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Vertical vibrations of a multi-span beam steel bridge induced by a superfast passenger train

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Abstract. Transient and quasi-steady-state vertical vibrations of a multi-span beam steel bridge located on a single-track railway line are considered, induced by a superfast passenger train, moving at speed 120-360 km/h. Matrix dynamic equations of motion of a simplified model of the system are formulated partly in the implicit form. A recurrent-iterative algorithm for solving these equations is presented. Excessive vibrations of the system in the resonant zones are reduced effectively with passive dynamic absorbers, tuned to the first mode of a single bridge span. The dynamic analysis has been performed for a series of types of bridges with span lengths of 10 to 30 m, and with parameters closed to multi-span beam railway bridges erected in the second half of the 20th century.

Key words: railway bridge; multi-span bridge; beam steel bridge; superfast train; moving load; transient vibrations; quasi-steady-state vibrations; passive dynamic absorbers; vibration control.

1. Introduction

Railway bridges under trains moving at high speeds create systems of a fast-varying configuration. Fryba (1972) presented the fundamental problems in dynamics of structures under moving load. Barchenkov (1976) considered the vehicle-bridge system and formulated dynamic equations of motion analytically and partly in the implicit form. Papers by Matsuura (1970, 1974, 1979) are of important performance in the vibration theory of single-span railway bridges. Fryba, Barchenkov and Matsuura applied the Galerkin method, without using a matrix calculus.

The fundamentals of dynamics of single-track single-span railway steel bridges, under action of trains moving at high speeds, developed (Klasztorny 1987). The author considered beam bridges with a closed or open platform and took into account the local flexibility of the track, both on the bridge and in the zones of access. He formulated models of rail-vehicles with single and double suspensions. Applying the Ritz-FEM combined method, he developed a concept of formulating matrix dynamic equations of motion, partly in the implicit form.

Klasztorny and Langer (1990) formulated a general matrix equation of motion, in the explicit form, governing vibrations of a beam bridge under a stream of moving oscillators. The authors discussed the basic phenomena ocurring in such a system, i.e., transient vibrations, steady-state vibrations, damped forced resonances, parametric-forced resonances and dynamic stability.

A theoretical analysis of dynamic behaviour of a single-track multi-span railway beam bridge, under a superfast passenger train moving at high speed of 120-360 km/h, is the basic goal of this

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study. The analysis is limited to the vertical transient and quasi-steady-state vibrations of the system. Matrix equations of motion of the subsystems are formulated partly in the implicit form and a computer algorithm for solving these equations is developed. In order to reduce excessive quasi-steady-state vibrations of the bridge in the forced resonance zones, the optimally tuned passive dynamic absorbers are applied (Klasztorny 1995). The analysis has been performed for a series of types of four-span steel bridges, with a repeatable span length of 10 to 30 m.

The shortened version of this paper has been presented at the 4^{th} European Conference on Structural Dynamics (Klasztorny 1999).

2. Physical model of vibrating system

In this study, the following assumptions are adopted:

- A single-track railway steel bridge consists of a number of repeatable spans. The spans are simply supported and consist of thin-walled main girders and a closed heavy platform.
- The rails are continuous beams, with rectilinear axes when unloaded.
- Damping of vibrations of each span is of an external type and satisfies the constant-decrement-damping model (Langer 1980).
- A superfast passenger train consists of a number of repeatable four-axle double-suspended rail-vehicles (Matsuura 1970, 1974, 1979).
- The bridge-moving train system is linear, both physically and geometrically.

Dynamic analysis of the system is focussed on the influence of repeatable bridge spans, repeatable rail-vehicles and high speeds of the train on dynamic response of the system.

The analysis will be performed on a simplified model of the vibrating system. A multi-span bridge is modelled as a set of simply supported Euler-Bernoulli beams, as shown in Fig. 1. A planar model of a rail-vehicle is assumed as a system of three sprung disks, reflecting the body and two trucks, joined with viscoelastic suspensions of the first and second stage, all reflected in Fig. 2. Weights of the wheel sets are also included, whereas small vertical inertia forces of these sets are neglected.

3. Dynamic equations of motion of subsystems

Following the Ritz method, the vertical deflections of each span can be approximated globally,

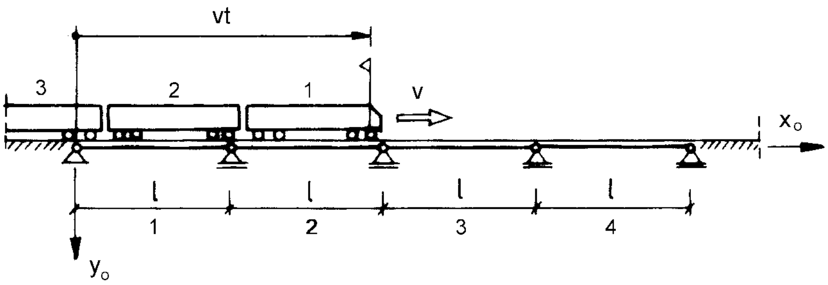

Fig. 1 A pictorial diagram of the multi-span bridge-moving train system

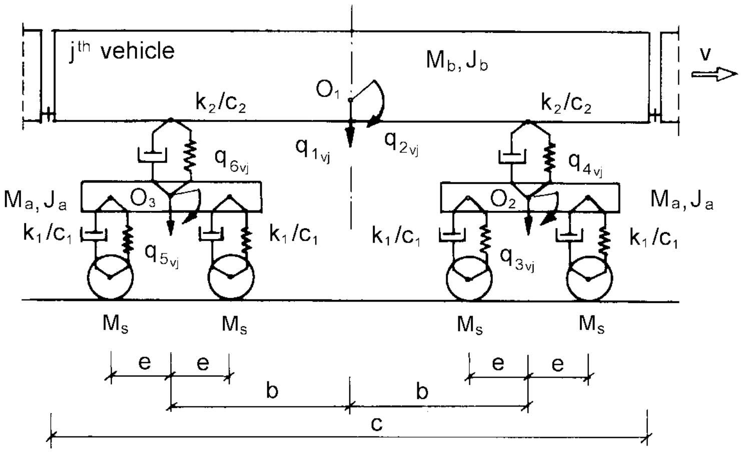

Fig. 2 A planar model of a rail-vehicle with 6 degrees of freedom

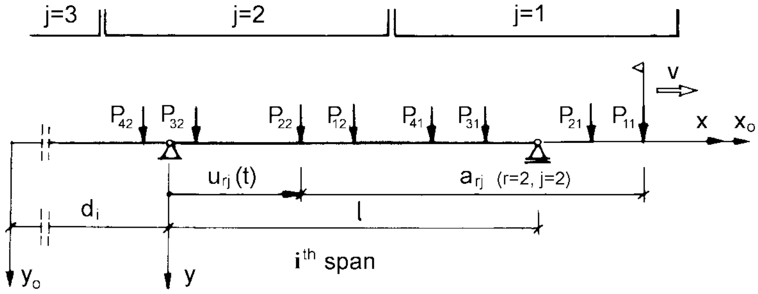

Fig. 3 A stream of moving forces $P_{rj}(t)$ partly acting on the i-th bridge span

using a complete set of kinematically admissible functions. In case of rolling supports, a sine series is in common use (Fryba 1972). For the i^{th} span one obtains (Fig. 3)

$$
\begin{gathered}
w_i(x,t)=\boldsymbol{q}_i^T(t)\boldsymbol{s}(x), \quad i=1,2,\ ...,\ N_b \\
\boldsymbol{q}_i=\mathrm{col}(q_{1i},q_{2i},...,q_{n_b i}),\ \boldsymbol{s}(x)=\mathrm{col}(s_1,s_2,...,s_{n_b}) \\
s_k(x)=\sin(k\pi x/l),\ k=1,2,...,\ n_b
\end{gathered}
\tag{1a-f}
$$

where w_i is the vertical deflection and $\boldsymbol{q}_i$ is a vector of the generalised coordinates, both for the i^{th} span; t is a time variable, n_b is a number of the aproximating functions $\boldsymbol{s}$ for each span, and N_b is a number of the bridge spans, each of length l. A matrix dynamic equation of motion for the i^{th} span has the following form (Klasztorny 1987)

$$
\begin{gathered}
\boldsymbol{B}_i\ddot{\boldsymbol{q}}_i+\boldsymbol{C}_i\dot{\boldsymbol{q}}_i+\boldsymbol{K}_i\boldsymbol{q}_i=\boldsymbol{F}_i, \quad i=1,2,...,\ N_b \\
\boldsymbol{B}_i=(ml/2)\boldsymbol{I},\ \boldsymbol{K}_i=(EI/2l^3)\{\boldsymbol{d}^4\},\ \ \boldsymbol{C}_i=(\gamma\sqrt{EIm}/l)\{\boldsymbol{d}^2\},\ \ \{\boldsymbol{d}\}=\mathrm{diag}(\pi,\ 2\pi,\ ...,\ n_b\pi)
\end{gathered}
\tag{2a-f}
$$

where EI, m, l, γ respectively denote the flexural rigidity, mass per unit length, span length and damping ratio for the beam modelling of a single bridge span, with γ_{cr}=1. The symbols $\boldsymbol{B}_i$, $\boldsymbol{C}_i$, $\boldsymbol{K}_i$

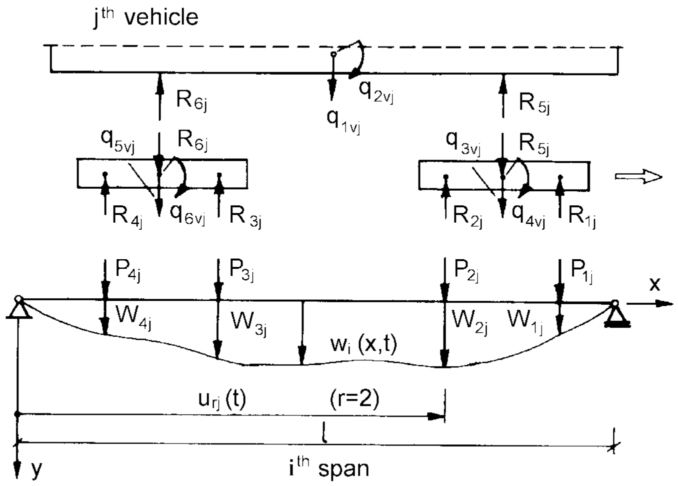

Fig. 4 A set of dynamic interactions carried by the first- and second-stage suspensions of the *j*-th rail-vehicle

denote the mass, damping and stiffness matrices for the i^{th} span. Differentiation with respect to time is marked with a dot, and symbol $\boldsymbol{I}$ denotes an identity matrix. The load vector for the i^{th} span, $\boldsymbol{F}_i$, is calculated from the formulae (Figs. 3 and 4, Klasztorny 1987)

$$\boldsymbol{F}_i(t)=\sum_{j=1}^{N_v}\sum_{r=1}^{4}P_{rj}(t)\boldsymbol{S}_{rj}(t)$$

$$P_{rj}(t)=G_j+R_{rj}(t),\ \boldsymbol{S}_{rj}(t)=\left\{\begin{array}{l} s[u_{rj}(t)],u_{rj}(t)\in(0,l) \\ \boldsymbol{0}\ \ u_{rj}(t)\notin(0,l)\end{array}\right\},\ u_{rj}(t)=vt-a_{rj}-d_i,\ \ d_i=(i-1)l \quad \text{(3a-e)}$$

where P_{rj} is the dynamic pressure of the r^{th} wheel axle of the j^{th} rail-vehicle on the track, G_j is the static pressure of this axle on the track, a_{rj} is a distance of this axle from the load head, R_{rj} is the dynamic interaction carried by the first-stage suspension, N_v is a number of rail-vehicles, v is a service velocity of the train.

Vertical vibrations of the j^{th} moving vehicle are measured from the static equilibrium level, determined on a rectilinear (unloaded) track and described by six coordinates, drawn in Figs. 2 and 4, creating the vector $\boldsymbol{q}_{vj}(t)=\mathrm{col}(q_{1vj},q_{2vj},...,q_{6vj})$. Matrix dynamic equation of motion of the j^{th} rail-vehicle, partly in the implicit form, can be written as (Figs. 2 and 4; Klasztorny 1987)

$$\boldsymbol{B}_{vj}\ddot{\boldsymbol{q}}_{vj}=\boldsymbol{F}_{vj},\ j=1,2,....,N_v$$

$$\boldsymbol{B}_{vj}=\mathrm{diag}(M_b,\ J_b,\ M_a,\ J_a,\ M_a,\ J_a),\ \ \boldsymbol{F}_{vj}=\mathrm{col}(F_{1vj},\ F_{2vj},...,\ F_{6vj})$$

$$F_{1vj}=-R_{5j}-R_{6j},\ F_{2vj}=(R_{6j}-R_{5j})b,\ F_{3vj}=R_{5j}-R_{1j}-R_{2j},\ F_{4vj}=(R_{2j}-R_{1j})e$$

$$F_{5vj}=R_{6j}-R_{3j}-R_{4j},\ F_{6vj}=(R_{4j}-R_{3j})e \quad \text{(4a-j)}$$

where $\boldsymbol{B}_{vj}$, $\boldsymbol{F}_{vj}$ are respectively the mass matrix and the load vector for the j^{th} vehicle, M_b, M_a are the body and the truck masses, J_b, J_a are the mass polar inertia moments of the body and the truck. The distance between the second-stage suspensions equals $2b$, whereas the axle base in each truck equals $2e$.

Dynamic interactions carried by suspensions in the j^{th} rail-vehicle are expressed by the following formulae (see Figs. 3 and 4)

$$R_{rj}=k_1(V_{rj}-W_{rj})+c_1(\dot{V}_{rj}-\dot{W}_{rj}),\ \ r=1,2,3,4$$

$$V_{1j}=q_{3vj}+eq_{4vj},\ V_{2j}=q_{3vj}-eq_{4vj},\ V_{3j}=q_{5vj}+eq_{6vj},\ V_{4j}=q_{5vj}-eq_{6vj}$$

$$W_{rj}(t)=\boldsymbol{q}_i^T\boldsymbol{S}_{rj},\ \dot{W}_{rj}(t)=\dot{\boldsymbol{q}}_i^T\boldsymbol{S}_{rj}+\boldsymbol{q}_i^T\dot{\boldsymbol{S}}_{rj},\ \dot{\boldsymbol{S}}_{rj}(t)=\left\{\begin{array}{ll}\frac{v}{l}\{\boldsymbol{d}\}\boldsymbol{c}[u_{rj}(t)], & u_{rj}(t)\in(0,l)\\ \boldsymbol{0}, & u_{rj}(t)\notin(0,l)\end{array}\right\}$$

$$\boldsymbol{c}(x)=\mathrm{col}(c_1,c_2,...,c_{n_b}),\ c_k(x)=\cos\left(\frac{k\pi x}{l}\right),\ \ k=1,2,...,n_b$$

$$R_{5j}=k_2(q_{1vj}+bq_{2vj}-q_{3vj})+c_2(\dot{q}_{1vj}+b\dot{q}_{2vj}-\dot{q}_{3vj})$$

$$R_{6j}=k_2(q_{1vj}-bq_{2vj}-q_{5vj})+c_2(\dot{q}_{1vj}-b\dot{q}_{2vj}-\dot{q}_{5vj}) \tag{5}$$

where k_1, k_2, c_1, c_2 are respectively stiffnesses and viscous damping coefficients of the first- and second-stage suspensions.

Eqs. (2a) and (4a), formulated partly in the implicit form, describe vertical transient and quasi-steady-state vibrations of the multi-span bridge-moving train system. For a sine approximation of the displacements of each span, the vertical displacement following the r^{th} wheel axle of the j^{th} rail-vehicle, W_{rj}, is continuous, while the second component of the vertical velocity following this axle, $\dot{W}_{rj}$, is discontinuous over the supports. This fact contradicts the reality. In order to decrease this discontinuity, to protect convergence of numerical integration of the equations of motion, vector $\dot{\boldsymbol{S}}_{rj}$ has been interpolated in short sections joining the supports, each of length of a_s, according to the formulae

$$\dot{\boldsymbol{S}}_{rj}=\dot{\boldsymbol{S}}_{rj}^0+\frac{u_{rj}(t)}{a_s}(\dot{\boldsymbol{S}}_{rj}^1-\dot{\boldsymbol{S}}_{rj}^0),\ \ u_{rj}(t)\in(0,a_s)$$

$$\dot{\boldsymbol{S}}_{rj}^0=\begin{cases}\boldsymbol{0} & ,i=1\\ \frac{v}{l}\{\boldsymbol{d}\}\boldsymbol{c}(L),i>1\end{cases},\ \dot{\boldsymbol{S}}_{rj}^1=\begin{cases}\frac{v}{l}\{\boldsymbol{d}\}\boldsymbol{c}(a_s),i\le N_s\\ \boldsymbol{0} & ,i=N_s+1\end{cases} \tag{6}$$

4. Vibration control with dynamic absorbers

In a multi-span bridge-moving train system forced resonances appear, as a result of periodicity of the moving load. The bridge can be protected from excessive forced vibrations by attaching light

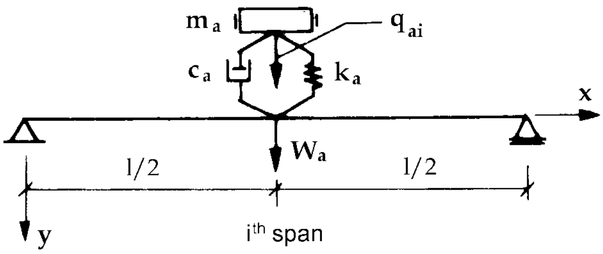

Fig. 5 The optimal location of the dynamic absorber in the *i*-th bridge span

passive dynamic absorbers to the spans, provided that the absorbers are tuned to the selected modal system of a single bridge span. The introductory analysis pointed that practically the resonances may only concern the fundamental modal system of each span. Thus the optimal location of an absorber device is in the mid-span of each bridge span (Fig. 5; Klasztorny 1995).

The author developed a general theory of reducing excessive vibrations of light damped structures, using passive dynamic absorbers (Klasztorny 1995). The matrix dynamic equation of motion (Eq. 2a), governing vibrations of the i^{th} bridge span equipped with the dynamic absorber in the mid-span, modifies to the form

$$\boldsymbol{B}_{bi}\ddot{\boldsymbol{q}}_{bi}+\boldsymbol{C}_{bi}\dot{\boldsymbol{q}}_{bi}+\boldsymbol{K}_{bi}\boldsymbol{q}_{bi}=\boldsymbol{F}_{bi},\quad i=1, 2, ..., N_b$$

$$\boldsymbol{B}_{bi}=\begin{bmatrix}\boldsymbol{B}_i & \boldsymbol{0}\\ \boldsymbol{0}^T & m_a\end{bmatrix},\ \boldsymbol{C}_{bi}=\begin{bmatrix}\boldsymbol{C}_i+c_a\boldsymbol{S}\boldsymbol{S}^T & -c_a\boldsymbol{S}\\ -c_a\boldsymbol{S}^T & c_a\end{bmatrix}$$

$$\boldsymbol{K}_{bi}=\begin{bmatrix}\boldsymbol{K}_i+k_a\boldsymbol{S}\boldsymbol{S}^T & -k_a\boldsymbol{S}\\ -k_a\boldsymbol{S}^T & k_a\end{bmatrix}$$

$$\boldsymbol{F}_{bi}=\begin{bmatrix}\boldsymbol{F}_i\\ 0\end{bmatrix},\ \boldsymbol{q}_{bi}=\begin{bmatrix}\boldsymbol{q}_i\\ q_{ai}\end{bmatrix}$$

$$\boldsymbol{0}=\text{col}(0,0,0,...),\qquad \boldsymbol{S}=\text{col}(1,0,-1,0,1,0,-1,0,...)\tag{7a-i}$$

where q_{ai} is the vertical displacement of the absorber mass m_a, whereas k_a, c_a are respectively stiffness and viscous damping coefficient of the absorber. The absorber parameters are calculated from the formulae (Klasztorny 1995)

$$m_a=\mu\frac{ml}{2},\ k_a=\alpha^2\left(\frac{\pi}{l}\right)^4\frac{EI}{m}m_a,\ c_a=2\gamma_a\sqrt{k_a m_a}\tag{8}$$

where μ, α, γ_a are dimensionless parameters of the den Hartog problem. Weights of light absorbers attached to the spans are neglected.

5 Other problems

Dynamic equations governing vibrations of the subsystems (Eqs. 2a, 4a, 7a) can be solved only numerically. In this study, a conditionally stable variant of the Newmark method is used, without numerical damping and with the parameter β=1/12 (Newmark 1959). This variant minimizes the period error. Since the equations are partly in the implicit form, linear algebraic equations with constant matrix coefficients are solved in each time step of length h. The integration algorithm is of a recurrent-iterative type. In the i^{th} time step, dynamic interactions carried by suspensions of the j^{th} rail-vehicle, $\boldsymbol{R}_j(t)=\text{col}(R_{1j}, R_{2j}, ..., R_{6j})$, are extrapolated parabolically (predictor), i.e.,

$$\boldsymbol{R}^p_{j,i+1}=3(\boldsymbol{R}_{j,i}-\boldsymbol{R}_{j,i-1})+\boldsymbol{R}_{j,i-2}\tag{9}$$

with $\boldsymbol{R}_{j,i}=\boldsymbol{R}_j(ih)$. The corrector $\boldsymbol{R}^c_{j,i+1}$ is determined from Eqs. (5). First three iterations are obligatory. The iteration process in a given step is finished if the following conditions of accuracy are satisfied:

$$\left|R^{c}_{rj,i+1} - R^{p}_{rj,i+1}\right| < \varepsilon, \qquad r=1,2,...,6, \qquad j=1,2,...,N_v \tag{10}$$

The critical velocities of the moving train can be estimated from a condition of equating the i^{th} natural period of a single span and the j^{th} harmonic component of the parametric-forced excitation (Klasztorny and Langer 1990), i.e.,

$$T_i=\frac{1}{i^2 f_1}, \;\; \overline{T}_j=\frac{c}{jv}, \;\; T_i=\overline{T}_j \Rightarrow v_{cr,\,i,\,j}=\frac{i^2 f_1 c}{j} \tag{11}$$

where f_1 is the fundamental natural frequency of a single span, and c is the total length of a repeatable vehicle.

Normal stresses in the bottom fibres of the superstructure of the i^{th} bridge span are calculated from the well-known formula

$$\sigma_i(x,t)=-\frac{EI}{U}\frac{\partial^2 w_i}{\partial x^2}=\frac{EI}{Ul^2}\boldsymbol{q}_i^T(t)\{\boldsymbol{d}^2\}\boldsymbol{s}(x) \tag{12}$$

where U is a bending index. The deflection $w_i(x, t)$ is defined by Eq. (1a).

The dynamic solutions are backgrounded by the quasi-static solutions determined from the folowing formulae describing the quasi-static passage of the train:

$$\boldsymbol{K}_i\boldsymbol{q}_{si}=\boldsymbol{F}_{si}, \;\; \boldsymbol{F}_{si}=\sum_{j=1}^{N_v}\sum_{r=1}^{4} G_j\boldsymbol{S}_{rj}(t) \Rightarrow \boldsymbol{q}_{si}=\boldsymbol{K}_i^{-1}\boldsymbol{F}_{si}$$

$$w_{si}(x,t)=\boldsymbol{q}_{si}^T(t)\boldsymbol{s}(x), \;\; \sigma_{si}(x,t)=\frac{EI}{Ul^2}\boldsymbol{q}_{si}^T(t)\{\boldsymbol{d}^2\}\boldsymbol{s}(x) \tag{13}$$

with the vector $\mathbf{S}_{rj}(t)$ calculated from Eq. (3c). Eqs. (13) directly result from Eqs. (1), (2), (3) and (12).

Relative, dynamic and quasi-static, time histories of the deflections and normal stresses and the dynamic coefficients for the i^{th} bridge span are calculated from well-known formulae

$$\tilde{w}_i=\frac{w_i}{\|w_i\|}, \;\; \tilde{w}_{si}=\frac{w_{si}}{\|w_i\|}, \;\; \tilde{\sigma}_i=\frac{\sigma_i}{\|\sigma_i\|}, \;\; \tilde{\sigma}_{si}=\frac{\sigma_{si}}{\|\sigma_i\|}$$

$$\|w_i\|=\max_t w_{si}, \;\; \|\sigma_i\|=\max_t \sigma_{si}$$

$$\varphi_{w_i}(x)=\max_t \tilde{w}_i(x,t), \;\; \varphi_{\sigma_i}(x)=\max_t \tilde{\sigma}_i(x,t) \tag{14}$$

The relative time histories of the dynamic pressures of the wheel sets on the track and the loading and unloading coefficients are calculated from the formulae

$$\tilde{P}_{rj}(t)=\frac{P_{rj}(t)}{G_j}, \;\; \varphi_{P_{rj}}=\max_t \tilde{P}_{rj}(t), \;\; \rho_{P_{rj}}=\min_t \tilde{P}_{rj}(t) \tag{15}$$

The unloading coefficients can be directly used for the assessment of the derailment risk of the superfast train.

Table 1 The parameters of a type of series of railway beam steel bridges

Bridge Code	N_b	l [m]	m [kg/m]	f_1 [Hz]	γ
B10	4	10	4800	12	0.03
B15	4	15	5000	8	0.03
B20	4	20	5200	6	0.03
B25	4	25	5400	4,5	0.03
B30	4	30	5600	4	0.03

6. Numerical analysis of the problem

Taking into account Eqs. (1)-(15), a computer algorithm has been formulated, written in Pascal and tested positively. The problem undertaken in the study has been analysed on a type of series of four-span steel bridges, with the parameters close to actual bridges with closed heavy platforms, erected in the second half of the 20th century. The parameters of the bridges forming a type of series are given in Table 1 (Klasztorny 1987).

The bridges are under action of a superfast passenger train (ST), with the parameters corresponding to a Shinkansen train (Matsuura 1970), of the following values

$$N_v=8,\ c=25\text{m},\ b=8.75\text{m},\ e=1.25\text{m}$$
$$k_1=2540000\ \text{N/m},\ c_1=19630\ \text{Ns/m},\ k_2=887000\ \text{N/m},\ c_2=43350\ \text{Ns/m}$$
$$M_s=2400\ \text{kg},\ M_a=4950\ \text{kg},\ J_a=3136\ \text{kgm}^2,\ M_b=36000\ \text{kg},\ J_b=1894000\ \text{kgm}^2$$

Excessive quasi-steady-state (QSS) vibrations of each bridge were suppressed with four passive dynamic absorbers (PDAs), located in the mid-spans. Mass of a single absorber has been assumed as 1.5% of the total mass of a single span. The dimensionless parameters of the absorbers, tuned to the first mode of the bridge, equal μ=0.03, α_{opt}=0.966, $\gamma_{a,opt}$=0.106 (Klasztorny 1995).

Accuracy of numerical integration of the dynamic equations of motion depends on a number of time steps per one cycle, $\tilde{N}$, related to the highest natural frequency of a single bridge span, as well as on a value of the parameter ε. The values of $\tilde{N}=10$, $\varepsilon=10^{-12}$ protect high accuracy of time histories of the output quantities; the relative errors do not exceed 10^{-5}.

For each passage of the ST train over the selected bridge, time histories of the vertical deflections and the bottom normal stresses were computed, in the cross-sections $x_1=0.50\,l$, $x_2=0.75\,l$. Moreover, time histories of the dynamic pressures of all moving wheel sets were derived.

The maximum values of dynamic coefficients for the displacements and bottom normal stresses, ocurring in the bridges described in Table 1, are collected in Table 2. These values correspond to the passages of the ST train moving at selected critical and non-critical velocities, belonging to the interval [120 km/h, 360 km/h]. The symbol B-T denotes the unmodified bridge-moving train system, while the symbol B-A-T denotes the bridge-moving train system modified with a set of passive dynamic absorbers.

Each bridge behaves differently when compared with the others. For the B10 bridge, only $v_{cr,1,3}$ can be reached for j =1, 2, 3. The PDAs protect effectively this bridge from excessive QSS vibrations at $v_{cr,1,3}$, by reducing dynamic increments in the displacements and stresses by 60%. The higher critical velocities less than 360 km/h do not cause valuable dynamic effects, since respective loading components are very small.

In the B15/ST system, critical velocities $v_{cr,1,2}$, $v_{cr,1,3}$ induce bigger dynamic increments, reduced by 35 to 60% with PDAs. The next bridge (B20) behaves save for both critical and non-critical

Table 2 Maximum values of dynamic coefficients of the vertical displacements and the bottom normal stresses, related to selected velocities of the ST train

Bridge Code	$v_{cr,i,j}$ [km/h] v [km/h]	max φ_w		max φ_σ	
		B-T	B-A-T	B-T	B-A-T
B10	v=180	1.13	1.10	1.12	1.10
	$v_{cr,1,5}$=216	1.07	1.08	1.07	1.07
	$v_{cr,1,4}$=270	1.29	1.22	1.29	1.22
	v=300	1.33	1.39	1.32	1.38
	$v_{cr,1,3}$ = 360	<u>2.31</u>	<u>1.54</u>	<u>2.23</u>	<u>1.49</u>
B15	$v_{cr,1,4}$=180	1.07	1.07	1.08	1.08
	$v_{cr,1,3}$=240	1.40	1.16	1.49	1.24
	v=300	1.06	1.05	1.21	1.19
	$v_{cr,1,2}$=360	<u>1.48</u>	<u>1.27</u>	<u>1.66</u>	<u>1.43</u>
B20	$v_{cr,1,3}$=180	1.13	1.10	1.14	1.11
	v=200	1.12	1.09	1.15	1.11
	$v_{cr,1,2}$=270	1.17	1.15	1.20	1.17
	v=360	<u>1.52</u>	<u>1.50</u>	<u>1.54</u>	<u>1.53</u>
B25	$v_{cr,1,3}$=135	1.17	1.11	1.18	1.12
	v=180	1.19	1.18	1.19	1.18
	$v_{cr,1,2}$=203	1.23	1.20	1.23	1.20
	v=300	1.41	1.48	1.41	1.49
	v=360	<u>2.21</u>	<u>2.09</u>	<u>2.21</u>	<u>2.08</u>
B25M	v=360	<u>1.23</u>	<u>1.24</u>	<u>1.24</u>	<u>1.24</u>
B30	$v_{cr,1,3}$=120	1.07	1.06	1.07	1.05
	$v_{cr,1,2}$=180	1.14	1.13	1.12	1.14
	v=240	1.19	1.16	1.18	1.15
	v=300	1.40	1.46	1.39	1.46
	$v_{cr,1,1}$=360	<u>2.13</u>	<u>1.61</u>	<u>2.17</u>	<u>1.67</u>

velocities. Since velocity v=360 km/h is non-critical, the absorber devices are inefficacious.

The B25 bridge exhibits unacceptable QSS vibrations at non-critical velocity v=360 km/h. In this case, the only possible way to suppress the vibrations to an acceptable level is to build a stronger bridge. For example, increasing the height of the main girders by 30% results in relatively small

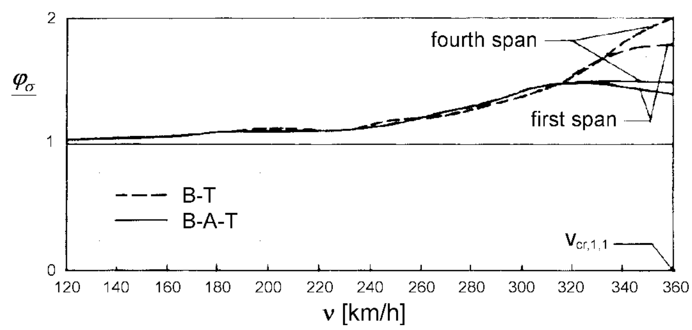

Fig. 6 Transient and quasi-steady-state vibrations of the B30/TS system. Dynamic coefficient $\varphi_\sigma(0.5l)$ versus velocity v [km/h]

Table 3 Extremum values of the loading and unloading coefficients for the wheel sets, related to selected velocities of the ST train

Bridge Code	$v_{cr,i,j}$ [km/h] v [km/h]	max φ_P		min ρ_P	
		B-T	B-A-T	B-T	B-A-T
B10	$v_{cr,1,3}=360$	1.07	1.05	0.92	0.95
B15	$v_{cr,1,3}=240$	1.08	1.06	0.93	0.94
	$v_{cr,1,2}=360$	1.12	1.10	0.88	0.90
B20	$v_{cr,1,3}=180$	1.03	1.03	0.97	0.97
	$v_{cr,1,2}=270$	1.05	1.05	0.94	0.94
	$v=360$	1.12	1.12	0.89	0.89
B25	$v_{cr,1,3}=135$	1.05	1.04	0.95	0.95
	$v_{cr,1,2}=203$	1.06	1.05	0.95	0.96
	$v=360$	1.25	1.21	0.82	0.84
B30	$v_{cr,1,3}=120$	1.05	1.05	0.94	0.94
	$v_{cr,1,2}=180$	1.04	1.04	0.95	0.95
	$v_{cr,1,1}=360$	1.22	1.13	0.79	0.84

values of the dynamic increments (see the B25M bridge in Table 2).

For the B30 bridge, the critical velocity $v_{cr,1,1}$ is reached, giving dangerous QSS vibrations of the system. After attaching the PDAs, the dynamic increments are suppressed by about 45% to an admissible level. At the remaining velocities the behaviour of the B30 bridge is safe.

A typical diagram of a selected dynamic coefficient versus velocity v is presented in Fig. 6, in which coefficient $\varphi_\sigma(0.5l)$ for the B30 bridge is presented for $v \in$ [120 km/h, 360 km/h]. For non-critical velocities the curves related to the B-T and B-A-T system slightly differ from each other. The values of relevant dynamic coefficients are very close for all spans. In the resonant zone around $v_{cr,1,1}$=360 km/h, the dynamic increments rise rapidly and the vibrations of the first span are substantially smaller in comparison with the others. As expected, the maximum dynamic effects occur in the last span. Moreover, one can observe suppressing excessive QSS vibrations with the

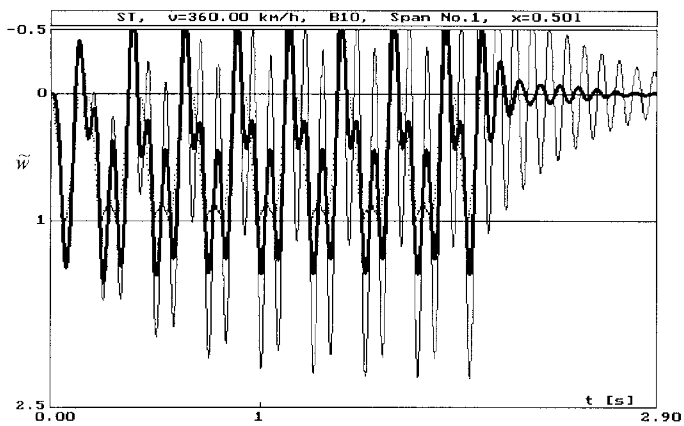

Fig. 7 The vertical deflection versus time, in the selected span and section of the B10 bridge, induced by the ST train moving at v=360 km/h

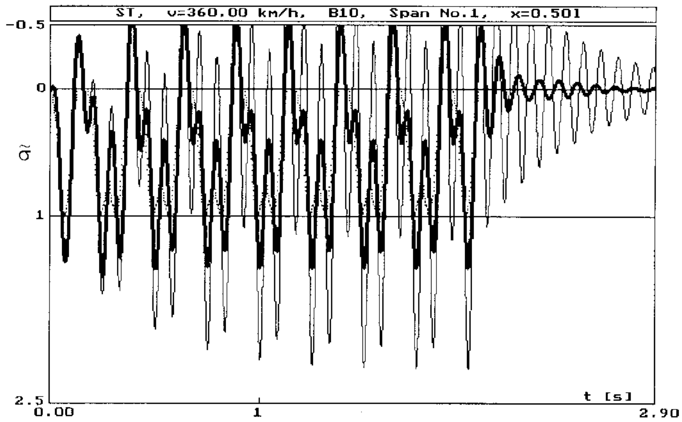

Fig. 8 The bottom normal stress versus time, in the selected span and section of the B10 bridge, induced by the ST train moving at v=360 km/h

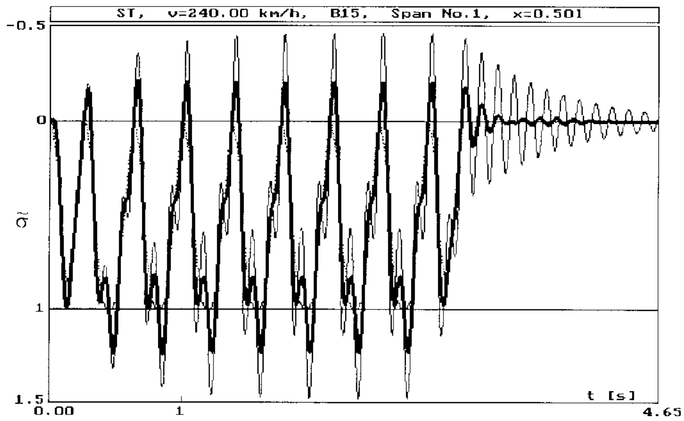

Fig. 9 The bottom normal stress versus time, in the selected span and section of the B15 bridge, induced by the ST train moving at v=240 km/h

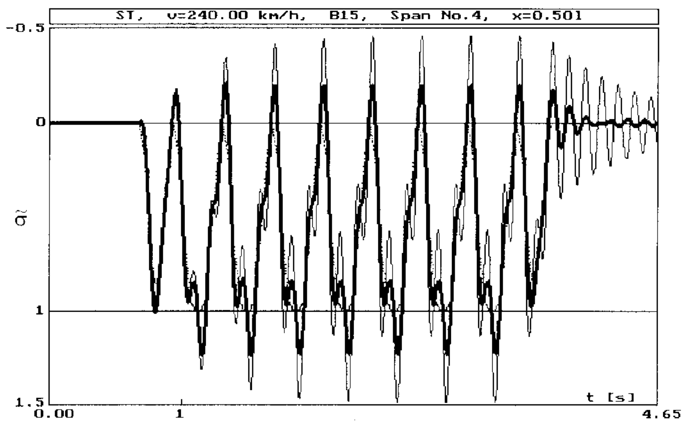

Fig. 10 The bottom normal stress versus time, in the selected span and section of the B15 bridge, induced by the ST train moving at v=240 km/h

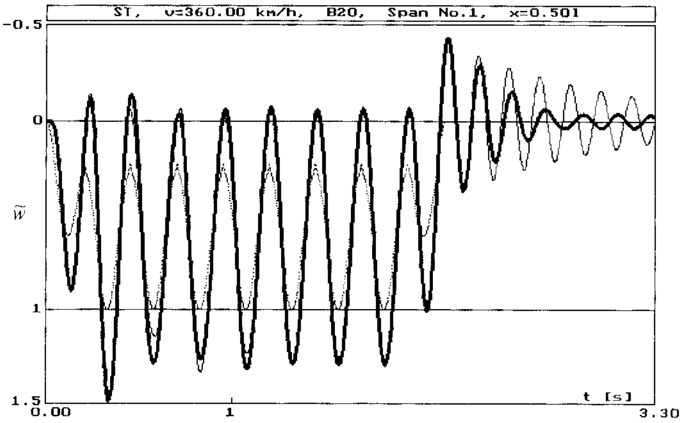

Fig. 11 The vertical deflection versus time, in the selected span and section of the B20 bridge, induced by the ST train moving at v=360 km/h

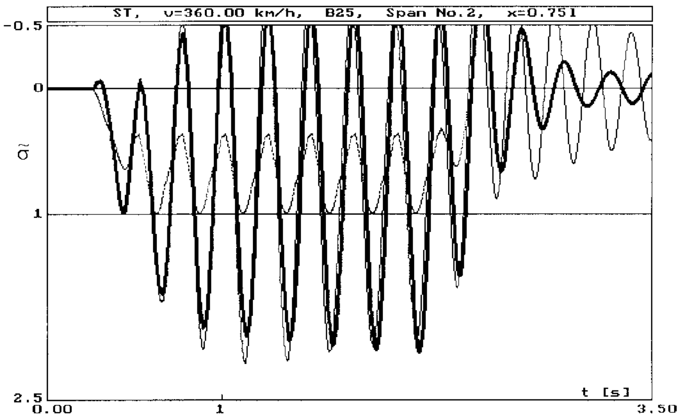

Fig. 12 The bottom normal stress versus time, in the selected span and section of the B25 bridge, induced by the ST train moving at v=360 km/h

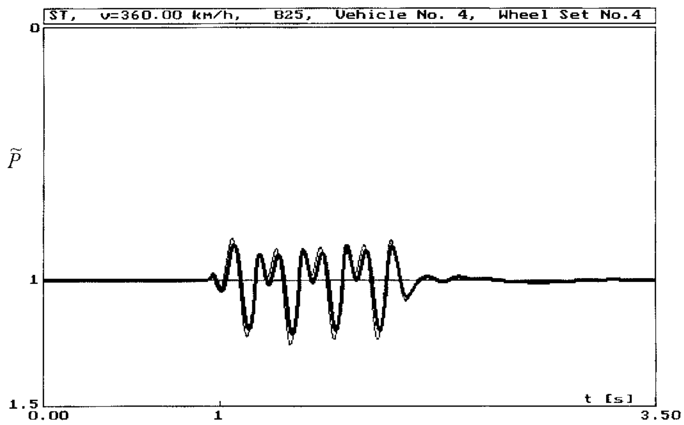

Fig. 13 The dynamic pressure of the selected wheel set versus time, during the passage of the ST train moving at v=360 km/h over the B25 bridge

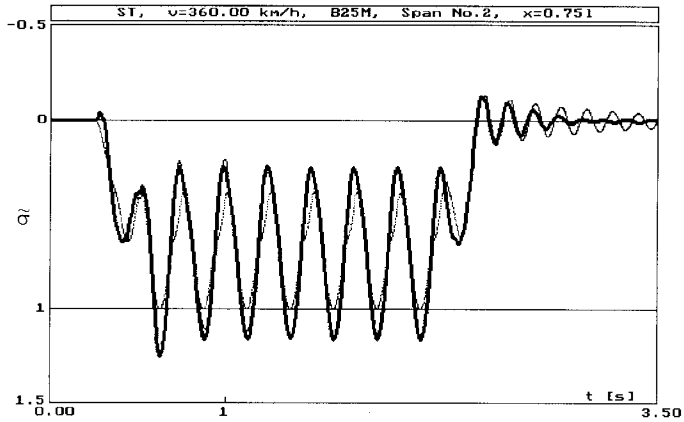

Fig. 14 The bottom normal stress versus time, in the selected span and section of the modified B25 bridge, induced by the ST train moving at v=360 km/h

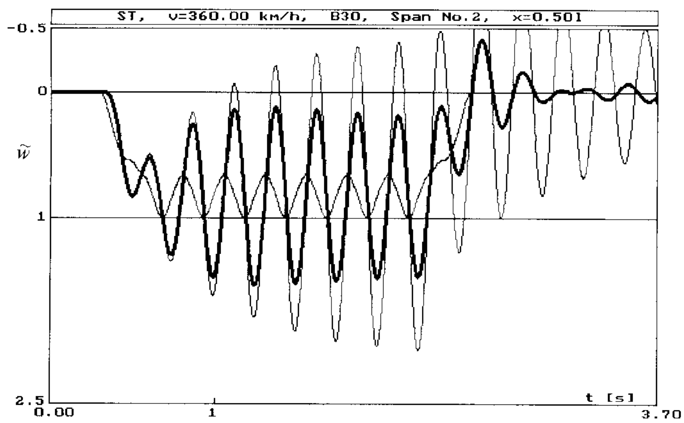

Fig. 15 The vertical deflection versus time, in the selected span and section of the B30 bridge, induced by the ST train moving at v=360 km/h

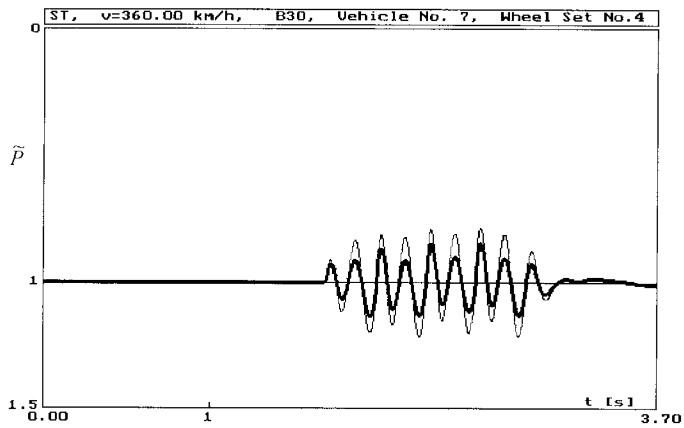

Fig. 16 The dynamic pressure of the selected wheel set versus time, during the passage of the ST train moving at v=360 km/h over the B30 bridge

PDAs to acceptable levels.

The extremum values of the loading and unloading coefficients for all wheel sets of the ST train, related to selected values of velocity *v*, are given in Table 3. As a rule, the transient vibrations of the rail-vehicles are increasing for subsequent vehicles. Motion of the ST train, moving at velocities up to 360 km/h, is safe and protected from derailment.

In order to illustrate dynamic phenomena in the multi-span bridge-moving train system, selected time histories of the output quantities are plotted in Figs. 7-16. In all figures, the dynamic passage of the train over the unmodified bridge is drawn with a solid thin line. The dynamic passage of the train over the bridge equipped with the absorbers is drawn with a solid thick line. The quasi-static passage of the train over a bridge is reflected with a dotted line.

7. Conclusions

The following conclusions result from the extended dynamic analysis of a type of series of four-span steel beam bridges loaded by a superfast passenger train:

1. The dynamic behaviour of repeatable spans of realistic multi-span single-track beam steel bridges is close to each other. Only in the parametric-forced resonance zones the dynamic effects may slightly increase in consecutive bridge spans.
2. Excessive quasi-steady-state vibrations occur in some bridges, when the train is moving at the selected critical velocities. These vibrations can be reduced to admissible levels by equipping all spans with light passive dynamic absorbers, tuned to the fundamental modal system of a single span. Vibration control with dynamic absorbers will elongate durability of the bridges considered in the study. This problem will be presented in detail in a separate paper.
3. When a dangerous velocity of the train is non-critical, excessive vibrations can only be suppressed by increasing the flexural stiffness of each span.
4. Derailment risk does not exist for a superfast passenger train like Shinkansen, moving over multi-span beam steel bridges at velocities 120-360 km/h, provided that the bridges are protected from excessive vibrations with passive dynamic absorbers. This conclusion has been formulated on the basis of the numerical results set up in Tables 2 and 3.

To the author's knowledge, the presented paper is the first theoretical research devoted to the parametric-forced vibrations of multi-span beam bridges under high speed moving trains. The study incorporates the following original concepts developed by the author (Klasztorny 1987, 1995):

- matrix formulation of the dynamic equilibrium equations of the bridge-moving train system,
- formulation of these equations partly in the implicit form, in terms of the dynamic interactions carried by the vehicle's suspensions,
- formulation of the recurrent-iterative algorithm for numerical integration of the matrix equations partly in the implicit form,
- vibration control of the bridge-moving train system, with properly tuned passive dynamic absorbers.

A technique of formulation and numerical integration of the equations of motion, developed in the study, has appeared very effective. Each series consumed only a few minutes on a PC computer (Pentium, 200 MHz).

The study is a simplified approach to the problem of the parametric-forced vibrations of the bridge-moving train system, since the secondary effects are neglected, such as, the local deformations of the track, snaking of the wheel sets, the vertical inertia forces of the moving wheel sets. These efffects are analysed in author's recent papers (Klasztorny 2000, Klasztorny and Podwórna 2001).

Acknowledgements

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References

Barchenkov, A.G. (1976), *Dynamic Analysis of Highway Bridges* (in Russian), Transport, Moscov.

Fryba, L. (1972), *Vibrations of Solids and Structures under Moving Load*, Academia, Prague.

Klasztorny, M. (1987), *Vibrations of Single-Track Railway Bridges under Trains Moving at High Speeds* (in Polish), Publishing House of Technical Univ. of Wroclaw, Wroclaw.

Klasztorny, M., and Langer, J. (1990), "Dynamic response of single-span beam bridges to series of moving loads", *Earthq. Eng. & Struct. Dyn*., **19**, 1107-1124.

Klasztorny, M. (1995), "Reduction of steady-state forced vibrations of structures with dynamic absorbers", *Earthq. Eng. & Struct. Dyn*., **24**, 1155-1172.

Klasztorny, M. (1999), "Vertical vibrations of a multi-span beam bridge under a train moving at high speeds", *Proc. 4th European Conf. on Structural Dynamics EURODYN99*, Prague, Czech Republic, (1999) *Structural Dynamics EURODYN99*, Fryba, L. and Naprstek, J., eds., Balkema, Rotterdam, **2**, 651-656.

Klasztorny, M. (2000), "Spatial vibrations of a multi-span railway steel bridge under a train moving at high speed", *Archiv. of Civ. Eng.*, **46**, 287-311.

Klasztorny, M., and Podwórna, M. (2001), "New computational algorithms in dynamic of bridge track moving train system", *CD Proc. 2nd European Conf. on Computational Mechanics ECCM-2001*, Cracow, Poland.

Langer, J. (1980), *Dynamics of Structures* (in Polish), Publishing House of Technical Univ. of Wroclaw, Wroclaw.

Matsuura, A. (1970), "A theoretical analysis of dynamic response of railway bridge to the passage of rolling stock", RTRI Quarterly Reports, **11**(1).

Matsuura, A. (1974), "Dynamic interaction between vehicles and girders in high speed railway", RTRI Quarterly Reports, **15**(3).

Matsuura, A. (1979), "Dynamic behaviour of bridge girder for high speed railway bridge", RTRI Quarterly Reports, **20**(2).

Newmark, N.M. (1959), "A method of computation for structural dynamics", *ASCE J. Eng. Mech. Div.* EM3.