identification of civil engineering structures.

Key words: system identification; closely spaced modal frequencies; modal damping ratio; the HHT method; the FFT method.

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1. Introduction

The identification of modal damping ratios of a civil engineering structure is an important task toward the accurate dynamic response prediction and the reliable dynamic design of the structures. For a linear lightly-damped structure with well-separated modal frequencies, the fast Fourier transform (FFT) method in conjunction with the bandwidth method are the most popular approach for identifying the modal damping ratios of the structure from the measured structural response time histories (Bendat and Piersol 1993). This approach, however, may encounter difficulties when applied to complicated civil engineering structures such as long span cable-supported bridges, large span space structures, tall buildings with flexible masts, and high television towers, for these

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structures may possess closely spaced modes of vibration.

Recently, Huang and his co-workers (1998) proposed the Hilbert-Huang transform (HHT) method, which consists mainly of the empirical mode decomposition (EMD) and the Hilbert transform (HT). The most powerful feature of the HHT method is its ability to analyse non-linear and nonstationary time histories in the frequency-time domain. Vincent *et al.* (1999) applied the EMD method for structural damage detection and compared the results with those obtained from the wavelet analysis (WA). They demonstrated that both the EMD method and the WA method could identify the time at which the structural stiffness was suddenly lost, but one of the advantages of using the EMD method was its adaptive nature. Yang and Lei (1999, 2000) successfully applied the HHT method together with the random decrement technique (RDT) to identify the modal parameters of linear structures based on the measured response data. They concluded that the HHT method offers a simple, effective and accurate tool for parametric identification of linear structures. However, the example structures considered in their work are generally those with well-separated modal frequencies. The applicability of the HHT method to identify the structures with closely spaced modes of vibration has not been investigated yet.

Thus, the objective of this paper is to explore the possibility of using the HHT method to identify the modal damping ratios of a structure with closely spaced modes of vibration. The principle of the HHT method is briefly introduced first. The procedure of using the HHT method for modal damping ratio identification suggested by Yang and Lei (1999) is adopted with some modification concerning an intermittency check. The dynamic response of a 2DOF system under an impact load is then computed for a wide range of dynamic properties from well-separated modal frequencies to very closely spaced modal frequencies. The natural frequencies and modal damping ratios identified by the HHT method are compared with the theoretical values and those identified using the FFT method. Finally, a 36-storey shear building with a 4-storey light appendage on its top subject to an ambient ground motion is analysed using both the HHT method and the FFT method to further examine the applicability of the HHT method for the system with closely spaced modes of vibration.

2. Hilbert-Huang transform (HHT)

The Hilbert-Huang transform (HHT) method is a two-step data analysing method (Huang *et al.* 1998). The first step is the empirical mode decomposition (EMD) method by which a complicated time history can be decomposed into a series of intrinsic mode functions (IMF) that admit well-behaved Hilbert transforms. This decomposition is based on the direct extraction of the energy associated with various intrinsic time scales of the time history itself. For instance, suppose $x(t)$ denotes the measured structural response time history to be analysed. The upper and lower envelopes of $x(t)$ are constructed by connecting the local maxima and minima of $x(t)$, respectively, using a cubic spline function. The upper and lower envelopes can also be constructed using other types of spline functions such as a taut spline function, but the improvement was found marginal (Huang *et al.* 1998). The mean of the two envelopes is then computed and subtracted from the original time history. This procedure is termed a sifting process. The difference between the original time history and the mean value, c_1, is called the first IMF if it satisfies the following two conditions: (1) within the data range, the number of extrema and the number of zero-crossings are equal or differ by one only; and (2) the envelope defined by the local maxima and the envelope defined by the local minima are symmetric with respect to the mean. The difference between $x(t)$ and c_1 is then treated as a new time history and subjected to the same sifting process, giving the second IMF. The EMD procedure continues until the residual response becomes so small that it is less than a predetermined value of consequence, or the residual response becomes a monotonic function. The original time history $x(t)$ is finally expressed as the sum of the IMFs plus the final residual,

$$x(t) = \sum_{j=1}^{N} c_j(t) + r(t)_N \tag{1}$$

where N is the number of IMF components; and r_N is the final residual.

To avoid any mode mixing during the sifting process, a criterion was suggested by Huang *et al*. (1999) to separate the waves of different periods into different modes based on the period length. The criterion, termed the intermittency check, is then designed as the lower limit of the frequency that can be included in any given IMF component. It can be achieved by specifying a cutoff frequency ω_c for each IMF during its sifting process, in which the data having frequencies lower than ω_c will be removed from the resulting IMF. The intermittency check, however, should be applied with care, for any addition criterion introduced in the sifting process implies an intervention with a subjective condition.

Having obtained the IMF components from the time history $x(t)$, the second step of the HHT method is implemented by performing the Hilbert transform (HT) to each IMF component. The Hilbert transform of a real-valued function $y(t)$ in the range $-\infty < t < \infty$ is a real-valued function $\tilde{y}(t)$ defined as (Bendat and Piersol 1986)

$$\tilde{y}(t) = H[y(t)] = \int_{-\infty}^{\infty} \frac{y(u)}{\pi(t-u)} du \tag{2}$$

where H denotes the Hilbert transform operator. An analytical signal associated with $y(t)$ can then be defined in complex terms as

$$z(t) = y(t) + i\tilde{y}(t) = A(t)e^{i\theta(t)} \tag{3}$$

$$A(t) = [y^2(t) + \tilde{y}^2(t)]^{1/2} \qquad \theta(t) = \tan^{-1}\left[\frac{\tilde{y}(t)}{y(t)}\right] \tag{4}$$

where $A(t)$ and $\theta(t)$ are defined as the amplitude and instantaneous phase angle of $y(t)$, respectively; and i is the imaginary unit. The instantaneous frequency $\omega(t)$ is then given by

$$\omega(t) = d\theta(t)/dt \tag{5}$$

It is pointed out by Huang *et al*. (1998) that the definition of instantaneous frequency for a signal has physical significance only if it is an IMF. This is the reason why a signal should be decomposed into the IMFs using the EMD method before applying the Hilbert transform.

After implementing the Hilbert transform to each IMF component $c_j(t)$ $(j = 1, 2, ..., N)$, the original time history, excluding the final residual, can then be expressed as the real part (*Re*) of the sum of the Hilbert transforms of all the IMF components.

$$X(t) = \boldsymbol{Re}\sum_{j=1}^{N} a_j(t)e^{i\int\omega_j(t)dt} \tag{6}$$

where $a_j(t)$ and $\omega_j(t)$ are the instantaneous amplitude and frequency of the i^{th} IMF component. Thus, the amplitude is not only a function of time but also a function of frequency. The frequency-time distribution of the amplitude is designated the Hilbert amplitude spectrum, $H(\omega, t)$, or simply the Hilbert spectrum, from which the inherent characteristics of a non-linear and/or nonstationary time history can be identified.

3. HHT for damping ratio identification

3.1 Linear SDOF systems

The Hilbert transform (HT) has long been used to study linear and non-linear dynamic systems and to identify their modal parameters in the frequency-time domain (Feldman 1985, 1994, Hammond 1987). For a linear SDOF system under the impulsive loading, the impulse displacement response function of the system $v(t) = 0$ for $t < 0$ and

$$v(t) = A_0 e^{-\xi\omega_0 t}\sin\omega_d t, \quad t \ge 0 \tag{7}$$

where ω_0 is the natural circular frequency of the system; ξ is the damping ratio; ω_d is the damped natural circular frequency; and A_0 is a constant that depends on the intensity of impulsive loading and the mass and frequency of the system. For convenience of applying the Hilbert transform, the impulse response function can be extended to the negative domain by considering its mirror image.

$$v(t) = A_0 e^{-\xi\omega_0|t|}\sin\omega_d t, \quad -\infty < t < \infty \tag{8}$$

By applying the HT method, the signal $z(t)$ for $v(t)$ can thus be obtained using Eq. (3).

$$z(t) = v(t) + i\tilde{v}(t) = A(t)e^{-i\theta(t)} \tag{9}$$

For a special case in which ξ is small and ω_0 is large, the amplitude $A(t)$ and the phase angle $\theta(t)$ for the SDOF system can be obtained as follows (Yang and Lei 1999).

$$A(t) = A_0 e^{-\xi\omega_0 t} \tag{10}$$

$$\theta(t) = \omega_d t - \pi/2 \tag{11}$$

By introducing the logarithmic and differential operators to Eqs. (10) and (11) respectively, one obtains

$$\ln A(t) = -\xi\omega_0 t + \ln A_0 \tag{12}$$

$$\omega(t) = \frac{d\theta(t)}{dt} = \omega_d \tag{13}$$

Therefore, the damped natural circular frequency ω_d can be identified from the instantaneous frequency $\omega(t)$. With the identified ω_d and the slope $-\xi\omega_0$ of the straight line of the decaying amplitude $A(t)$ in a semi-logarithmic scale, the damping ratio ξ can be identified from the function $\omega_d = \omega_0\sqrt{1-\xi^2}$. Considering that the instantaneous frequency may fluctuate around its mean value due to the amplitude variation of the signal (Huang 1998) and that the requirement for small damping ratio may limit the application of the HT method, Yang and Lei (1999) suggested the following procedures for the system identification of SDOF systems based on the HT method: (1) determine the damped frequency ω_d from the slope of the phase function $\theta(t)$ using a linear least-squares fitting technique; and (2) determine the damping ratio ξ by applying the linear least-squares fitting technique to the decaying amplitude $A(t)$ in a semi-logarithmic scale.

3.2 Linear MDOF systems

For a linear MDOF system, its dynamic displacement response $\boldsymbol{v}(t)$ caused by the external excitation $\boldsymbol{P}(t)$ can be expressed as the superposition of modal displacement responses.

$$\boldsymbol{v}(t) = \sum_{j=1}^{N} \boldsymbol{v}_j(t) = \sum_{j=1}^{N} \boldsymbol{\Phi}_j Y_j(t) \tag{14}$$

where N is the number of modes of vibration involved; $\boldsymbol{v}_j(t)$ is the j^{th} modal displacement response vector; $\boldsymbol{\Phi}_j$ is the j^{th} mode shape; and $Y_j(t)$ is the j^{th} modal response which can be computed from the following uncoupled modal equation.

$$\ddot{Y}_j(t) + 2\xi_j\omega_j\dot{Y}_j(t) + \omega_j^2 Y_j(t) = P_j(t)/M_j, \quad j = 1, 2, \ldots, N \tag{15}$$

where ξ_j and ω_j are the j^{th} modal damping ratio and natural circular frequency, respectively; and $P_j(t)$ and M_j are the jth modal force and modal mass, respectively.

To identify ξ_j and ω_j ($j = 1, \ldots, N$) of a MDOF system from the measured response time histories of the system under impulsive loading, the HHT method can be applied. The measured response time history is decomposed into the IMFs using the EMD method with the intermittency check to obtain modal response time histories. For the MDOF system with closely spaced modes of vibration, the cutoff frequencies used in the sifting process with the intermittency check are determined from the power spectrum of the measured response time history. Then, for each of identified modal response time histories, the HT identification procedure for a SDOF system can be applied to identify the modal parameters ξ_j and ω_j. If the response time histories of a MDOF system are measured under ambient excitation other than impulsive loading, the random decrement technique (RDT) is applied to each of the modal response time histories identified by the EMD method with the intermittency check to obtain the modal free response time histories. Then, the HT identification procedure for a SDOF system is applied to each of the modal free response time histories to identify the modal parameters ξ_j and ω_j.

4. Application to 2DOF systems

To explore the possibility of applying the aforementioned HHT method to identify the modal

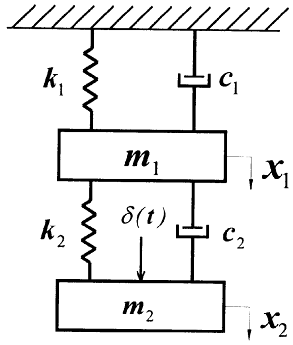


Fig. 1 Two-degrees-of-freedom (2DOF) system

damping ratios of a structure with closely spaced modal frequencies, a 2DOF system under an impulsive load, as shown in Fig. 1, is studied for a wide range of dynamic properties from well-separated modal frequencies to very closely spaced modal frequencies. For the simplicity, the impulsive load is applied to one mass only and the Rayleigh damping model is employed for the system. The equation of motion of the 2DOF system can be easily established as follows:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \left(a\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} + b\begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \right)\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ \delta(t) \end{Bmatrix} \tag{16}$$

where m_1 and m_2 are the two masses; k_1 and k_2 are the two stiffness coefficients; $\delta(t)$ is the impulsive load; it is equal to 1 when $t = 0$ and equal to zero when $t \neq 0$; and the damping factors a and b are given by

$$a = \frac{2\omega_1\omega_2}{\omega_1 + \omega_2}\xi, \quad b = \frac{2}{\omega_1 + \omega_2}\xi \tag{17}$$

where ω_1 and ω_2 are the two undamped natural circular frequencies of the 2DOF system. The two modal damping ratios are assumed the same for the two modes of vibration. Three typical levels of modal damping ratio ξ(1%, 3% and 5%) are considered in this investigation to see the effects of the modal damping ratio on the applicability of the HHT method. By changing the structural parameters m_1, k_1, m_2 and k_2 of the system, the difference (space) between the two natural frequencies are altered. To present a measure of the space (difference) between the two undamped natural frequencies f_1 and f_2, a space index (MSC NASTRAN 1983) is adopted here.

$$\gamma = (f_2 - f_1)/(f_2 + f_1) \tag{18}$$

The index γ varies in a [0, 1] range; a large value corresponds to well-separated modes of vibration while a small value denotes closely spaced modes of vibration. The maximum index considered in this application is 0.75 with f_1 = 1.10 Hz and f_2 = 7.71 Hz, and the minimum index is 0.046 with f_1 = 3.39 Hz and f_2 = 3.72 Hz. The space index less than 0.046 is hardly achieved for the 2DOF system investigated.

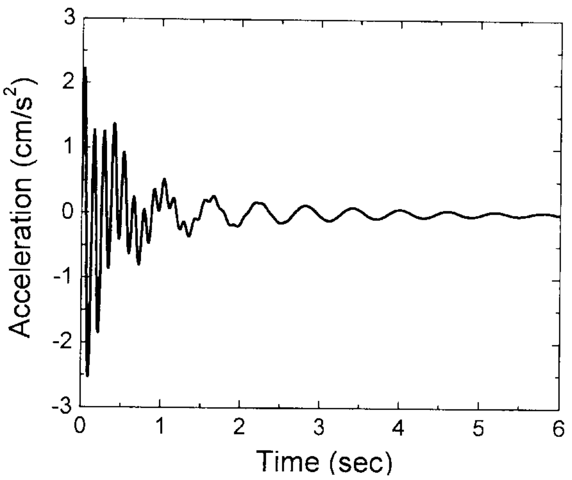


Fig. 2 Acceleration time history for well-separated modes of vibration

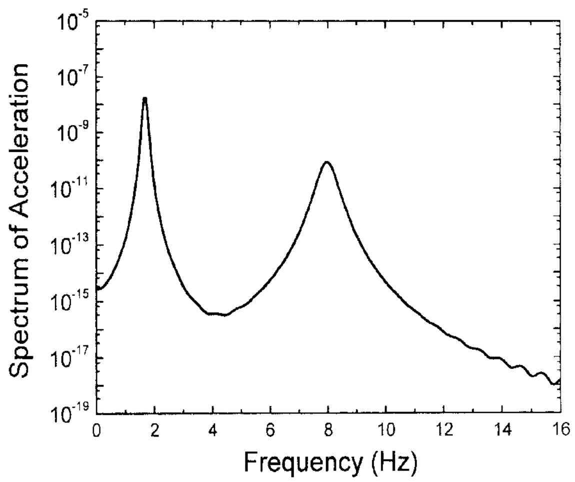


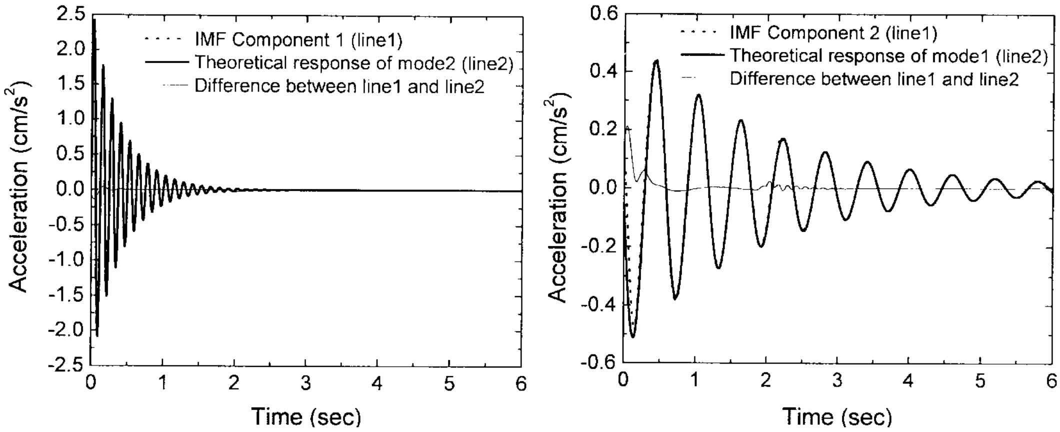
Fig. 3 Power spectrum for the time history shown in Fig. 2

The impulse responses of the two masses can be computed according to Eqs. (14) and (15). The HHT method is applied to the impulse acceleration response time histories of mass 2 to identify the two modal damping ratios, which are then compared with the preset theoretical values and those identified by the traditional FFT method. Since it is a 2DOF system, only one cutoff frequency for the first IMF component is designed in the implementation of intermittency check during the EMD. The cutoff frequency is decided in such a way that it corresponds to the trough of the response power spectrum between the two natural frequencies.

Let us first examine the case of the system with well-separated modes of vibration. The structural parameters are $m_1 = 1$ kg, $m_2 = 2$ kg, $k_1 = 2256$ N/m, and $k_2 = 250$ N/m. The two undamped natural frequencies are calculated as $f_1 = 1.68$ Hz and $f_2 = 7.99$ Hz, and accordingly the space index is 0.66. The preset theoretical value for the two modal damping ratios is 5%. Fig. 2 and Fig. 3 depict the acceleration response time history of mass 2 and its power spectrum. From the power spectrum, the two natural frequencies and modal damping ratios can be easily identified using the FFT method in conjunction with the bandwidth method. The results are listed in Table 1. To identify the modal damping ratios of the system using the HHT method, the EMD method with the intermittency check is applied to the acceleration response time history of mass 2 with the cutoff frequency of 4.39 Hz, resulting in a total of 5 IMF components. The first and second IMF components are plotted in Figs. 4a and 4b, respectively, together with the theoretical second and first modal responses of mass 2 of the

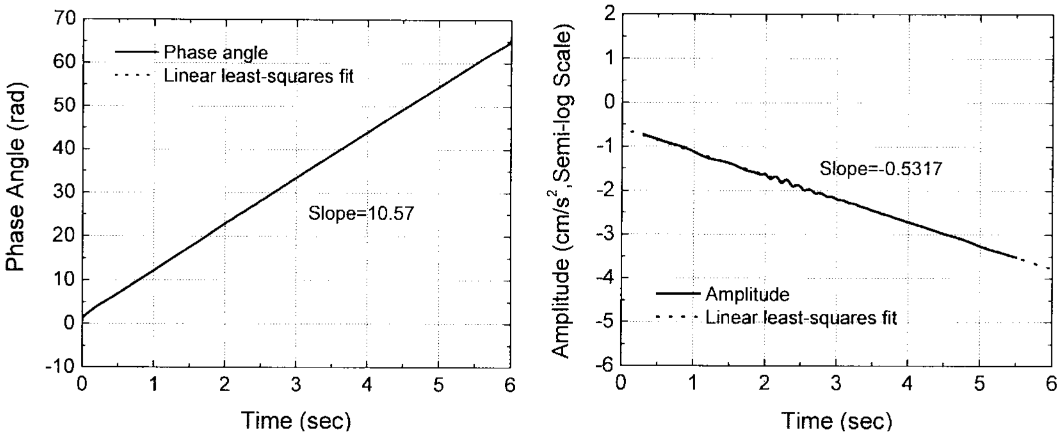
Table 1 2DOF system with well-separated modal frequencies

	Damped Frequency (Hz)			Damping ratio		
	Theoretical Value	FFT	HHT	Theoretical Value	Bandwidth Method	HHT
First Mode	1.68	1.71	1.68	5%	4.62%	5.02%
Second Mode	7.98	7.99	7.99	5%	4.81%	5.01%



(a) (b)

Fig. 4 The IMF components and the modal responses (well-separated modes): (a) The first IMF component and the second modal response, (b) The second IMF component and the first modal response



(a) (b)

Fig. 5 Instantaneous functions of the first IMF component (well-separated modes): (a) Phase angle and linear least-squares fit, (b) Amplitude and linear least-squares fit

system correspondingly. The differences between the IMF component and the corresponding model response are also plotted in Figs. 4a and 4b. It is noted that the first IMF component is the same as the second modal response of mass 2 of the system and the second IMF component is almost the same as the first modal response of mass 2 of the system. This implies that the first two IMF components identified by the EMD method have definite physical characteristics. Then, by applying the Hilbert transform to the first two IMF components, the instantaneous phase angle and amplitude of each IMF component can be obtained as a function of time. Figs. 5a and 5b display the instantaneous phase angle and amplitude of the first IMF component together with the fit lines using the linear least-squares fit technique. Finally, the modal damping ratios and natural frequencies can

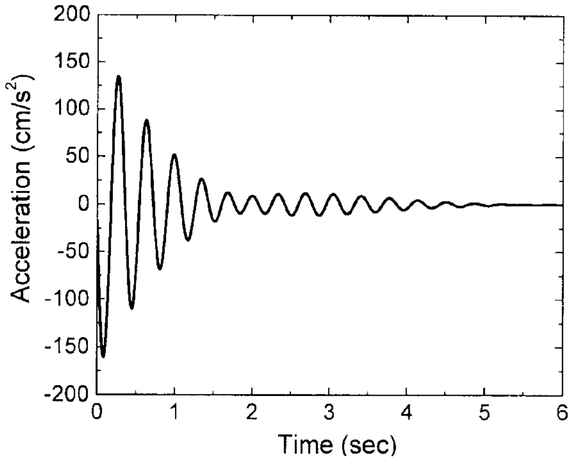


Fig. 6 Acceleration time history for closely spaced modes of vibration

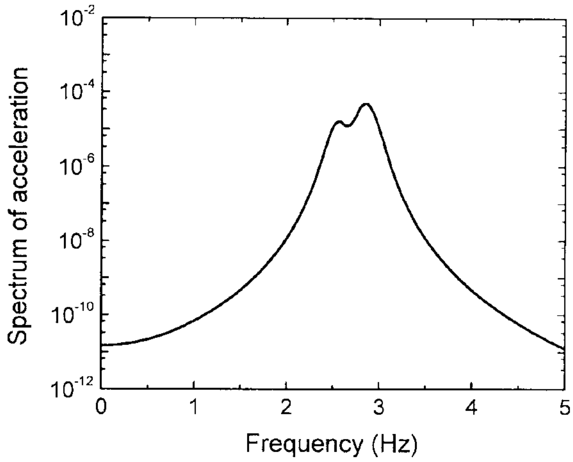


Fig. 7 Power spectrum for the time history shown in Fig. 6

be identified from the slopes of the two fit lines, and the results are listed in Table 1. It is seen from Table 1 that the natural frequencies identified by the HHT method and the FFT method both are very close to the theoretical values. The modal damping ratios identified by the FFT method are slightly smaller than the theoretical values, but the modal damping ratios identified by the HHT method are very close to the theoretical values.

Now, let us examine the system of very closely spaced modes of vibration. The structural parameters are selected as $m_1 = 11.6$ kg, $m_2 = 0.1$ kg, $k_1 = 3256$ N/m, and $k_2 = 30$ N/m. The two modal damping ratios are set as 5%. The two undamped natural frequencies are computed as $f_1 = 2.58$ Hz and $f_2 = 2.85$ Hz. The space index of the frequencies is thus 0.050. Fig. 6 and Fig. 7 show, respectively, the acceleration response time history of mass 2 of the system and its power spectrum. It is seen from the power spectrum that there are two peaks corresponding to the two damped natural frequencies. However, these two peaks are too close to each other to identify the two modal damping ratios using the bandwidth method. Thus, the HHT method is applied to the acceleration response time history of mass 2 to identify the modal damping ratios. The cutoff frequency used in the EMD is 2.68 Hz, which corresponds to the trough of the response spectrum between the two peaks. A total of 6 IMF components are obtained after having a proper sifting process in the EMD. The first and second IMF components are plotted in Figs. 8a and 8b, respectively, together with the second and first modal responses of mass 2. It is seen that due to the closely spaced modes of vibration, the first two IMF components deviate from the theoretical modal responses to some extent. The instantaneous phase angles and amplitudes obtained by the HHT method and their linear least-squares fits are shown in Figs. 9a and 9b for the first IMF component and in Figs. 10a and 10b for the second IMF component. Clearly, the phase angle for each mode is almost a linear function of time, but the amplitude as a function of time deviates from a linear function in the semi-logarithmic coordinate, in particular for the second IMF component. The natural frequencies and modal damping ratios identified by the HHT method are listed in Table 2 together with the theoretical values. It is seen that the natural frequencies identified by the HHT method and the FFT method are quite close to the theoretical values. The two modal damping ratios

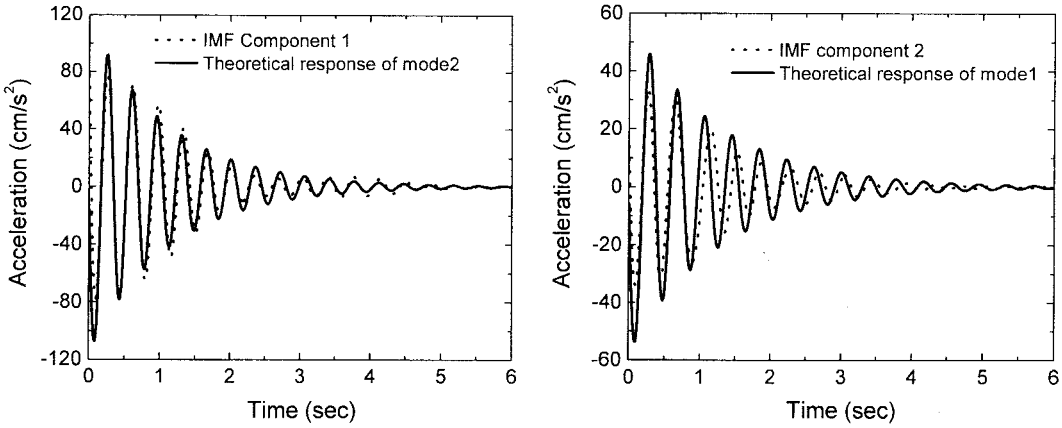


Fig. 8 The IMF components and the modal responses (closely spaced modes): (a) The first IMF component and the second modal response, (b) The second IMF component and the first modal response

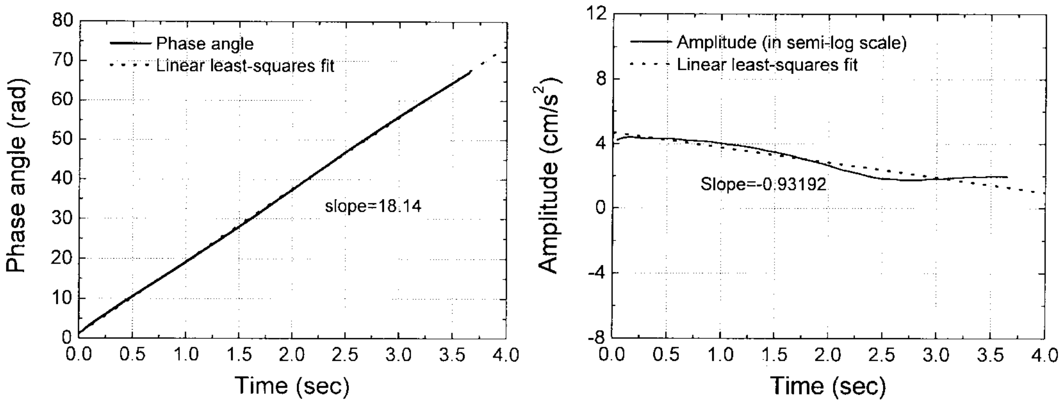
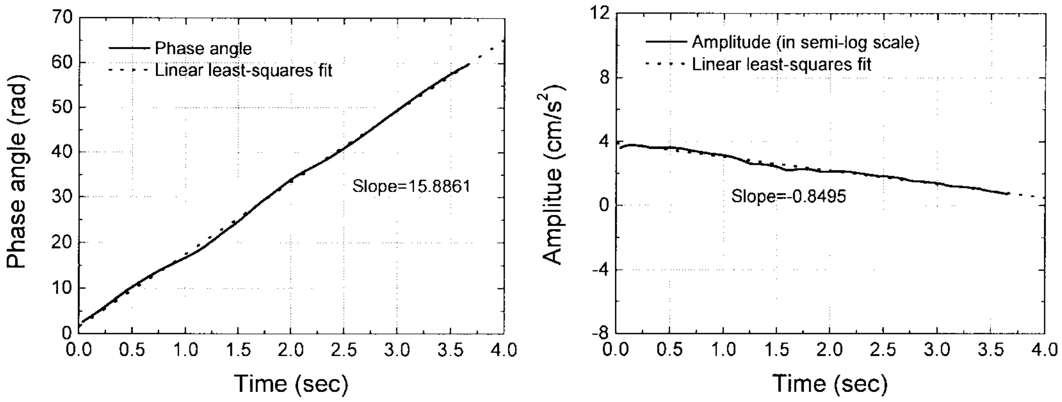


Fig. 9 Instantaneous functions of the first IMF component (closely spaced modes): (a) Phase angle and linear least-squares fit, (b) Amplitude and linear least-squares fit

identified by the HHT method are larger than the theoretical values but differ by less than 7%. The FFT method together with the bandwidth method, however, fails to identify the modal damping ratios.

It is interesting to know how sensitive the modal damping ratios identified by the HHT method are to the cutoff frequency. A series of cutoff frequencies are thus selected around 2.68 Hz for the system of very closely spaced modes of vibration and below 7.7 Hz for the system of well-separated modes of vibration, and the identified modal damping ratios are shown in Fig. 11a and Fig. 11b correspondingly. For the system of very closely spaced modes of vibration, it is encouraging to see that the cutoff frequency has almost no effects on the two modal damping ratios within a range from 2.65 to 2.72 Hz. However, when the cutoff frequency is out of this range, that is, when it further approaches to either the first natural of frequency of 2.58 Hz or the second

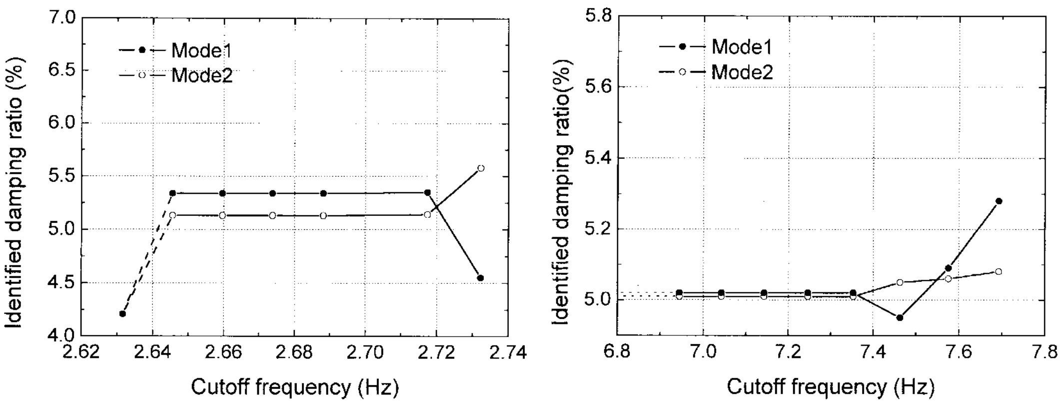


(a) (b)

Fig. 10 Instantaneous functions of the second IMF component (closely spaced modes): (a) Phase angle and linear least-squares fit, (b) Amplitude and linear least-squares fit

Table 2 2DOF system with very closely spaced modal frequencies

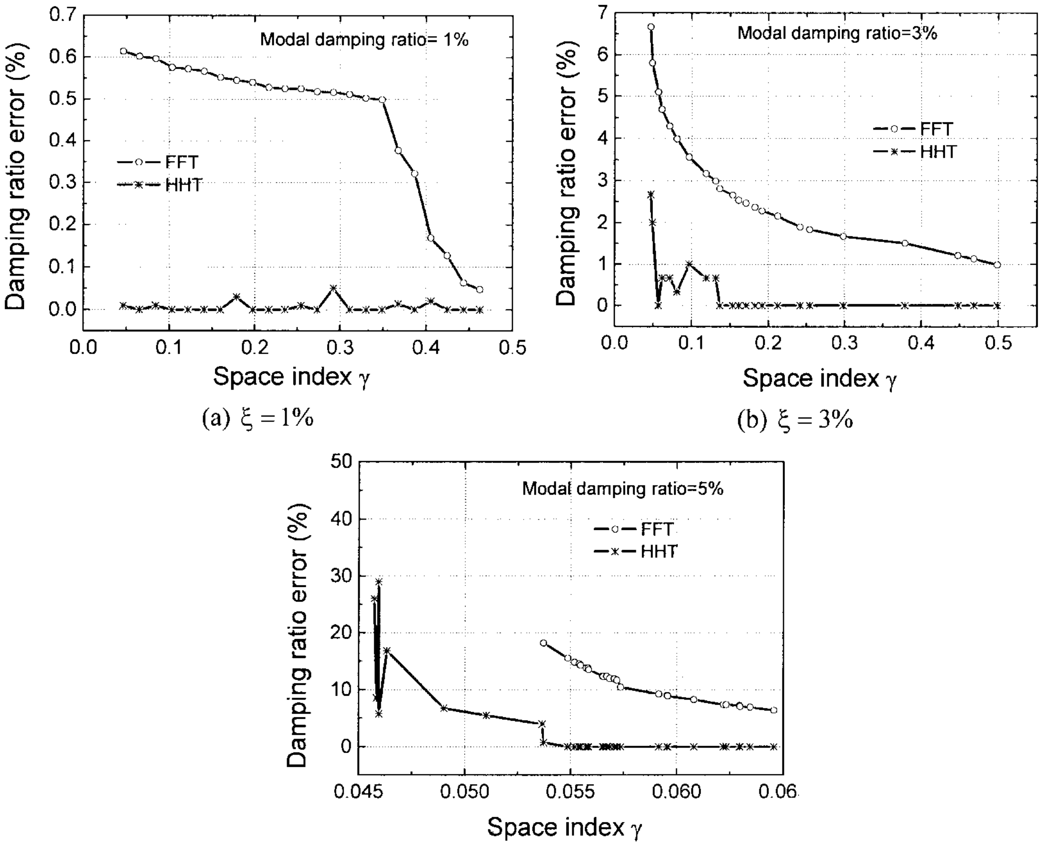
	Damped Frequency (Hz)			Damping ratio		
	Theoretical Value	FFT	HHT	Theoretical Value	Bandwidth method	HHT
First Mode	2.58	2.56	2.53	5%	-----	5.34%
Second Mode	2.85	2.87	2.88	5%	-----	5.13%



(a) (b)

Fig. 11 Sensitivity of the modal damping ratio identification to the cutoff frequency: (a) 2DOF system with closely spaced modes of vibration, (b) 2DOF system with well-separated modes of vibration

natural frequency of 2.85 Hz, the two modal responses cannot be successfully separated by the EMD and accordingly the two modal damping ratios cannot be satisfactorily estimated. For the system of well-separated modes of vibration, it is seen from Fig. 11b that when the cutoff frequency



(a) $\xi = 1\%$ (b) $\xi = 3\%$ (c) $\xi = 5\%$

Fig. 12 Variation of damping ratio error with space index; (a) $\xi = 1\%$, (b) $\xi = 3\%$, (c) $\xi = 5\%$

is less than 7.3 Hz, the cutoff frequency again has almost no effects on the modal damping ratios identified by the HHT method. Only when the cutoff frequency is greater than 7.4 Hz, that is, when it further moves towards the second natural frequency of 7.99 Hz, the influence of cutoff frequency on the two modal damping ratios become more and more significant.

To examine further the applicability of the HHT method for the identification of modal damping ratios, the first modal damping ratio of the 2DOF system with various spaced modal frequencies is identified using both the HHT method and the FFT method. The identified results are plotted, in terms of the relative damping ratio error against the space index, in Figs. 12a, 12b, and 12c for the cases of the theoretical modal damping ratio of 1%, 3% and 5%, respectively. The relative damping ratio error is defined as the ratio of the absolute value of the difference between the identified damping ratio and the theoretical value to the theoretical value. It is seen that for the case of 1% modal damping ratio, the modal damping ratio identified by the HHT method is almost the same as the theoretical value and independent of the space index. The relative damping ratio error caused by the FFT method, however, increases with the decrease of space index but the relative error is smaller than 0.7%. For the case of 3% modal damping ratio, the modal damping ratio identified by the HHT method is almost the same as the theoretical value if the space index is less than 0.13. Then, with the decrease of space index, the relative error increases but it is less than 3% within the concerned range. The deviation of the modal damping ratio identified by the FFT method increases with the decrease of space index but the maximum error in the concerned space index range is less than 7%. For the case of 5% modal damping ratio, the FFT-based bandwidth method fails to identify the modal damping ratio when the space index γ becomes less than 0.053 but the HHT method is still workable. Though the relative damping ratio error becomes large for the case of 5% modal damping ratio, it is less than 10% when γ is larger than 0.05 and when the HHT method is used.

The above results clearly demonstrate that the HHT method is superior to the FFT method in the identification of modal damping ratios of the structure with closely spaced modes of vibration. The above results also manifest that the error involved in the modal damping ratio identification is larger for the systems of closely spaced modes of vibration than for the systems of well-separated modes of vibration. For the systems of closely spaced modes of vibration, the error then increases with the increasing modal damping ratio. This is the inherent nature of the problem: when the two natural frequencies are too close and when the corresponding two modal damping ratios are larger, the vibration energy in these two modes of vibration is mixed and cannot be clearly separated. Moreover, in consideration of the complex nature of the modal damping in a real structure, a 10% error in the modal damping ratio may be still acceptable in practice for most cases.

5. Application to shear building with light appendage

Toward the real application of the HHT method to civil engineering structures, a 36-storey building with a 4-storey light appendage on is top (see Fig. 13) is analyzed. All the 36 floor units in the main building are assumed to be identical. The mass of the building is lumped at the horizontal rigid floors and the columns are assumed to be massless so that the building can be seen as a shear type of building for the simplicity of investigation. The structural parameters of the building are: the lumped mass at each floor is 1.29×10^6 kg; the elastic shear stiffness in each storey is 10^9 N/m. The

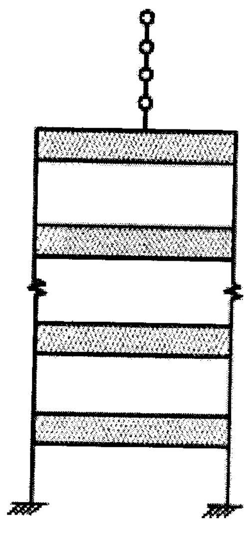


Fig. 13 Shear building with light appendage

light appendage is modeled as a lumped mass system of four-degrees-of-freedom. The four lumped masses are the same, each having a mass ratio of 0.02 to the lumped mass at the building floor, i.e., 2.58×10^4 kg. The stiffness of the appendage is 0.03% of the storey shear stiffness of the building, i.e., 3×10^5 N/m. The first four undamped natural frequencies of the system are found to be 0.184, 0.196, 0.542, 0.573 Hz. The space index is 0.032 for the first two natural frequencies and 0.028 for the third and fourth natural frequencies. For the structural damping of the system, an extended Rayleigh damping model is adopted and given by (Clough and Penzien 1993)

$$\boldsymbol{C} = \boldsymbol{M}\left[\sum_{j=1}^{N}\frac{2\xi_j\omega_j}{M_j}\boldsymbol{\Phi}_j\boldsymbol{\Phi}_j^T\right]\boldsymbol{M} \tag{19}$$

where $\boldsymbol{M}$ and $\boldsymbol{C}$ are the mass and damping matrix of the system, respectively; $\boldsymbol{\Phi}_j$ is the j^{th} mode shape vector; M_j, ξ_j and ω_j are the j^{th} modal mass, modal damping ratio, and undamped natural circular frequency of the system, respectively; N is the number of modes of vibration; and T denotes the matrix transposition. To achieve the damping matrix $\boldsymbol{C}$ for the system, the modal damping ratios in the first four modes of vibration ($N = 1, 2, 3, 4$) are assumed to be identical as 1% and the other modal damping ratios are taken as zero.

The shear building with a light appendage is subjected to ambient ground motion. The ambient ground motion is modeled as a white-noise random process, which is then converted to the ground motion time history with a peak acceleration of 0.04 m/s^2 (see Fig. 14a). The dynamic response of the system to the ground motion is computed. Fig. 14b depicts the acceleration response at the top of the appendage and Fig. 15 shows its power spectrum. It is seen from Fig. 15 that the first two spectral peaks are very close to each other and the third and fourth spectral peaks are also close to each other. Nevertheless, the bandwidth method can be still applied to the response spectrum to estimate the first four natural frequencies and modal damping ratios but with some degrees of uncertainty. The results are listed in Table 3.

To apply the HHT method, the acceleration response at the top of the appendage is decomposed into the IMFs using the EMD method with the intermittency check. The cutoff frequencies used for

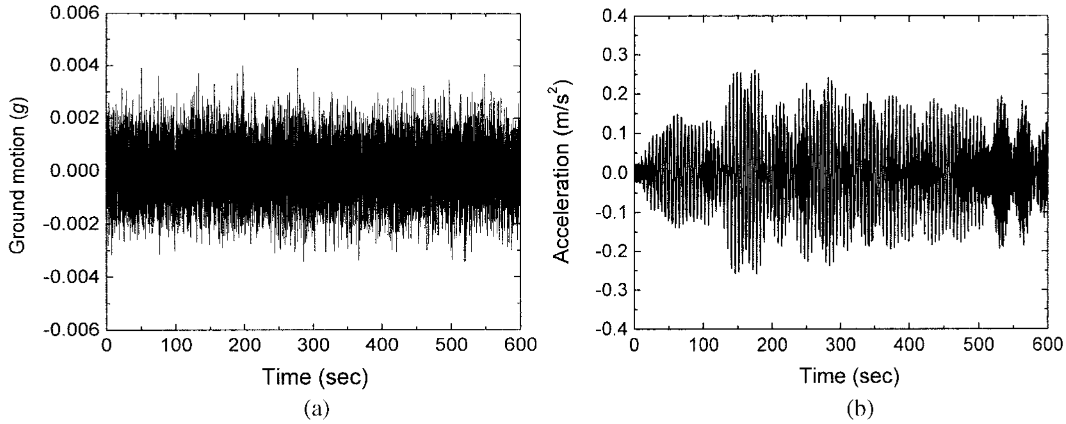


Fig. 14 Ground motion and acceleration response time histories: (a) Ground motion, (b) Acceleration response at the top of appendage

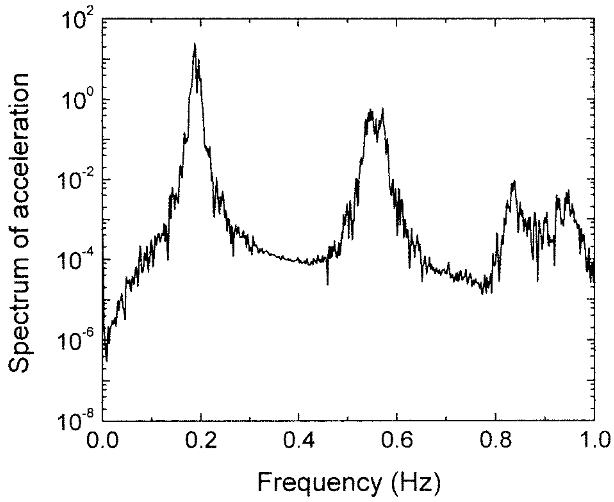


Fig. 15 Power spectrum of the time history shown in Fig. 14b

Table 3 Shear building with light appendage

	Damped Frequency (Hz)			Damping ratio		
	Theoretical Value (Hz)	FFT	HHT	Theoretical Value	Bandwidth method	HHT
Mode1	0.1836	0.1892	0.1833	1%	1.23%	1.01%
Mode2	0.1956	0.1953	0.1936	1%	1.52%	1.13%
Mode3	0.5419	0.5416	0.5505	1%	1.80%	1.09%
Mode4	0.5727	0.5722	0.5706	1%	1.86%	1.07%

the first 5 IMF components are 0.9, 0.7, 0.559, 0.4 and 0.1923 Hz in sequence, which are decided from the troughs of the Fourier spectrum shown in Fig. 15. According to the experience gained from the 2DOF systems, a relatively large variation of the cutoff frequency around 0.9, 0.7 and 0.4 Hz will have little effects on the identified results, but the allowable variation of the cutoff frequency around 0.559 and 0.1923 Hz is limited. The first four modal response time histories identified by the EMD method are plotted in Fig. 16. Since these modal responses are the total modal responses other than the free modal responses, the random decrement technique (RDT) is then applied to the total modal responses to obtain the free modal responses to which the HT method can be applied. When applying the RDT to each of the total modal responses, a threshold level should be selected to obtain N segments. The j^{th} free acceleration modal response $\ddot{x}_j^f(t)$ can be then obtained from the ensemble average of all segments as

$$\ddot{x}_j^f(i\Delta t) = \frac{1}{N}\sum_{k=1}^{N}\ddot{x}_j^t(t_k + i\Delta t); \quad i = 1, 2, \ldots, m \tag{20}$$

where $\ddot{x}_j^t(t)$ is the j^{th} total acceleration modal response; t_k is the starting time for each segment; and

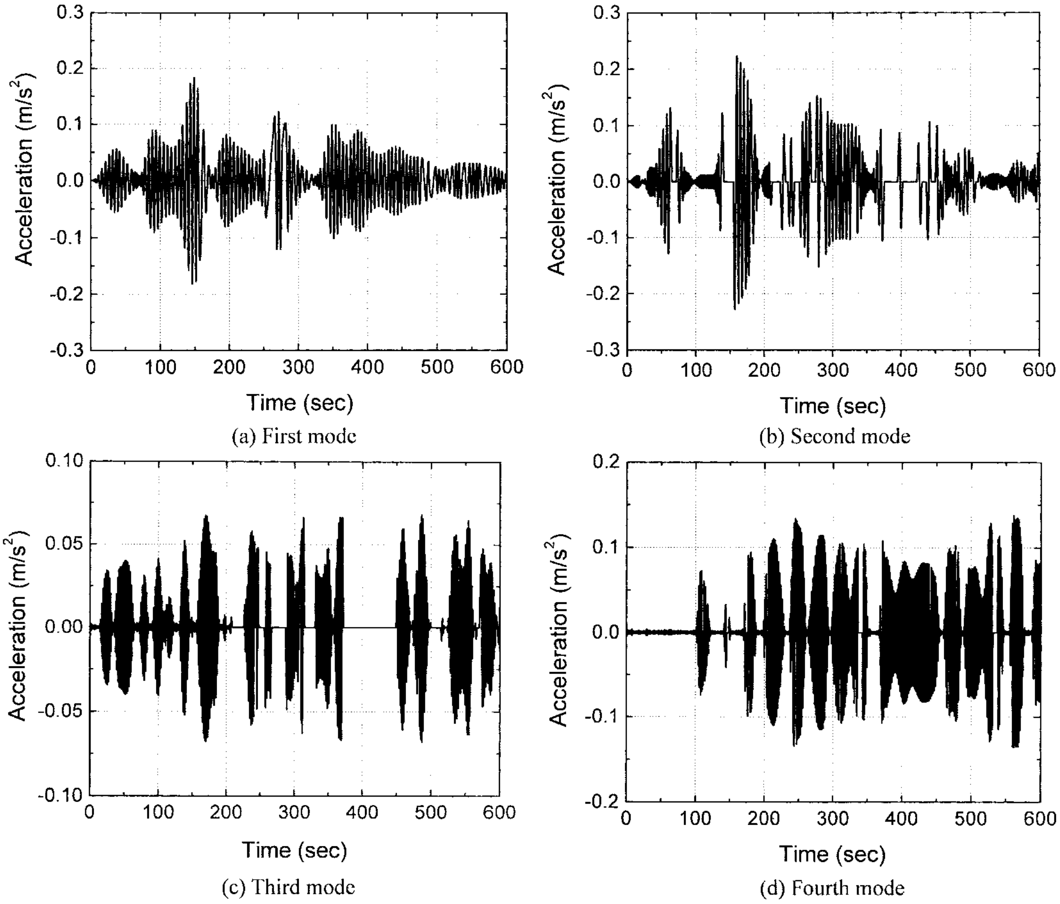


Fig. 16 The first four total modal responses: (a) First mode, (b) Second mode, (c) Third mode, (d) Fourth mode

$m\Delta t = t$ with t being the duration of each segment. For the particular shear building, the time duration used is 45 seconds for the first two modal responses and 15 seconds for the third and fourth modal responses. The number of segments is 125, 104, 338 and 364 for the first, second, third, and fourth modal responses, respectively. The time interval Δt is 0.01 second. Finally, the application of the HT method to each of the free modal response time histories yields the instantaneous phase angle and amplitude functions. Figs. 17a and 17b shows, respectively, the instantaneous phase angle and amplitude functions from the first free modal response and their linear least-squares fits. The damped natural frequencies and modal damping ratios of the building obtained using the HHT method are given in Table 3 together with the theoretical values and those identified using the FFT method.

It is seen that the first four natural frequencies of the building are satisfactorily identified by both the HHT method and the FFT method. The first four modal damping ratios of the building are also adequately identified by the HHT method with the maximum error of 13%. The FFT method, however, overestimates the modal damping ratios by more than 85%. Thus, it can be concluded that

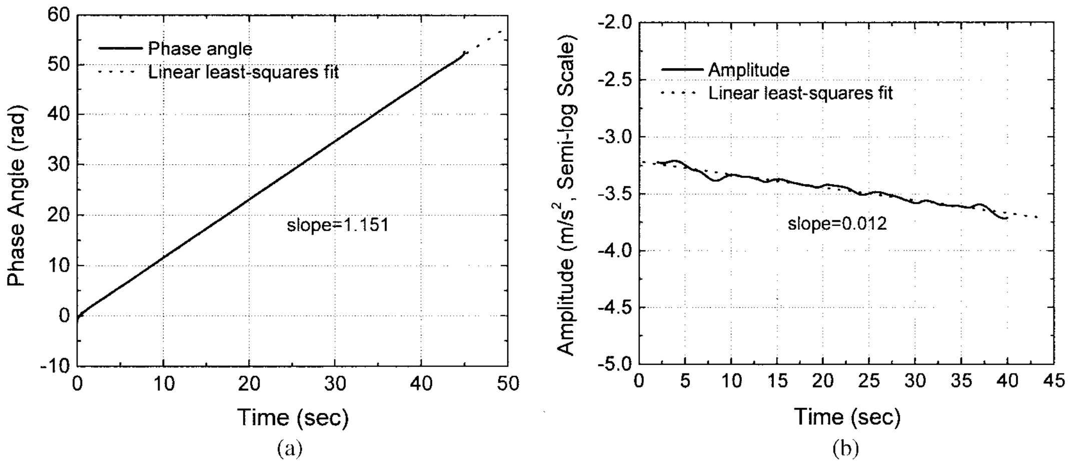


Fig. 17 Instantaneous functions of the first free modal response (shear building): (a) Phase angle and linear least-squares fit, (b) Amplitude and linear least-squares fit

the HHT-based method is superior to the FFT-based bandwidth method in the identification of modal damping ratios of the structures with closely spaced modes of vibration. The HHT method performing in the frequency-time domain seems to be a promising tool for system identification of civil engineering structures. As for the computation efforts required for the identification in this example, the HHT method needs more computational efforts than the FFT method.

6. Conclusions

The application of the Hilbert-Huang transform to the identification of modal damping ratio of the structures with closely spaced modes of vibration have been investigated using different types of structures of a variety of dynamic properties under different types of excitations. The natural frequencies and modal damping ratios identified by the HHT method are compared with the preset theoretical values and those from the FFT-based bandwidth method. The results show that the FFT-based bandwidth method sometimes fails to identify modal damping ratios when the two modal frequencies are too close to each other but the HHT method is still workable. The modal damping ratios identified by the HHT method are more accurate than those from the FFT-based bandwidth method. The HHT method performing in the frequency-time domain seems to be a promising tool for the identification of modal damping ratios of the structures with closely spaced modes of vibration.

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