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Influence of end fixity on post-yield behaviors of a tubular member

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Abstract. For the evaluation of the capability of a tubular member of an offshore structure to absorb the collision energy, a simple method can be employed for the collision analysis without performing the detailed analysis. The most common simple method is the rigid-plastic method. However, in this method any characteristics for horizontal movement and rotation at the ends of the corresponding tubular member are not included. In a real structural system of an offshore structure, tubular members sustain a certain degree of elastic support from the adjacent structure. End fixity has influences in the behaviors of a tubular member. Three-dimensional FEM analysis can include the effect of end fixity fully, however in viewpoints of the inherent computational complexities of the 3-D approach, this is not the recommendable analysis at the initial design stage. In this paper, influence of end fixity on the behaviors of a tubular member is investigated, through a new approach and other approaches. A new analysis approach that includes the flexibility of the boundary points of the member is developed here. The flexibility at the ends of a tubular element is extracted using the rational reduction of the modeling characteristics. The property reduction is based on the static condensation of the related global stiffness matrix of a model to end nodal points of the tubular element. The load-displacement relation at the collision point of the tubular member with and without the end flexibility is obtained and compared. The new method lies between the rigid-plastic method and the 3-demensional analysis. It is self-evident that the rigid-plastic method gives high strengthening membrane effect of the member during global deformation, resulting in a steeper slope than the present method. On the while, full 3-D analysis gives less strengthening membrane effect on the member, resulting in a slow going load-displacement curve. Comparison of the load-displacement curves by the new approach with those by conventional methods gives the figures of the influence of end fixity on post-yielding behaviors of the relevant tubular member. One of the main contributions of this investigation is the development of an analytical rational procedure to figure out the post-yielding behaviors of a tubular member in offshore structures.

Key words: tubular member; end fixity; post-yielding; FEM analysis.

1. Introduction

The potential hazards of a collision between a ship, and any other moving object at sea and an offshore structure can be so large that an understanding of this collision problem is certainly of importance.

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For the evaluation of the capability of a tubular member of an offshore structure to absorb the collision energy, 3-demensional numerical model approach can be used (Dexter *et al.* 1996, Hyde *et al.* 1999, Kitamura 1997). However, in view of the inherent computational complexities of the 3-D approach, a simple method is rather employed for the collision analysis without performing the detailed analysis. The most common simple method is the rigid-plastic method (Soreide 1981). In this method any characteristics for horizontal movement and rotation at the ends of the corresponding tubular member are not included. In a real frame system of an offshore structure the tubular element sustains a certain degree of elastic support from the adjacent structure. Consequently, both simple method and 3-D analysis have inherent difficulties in the collision analysis. In this paper a new approach that is simple and effective, is presented. In the method, the flexibility at the ends of a tubular element is extracted using the rational reduction of the modeling characteristics. The property reduction is based on the static condensation of the related global stiffness matrix of a model to end nodal points of the tubular element (Cho 1989). The load-displacement relation at the collision point of the tubular member with the extracted end flexibility is obtained through the procedure described in the following sections.

For the comparison of the analysis results by the new approach, the collision analysis of a typical semi-submersible rig are carried out by the simple method and 3-D numerical approach using NOAMAS program developed by Century Research Corporation (Kahaner 1990). Thus, the influence of end fixity of a tubular member is demonstrated through the analysis.

2. Rigid-plastic method

The rigid-plastic method of the analysis provides relatively simple analytical results, often with acceptable accuracy (Soreide and Amdahl 1982). Thus, the method is appropriate for early design situations, for planning experimental and numerical studies, and for checking numerical results. Numerical methods usually provide more accurate results. However, their cost, which includes program development, data preparation, computer utilization, and output analysis make their application more adequate to the final analysis of structures with relevant tests (Ludolphy and Boon 2000).

The simplest approach to the beam type of deformation is employed here to investigate the possibility of using this kind of simple approach for the collision analysis without performing the detailed analysis. The basic assumptions within the rigid-plastic method are the stress-strain

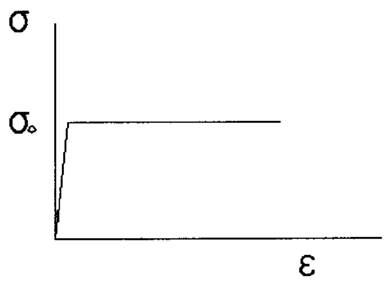

Fig. 1 Elastic perfectly plastic material idealization

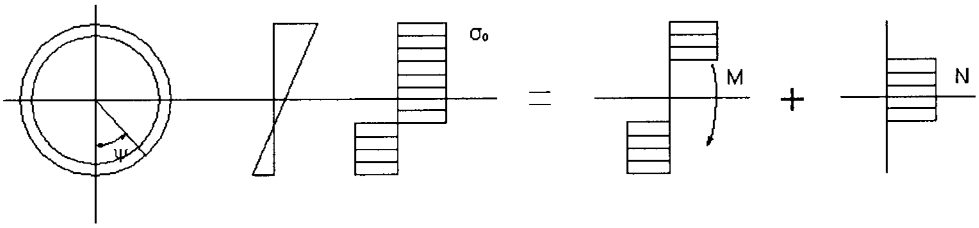

Fig. 2 Strain and stress distribution of a circular element subjected to bending and tension

behavior, indicated in Fig. 1.

Based on this assumption, consider the circular section shown in Fig. 2.

Assume that the section is subjected to a fully plastic stress distribution resulting from a combination of an axial force N and a bending moment M. The resultants of the assumed stress distribution are given by

$$N = 4\sigma_0 \int_{\psi}^{\pi/2} t(D/2)d\psi = 2\sigma_0 Dt(\pi/2 - \psi) \tag{1}$$

$$M = 4\sigma_0 \int_0^{\psi} t(D/2 \cdot d\psi)D/2\cos\psi = \sigma_0 D^2 t\sin\psi \tag{2}$$

For the limiting case of $\psi = 0$, the fully plastic axial load is given by

$$No = \sigma_0 \pi Dt \tag{3}$$

The fully plastic moment occurs for $\psi = \pi/2$

$$M_0 = \sigma_0 D^2 t \tag{4}$$

For other combinations of axial force and bending moments, the interaction relation can be obtained.

$$M/M_0 - \cos\pi N/2N_0 = 0 \tag{5}$$

The load-deflection behavior of an axially restrained tube can be obtained using the Eqs. (1) to (5) and applying the Principle of Virtual Work. The load-deflection relation for the clamped case reads:

$$P/P_0 = \sqrt{1-(W/D)^2} + W/D \cdot \arcsin W/D \quad ; \quad W/D \le 1 \tag{6}$$

$$\frac{P}{P_0} = \frac{\pi}{2} \cdot \frac{W}{D} \quad ; \quad \frac{W}{D} > 1 \tag{7}$$

Where W is the central deflection at the point of impact and D is the tube diameter. P_0 is the plastic collapse load of a circular tube in bending:

$$P_0 = \frac{8M_0}{l} = \frac{8\sigma_0 D^2 t}{l} \tag{8}$$

The above equations are based on the assumption that the ends have full axial restraints. In a real system, the tubular element sustains a certain degree of elastic support from the adjacent elements. For this clamped, ideally plastic element the absorbed energy at any level of deflection W is found by integration of the load-displacement expressions in Eqs. (6), (7). The following energy expression emerges:

$$E = P_0 D\left(\frac{3}{4}\cdot\frac{W}{D}\sqrt{1-\frac{W^2}{D^2}} + \frac{1+2W^2/D^2}{4}\arcsin\frac{W}{D}\right) \quad ; \quad \frac{W}{D} \le 1 \tag{9}$$

$$E = \frac{\pi}{8}P_0 D\left(1+2\frac{W^2}{D^2}\right) \quad ; \quad \frac{W}{D} > 1 \tag{10}$$

3. A new method with end flexibility

3.1 End flexibility extraction scheme

In the method developed here, the global stiffness of the related system is condensed to the nodal points of a tubular element of interest using a static condensation procedure. The equilibrium equations of the model can be written in matrix-partitioned form as,

$$\begin{bmatrix} K_{nn} & K_{nm} \\ K_{mn} & K_{mm} \end{bmatrix} \begin{Bmatrix} X_n \\ X_m \end{Bmatrix} = \begin{Bmatrix} F_n \\ F_m \end{Bmatrix} \tag{11}$$

Here, X_m represents the degrees of freedom to be condensed to the corresponding tubular member nodal points. X_n represents the residual degrees of freedom of bracing member nodal points. Solving Eq. (11) for X_m, we obtain,

$$\{X_m\} = -[K_{mm}]^{-1}[K_{mn}]\{X_n\} + [K_{mm}]^{-1}\{F_m\} \tag{12}$$

Substituting this result into Eq. (11) and collecting terms, we can write the condensed equilibrium equations.

$$\{[K_{nn}] - [K_{nm}]^{-1}[K_{mn}]\}\{X_n\} = \{F_n\} - [K_{nm}][K_{mm}]^{-1}\{F_m\} \tag{13}$$

Where L.H.S. stiffness matrix is the effective stiffness matrix. The condensed stiffness matrix is used to calculate the stiffness of the "end spring" corresponding to a certain degree of elastic support from the adjacent structure.

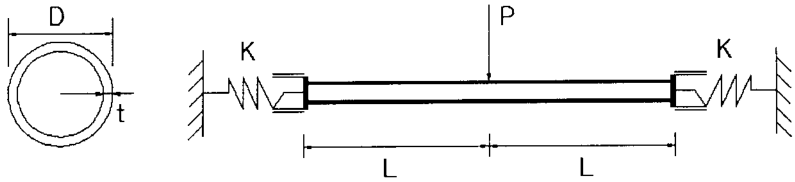

Fig. 3 Tubular beam with elastic horizontal restraints

This "end spring" restrictions can be included in the analysis of a tubular element subjected to lateral collision load.

3.2 Post-yield behavior of a tubular element with partial end restraints

Here the influence of partial end restraints in a bracing member subjected to lateral load is investigated by extending Hodge's theoretical study (Hodge 1974). To apply the method to our tubular member, we consider a tubular member, which is supported as shown in Fig. 3.

The yield condition for positive moment for the tubular member without any local indentation and buckling or crumpling phase is

$$m - \cos\frac{\pi}{2}n = 0 \tag{14}$$

Where $m = M/M_0 = M/D^2 t\sigma_0$

$$n = N/N_0 = N/\pi D t\sigma_0 \tag{15}$$

M : Bending moment
M_0 : Plastic capacity for bending moment
N : Axial force
N_0 : Plastic capacity for axial force
D : Diameter of the member
t : Thickness of the member
σ_0 : yield stress

It is also assumed that the ends of the member is supported by springs so that

$$F = KU \tag{16}$$

The tubular member is assumed to reach the yield-point value of $4M_0/L$ without any motion. The hinge is assumed to form in the center of the member. The extension Λ and the rotation θ of the hinge are related as shown in Fig. 4.

The small parameter d and dimensionless quantities are defined as shown below.

$$d = D/L,\ w = W/d,\ u = UL/d^2,\ \lambda = \Lambda N_0/M_0 \tag{17}$$

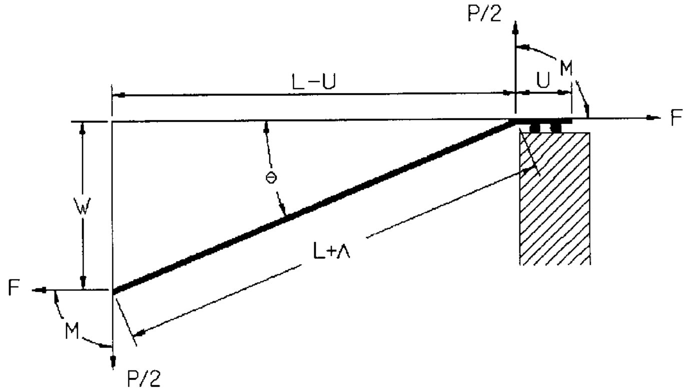

Fig. 4 Configuration of the tubular member

The kinematic quantities can be written in the form

$$\lambda = \frac{\pi}{2}d\left[(w^2 - 2u) + d^2w^2\left(u - \frac{w^2}{4}\right) + \text{higher order terms}\right]$$

$$\theta = dw[1 + d^2(u - w^2/3) + \text{higher order terms}] \tag{18}$$

The equilibrium condition gives

$$m = p - \frac{\pi}{2}wf - d^2pu$$

$$n = \cos\theta\left(f + \frac{2}{\pi}dp \quad \tan\theta\right) \tag{19}$$

Where $f = F/N_0,\ p = PL/4M_0$ (20)

From Eq. (16),

$$u = f/2c \tag{21}$$

Where we defined the dimensionless constant

$$c = K\frac{d}{2\pi L t\sigma_0} \tag{22}$$

For the case $n < 1$, the slope of the strain vector is perpendicular to the tangent to the interaction curve.

This condition gives,

$$\dot{\lambda} = \frac{\pi}{2}\sin\left(\frac{\pi}{2}n\right)\dot{\theta} \tag{23}$$

For simplicity we keep only leading terms. The equilibrium equation becomes,

$$m = p - \frac{\pi}{2}wf$$
$$n = f \tag{24}$$

Using Eqs. (18), (21), (23), we obtain

$$\frac{dn}{dw} + c\ \sin\left(\frac{\pi}{2}n\right) = 2cw \tag{25}$$

For $n<1$, we get load-displacement relation by solving Eq. (25) after combining Eqs. (14), (24).

4. Evaluation of the influence of end fixity

4.1 Description of numerical model

A semi-submersible drilling rig consists of two pontoons, four columns, four internal columns and additional tubular member are chosen for a numerical analysis. The ship is assumed to collide head to the weakest brace of the semi-submersible. The weakest tubular element dimensions from the model are, D = 1.8 m, t = 28 mm, L = 19 m.

The semi-submersible model using equivalent tubular structural units is shown in Fig. 5

4.2 Numerical results by the new method

The static condensation method described earlier, Eq. (13) is applied and the resulting adjacent spring constant K, representing the end flexibility is obtained.

The Eqs. (14), (24), (25) are employed to get post-yielding load-displacement relations with the

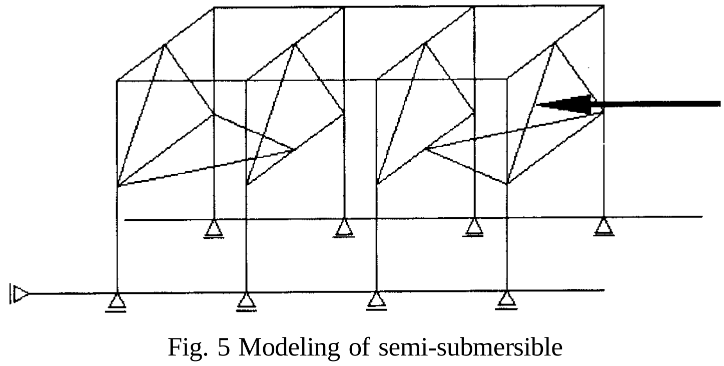

Fig. 5 Modeling of semi-submersible

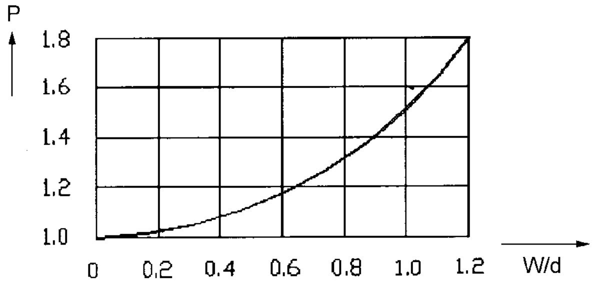

Fig. 6 Load-displacement curve for tubular member with elastic horizontal restraints

calculated end flexibility. The load-displacement relation with the corresponding end flexibility is shown in Fig. 6. This relation is based on the computation by solving the equations in Chapter 3, with corresponding section properties. The value of c used here is 1.2602. This figure shows the load-displacement relation of the member after the load exceeds the plastic collapse load P_0.

4.3 Numerical analysis by NOAMAS

For the detailed 3-D analysis NOAMAS program is used (Kahaner 1990, NOAMAS 1983). The structure is divided into tubular structural units and the incremental load method is used. First, the load-free structure is considered. The stiffness matrix of each structural unit is constructed and transformed into the global coordinates. The global stiffness matrix of the whole structure is then assembled. After the boundary conditions are introduced the first load increment is applied. The deformation of the structure is obtained and the internal forces in each structural unit are evaluated. Each structural unit is then checked for buckling and/or the occurrence of plastic deformation (Ricles and Bruin 1998).

Since the stiffness matrix of the tubular structural unit is dependent on the deformation, a new stiffness matrix is constructed and transformed into the global coordinates for each structural unit after each load increment. The global stiffness matrix is reassembled and the next increment of loads is applied.

When buckling and/or plastic deformation of a structural unit or more are detected within a loading step, the load increment is scaled down to that value just necessary to cause such a failure.

4.3.1 Modeling of a semi-submersible

The structure is divided into "tubular structural units" (Ueda and Rashed 1984), each of which is a complete tubular member running between two joints. The semi-submersible is modeled using equivalent tubular structural units as shown before. In this modeling, the tubular structural units are connected through pin joints connecting all translations and rotations. Since the NOAMAS uses the tubular structural unit, the actual structure elements, which are not in the tubular form, should be transformed into the tubular formed elements equivalently.

4.3.2 Collision loads and boundary conditions

A ship is assumed to collide head to the weakest tubular member of the semi-submersible. The collision force is applied at the middle of this member perpendicularly with respect to the element span. In order to include the effect of the inertia forces of semi-submersible, distributed loads, total amount of which is same as the collision load, through the nodal points are imposed in opposite direction with respect to the collision load. Thus rigid body motion in the X direction is prevented.

Total number of 27 load increments are imposed which are thought to be sufficient increments for this analysis. To prevent rigid body motions, constrains in the Y and Z directions are imposed through the corresponding nodal points. The collision loads are increased step by step up to the point at which the element is supposed to collapse.

4.3.3 NOAMAS and simple method

The rigid-plastic method is employed for the comparison purpose with NOAMAS results and the new method. Evidently, the load-displacement curve by the rigid-plastic method should coincide with the load displacement curve by the new approach when we set the value of c to be infinite value i.e., very large value of the stiffness. The load-displacement curve by NOAMAS should lie under the 2 curves, since the real structural modeling was intended.

5. Comparison among 3-D, rigid plastic, and new method

Fig. 7 is the load-displacement curves from the 3-dimensional modeling approach and the rigid plastic approach and the new approach developed here. Because of the end restraints of the element, the membrane forces are activated during global deformation resulting in the strengthening of the member in rigid plastic method. The new method lies between the rigid-plastic method and the 3-dimensional analysis as expected. It is self-evident that the rigid-plastic method gives high strengthening membrane effect of the member during global deformation resulting in the steeper slope than the present's method. On the while full 3-dimensional analysis gives less strengthening membrane effect of the member since there are deformations of the adjacent members, resulting in the slow going load-displacement curves. Differences between the result by rigid plastic method and the new approach are solely due to the influence of the end fixity. In this case, the effect of end fixity is about 5% difference with other methods.

Comparison of the curve by the new approach developed here with those by conventional methods shows that the subject method provides economic and more precise results.

6. Collision energy calculation

Conservation of the energy requires that the kinetic energy of the impacting ship is transferred into the elastic deformation energy and the plastic dissipation of energy in ship and platform. The total kinetic energy can be assumed to be dissipated in the platform itself. The absorbed energy is calculated by integrating the area under the curve in the load-displacement relations obtained. The resulting curve, which provides the collision energy at various displacement steps, was to be satisfactory in general.

7. Discussions

The load-displacement relations at the collision point are obtained with 3 different approaches as shown in Fig. 7. This figure shows the elasto-plastic load-displacement relations for the collision

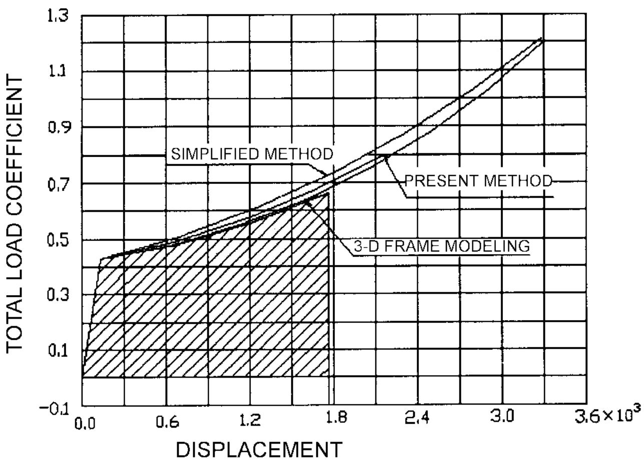

Fig. 7 Load-displacement curves by various approaches

Table 1 Comparison of 3 methods

Lateral displacement	Calculated absorbed collision energy (MJ)		
	Rigid-plastic method	New approach	3-D Frame modeling (NOAMAS)
1.736	12.72	12.48	12.0
2.057	16.56	16.02	15.36
2.365	20.88	19.98	19.20
2.667	25.68	24.65	23.52
2.959	30.96	30.12	28.32
3.245	35.76	34.06	32.64

Table 2 Collision energy, elongation, displacement relations

Longitudinal elongation, %	Lateral displacement, *m*	Collision energy, M Joule
5	1.735	12.0
6	2.057	15.4
7	2.365	19.2
8	2.667	23.5
9	2.959	28.3
10	3.245	32.6

point. This load-displacement curve includes geometrical and material nonlinearities. The material nonlinearity means the nonlinear "stress-strain" relations and the geometrical, nonlinearity means nonlinear "strain-displacement" relations in general, respectively. The area under the curve in the load-displacement relation represents the absorbed energy. The absorbed energy is calculated from the shaded area in the Fig. 7. The amount of the absorbed energy is 12 MJ by NOAMAS and the corresponding vertical displacement at the collision point is 1.735 m. The longitudinal elongation of the bracing member when the vertical displacement is 1.735 m is sufficiently within the generally accepted longitudinal elongation.

The collision energy recommended by the DnV MOU Rule (Det 2000) is 11 MJ. Thus the result of the collision analysis by the 3 methods shows that the structure is safe enough in this specific collision circumstance.

To get some understanding/feeling of the magnitude of the deformation obtained by NOAMAS, the longitudinal elongation of the tubular member at which the collision occurred is calculated and tabulated in Table 2. Table 2 also shows the lateral displacement of the bracing element and the corresponding collision energy for better understanding of the relation among them.

8. Conclusions

For the evaluation of the capability of a tubular member of an offshore structure to absorb the collision energy, the influences of end fixity on the behaviors of a tubular member is investigated, through a new approach and other approaches. A new analysis approach that includes the flexibility of the boundary points of the member is developed here. Comparison of the load-displacement curves by the new approach with those by conventional methods gives the figures of the influence of end fixity on post-yielding behaviors of the relevant tubular member.

The main contribution of this investigation is the development of an analytical rational procedure to solve the post-yielding behavior of a tubular member that is subjected to the lateral loads. The new approach determines the end flexibility effect quantitatively as well as qualitatively. The method also provides an economic tool for practical collision analysis of the typical offshore structure without performing detailed computations.

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