Structural Engineering and Mechanics, Vol. 12, No. 4 (2001) 363-376
DOI: http://dx.doi.org/10.12989/sem.2001.12.4.363

A General approach to the wrinkling instability of sandwich plates

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Abstract. Sandwich plates are widely used in lightweight design due to their high strength and stiffness to weight ratio. Due to the heterogeneous structure of sandwich plates, they can exhibit local instabilities (wrinkling), which lead to a sudden loss of stiffness in the structure. This paper presents an analytical solution to the wrinkling problem of sandwich plates. The solution is based on the Rayleigh-Ritz method, by assuming an appropriate deformation field. In contrast to the other approaches up to now, this model takes arbitrary and different orthotropic face layers, finite core thickness and orthotropic core material into account. This approach is the first to cover the wrinkling of unsymmetric sandwiches and sandwiches composed of orthotropic FRP face layers, which are most common in advanced lightweight design. Despite the generality of the solution, the computational effort is kept within bounds. The results have been verified using other analytical solutions and unit cell 3D FE calculations.

Key words: sandwich; instability; wrinkling; elastic foundation; shell; plate; buckling; orthotropy.

1. Introduction

Wrinkling (i.e. local short wavelength buckling of the face layers) is a common local stability problem of sandwich plates and shells under compressive or bending loads (see Allen 1969, Plantema 1966). The post buckling behaviour in typical sandwich applications is unstable (Stiftinger and Rammerstorfer 1997, Martikainen and Hassinen 1996), leading to an abrupt loss of stiffness in the affected region. Therefore, the occurrence of wrinkling is likely to cause global failure of a sandwich structure.

Due to the local nature of the problem, the wrinkling failure mode is not detected in a FE analysis of a sandwich structure using sandwich shell or beam elements, respectively. A sensible approach to overcome this problem has been presented in Starlinger and Rammerstorfer (1992). There, it is checked on integration point level whether the critical wrinkling load has been exceeded or not, and correspondingly the stiffness matrix of the element is adapted. This approach proved to be sensible, but the wrinkling calculation itself is practically limited to isotropic symmetric sandwiches. The objective of this work is to derive a most general solution for the critical wrinkling load of sandwich plates. It is intended to implement this approach into FE codes in the way mentioned above.

There are two main analytical approaches to calculate the bifurcation load at which wrinkling will take place. On the one hand the energy approach is used, where the total potential energy consisting

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of the strain energy of the two face layers and the core and the potential of the external forces is minimised. On the other hand the differential equation of a plate on an elastic foundation is used. Here, one face layer is modeled as a plate and the core with the other face layer is regarded as a foundation of the one face layer.

The classical solutions (Allen 1969, Plantema 1966) divide the wrinkling problem into two different cases. The first case is the "thick sandwich" case, where it is assumed, that the thickness of the sandwich is big enough to preclude interaction between the two face layers (Gough *et al.* 1940, Yusuff 1955, Hwang 1988). The second case is the "thin sandwich" case, where interaction between the face layers is included (Hoff and Mautner 1945, Yusuff 1955, Gutierrez and Webber 1980, Stiftinger and Rammerstorfer 1997). However, the thin sandwich solutions that are available up to now assume symmetric sandwiches, i.e., the two face layers have to be identical. Moreover, orthotropic face layers are only covered by these approaches in the case of compression in direction of the axes of orthotropy. Thus, the solutions available up to now comprise suppositions that are insufficient to capture the wide range of sandwiches in lightweight design. However, they are useful for analytical estimations of bounds to the wrinkling load, but they show limitations when used within an automated FE sandwich structure analysis.

The authors chose the approach of using the differential equation of an elastic plate on an elastic foundation, which has already been solved in a very general manner for the case of thick cores (Vonach and Rammerstorfer 1999, 2000). Within this contribution, the work is extended to thin cores, i.e., the interaction between the two face layers is included. A more detailed description related to the problems and effects considered within this paper is given in the PhD thesis of the leading author (Vonach 2001).

2. Analytical approach

Wrinkling is a very localized short wave buckling phenomenon. For most applications, the face

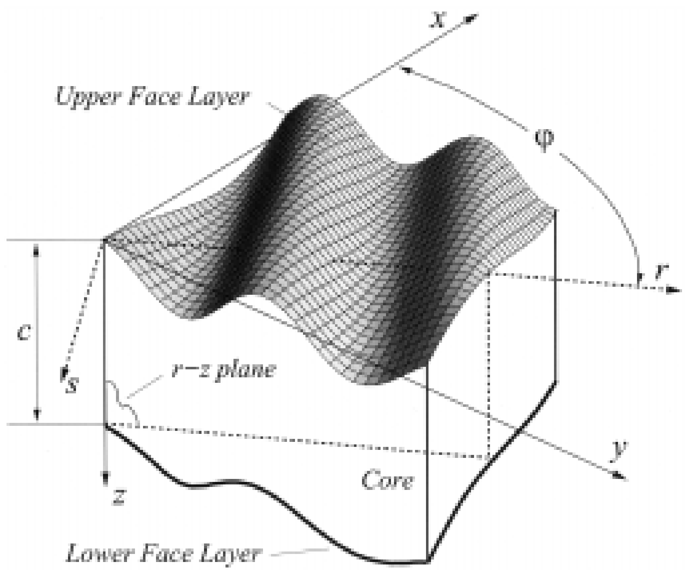

Fig. 1 Geometry of the buckling pattern

layers may, therefore, be considered as being under a homogeneous membrane stress state prior to buckling, and the length and width of the sandwich plate can be assumed to be infinite. Perfect bonding between face layers and core is assumed, the deformations of the face layers and the core are identical at $z = 0$ and $z = c$, where c denotes the core thickness (see Fig. 1).

2.1 Upper face layer

The upper face layer is supported by the core and, via the core, by the lower face layer. The buckling deformation pattern of the upper face layer w^u is assumed to be sinusoidal in r-direction and constant in s-direction (see Fig. 1):

$$w^u = w^{u*} \sin\left(\frac{\pi r}{a}\right) = w^{u*} \sin\left(\frac{\pi(x \cos\varphi + y \sin\varphi)}{a}\right) \tag{1}$$

The superscript u refers to the upper face layer, c to the core, l to the lower face layer, and * to amplitude values. Eq. (1) is a Rayleigh-Ritz ansatz for the wrinkling pattern with two free variables, namely the half-wavelength a and the direction of the buckling pattern φ. Both of these variables are a priori unknown. The geometry of the problem is shown in Fig. 1. The xyz coordinate system denotes the global coordinate system defined by the material axes of the upper face layer, and the rsz system is defined by the angle of the sinus wave φ.

The orthotropic upper face layer is described using the differential equation of a thin elastic orthotropic plate on an elastic foundation

$$B_x^u \frac{\partial^4 w^u}{\partial x^4} + 2B_{xy}^u \frac{\partial^4 w^u}{\partial x^2 \partial y^2} + B_y^u \frac{\partial^4 w^u}{\partial y^4} + P_x^u \frac{\partial^2 w^u}{\partial x^2} + 2P_{xy}^u \frac{\partial^2 w^u}{\partial x \partial y} + P_y^u \frac{\partial^2 w^u}{\partial y^2} + w^u k^{\text{thin}} = 0 \tag{2}$$

where the individual B's are the plate bending stiffness terms, the P's are the compressive membrane forces and k^{thin} denotes the elastic foundation stiffness, which also depends on the wrinkling mode.

Using the deformation pattern in Eq. (1), the plate equation is reduced to the equation of a beam (in r-direction) with unit-width on an elastic foundation.

$$B_r^u(\varphi) \frac{\partial^4 w^u}{\partial r^4} + P_r^u(\varphi) \frac{\partial^2 w^u}{\partial r^2} + w^u k^{\text{thin}} = 0 \tag{3}$$

The beam bending stiffness (B_r^u) and section force P_r^u are formulated in r-direction which makes them dependent on the angle φ. The loading is defined by ρ^u, ξ^u and ζ^u relative to the global load level P.

$$\begin{aligned} B_r^u(\varphi) &= B_x^u \cos(\varphi)^4 + 2B_{xy}^u \sin(\varphi)^2 \cos(\varphi)^2 + B_y^u \sin(\varphi)^4, \\ P_r^u(\varphi) &= P(\rho^u \cos(\varphi)^2 + 2\zeta^u \sin(\varphi)\cos(\varphi) + \xi^u \sin(\varphi)^2), \\ \rho^u &= P_x^u/P,\ \xi^u = P_y^u/P \text{ and } \zeta^u = P_{xy}^u/P \end{aligned} \tag{4}$$

Inserting Eq. (1) into Eq. (3) yields

$$B_r^u(\varphi)\left(\frac{\pi}{a}\right)^4 - P_r^u(\varphi)\left(\frac{\pi}{a}\right)^2 + k^{\text{thin}} = 0 \tag{5}$$

Apart from the two free variables a and φ, the only remaining unknown in this equation is the

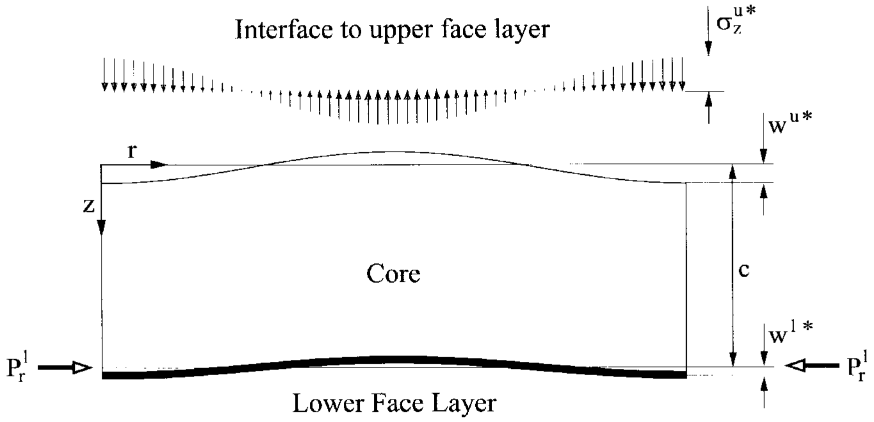

Fig. 2 Loading and deflection of the foundation model

foundation (spring) stiffness k^{thin} which is derived in the following section.

2.2 Core and lower face layer

The foundation stiffness k^{thin} is defined as

$$k^{\text{thin}} = \frac{-\sigma_z^u}{w^u} \tag{6}$$

where σ_z^u is the stress in z-direction and w^u is the deflection in z-direction, both at the interface between the core and the upper face layer.

The deflection w^u is defined in Eq. (1). It can be shown that this sinusoidal deflection leads to a sinusoidal distribution of the normal stresses in z-direction σ_z^u at the interface. To derive the foundation stiffness according to Eq. (6), the core and the lower face layer, which is bonded to the core, are loaded with this (self equilibrating) sinusoidal stress distribution:

$$\sigma_z^u = \sigma_z^c(z = 0) = \sigma_z^{u*}\sin\left(\frac{\pi r}{a}\right) \tag{7}$$

The amplitude of this load σ_z^u is arbitrary, since a linear elastic response is expected. The deflections w^u of the core which result from this loading are calculated using linear elasticity and thus k^{thin} can be determined from Eq. (6). This calculation will now be discussed in detail.

In order to allow for an efficient analytical solution, it is assumed, that the deformation w of the core and the lower face layer is also a single sinus wave, having the same wavelength a and wave direction φ as the upper face layer, but an amplitude $w^*(z)$ which is depending on z:

$$w^c(z) = w^{c*}(z)\sin\left(\frac{\pi r}{a}\right) = w^{c*}(z)\sin\left(\frac{\pi(x\cos\varphi + y\sin\varphi)}{a}\right),$$
$$\text{with } w^u = w^c(z = 0) \text{ and } w^l = w^c(z = c)$$

This assumption is the key to the further derivations and is, therefore, discussed in more detail. Earlier work of the authors (Vonach and Rammerstorfer 1999, 2000) has shown that this assumption is leading to the exact solution as long as thick sandwiches with transversely isotropic cores are considered. The most common sandwich core materials, with the exception of honeycomb, can be regarded as being transversely isotropic, such as foams and balsa. In the case of honeycomb it is desirable to use a fully orthotropic material law. This has been shown to lead to impreciseness in combination with Eq. (8), but the accuracy of the results is only affected to a minor degree in the parameter range of commercial honeycomb cores.

However, this new approach is derived to cover the area of thin sandwiches as well. As mentioned before, the analytical thin core solutions available for wrinkling analyses are limited to symmetric sandwiches with symmetric or antimetric loading. Within this area of sandwiches it is shown that Eq. (8) holds as well. However, there is a priori no information available to which extent the deformation pattern of an unsymmetric sandwich under general loading conditions differs from the constraint defined in Eq. (8). Therefore, FE unit cell calculations have been performed to investigate this matter.

Regarding the deformation pattern in Eq. (8) it can be stated that the deformations are constant in *s*-direction (see Fig. 1). Therefore it is sufficient to regard the core and the lower face layer only as a 2D plane strain problem in the *r*–*z* plane as displayed in Fig. 2.

The core is modeled by the Airy stress function in the *r*-*z* plane:

$$\frac{1}{D_z^c}\frac{\partial^4 F}{\partial r^4} + \frac{2}{D_{rz}^c}\frac{\partial^4 F}{\partial r^2\,\partial z^2} + \frac{1}{D_r^c}\frac{\partial^4 F}{\partial z^4} = 0,$$

$$\sigma_r^c = \frac{\partial^2 F}{\partial z^2},\ \sigma_z^c = \frac{\partial^2 F}{\partial r^2},\ \sigma_{rz}^c = \frac{\partial^2 F}{\partial r\,\partial z} \tag{9}$$

with the membrane stiffness terms (D^c):

$$D_r^c = E_r^{c'},\quad D_{rz}^c = \frac{2E_r^{c'}G_{rz}^c}{-2\nu_{rz}^{c'}G_{rz}^c + E_r^{c'}},\quad D_z^c = E_z^{c'} \tag{10}$$

Using the Airy stress function the full 2D plane strain problem is solved, and the influence of the in-plane core stiffness E_r^c is included. This is of high importance when strongly orthotropic core materials, such as honeycomb, are used (Vonach and Rammerstorfer 1988, see also Fig. 7). The Airy stress function is normally used to solve plane stress problems. In order to transform the Airy stress function equations from plane stress to plane strain, modified stiffness values (see Timoshenko and Goodier 1988) are used. These modified stiffness values of the core are marked with $'$ in Eq. (10).

In sandwich construction, the *z*-axis is generally a material axis of the core which is therefore also assumed here. The orientation of the material axes of the core in the *xy*-plane is defined by an additional angle (ϑ) which is used together with φ to calculate the core stiffness parameters ($E_r^{c'}, E_z^{c'}, \nu_{rz}^{c'}, G_{rz}^c$) which are needed in the *rsz*-coordinate frame.

Using the sinusoidal stress distribution given in Eq. (7), the full solution of Eq. (9) is:

$$F = \sum_{i=1}^{4} C_i\left(\frac{a}{\pi}\right)^2 \exp\left(\frac{\mu_i \pi z}{a}\sqrt[4]{\frac{D_r^c}{D_z^c}}\right)\sin\left(\frac{\pi r}{a}\right) \tag{11}$$

with the following four coefficients:

$$\mu_{1,3} = \pm\sqrt{\xi + \sqrt{\xi^2 - 1}}, \quad \mu_{2,4} = \pm\sqrt{\xi - \sqrt{\xi^2 - 1}}, \quad \xi = \frac{\sqrt{D_r^c D_z^c}}{D_{rz}^c} \tag{12}$$

In order to calculate the four constants C_i, four boundary conditions have to be used. Two boundary conditions are imposed at the interface to the upper face layer ($z = 0$). The first is already defined in Eq. (7). The second is derived from the common sandwich approximation, that the strains in the core in r-direction are set to zero. This assumption is based on the fact, that the facing material is typically much stiffer than the core material. This leads to the second boundary condition:

$$\varepsilon_r^c = \frac{1}{E_r^{c'}}(\sigma_r^c - \nu_{rz}^{c'}\sigma_z^c) = 0 \text{ at } z = 0 \tag{13}$$

The remaining two boundary conditions are defined analogously at the interface between the lower face layer and the core which is located at $z = c$:

$$\sigma_z^l = \sigma_z^c(z = c) = \sigma_z^{l*}\sin\left(\frac{\pi r}{a}\right), \tag{14}$$

$$\varepsilon_r^c = \frac{1}{E_r^{c'}}(\sigma_r^c - \nu_{rz}^{c'}\sigma_z^c) = 0 \text{ at } z = c \tag{15}$$

These four boundary conditions lead to the four constants C_i:

$$C_1 = \frac{(exp\,a\,\sigma_z^{l*} - \sigma_z^{u*})X_3}{-1 + exp\,a^2} \qquad C_3 = \frac{exp\,a(-\sigma_z^{l*} + exp\,a\,\sigma_z^{u*})X_3}{-1 + exp\,a^2}$$

$$C_2 = -\frac{(exp\,b\,\sigma_z^{l*} - \sigma_z^{u*})X_4}{-1 + exp\,b^2} \qquad C_4 = -\frac{exp\,b(-\sigma_z^{l*} + exp\,b\,\sigma_z^{u*})X_4}{-1 + exp\,b^2} \tag{16}$$

with:

$$X_3 = \frac{\mu_2^2 + \nu_{rz}^{c'}\sqrt{\frac{D_z^c}{D_r^c}}}{\mu_1^2 - \mu_2^2} \qquad X_4 = \frac{\mu_1^2 + \nu_{rz}^{c'}\sqrt{\frac{D_z^c}{D_r^c}}}{\mu_1^2 - \mu_2^2}$$

$$exp\,a = \exp\left(\frac{c}{a}\mu_1\pi\sqrt[4]{\frac{D_r^c}{D_z^c}}\right) \qquad exp\,b = \exp\left(\frac{c}{a}\mu_2\pi\sqrt[4]{\frac{D_r^c}{D_z^c}}\right) \tag{17}$$

At this stage, the stress and strain field in the core is known as a function of a, φ, the upper interface stress amplitude σ_z^{u*}, and the lower interface stress amplitude σ_z^{l*}. The upper interface stress amplitude represents the loading at which the deflections have to be calculated to solve Eq. (6). The lower interface stress amplitude leads to a deformation of the lower face layer. This is calculated using the constitutive equation for the lower face layer.

The lower face layer is modeled exactly the same way as the upper face layer, i.e., using the differential equation of a thin elastic plate on an elastic foundation, which is again reduced to the form equivalent to Eq. (3):

$$B_r^l(\varphi, \gamma)\frac{\partial^4 w^l}{\partial r^4} + P_r^l(\varphi)\frac{\partial^2 w^l}{\partial r^2} + \sigma_z^{l*}\sin\left(\frac{\pi r}{a}\right) = 0 \tag{18}$$

The orientation of the material axes of the lower face layer with respect to the *xyz* coordinate frame is defined by the angle γ which is used together with φ to calculate the beam bending stiffness in *r*-direction B_r^l. The loading is formulated relative to the same global loading parameter *P* as in the case of the upper face layer (see Eq. 4).

The stresses and strains in the core and the lower face layer are now defined dependent on a, φ, σ_z^{u*} and σ_z^{l*}. However, given a, φ and σ_z^{u*}, i.e., given deflection geometry and load, will always results in a certain σ_z^{l*}, which is nothing but a stress value in the interior of the foundation. Thus, σ_z^{l*} can be calculated by minimising the strain energy of the foundation (i.e. core and lower face layer).

The calculation and minimization of the strain energy of the core and the lower face layer is a basic but arduous procedure. Since the results are long and cumbersome equations, they will not be given here. The minimisation of the strain energy leads to σ_z^{l*}, which is proportional to σ_z^{u*} and depends on the free parameters a and φ as well as on the geometric, material and load input parameters.

Using this result, the strains in the core in *z*-direction can be calculated and integrated over the core thickness, which gives the deflection of the core. Together with the resulting deflection of the lower face layer w^l (calculated from Eq. 18), the total deflection of the foundation under a given load σ_z^{u*} is derived (see Fig. 2):

$$w^u = \int_c^0 \varepsilon_z^c dz + w^l = H(a, \varphi)\sigma_z^u = H(a, \varphi)\sigma_z^{u*}\sin\left(\frac{\pi r}{a}\right) \tag{19}$$

Inserting this result in Eq. (6), k^{thin} is derived as an explicit function of a and φ and of the material, geometry and loading parameters of the core and the lower face.

2.3 Calculation of the critical load level

Inserting the result for k^{thin} into Eq. (3), and solving the resulting equation for a non-trivial equilibrium ($w^u \neq 0$) leads to a quadratic equation for the load level *P* as a function of a and φ. Thus, for each combination of wavelength a and wave angle φ two load levels $P_{1,2}$ are calculated, the lower being decisive.

The reason for the two results for *P* is, that the load level *P* enters the model in the upper and in the lower face layer equation via Eq. (3) and k^{thin}, respectively. Regarding very thick sandwiches, where the upper and the lower face layer are uncoupled, it is obvious that there have to be two possible load levels *P* with the same wavelength a and wave angle φ, one corresponding to the buckling of each face layer. These two solutions have different deformation patterns in the sense that the amplitude of the deformation $w^*(z)$ varies differently throughout the core thickness while a and φ are identical.

It is an important aspect of this solution, that the resulting load levels are independent of the choice which of the two face layers is taken as the upper face layer. Regarding the derivation it becomes clear that, although the face layer on a foundation approach is used, both face layers are modeled the same way using equivalent boundary conditions. Therefore the solution is symmetric in the sense that the solution is independent of whether a face layer is the upper or the lower face.

In order to calculate the critical load level P^{crit}, $P(a, \varphi)$ has to be minimised with respect to a and

φ. This minimisation has to be done numerically, and two different methods have been employed. In the rather simple case of isotropic symmetric sandwiches it is sufficient to use a local search algorithm. In this case, only two local minima of $P(a, \varphi)$ exist and they are both easily found by calculating estimates for the starting points and using a sequential quadratic programming algorithm.

However, in the case of orthotropic sandwiches with different loading and material parameters for the two face layers, the situation is more complex. Fig. 3 shows the value of P as a function of a and φ. There are several local minima of $P(a, \varphi)$ in vicinity of each other. Although some effort was put into the calculation of appropriate starting values for different local search algorithms, it was not possible to calculate all different local minima by this method, and thus the global minimum was not always found using this approach. An example of these results is displayed in Fig. 4 which shows the critical load level depending on the core thickness of the sandwich. The dashed line represents the result for the local search algorithm and the saw teeth show that the local algorithm is sometimes not able to find the global minimum.

Consequently, a stochastic global optimisation code based on Boender *et al*. (1982) is employed. This routine calculates the value of $P(a, \varphi)$ at a number of different stochastic values of a and φ and starts a local search algorithm from the lowest values of P found so far. This global and local search is repeated until no new local minimum is found. Moreover a clustering algorithm prevents the code from evaluating the same minimum too often. Using a logarithmic scale for a (as in Fig. 3), the use of this algorithm to find all local minima and thus the global minimum of $P(a, \varphi)$ proved to be robust and reliable. The computational effort is kept to about 1000 evaluations of $P(a, \varphi)$ to find all minima, and the computing time needed is relatively low. Besides, the different local minima found by this code can all be related to buckling patterns such as symmetrical wrinkling, antimetrical wrinkling and shear buckling in different directions (see Fig. 4).

It is interesting to note that the result of $P(a \to \infty)$ corresponds to the shear buckling load which is derived by a simple and common model (see e.g., Zenkert 1995). This coincidence can not be

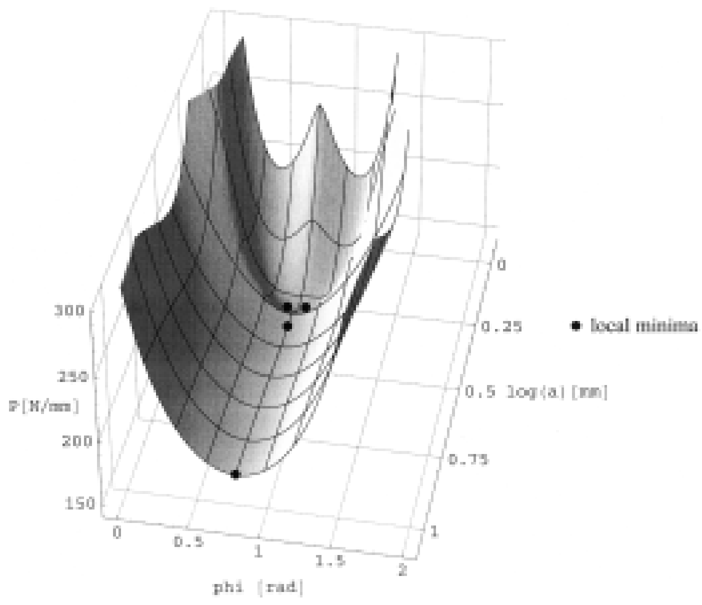

Fig. 3 Load level P as a function of a and φ

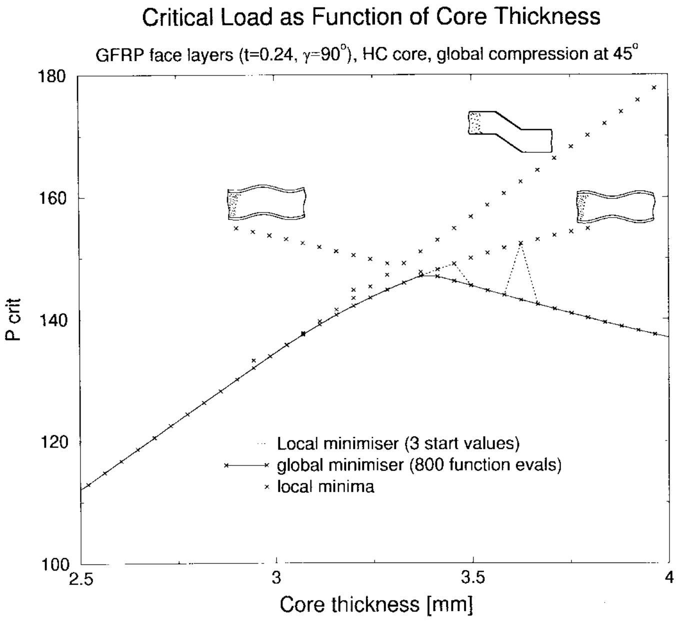

Fig. 4 Global and local optimisation

explained mechanically. Since this approach has been set up to investigate the local phenomenon of wrinkling under the assumption that the wavelength a is much smaller than the overall dimensions of the sandwich, a result for an infinite wavelength a is not within the scope of this approach. However, from the mathematical point of view it can be stated that the strain energy due to bending of the faces and the core vanishes compared to the strain energy due to shear deformation of the core as a approaches infinity. Since the shear buckling load is independent from the wavelength a and the buckling shape, the case $a \to \infty$ represents the case of pure shear buckling of the core. This shear buckling load is regarded as a conservative estimate for a local failure load, since $a < \infty$ leads to a higher critical load. It is obvious that for a global failure mode the interaction with Euler buckling has to be regarded, but this is not within the scope of this contribution.

3. Verification

Due to the formulation of all loading parameters relative to the load level P, the resulting P^{crit} represents the factor of safety for a given configuration and nominal load. In the following figures, the loading is chosen to be a unit-load so that P^{crit} equals the critical section force.

This new approach has been compared to the other analytical approaches mentioned in the introduction, and the agreement was found to be very good. However, these comparisons are limited to simple sandwich configurations due to the suppositions of the other analytical approaches.

A basic and conclusive new result is shown in Fig. 5 which displays the critical load level leading to wrinkling under pure bending load as a function of the core thickness. In this case of pure bending of a sandwich beam it has usually been assumed up to now that the face layer which is under tension remains perfectly flat. This case is shown as the result of Stiftinger and

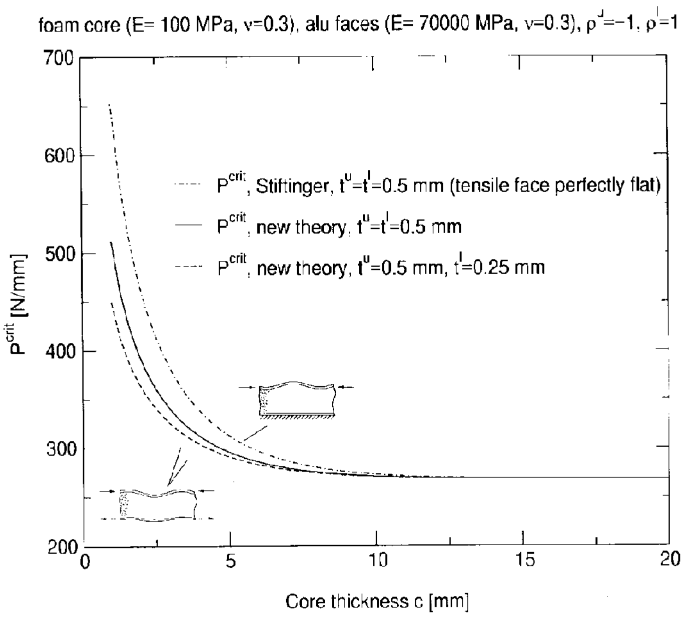

Fig. 5 Critical load level under pure bending

Rammerstorfer (1997) in the figure (t denotes the face layer thickness). In contrast to this assumption, the new model allows for deformations of the face layer which is under tension. As long as the core thickness is rather high, the results coincide. However, as the core thickness gets smaller, the interaction between the two face layers grows and the face layer which is under tension starts to deform during wrinkling. Therefore, the critical load level is lower than calculated by Stiftinger. Moreover, it is common practice to use a thinner face layer for the tensile face because local stability problems are only expected at the compressed face. As shown by the dashed line in Fig 5, this has also as an influence on the critical load level, due to face layer interaction in the case of rather thin cores.

Another basic problem that has not been solved up to now by analytical approaches is the transition between bending and compression of sandwich beams. In the paper of Stiftinger and Rammerstorfer (1997), FE calculations were compared to their analytical approach and the agreement was not satisfying. Fig. 6 shows their results as well as the new result. The model Stiftinger used calculates a foundation stiffness for pure compression and a foundation stiffness for pure bending (with a flat face as mentioned above). The results in the transition regime between bending and compression are then calculated by interpolating the foundation stiffness between the bending and the compression value (Stamm and Witte 1974). However, the FE results as well as the new results indicate that, in the considered example, there is a mode change between the failure under bending load, which is wrinkling, and the failure under compressive load, which is shear buckling. Therefore, the interpolation between these two results is not permissible.

Regarding the failure mode of shear buckling it is known (see Zenkert 1995) that this mode interacts with Euler buckling of the global structure. Using the appropriate interaction formula, the Euler buckling influence can be eliminated from the Stiftinger FE results and these modified FE results correspond to the shear buckling load which is the local failure load. The agreement between the new model and the FE results in Fig. 6 is shown to be very good. The discrepancy between the

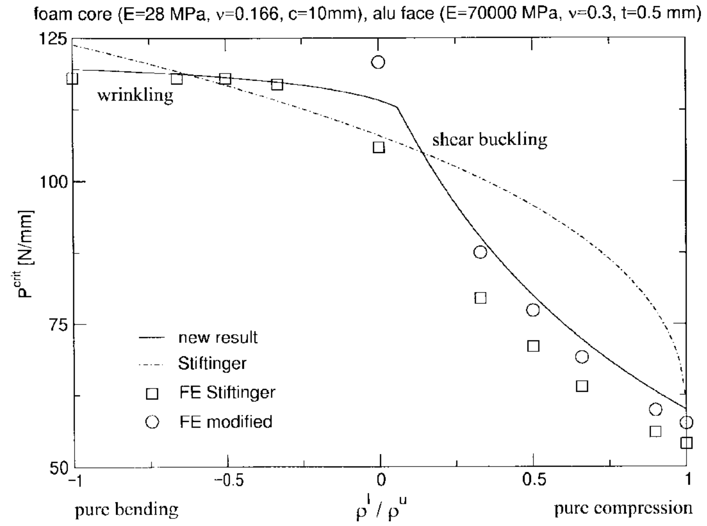

Fig. 6 Transition bending-compression

FE results and the new model for the case ρ^l/ρ^u=0 is due to the fact that the FE model calculated the critical mode as Euler buckling with shear influence. Therefore, the corresponding modified FE result is on the extension of the shear buckling curve of the new model. However, the local failure mode has already switched to wrinkling.

In cases similar to Fig. 6 where no mode change takes place, it is also observed that the new model fits FE calculations far better than the Stiftinger model. Thus, the interpolation method proposed by Stamm and Witte should not be applied.

So far, the verification has been confined to simple sandwich beam configurations where other analytical approaches exist. It is the aim of this work to present a wrinkling calculation which is capable of much more complex sandwich plate configurations and loading conditions. In order to verify the new model for these cases, it was chosen to use a periodic FE unit cell approach. This approach proved to be complex but sensible (Vonach and Rammerstorfer 2001), and by using several verification configurations it can be concluded, that the agreement between the FE model and the new model is also good in the case of differently oriented orthotropic face layers and general loading conditions. An example of the results obtained is shown in Fig. 7. This agreement deteriorates as the stiffness of the core increases relative to the stiffness of the face layers. However, such configurations are very unlikely to fail due to wrinkling since the wrinkling load exceeds the material strength for common sandwich materials such as aluminum, GFRP, CFRP and steel under these circumstances.

4. Conclusions

A new and most general model for the calculation of the wrinkling load of sandwiches has been set up. The model is based on the assumption of a single sinus wave pattern, which simplifies the 3D problem to the problem of a 2D section, thus being efficient in the analytical derivation.

The variety of wrinkling patterns in general orthotropic sandwich configurations enforces the use

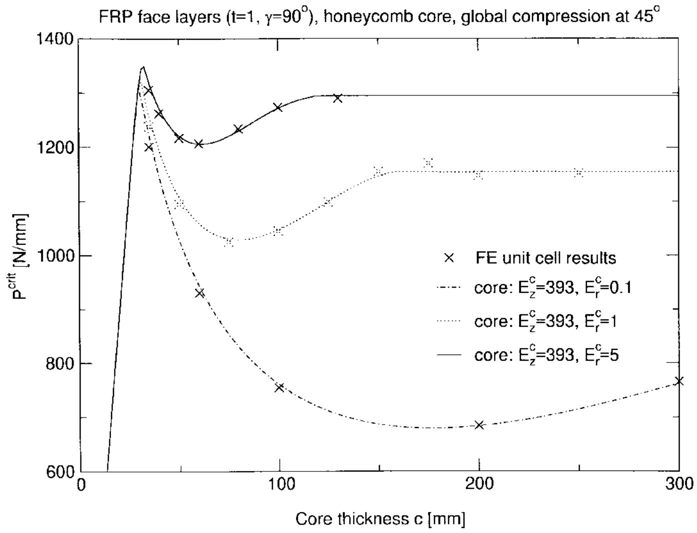

Fig. 7 Results of general verification, influence of E_r^c

of a stochastic global optimisation code. The computing time needed for these calculations is not very high, and it seems sensible to include this model in FE shell elements on integration point level.

The new model agrees very well with other analytical approaches. In the case of wrinkling under bending loads or combinations of bending and compression it is shown that the new model provides much better and more conclusive results than other approaches. In the case of general orthotropic sandwich plates under general loading conditions the model has been verified using FE unit cell calculations. The results proved to be in good agreement.

This new model is capable of calculating wrinkling loads for sandwich configurations and loading situations which, regarding the global behaviour, can be modeled using common FE sandwich shell elements. It can therefore be implemented into FE sandwich shell elements or into post processing routines leading to a reliable and automated wrinkling calculation for FE users.

Acknowledgements

The FORTRAN code of the global optimisation package is supplied as freeware by Dr. T. Csendes of József Attila University in Szeged, Hungary, which is hereby gratefully acknowledged.

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Notation

a : wavelength of the deformation pattern [mm]
B : plate bending stiffness per unit width [Nmm]
c : core thickness [mm]
D : core membrane stiffness per unit thickness (plane strain) [N/mm^2]
E : elastic modulus [N/mm^2]
F : Airy's stress function [N]
G : shear modulus [N/mm^2]
k^{thin} : foundation stiffness [N/mm^3]
P : global load level [N/mm]
P_j^i : compressive membrane section force per unit width [N/mm]
w : deflection in z - Direction [mm]
ε : strain [ ]
φ : wave angle of deformation pattern [ ]
ρ, ξ, ζ : dimensionless load factors [ ]
ν : poisson number [ ]
σ : stress [N/mm^2]

Superscripts

c : core
l : lower face layer
u : upper layer
crit : critical value
* : amplitude value

Subscripts

x,y,z,r,s : coordinate directions (Fig. 1)