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Mass perturbation influence method for dynamic analysis of offshore structures

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Abstract. The current work presents an analysis algorithm for the modal analysis for the dynamic behaviors of offshore structures with concepts of mass perturbation influence term. The mass perturbation concept by using the term, presented in this paper offers an efficient solution procedure for dynamical response problems of offshore structures. The basis of the proposed method is the mass perturbation influence concepts associated with natural frequencies and mode shapes and mass properties of the given structure. The mathematical formulation of the mass perturbation influence method is described. New solution procedures for dynamics analysis are developed, followed by illustrative example problems, which deal with the effectiveness of the new solution procedures for the dynamic analysis of offshore structures. The solution procedures presented herein is compact and computationally simple.

Key words: dynamic response; offshore structure; mass perturbation; modal analysis.

1. Introduction

Determining of the dynamic characteristics of a structure can be done by the normal mode method. The deflection of a point in the structure is expressed in terms of normal modes of the structure and the normal coordinates.

Mode superposition method for determining the motion characteristics of offshore structures such as offshore platforms is well established. The method to figure out the forced response of the structure is based on the natural frequencies and corresponding mode shapes obtained from an eigenvalue analysis of the structure. The equations of the motion in uncoupled form by using the calculated natural frequencies and mode shapes can be derived and solved. The forced response of the given structure can be obtained by superposition of the selected number of mode shapes. However, it is difficult and ambiguous to decide the number of participating mode shapes in the mode superposition method in the response analysis.

In order to overcome these difficulties, an efficient analysis algorithm for the modal analysis for the dynamic behaviors of offshore structures is proposed. The basis of the proposed method is the mass perturbation influence concepts.

The mass perturbation influence term provides information of the dynamic characteristics of the structure. The term indicates how much the corresponding mode influence into the response of the structure. Large value of the term with a specific element means that small structural change is associated with the motion. Small value of the term with a specific element means that large

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structural change is associated in the motion response. Thus, the value of mass perturbation influence term associated with specific mode can be the indication of the participation of that mode in the forced response. The method discussed herein requires only calculation of natural frequencies and mode shapes from an eigenvalue analysis of the objective structure, which is supposed to be dynamically responded due to the exciting forces.

The modal contribution depends on the system's natural frequencies as well as the excitation frequency. The failure of offshore structures is an awesome possibility under resonance due to the exciting frequency. Structures are designed to escape the resonance in the early design stage. Thus in this paper the effects of the system's natural frequency and mode shapes are mainly considered (API RP 2A, 1993).

In the following section, the mathematical formulation of the mass perturbation influence method is described. Solution procedures for the dynamic analysis are developed, followed by illustrative numerical examples.

2. Theory

2.1 Background

The differential equations of motion including the offshore structure's motion, in the x coordinate system in the matrix form can be written as,

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \{F(t)\} \tag{1}$$

Where $[M]$: mass matrix of structure
$[C]$: damping matrix of structure
$[K]$: stiffness matrix of structure
$\{F(t)\}$: column matrix of external forces
$\{x\}$: column matrix of displacement of structure
$\{\dot{x}\}$: column matrix of velocity of structure
$\{\ddot{x}\}$: column matrix of acceleration of structure

Eq. (1) represents a system of differential equations, the solution of the differential equations can be obtained by two methods. They are direct integration procedure and mode superposition procedure. However, the procedures for the solution of differential equations can become very expensive when the order of the matrices is large.

The mode superposition method is characterized by the fact that the differential equations of motion are decoupled when the displacements are expressed in terms of the normal modes. Thus, the algebra required in the solution is considerably reduced. Furthermore, proper selection of participating modes in the displacements results in a drastic reduction of computation efforts without losing the precision. The question is how to decide the number of participating mode shapes in the mode superposition method in the response analysis.

2.2 Mathematical development

An eigenvalue analysis of a structural system including offshore structure can be defined by

$$[M]\{\ddot{x}(t)\} + [K]\{x(t)\} = 0 \tag{2}$$

which produces the eigenvectors $\{\Phi\}$ and the natural frequencies $\{\omega^2\}$

The modal perturbation method offers an approximate solution of the redesign problem (Bernitsas and Kang, Cho 1999, 1994, 1989).

The linearized modal dynamic perturbation equation can be derived as below, where apostrophe prime indicates the structural properties of the modified structure.

$$[\Phi]^T[\Delta K]\,[\Phi] - [\Phi]^T[\Delta M]\,[\Phi]\{\omega^2\} = [\Delta] \tag{3}$$

where $[\Delta] = \begin{cases} M_i \Delta\omega_i^2 & i = j \\ M_j C_{ij}(\omega_i^2 - \omega_j^2) & i \neq j \end{cases}$

$$[M'] = [M] + [\Delta M]$$

$$[K'] = [K] + [\Delta K]$$

$$[\omega'^2] = [\omega^2] + [\Delta\omega^2]$$

$$[\Phi'] = [\Phi] + [\Delta\Phi] = [\Phi]([I] + [C])^T$$

Perturbation influence terms can be defined

$$P_e^k = \{\Phi_j\}^T[K_e]\{\Phi_i\},$$
$$P_e^m = -\omega_i^2\{\Phi_j\}^T[M_e]\{\Phi_i\} \tag{4}$$

Where $[K_e]$ $[M_e]$ represent the stiffness matrix of element *e* and the mass matrix of element *e*, used for static/dynamic redesign problems, respectively (Kim 1988). The algorithm developed here is based on the fact that the perturbation influence terms provide important information for redesign process. Large value for *P* with a specific element means that small structural change would be required to meet the design specifications. Small values for *P* means that large structural changes would be required.

The perturbation influence term can be used to identify structural elements, which have any effect on the modal changes. For large structures including offshore structures, it is important to figure out how many modes should be included for the normal mode analysis. Generally the lower numbered modes correspond to the smaller eigenvalues dominate the dynamic solution. The higher modes that are truncated correspond to high frequency motion that is not easily excited (Temarel *et al*. 2000, Payer 1994).

2.3 Methodology

To use the perturbation idea in the modal analysis, we can define new mass perturbation influence term, which provides information of the dynamic characteristics of the structure.

$$P_i^m = -\omega_i^2\{\Phi_j\}^T[M]\{\Phi_i\} \tag{5}$$

The term indicates how much the corresponding mode influence into the response of the structure.

Large value of the term with a specific element means that small structural change is associated with the motion. Small value of the term means that large structural change is associated in the motion response. The value of mass perturbation influence term associated with specific mode can be the indication of the participation of that mode in the forced response.

The amount of contribution of the specific mode in the total motion response can be represented

$$E_i = \frac{\left|(P^m)_i^{-1}\right|}{\left|\sum_{r=1}^{n} [(P^m)^{-1}]_r\right|} \tag{6}$$

Thus this equation gives the guidelines to decide the number of participating mode shapes in the normal mode method in the response analysis of offshore structures.

3. Applications

3.1 Jacket structure

A simple model (Yun *et al.* 1985) of an offshore tower is used to evaluate the application of the mass perturbation influence method in the response analysis of the structure.

In modeling the jacket structure, we have $m_1 = 9.6 \times 10^5$, $m_2 = 1.16 \times 10^5$, $m_3 = 1.5 \times 10^5$, $m_4 = 1.76 \times 10^5$ (slugs), and corresponding mass and stiffness matrix of the modal are as follows.

$$[M] = \begin{bmatrix} 9.6 & 0 & 0 & 0 \\ 0 & 1.16 & 0 & 0 \\ 0 & 0 & 1.5 & 0 \\ 0 & 0 & 0 & 1.76 \end{bmatrix} \times 10^5 \quad \text{(slugs)}$$

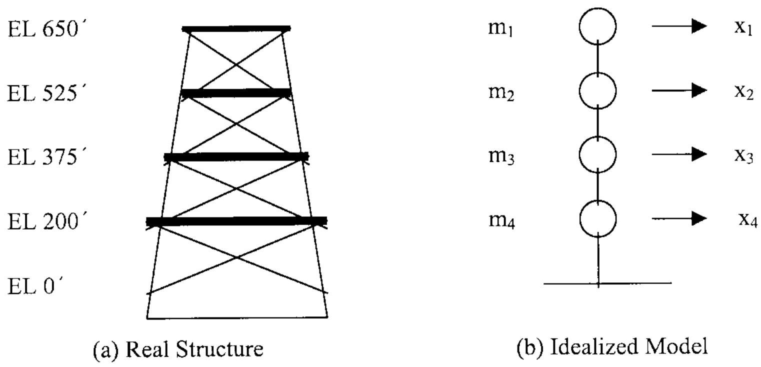

Fig. 1 Jacket structure model

$$[K] = \begin{bmatrix} 1.961 & -2.742 & 0.5046 & 0.2503 \\ -2.742 & 5.597 & -3.042 & 0.1698 \\ 0.5046 & -3.042 & 5.063 & -2.568 \\ 0.2503 & 0.1698 & -2.568 & 4.691 \end{bmatrix} \times 10^7 \text{ (lb/ft)}$$

Eigenvalue analysis gives the following results.

$$T_1 = 3.82 \text{ sec}, \quad T_2 = 0.68 \text{ sec}, \quad T_3 = 0.34 \text{ sec}, \quad T_4 = 0.24 \text{ sec}$$

$$[\Phi] = [\{\Phi\}_1, \{\Phi\}_2, \{\Phi\}_3, \{\Phi\}_4]$$

$$= \begin{bmatrix} 1.00 & -0.1303 & 0.0774 & -0.0475 \\ 0.7068 & 0.5545 & -0.8453 & 1.00 \\ 0.3998 & 1.00 & -0.4977 & -0.7211 \\ 0.1414 & 0.7882 & 1.00 & 0.2722 \end{bmatrix}$$

With API Recommended Spectrum and RMS (Root Mean Square) Methods, we have a typical design response as follows.

$$\begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{Bmatrix} = \begin{Bmatrix} 4.831 \\ 3.420 \\ 2.020 \\ 0.825 \end{Bmatrix} \quad \text{(inch)}$$

For the above model, the responses for each mode can be obtained.

1st mode, $$\{X\}_1 = \begin{Bmatrix} 4.83 \\ 3.41 \\ 1.93 \\ 0.68 \end{Bmatrix} \quad \text{(inch)}$$

2nd mode, $$\{X\}_2 = \begin{Bmatrix} -0.07 \\ 0.31 \\ 0.58 \\ 0.46 \end{Bmatrix}$$

3rd mode, $$\{X\}_3 = \begin{Bmatrix} 0.005 \\ -0.06 \\ -0.03 \\ 0.07 \end{Bmatrix}$$

Table 1 Mass perturbation terms and its contribution for Jacket analysis

10^7	$\left\|(P^m)_i^{-1}\right\|$	$\left\|\sum(P^m)_r^{-1}\right\|$	$\left\|\dfrac{(P^m)_r^{-1}}{\sum(P^m)_r^{-1}}\right\|$
1 st mode	139.6	161.03	86.69%
2 nd mode	14.85	161.03	9.22%
3 rd mode	3.83	161.03	2.38%
4 th mode	2.75	161.03	1.71%

4th mode, $$\{X\}_4 = \begin{Bmatrix} -0.0005 \\ 0.01 \\ -0.007 \\ 0.003 \end{Bmatrix}$$

In this case, it is found that the precise real response can be obtained just by employing only 1st and 2nd modes in the modal analysis. Relevant mass perturbation influence terms $(P^m)_i^{-1}$ corresponding to each mode can be obtained based on the method developed here (Table 1). In this case the mass perturbation influence terms indicate that 96% of the forced response may be obtained by using 1st and 2nd modes. Real response indication of this problem is 94.25% with 1st and 2nd modes participation in the modal analysis.

This means that the mass perturbation influence term can be an indication of the mode participation in the modal analysis.

3.2 Two degrees of freedom spring-mass system

A two degrees of freedom spring-mass system is used to demonstrate the usage of the method. The equation of motion is as follows.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{Bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \end{Bmatrix} + 0.01\begin{bmatrix} 3 & 2 \\ 2 & 6 \end{bmatrix}\begin{Bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{Bmatrix} + \begin{bmatrix} 2 & 2 \\ 2 & 5 \end{bmatrix}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} f_1(t) \\ f_2(t) \end{Bmatrix}$$

Eigenvalue analysis gives

$$\omega_1^2 = 1 \qquad \Phi_1 = \begin{Bmatrix} 1 \\ -0.5 \end{Bmatrix}$$

$$\omega_2^2 = 6 \qquad \Phi_2 = \begin{Bmatrix} 0.5 \\ 1 \end{Bmatrix}$$

Modal analysis gives the response of the system,

$$\begin{Bmatrix} X_1(t) \\ X_2(t) \end{Bmatrix} = q_1(t)\{\Phi\}_1 + q_2(t)\{\Phi\}_2$$

Table 2 Two d. o. f. mass-spring model and mass perturbation terms and contributions

	$\left\|(P^m)_i^{-1}\right\|$	$\left\|\sum(P^m)_r^{-1}\right\|$	$\left\|\dfrac{(P^m)_r^{-1}}{\sum(P^m)_r^{-1}}\right\|$
1 st mode	0.80	0.93	86%
2 nd mode	0.13	0.93	14%

And the decoupled equation of motion is,

$$\ddot{q}_1(t) + 0.02\dot{q}_1(t) + q_1(t) = \frac{1}{\sqrt{5}}(2f_1(t) - f_2(t))$$

$$\ddot{q}_2(t) + 0.07\dot{q}_2(t) + q_2(t) = \frac{1}{\sqrt{5}}(f_1(t) + 2f_2(t))$$

For a specific value of $f_1(t)$, $f_2(t)$, the total response of the system can be,

$$\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 1.25 \\ 7.5 \end{Bmatrix}$$

For this model, the response for each mode can be expressed,

1st mode, $$\{X\}_1 = \begin{Bmatrix} 1.2096 \\ 0.6075 \end{Bmatrix}$$

2nd mode, $$\{X\}_2 = \begin{Bmatrix} 0.135 \\ 0.27 \end{Bmatrix}$$

In this case, it is found that the precise response can be obtained by using 1st mode only. Relevant mass perturbation terms, $(P^m)_i^{-1}$ corresponding to each mode can be obtained (Table 2). In this case mass perturbation influence terms indicate that 86% of the forced response is obtained by using 1st mode only. Real response indication of the problem is to be 80% with 1st mode participation in the modal analysis. This example shows that the precision is a little bit going down.

4. Conclusions

The current work develops an analysis scheme for the modal analysis of dynamic behaviors of offshore structures with many degrees of freedom. The basis of the proposed scheme is the application of the mass perturbation influence concepts associated with natural frequencies and mode shapes and mass matrix of the given structures. It is found that the mass perturbation influence term gives the guideline for selecting the necessary mode shapes in carrying out the modal analysis. The mathematical formulation of the mass perturbation influence method is developed. New solution procedures for the typical structural problems are demonstrated. The solution procedure presented herein is compact and computationally much simpler. Example problems, which demonstrate the effectiveness of the new procedures, were followed, and the method is proved to be efficient and reliable.

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