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Analysis and design for torsion in reinforced and prestressed concrete beams

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Abstract. This paper presents a simplified method for the design and analysis of non-prestressed, partially prestressed, and fully prestressed concrete beams subjected to pure torsion. The proposed model relates the torsional strength to the concrete compressive strength and to the amounts of transverse and longitudinal reinforcement. To check the adequacy of this simple method, the calculated strength and mode of failure are checked against the experimental results of 17 prestressed concrete 66 reinforced concrete beam tests available in the literature, and very good agreement is found. The simplicity of the method is illustrated by two examples, one for design and another for analysis.

Key words: beams; code methods; design; mode of failure; prestressed concrete; reinforced concrete; shear; strength; torsion.

1. Introduction

Many structural components in bridges and buildings are subjected to significant torsional moments that are critical in design. Box girder bridges, beams in eccentrically loaded frames of multi-deck bridges, edge members in shells, and spandrel beams in buildings are typical examples of such elements.

Research on the torsional behavior of reinforced concrete began in 1929 with Rausch, who derived torsional strength equations based on the space truss model (Rausch 1929). In the 1960s, computers made the calculation of the torsional moments in space frames possible, leading to a significant interest in research on torsion.

Rahal and Collins (1996) categorized the currently available methods for computing the ultimate torsional strength into two main categories. Methods in the first category use semi-empirical equations chosen to fit the available experimental data. The main advantage of these methods is their simplicity. Methods in the second category use procedures based on more rational models such as the space truss analogy. These models are generally more time demanding, and suitable for microcomputers or programmable calculators. Their strength comes from their rationality, and from their ability to give the engineer a feel for the behavior of the structural member being designed.

The American building code (ACI 1995) and the Australian code (AS3600 1994) are based on semi-empirical models. On the other hand, the current American bridge code (AASHTO 1998) and Canadian building code (CSA 1994) offer two alternative design methods for shear and torsion. The

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first approach, referred to as the “simplified method”, is based on the traditional semi-empirical models. The second approach, referred to as the “General Method”, is a simplification of a more rational model named the Modified Compression Field Theory (MCFT). The MCFT is a powerful rational model capable of calculating the full response of sections subjected to shear, axial load, bending and torsional moments (Vecchio and Collins 1986, 1988, Collins and Mitchell 1991, Rahal and Collins 1995). The General Method is not as popular as the traditional methods because in some cases, it could be more time consuming. In conclusion, there is a lack of a unified approach, which blends the simplicity of the traditional semi-empirical approach and the rationality of the alternative approach.

A recently developed simplified model (Rahal 2000a) was shown to be an accurate and rational tool for calculating the strength of membrane elements subjected to shear. This model is based on the equations of the Modified Compression Field Theory (MCFT). The new model was able to cast the results of the rational MCFT into a simple procedure. The applicability of the model was extended (Rahal 2000b) to cover non-prestressed concrete beams subjected to shearing and axial forces and bending moments. The effects of axial loads and bending moments on the shear strength were accounted for by a simplified superposition procedure. The model was also generalized to apply to non-prestressed concrete beams subjected to pure torsion (Rahal 2000c), and it was shown that the assumptions adopted for the torsion application did not affect the rationality nor the accuracy of the model.

This method was proposed (Rahal 2000a) as the basis for a simple and unified approach for the design of reinforced concrete sections. An attractive method should also account for the effects of prestressing in a rational and simplified manner.

In this paper, the applicability of the new method is extended to prestressed sections subjected to pure torsion. The adequacy of the new method is checked by comparing the calculated torsional strength and mode of failure (MOF) results with experimental results of 17 prestressed and 66 non-prestressed reinforced concrete beams available in the literature. The use and simplicity of the model are illustrated by two examples, one for analysis and one for design.

2. Hollow tube analogy for torsion

Torsional moments acting across a beam cross section cause shearing stresses that circulate near the periphery as shown in Fig. 1(a). For this reason, torsion design has typically been linked to shear design, and the design provisions for both stress resultants are addressed in the same chapter in the building and bridge design codes (CSA 1994, AASHTO 1998, ACI 1995, AS3600 1994). The proposed method for torsion is an extension of the method for shear.

In a comprehensive experimental testing program, Hsu (1968) showed that hollow and solid beams have similar ultimate torsional strength. This indicates that the contribution of the inside core of the concrete to the capacity of the section is not significant. Similarly, strain measurements by Mitchell and Collins (1974) showed that the principal compressive strains between the cracks vary in a linear manner, with the maximum near the surface, and zero at a distance t_d below the surface. This enforces the conclusion from Hsu’s tests regarding the contribution of the inside core of concrete.

The hollow tube analogy assumes that near ultimate conditions, a concrete beam can be idealized as a hollow tube with the same outer dimensions and with a thickness t_d (see Fig. 1b). A field of

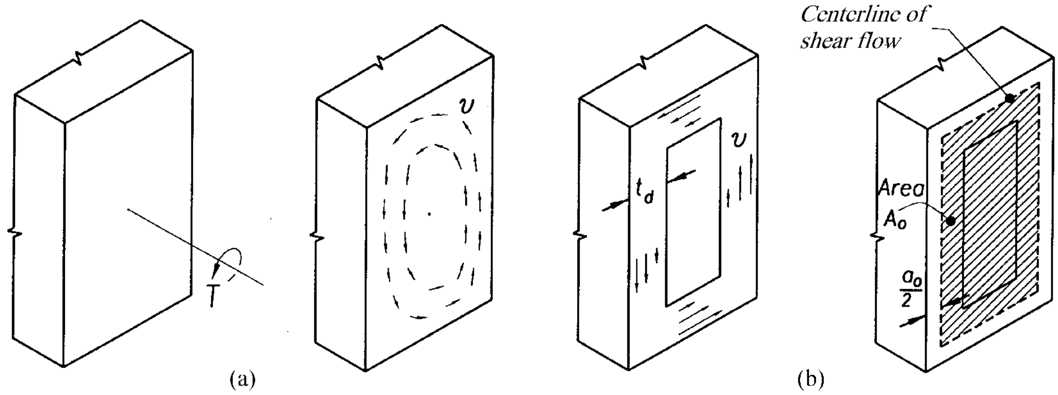

Fig. 1 (a) Shearing stresses across cross section, (b) Equivalent hollow tube and centerline of shear flow

shearing stresses v circulating in the tubular section resists the torsional moment. These stresses vary from zero on the inside face to a maximum at the outer face of the tube as shown in Fig. 1(b). Similar to the concept of equivalent compressive stress block in flexure, an equivalent field of constant shear flow q acting over a thickness a_o can be obtained (Collins and Mitchell 1991). The area enclosed by the centerline of the shear flow is named shear flow area A_o. For thin walled closed sections, the relationship between the ultimate torque T_u, the shear flow q_u, and the shear flow area A_o is given by:

$$T_u = 2q_u A_o \tag{1}$$

The shear flow q_u and the nominal shear stress v_u can be related by:

$$q_u = a_o v_u \tag{2}$$

The thickness of the tube depends on the amounts of longitudinal and transverse reinforcement, the concrete strength, and the geometry of the section (Mitchell and Collins 1974). Based on a simplified model, Rahal and Collins (1996) found that the average value of the thickness of the concrete that is effective in resisting the torsional moment was:

$$t_d = 0.5\frac{A_c}{p_c} \tag{3}$$

where

A_c = area enclosed by outer concrete dimensions
p_c = perimeter of outer concrete dimensions

A similar relationship but in terms of the stirrup dimensions can be found in the Canadian code (CSA 1994) as the minimum required thickness of hollow sections for which the presence of the void does not reduce the torsional capacity.

For normal strength concrete (below 50 MPa), the stress-strain relationship in compression can be represented by a parabola. Assuming a parabolic stress-strain relationship of concrete in compression and a linear variation of the concrete principle compressive stress along t_d, the depth of the equivalent stress block a_o can be taken as (Collins and Mitchell 1991):

$$a_o = 0.833\ t_d \tag{4}$$

The assumption that the principal compressive strains vary linearly from a maximum at the surface to zero at a depth t_d was first proposed by Mitchell and Collins (1974) based on their experimental findings.

Based on the results of the simplified approach proposed by Rahal and Collins (1996), the area enclosed by the centerline of the shear flow A_o can be approximated as:

$$A_o = 0.8\ A_c \tag{5}$$

Combining Eqs. (1) to (5) gives:

$$T_u = 0.67\frac{A_c^2}{p_c}v_u \tag{6}$$

Eq. (6) gives a simple relationship between the ultimate torsional moment T_u and the ultimate shearing stress in the walls of the equivalent tube.

It is to be noted that due to the simplifying assumptions, Eq. (6) does not depend on the shape of the cross section. Consequently, it can be applied to a wide range of cross section shapes such as square, rectangular, circular and multi-cellular sections and *T*, *L*, and *I* shaped sections.

Eq. (6) calculates the torque at high strains after considerable cracking and non-linearity in the behavior of concrete. The torsional strength is not to be smaller than the cracking torsional moment T_{cr}:

$$T_{cr} = 0.4\frac{A_c^2}{p_c}\sqrt{f_c'}\sqrt{1 + \frac{f_{pc}}{0.4\sqrt{f_c'}}} \tag{7}$$

where f_{pc} is the compressive stress in the concrete due to the prestressing operation, and f_c' is the compressive strength of concrete. Eq. (7) is adopted from the Canadian building code (CSA 1994).

If the nominal shearing capacity of the reinforced concrete in the walls of the tubular section can be calculated, the ultimate torsional moment can be easily obtained using Eq. (6). In the proposed method, the walls of the section can be idealized as reinforced concrete membrane elements subjected to pure in-plane shearing stresses. Eqs. (6) and (7) are combined with the simplified method for shear in membrane elements (Rahal 2000a) to calculate the torsional capacity of reinforced and prestressed concrete cross sections.

3. Proposed method

The simplified method was originally developed to calculate the ultimate shear strength of non-prestressed membrane elements (Rahal 2000a). This section presents a summary of the original method, and the proposed modifications required in order to extend its applicability to prestressed and non-prestressed beams subjected to pure torsion.

3.1 Membrane shear method

The ultimate in-plane shear stress is related to the following two non-dimensional indexes:

$$\omega_t = \frac{\rho_t f_{yt}}{f_c'} \tag{8}$$

$$\omega_L = \frac{\rho_L f_{yL}}{f_c'} \tag{9}$$

where

ρ_L = ratio of longitudinal steel per unit area of concrete
ρ_t = ratio of transverse reinforcing steel per unit area of concrete
f_{yL} = yield strength of non-prestressed longitudinal steel
f_{yt} = yield strength of transverse steel

The indexes are commonly referred to as the reinforcing indexes. The subscript letters L and t in the terms ω_L and ω_t refer to longitudinal and transverse respectively. The relationship between the indexes and the normalized shear resistance v_u / f_c' is shown in Fig. 2, and is based on the equations of the MCFT. Rahal (2000c) describes how the curves in Fig. 2 were developed and the assumptions taken.

Each curve in Fig. 2 represents the relationship between v_u / f_c' and ω_t at a given ω_L. It is shown that as ω_t increases, the strength v_u / f_c' increases. At low ω_t values, the strains in the transverse steel exceed the yielding strains before the ultimate capacity is reached. After a specific level of reinforcement ω_t, the concrete crushes before yielding. Fig. 2 shows a curve passing in those points past which the section is over-reinforced where crushing of the concrete takes place before yielding of the reinforcement. A similar curve can be obtained for the yielding of the longitudinal reinforcement.

It is to be noted that ω_t, ω_L and v_u / f_c' are dimensionless, and can be applied in SI as well as American Customary Units. Due to symmetry of the shear element problem, the longitudinal and transverse indexes can be interchanged in Fig. 2 without affecting the results.

3.2 Application to torsion problem

For the case of torsion in the equivalent hollow tube, the reinforcement ratios can be calculated using the following equations:

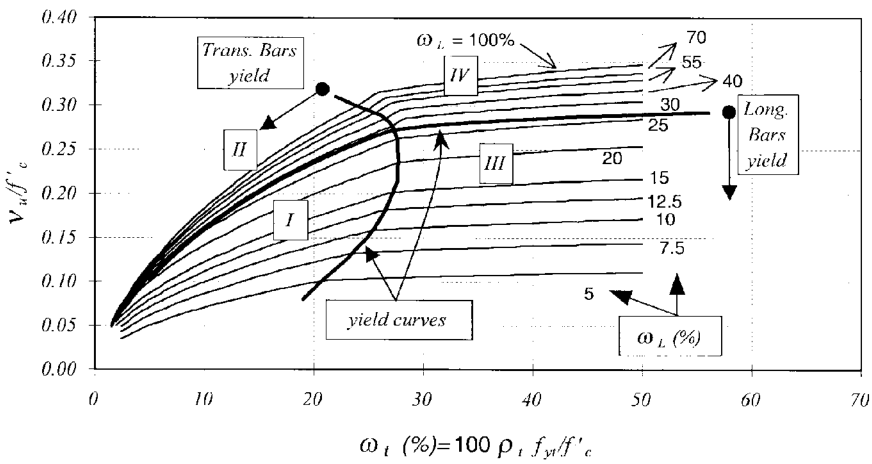

Fig. 2 Normalized shear strength curves for reinforced concrete

$$\rho_t = \frac{A_t}{s\ a_o} \tag{10}$$

$$\rho_L = \frac{A_L}{p_o\ a_o} \tag{11}$$

where

A_L = total area of non-prestressed longitudinal steel
A_t = area of stirrups within a distance s of a single wall
s = spacing of the stirrups measured along the length of the beam

In Eq. (11), it is assumed that the longitudinal steel is distributed over an effective area of thickness a_o, and of length p_o, where p_o is the perimeter of the centreline of the shear flow. Based on the results of their simplified method, Rahal and Collins (1996) proposed that the term p_o can be approximated as:

$$p_o = 0.9\ p_c \tag{12}$$

By combining Eqs. (3), (4), (9), (11) and (12), the term ω_L for the case of torsion can be calculated using:

$$\omega_L = \frac{A_L\ f_{yL}}{0.375\ A_c\ f_c'} \tag{13}$$

and by combining Eqs. (3), (4), (8), and (10), the term ω_t can be calculated using:

$$\omega_t = \frac{A_t\ f_{yt}\ p_c}{0.42\ s\ A_c\ f_c'} \tag{14}$$

Mitchell and Collins (1974) have shown that the prestress does not affect the ultimate strength if the prestressing steel yields at ultimate conditions. To account for the effects of prestressing, it is proposed to modify the nominator of Eq. (13) to include the contribution of the prestressing steel to the total amount of longitudinal steel. Hence, the longitudinal index can be approximated as:

$$\omega_L = \frac{A_L\ f_{yL} + A_p\ f_{yp}}{0.375\ A_c\ f_c'} \tag{15}$$

where

A_p = total area of prestressed longitudinal steel
f_{yp} = yield strength of prestressed longitudinal steel

Along with reinforced concrete, Eq. (15) can be applied to partially or fully prestressed concrete beams.

3.3 Mode of failure

The two solid “yield” curves divide Fig. 2 into four regions. The relative position of a point of coordinates (ω_L, ω_t) with respect to these curves or regions, indicates the expected mode of failure (MOF) of an element with these reinforcement indexes. Four modes of failure are possible. The first mode is for completely under-reinforced sections where the longitudinal and transverse steel yield (marked I in Fig. 2). The second and third modes are for partially over-reinforced sections: only transverse steel yields mode (marked II in Fig. 2), and: only longitudinal reinforcement yields (marked III in Fig. 2). The fourth mode is for completely over-reinforced sections where the concrete crushes before steel yielding (marked IV in Fig. 2). The mode of failure of the beam subjected to torsion is similar to that of the walls subjected to pure shear.

3.4 Use of curves for analysis

The following steps are followed to calculate the strength and the mode of failure of a reinforced or prestressed concrete cross section:

1. Calculate the reinforcement indexes ω_t and ω_L using Eqs. (14) and (15) respectively.
2. Use Fig. 2 to obtain v_u/f_c' and mode of failure (the relative position of a point of coordinates ($\omega_L = \omega_t$) with respect to the yield curves indicates the expected mode of failure).
3. Calculate T_u from Eq. (6).
4. Ensure T_u is larger than T_{cr} calculated using Eq. (7).

Appendix A provides an example where the torsional capacity and the mode of failure of a prestressed concrete cross section is calculated.

3.5 Use of curves for design

The following steps are followed to design the reinforcement of a section subjected to a torsional moment:

1. Given the design torque, calculate v_u/f_c' using Eq. (6).
2. Use Fig. 2 to obtain the reinforcement indexes ω_t and ω_L (select one and obtain the other from the figure).
3. Calculate the reinforcement areas from Eqs. (14) and (15).
4. To ensure ductility, a minimum level of reinforcement or reserve strength past the cracking torque should be provided.

If the value of v_u/f_c' calculated in step 1 can not be achieved with an under-reinforced section, larger sectional dimensions or higher concrete strength is required.

In step 2, the selection of the reinforcement indexes is not unique as would be expected in a design situation. The most straight forward solution is to select $\omega_t = \omega_L$, which, for the case of under reinforced sections gives $\omega_t = \omega_L = v_u/f_c'$ as will be shown later. In many situations, manufacturing and placing transverse reinforcement is more expensive than longitudinal reinforcement. In those cases, a larger longitudinal index is selected to minimize the required transverse reinforcement. In other cases such as in some prestressed concrete beams, the amount of longitudinal reinforcement and prestressing is near the maximum that the section can fit. In such situation, it is more suitable to select a larger transverse index to minimize the longitudinal steel requirements.

To show how the proposed method is used for design, Appendix B provides an example where the reinforcement in a concrete cross-section is selected to resist a specified torque.

4. Experimental verification

The proposed method is used to calculate the strength and mode of failure of 83 test specimens available in the literature (66 non-prestressed, and 16 partially prestressed, and 1 fully prestressed

Table 1 Comparison between experimental and theoretical results

	Torsion T_{exp}/T_{calc}–83 beams				
	Average	Std. Dev.	COV (%)	Maximum	Minimum
Proposed method	1.03	0.114	11.1	1.37	0.76
General method	1.38	0.234	17.0	2.06	1.01
ACI code	1.40	0.271	19.3	2.15	0.95
	Shear v_{exp}/v_{calc}–46 shear panels (Rahal 2000a)				
	Average	Std. Dev.	COV (%)	Maximum	Minimum
Proposed method	1.01	0.126	12.5	1.36	0.83
General method	1.07	0.142	13.3	1.49	0.83
ACI code	1.13	0.372	32.9	2.27	0.60

beams). Fifty-three of these specimens were tested at the PCA labs (Hsu 1968), 22 at the University of Calgary (McMullen and Rangan 1978, El-Degwy and McMullen 1985), and 8 at the University of Toronto (Mitchell and Collins 1974, Mardukhi and Collins 1974).

4.1 Ultimate torsional strength

Table 1 compares the observed ultimate torsional moments and the calculations of the proposed method. The mean of the ratios T_{exp}/T_{calc} of the 83 test results is 1.03 and the coefficient of variation is 11.1%. The ratios ranged from 0.76 to 1.37.

Table 1 also shows the results obtained using the "General Method" (CSA 1994, AASHTO 1998) and the ACI method (ACI 1995). The mean and the coefficient of variations were 1.38 and 17.0% for the "General Method" and 1.40 and 19.3% for the ACI method. Table 1 also shows a larger scatter in the two code methods relative to that in the proposed method. The ratios were between 1.01 and 2.06 for the General Method, and 0.95 and 2.15 for the ACI method.

Fig. 3 shows a plot between the T_{exp}/T_{calc} ratio and the concrete compressive strength for the three methods. It shows that the accuracy of the method was resonably uniform for the wide range of concrete strength used in the 83 specimens (14 to 46 MPa). Fig. 3 also shows no significant loss of accuracy if the longitudinal steel is prestressed. Similar conclusions can be drawn regarding the performance of the code methods. The proposed method however shows a narrower range of scatter

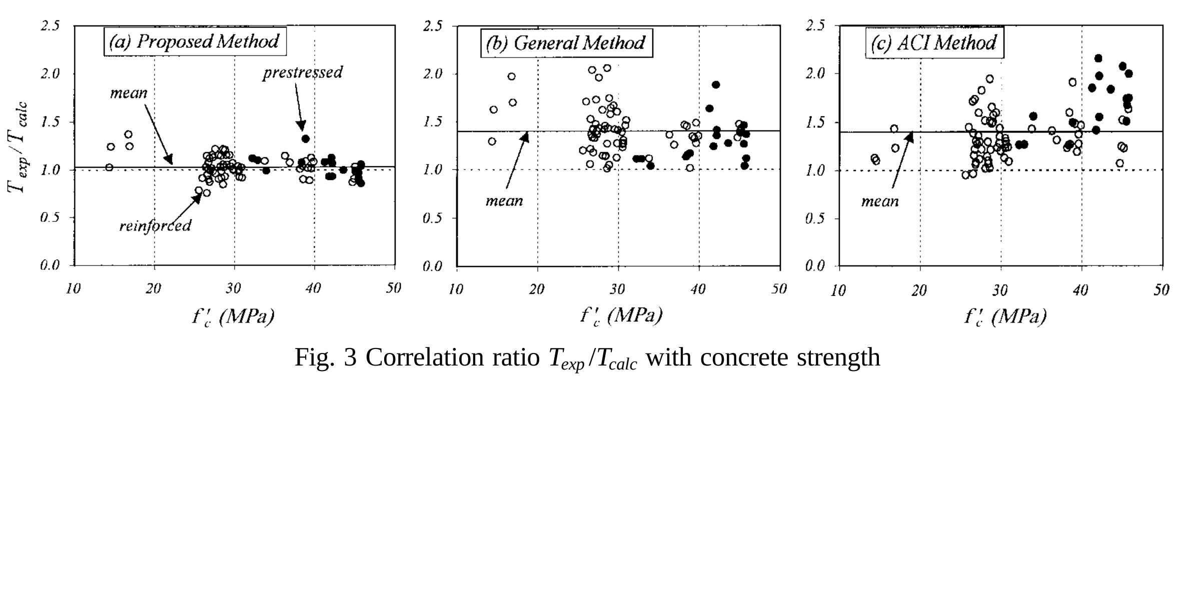

Fig. 3 Correlation ratio T_{exp}/T_{calc} with concrete strength

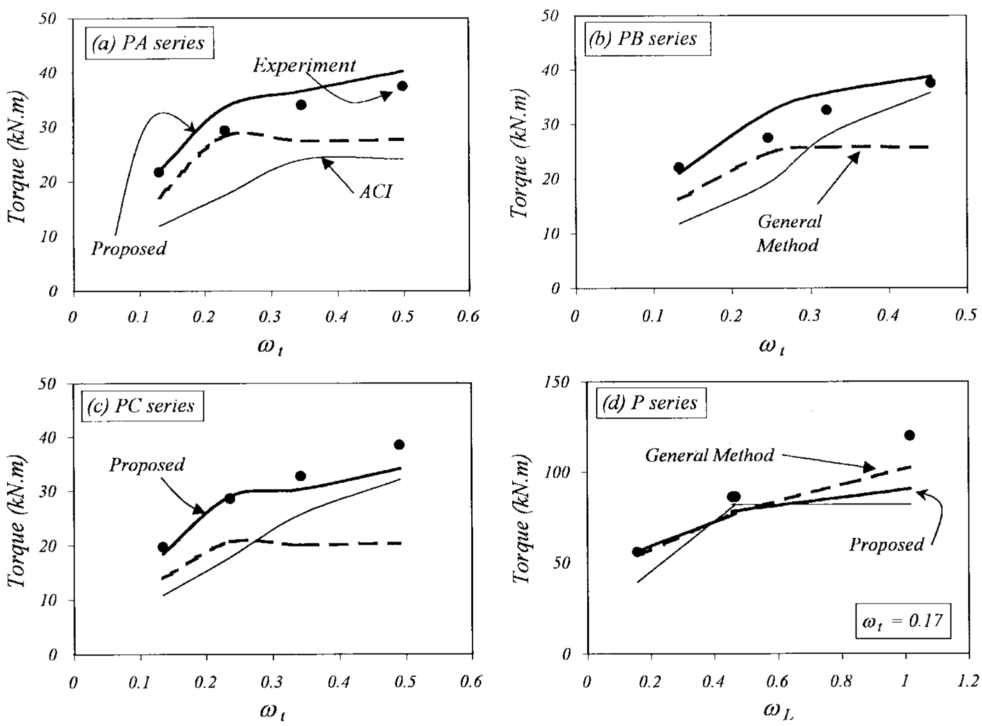

Fig. 4 Effects of amount of reinforcement on torsional strength

at different levels of concrete compressive strengths.

El-Degwy and McMullen (1985) tested 12 partially prestressed beams to study the effects of the amount of reinforcement and the aspect ratio. The beams were divided into three groups, with a different aspect ratio in each group. Both the longitudinal and transverse reinforcement were proportionally increased in the beams of the same group. These test results are used to verify the ability of the method to accuratly capture the effects of the amount of reinforcement and the aspect ratio.

Beams PA1R, PA2, PA3 and PA4 of series PA were solid and square with 254 mm (10 in) outer dimension, and the reinforcement index ω_t ranged from 0.13 to 0.5 ($\omega_L \cong 2\omega_t$). Fig. 4(a) compares the experimental results from these beams with the calculations of the proposed method, and good agreement is observed. The calculations of the two code methods are significantly conservative.

Beams PB1, PB2, PB3 and PB4 of series PB were solid and rectangular with 178 mm × 356 mm outer dimensions, and the reinforcement index ω_t ranged from 0.13 to 0.45 ($\omega_L \cong 2\omega_t$). Beams PC1, PC2, PC3 and PC4 of beam series PC were solid and rectangular with 146 mm × 438 mm outer dimensions, and the reinforcement index ω_t ranged from 0.13 to 0.45 ($\omega_L \cong 2\omega_t$). Figs. 4(b) and 4(c) compare the experimental results from the beams of series PB and PC respectively with the calculations of the proposed method, and good agreement is observed. The calculations of the two code methods are conservative.

In specimens P3, P1, P2, and P5 tested by Mitchell and Collins (1974), the level of transverse reinforcement remained practically unchanged ($\omega_t \cong 0.17$) while the longitudinal steel index ω_L increased from 0.16 to 1.02. Fig. 4(d) compares the experimental results from these beams with the calculations of the proposed method and the two code methods, and good agreement is observed.

Similar comparisons for non-prestressed members were reported by the author (Rahal 2000c), and similar agreement between the experimental and calculated results was observed.

The aspect ratio is the ratio of the longer to smaller outer dimension of the concrete section. The aspect ratio of specimens PA2 (254 × 254 mm), PB2 (178 × 356 mm), and PC2 (146 × 438 mm) tested by El-Degwy and McMullen (1985) were 1, 2, and 3 respectively. The level of reinforcement remained practically unchanged in all three specimens. Fig. 5(a) shows a plot between the aspect ratio and the observed and calculated shear strength ratio $Tp_o/A_c^2/f_c'$ (equal to $v_u/f_c'/1.5$ as given by Eq. 6). Fig. 5(b) shows a similar comparison for beams PA3, PB3, and PC3 which were similar to the specimens shown in Fig. 5(a) except that the reinforcement levels were higher. A good agreement is observed. The calculations of the code methods are also shown in Fig. 5, and they are significantly conservative.

Figs. 5(c) and 5(d) show similar comparisons for reinforced beams tested by Hsu (1968). In this comprehensive series of tests on the torsional strength of reinforced concrete beams, Hsu studied the effects of numerous variables including the aspect ratio. Fig. 5(c) shows the comparison for specimens C1, B1, G2, G6, and K1 which had similar reinforcement levels and concrete strength, but aspect ratios ranging from 1 to 3.25, and cross-section dimensions ranging from 254 × 254 mm to 152 × 495 mm. These specimens failed in an under-reinforced mode.

Fig. 5(d) shows a similar plot for specimens C4, B4, G5, and K3 (254 × 254 mm to 152 × 495 mm), which had larger reinforcement levels causing specimens C4 and G5 to fail in an over-reinforced mode. Again, good agreement is observed between the experimental results and the calculations of

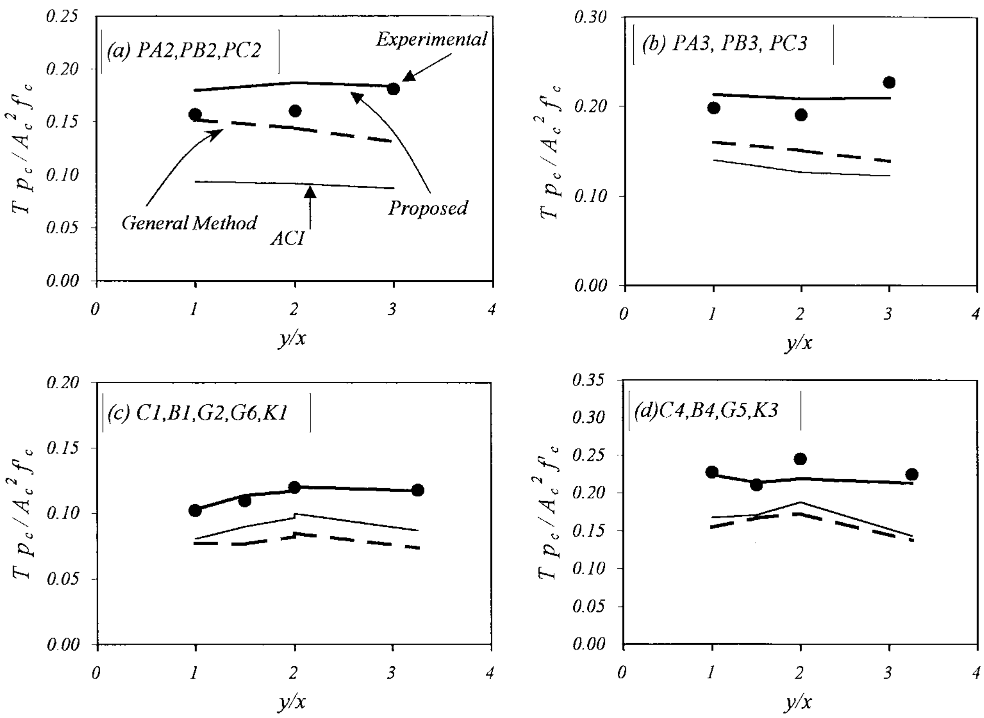

Fig. 5 Effect of aspect ratio on torsional strength

the proposed method. The results of the code methods were conservative.

From Fig. 5, it is shown that the normalized shear strength calculated using Eq. (6) is practically independent of the aspect ratio, and that the proposed method and the code methods captured this phenomenon. The calculations of the proposed method were closer to the experimental results than the two code methods.

4.2 Mode of failure

Even though the proposed method clearly differentiates between under-reinforced and over-reinforced sections, experimental results are not always this simple to interpret. Steel strains in experiments are normally measured using strain gauges fixed to the reinforcement at different locations. The strain reading depends significantly on the location of the gauge with respect to the crack location, with larger strain measured at a crack. Experimental readings hence give measurement of local instead of average strains. To simplify the comparison, it will be assumed that a section is under-reinforced if the strain in at least one gauge exceeded the strain at yield.

In the 83 torsion tests used in this study, the measured transverse strains exceeded the yield strains in 57 specimens while the measured longitudinal strains exceeded this limit in 46 specimens.

The proposed method correctly predicted the state of transverse stress (yielding versus non-yielding) in 51 out of the 83 beams. The corresponding numbers for the General Method and the ACI method were 53 and 48 respectively.

The proposed method also correctly predicted the state of longitudinal stress (yielding versus non-yielding) in 55 out of the 83 beams. The corresponding number for the General Method was 64. The ACI code does not give a clear procedure to check if the section is under-reinforced in the longitudinal direction, especially if the steel is prestressed. In fact, the minimun longitudinal steel equation in ACI caused the calculations of the ACI method to be excessively conservative for prestressed for prestressed beams as shown in Figs. 4 and 5.

5. Nominal shear stress

There is a lack of a unified equation to calculate the maximum torsional shear stress and its distribution across the thickness of the equivalent tube. It is however commonly accepted that the shearing stresses are largest near the surface of the concrete cross section and that they decrease to zero at the inside face of the tube. The North American codes give different equations for the "nominal" shear stress due to torsion. The "General Method" (CSA 1994, AASHTO 1998) assumes:

$$v_u=\frac{T_u\, p_h}{A_{oh}^2} \tag{16}$$

where p_h and A_{oh} are respectively the perimeter and area enclosed by the stirrups. The ACI code (ACI 1995) assumes:

$$v_u=0.59\frac{T_u\, p_h}{A_{oh}^2} \tag{17}$$

Based on Eq. (6), the proposed method uses:

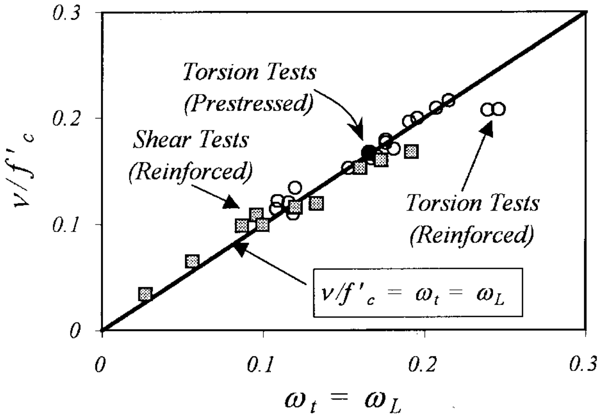

Fig. 6 Strength of under-reinforced sections with $\omega_t = \omega_L$

$$v_u = 1.5\frac{T_u\, p_c}{A_c^2} \tag{18}$$

It is critical to prove that the assumptions adopted to develop Eq. (6) and (18) on one side and Eqs. (14) and (15) on the other side are compatible.

It was suggested by the author (Rahal 2000a) that for under-reinforced sections with equal reinforcement indexes ($\omega_L = \omega_t < 0.27$), the normalized strength v_u / f_c' can be taken equal to ω_t and ω_L. This is correctly reflected in Fig. 2. For example, at $\omega_L = 0.2$ and $\omega_t = 0.2$, it can be found that $v_u / f_c' = 0.2$. Fig. 6 compares v_{exp} / f_c' and $\omega_L = \omega_t$ for 46 reinforced concrete shear panels (Rahal 2000a), and 20 PCA reinforced concrete beam torsion tests. A line representing the equality $v_{exp}/ f_c' = \omega_L = \omega_t$ (theoretical calculations) is also shown, and very good agreement was observed.

Fig. 6 shows a similar comparison for one prestressed beam tested at the University of Toronto. Again a good agreement is observed. This agreement shown for reinforced members subjected to torsion indicates that the assumptions adopted to develop Eqs. (6) and (18) on one side and Eqs. (14) and (15) on the other side are compatible. It is to be noted that Eqs. (6), (14), and (15) did not include any empirical modifications to improve the accuracy of the proposed method. Further test results are needed to draw a similar conclusion for prestressed concrete beams.

6. Conclusions

A simple method for the calculation of torsional strength of prestressed and reinforced concrete beams was presented. The proposed method is a generalization of a recently developed simplified approach to the calculations of the ultimate strength of (reinforced) shear panels. This method was also extended to reinforced concrete beams subjected to shearing and axial forces, and bending moment, and to non-prestressed reinforced beams subjected to torsion.

The results of the method were compared with the experimental results from 17 prestressed and 66 reinforced concrete beams, and good agreement was obtained. The accuracy of the method for the cases of pure torsion and pure shear were very similar, which points to the adequacy of the assumptions adopted to apply the method to the torsion problem.

The experimental results were also compared with the results from the current ACI code method and the current CSA and AASHTO code methods. It was shown that the results of the proposed method agreed better with the experimental results. Another advantage of the method is that it combines the rationality of the General method (CSA and AASHTO code) and the ACI method.

It is suggested that the proposed method can serve as a unified and rational basis for the design and capacity calculations of reinforced and prestressed beams and panels subjected to combined stress resultants.

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Appendix A: Capacity calculation example

The torsional strength of prestressed specimen P2 tested by Mitchell and Collins (1974) at the University of Toronto labs is calculated to illustrate the simplicity of the proposed method.

Problem statement: Calculate the strength of the prestressed concrete hollow beam P2 shown in Fig. A1. Given b=356 mm, h=432 mm, f_c'=32.9 MPa, A_t=71 mm^2, f_{yt}=327.6 MPa, s=96.5 mm, A_L=568 mm^2, f_{yL} = 327.6 MPa, A_p=463 mm^2, f_{yp}=1476 MPa, f_{pc}=3.5 MPa.

Solution:

p_c=1567 mm, A_c=153,790 mm^2.

From Eq. (15) $$\omega_L=\frac{A_L f_{yL}+A_p f_{yp}}{0.375A_c f_c'}=\frac{(568)(327.6)+(463)(1476)}{(0.375)(153{,}790)(32.9)}=0.46$$

From Eq. (14) $$\omega_t=\frac{A_t f_{yt} p_c}{0.42\, s\, A_c f_c'}=\frac{(71)(327.6)(1567)}{(0.42)(96.5)(153{,}790)(32.9)}=0.177$$

From Fig. 2, v_u/f_c'=0.24. The calculated ultimate torque is given by Eq. (6):

$$T_u=0.67\frac{A_c^2}{p_c}v_u=0.67\frac{(153{,}790)^2}{1567}(0.24)(32.9)=79.85\ \text{kN.m}$$

The observed T_{exp}=86.2 kN.m. The cracking torque calculated using Eq. (7) is 54.6 kN.m and does not govern the results. The ratio T_{exp}/T_{calc} is hence 86.2/79.9=1.08.

Mitchell and Collins (1974) reported transverse strains ranging from 0.00278 to 0.00445 (yielding) and longitudinal strains ranging from 0.00042 to 0.0013 (not yielding). The relative position of the point (ω_L= 0.46; ω_t=0.177) in Fig. 2 indicates that the method correctly predicts that only the transverse steel yields at

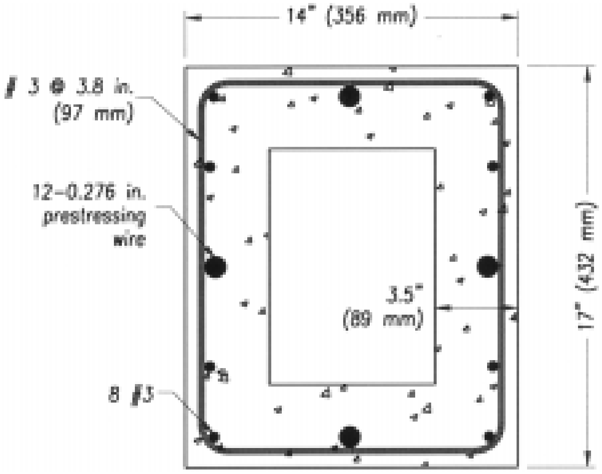

Fig. A1 Cross-section of beam P2 tested by Mitchell and Collins (1974) ultimate conditions.

Appendix B: Design example

To provide a basis of comparison between the proposed method and other available methods, the design example is adopted from Hsu and Mo (1985) after changing the example to SI units.

Problem statement: Design a reinforced concrete hollow beam to resist a torsional moment of 836 kN.m (7400 in.-kips). The material properties are: f_c' =27.6 MPa (4000 psi), and $f_{yt} = f_{yL}$ =414 MPa (60,000 psi). The cross-section is shown in Fig. B1.

Solution:

p_c =3990 mm, A_c =975,500 mm^2.

The cracking torque for the cross section is given by Eq. (7):

$$T_{cr}=0.4\frac{A_c^2}{p_c}\sqrt{f_c'}\sqrt{1+\frac{f_{pc}}{0.4\sqrt{f_c'}}}=0.4\frac{(975{,}500)^2}{3990}\sqrt{27.6}=501.2\ \text{kN.m.}$$

which is significantly less than the design torque. Hence, the reinforcement required to resist the design torque will provide adequate ductility past the cracking level.

From Eq. (6), the normalized shear stress is:

$$\frac{v}{f_c'}=\frac{T_u p_c}{0.67\ A_c^2 f_c'}=\frac{836{,}000{,}000\ 3{,}990}{0.67\ (975{,}500)^2\ 27.6}=0.19$$

From Fig. 2, selecting equal longitudinal and transverse reinforcement levels (ω_L=ω_t) gives ω_L=0.19 and ω_t =0.19. From Eq. (15) and (14), the amounts of longitudinal and transverse reinforcement are:

$$A_L=\frac{0.375\ A_c f_c'}{f_{yL}}\omega_L=\frac{(0.375)(975{,}500)(27.6)}{414}0.19=4{,}635\ \text{mm}^2$$

$$A_t/s=\frac{0.42\ A_c f_c'}{f_{yt} p_c}\omega_t=\frac{(0.42)(975{,}500)(27.6)}{(414)(3990)}0.19=1.30\ \text{mm}^2/\text{mm}$$

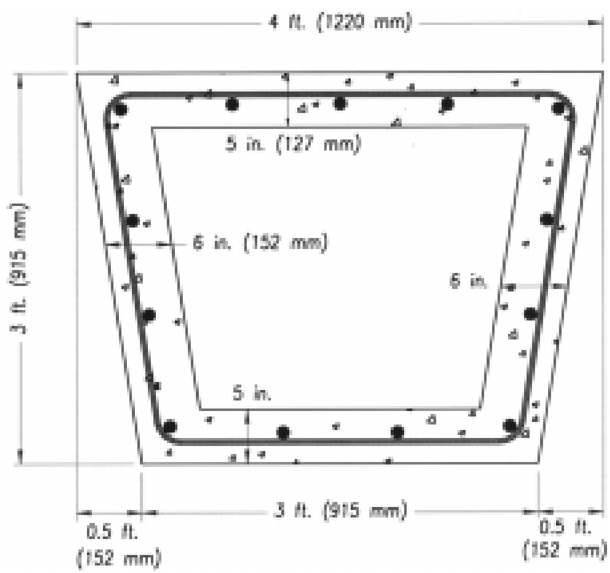

Fig. B1 Cross-section of beam designed by Hsu and Mo (1985)

Appropriate bar details can now be selected in accordance with general code requirements. It is to be noted that the method proposed by Hsu and Mo (1985) required A_L=4760 mm^2 and A_t/s=1.37 mm^2/mm in a more time demanding procedure.

Notation

a_o = depth of equivalent stress block
A_o = area enclosed by shear flow resultant
A_{oh} = area enclosed in closed stirrup
A_c = gross area of within concrete outer dimensions
A_L = area of non-prestressed longitudinal steel in section
A_p = area of prestressed longitudinal steel in section
A_t = area of one leg of transverse reinforcement within a distance s
f_c' = specified compressive strength of concrete
f_{pc} = compressive stress in the concrete due to the prestressing operation
f_{yL} = yield strength of non-prestressed longitudinal bars in section
f_{yp} = yield strength of prestressed longitudinal bars in section
f_{yt} = yield strength of the stirrups
p_o = perimeter of shear flow resultant
p_c = perimeter of outer concrete dimensions
p_h = perimeter of closed stirrup
q_u = shear flow at ultimate
s = spacing of the stirrups measured along the length of the beam
T_{calc} = calculated ultimate torsional moment
T_{cr} = cracking torsional moment
t_d = thickness of tube resisting torsion
T_{exp} = experimentally measured ultimate torsional moment
T_u = ultimate torsional moment
ω_L = non-dimensional longitudinal reinforcement index
ω_t = non-dimensional transverse reinforcement index
ρ_L = ratio of total longitudinal reinforcement
ρ_t = ratio of transverse reinforcing steel
v_{exp} = ultimate experimental shearing stress resistance
v_u = ultimate shearing stress resistance