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Role of membrane forces in seismic design of reinforced concrete liquid storage structures

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Abstract. To prevent major cracking and failure during earthquakes, it is important to design reinforced concrete liquid storage structures, such as water and fuel storage tanks, properly for the hydrodynamic pressure loads caused by seismic excitations. There is a discussion in recent Codes that most of the base shear applied to liquid containment structures is resisted by inplane membrane shear rather than by transverse flexural shear. The purpose of this paper is to underline the importance of the membrane force system in carrying the base shear produced by hydrodynamic pressures in both rectangular and cylindrical tank structures. Only rigid tanks constrained at the base are considered. Analysis is performed for both tall and broad tanks to compare their behavior under seismic excitation. Efforts are made to quantify the percentage of base shear carried by membrane action and the consequent procedures that must be followed for safe design of liquid containing storage structures.

Key words: membrane forces; reinforced concrete tanks; seismic loading; specifications.

1. Introduction

Liquid containing storage structures such as water tanks and fuel storage tanks may fail during earthquakes causing disruption of services that are necessary for public safety and well being or the release of a hazardous material into an already hazardous environment. Hence, the design of these structures to withstand the seismic forces has received significant attention from the research profession for the last several decades. Most of this attention however has been directed to steel tanks. Reinforced concrete tanks have not been subject to the same scrutiny.

The seismic design of liquid storage tanks is quite complex due to the interaction of the fluid with the structure during seismic excitation. Design loads are normally computed based on a rigid structure while design forces are normally calculated on the basis of a linearly elastic model. Design forces for concrete tanks are covered by the AWWA and ACI Specifications. ACI Committee 350 is completing a Code and Commentary to act as a supplement ACI 318's Code and Commentary which is essentially for buildings. ACI 350 is to stipulate the specific requirements pertinent to liquid containing structures (ACI 1999). As ACI is not in the load definition business, 350 has developed a supplement to its Code and Commentary, 350.3, to specifically recommend seismic loadings states. To make the ACI 350 requirements compatible with IBC 2000 and the NEHRP 1997 edition, the loading specification has just been revised to follow the directions of the IBC and NEHRP specifications.

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The transformation of the specified loads into the required resistance for tanks requires determining the proper load path. For rectangular tanks the wall perpendicular to the direction of the earthquake can be designed as a flat slab subjected to horizontal pressure. This is a definite simplification of the actual design situation, where the slab reactions load the walls parallel to the ground motion with inplane forces which are resisted as membrane forces. In other words, the force resisting mechanism played by the membrane forces are significant contributors in the overall resistance system for these liquid storage structures.

In this paper, the load-resisting mechanism by membrane action is examined by considering seismically loaded rigid rectangular and cylindrical tanks fixed at the base. Both tall and broad tanks are considered. The effect of wall flexibility on the hydrodynamic pressure is ignored. The tanks are analyzed by finite element analysis using quasi-static loading and conclusions are drawn on the percentage of base shear and the overturning moment carried by membrane forces relative to those from slab bending. Finally, recommendation are made on the safe design of storage tanks.

2. Overview of literature

The damage to storage tanks due to recent earthquakes has been extensively studied (Jennings 1971, Hanson 1973, and Monos and Clough 1985). These tanks are mainly steel tanks whose failure modes are edge effects in the form of elephant foot buckling at the base. Housner (1957) first considered the hydrodynamic pressure distribution developed in rigid tanks during horizontal base excitation. He formulated a dynamic model for estimating the liquid response in seismically excited rigid, rectangular and circular tanks. The effect due to shell flexibility was later incorporated in the model by Veletsos and Yang (1976), Nash *et al.* (1978), Haroun and Housner (1980). Haroun and Tayel (1984) have investigated the effect of soil-structure interaction. Yang (1976), Veletsos and Tang (1986) and Luft (1984) have considered the effect of vertical excitation on the hydrodynamic pressures. Haroun and Chen (1989) have investigated the nonlinear sloshing behavior in rectangular tanks by considering large amplitude sloshing.

The finite element analysis of the liquid-tank system is studied by Haroun and Housner (1981). Several studies were also carried out to investigate that dynamic interaction between deformable wall of the tank and the liquid using finite element analysis. ASCE (1984a) comprehensively discusses the effect of fluid-structure interaction on the hydrodynamic pressures and ASCE (1984b) provides excellent guidelines for the analysis and design of liquid storage structures.

3. Seismic loads

The loads on liquid storage structures consist of hydrodynamic pressures generated by the fluid accelerated by the earthquake and acting on the wall. A certain portion of the fluid acts as a rigid mass as if it were in contact with the wall. This mass exerts a force proportional to the acceleration of the tank base and is termed impulsive force, Fig. 1. The seismic excitation also induces the oscillation of the fluids, called sloshing, contributing additional dynamic pressure on the tank wall, which is termed convective forces. These forces are a function of tank height to diameter ratio, depth of liquid, amplitude and frequency of the ground motion.

Design is computed using the maximum considered earthquake (MCE) which is ground motion

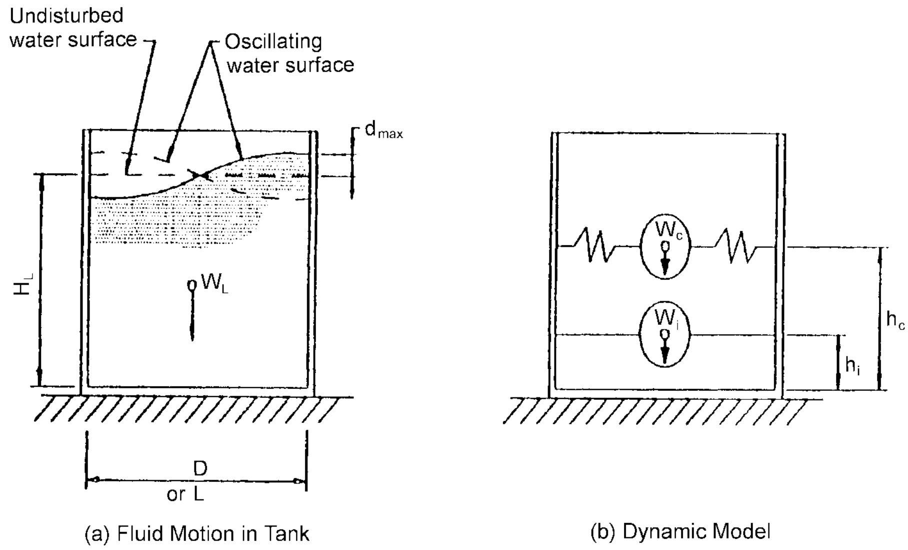

Fig. 1 Dynamic model of liquid-containing tank rigidly supported

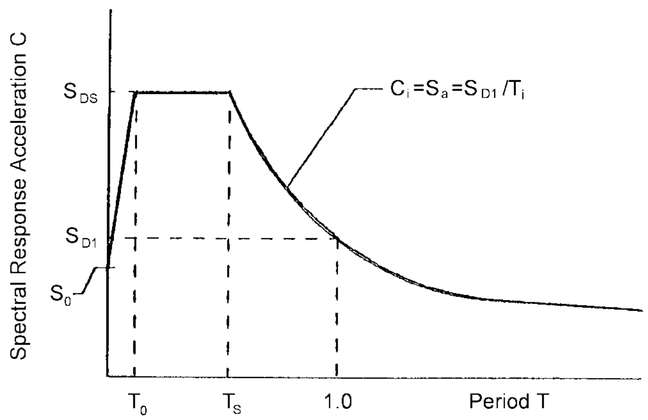

Fig. 2 Design response spectrum

defined with a uniform likelihood of exceedance of 2 percent in 50 years (return period of about 2500 years). The design response spectrum that is considered, if site specific procedures are not being used, is as shown in Fig. 2. The designations C_i, C_c, and C_v correspond to impulsive connective and vertical accelerations. They correspond to spectral response acceleration S_a in NEHRP.

The spectral response acceleration coefficients are determined as follows:

The impulsive component

For $T_i<T_0$ $$C_i=S_{DS}\left[3\frac{T_i}{T_s}+0.4\right]$$

For $T_0<T_i<T_s$ $$C_i=S_{DS}=\frac{S_{D1}}{T_s} \tag{1}$$

For $T_i > T_S$ $$C_i = \frac{S_{D1}}{T_i}$$

but C_i is not to exceed S_{DS}. S_{DS} is the design spectral response acceleration at short periods. S_{DS}=2/3 $S_S F_a$. S_{D1} is the design spectral response acceleration at a 1 second period. S_{D1}=2/3 $S_1 F_V$ with S_S and S_1 being the mapped spectral accelerations obtained from maps provided with NEHRP. F_a and F_v being NEHRP tables to adjust the spectral accelerations for short and 1 second periods for site class effects.

$$T_S = \frac{S_{D1}}{S_{DS}}$$

and $$T_0 = 0.2 T_s = 0.2 \frac{S_{D1}}{S_{DS}}$$

The vertical component C_v follows the same format as the impulsive.
The convective component (a sloshing contribution with long periods)

For $T_c < 4.0s$ $$C_C = \frac{1.5 S_{D1}}{T_c}$$

For $T_c > 4.0s$ $$C_c = \frac{6 S_{D1}}{T_c^2}$$

To calculate the liquid hydrodynamic convective pressures, the liquid sloshing heights are computed using small amplitude motions of the liquid surface. Under this assumption, the theoretically computed stresses due to sloshing are small compared to those due to impulsive pressure. Sloshing increases the fluid pressures on one side and decreases on the other side of the tank walls, which tend to overturn the tank. Normally this is not important in case of reinforced concrete tanks due to their dead weight. Kelly and Mayes (1989) have found that the maximum forces caused by sloshing were from 4 percent to 6 percent of the impulsive forces caused under 0.3 g maximum credible earthquake and therefore the effect of the sloshing modes on the total design base shear is negligible.

4. Hydrodynamic wall pressures in rigid tanks

4.1. Rectangular tanks

A rectangular tank of length L, width B and height H filled with a liquid of density ρ is considered in Fig. 3. A Cartesian coordinate system is considered with the origins being at the center of the base. The ground motion has a known displacement history $x_g(t)$ along the horizontal axis. The motion is transferred to the tank through a rigid foundation and base plate. The liquid is assumed to be incompressible, homogeneous, and frictionless. The fluid velocity is small and the structural motions remain within the linear elastic range of response. There is assumed to be no separation or cavitation between the liquid and the tank.

The dynamic model adopted in the present study is that of Housner (1963) as implemented by ACI Committee 350 in their recommendations, as shown in Fig. 1. In the model W_i represents the

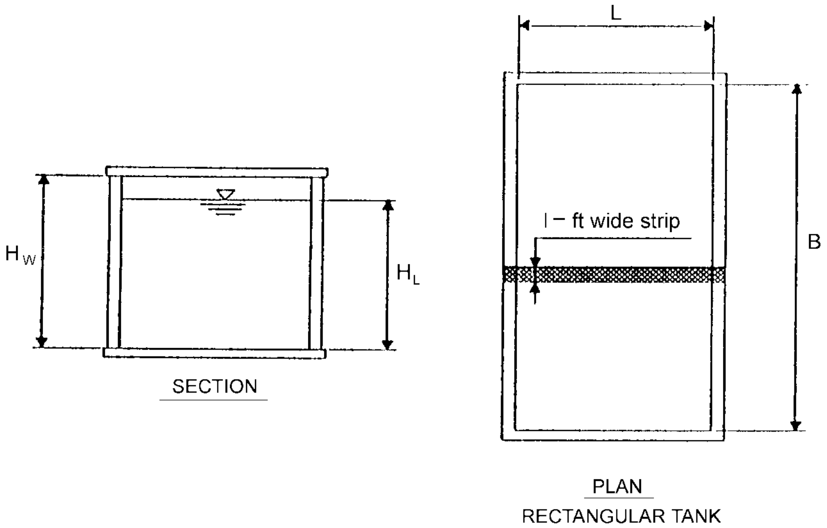

Fig. 3 Dimensions of rectangular tank

resultant effect of impulsive seismic pressure on the tank walls and W_c represents the resultant of the sloshing fluid pressures. W_i acts at a height of h_i corresponding to the location of the resultant impulsive force P_i. During an earthquake P_i, changes direction several times per second, corresponding to the change in the direction of the base acceleration, thus making it ineffective in overturning the tank. W_c represents the equivalent weight of the oscilating fluid that produces convective pressures on the tank walls with the resultant force P_c, which acts at a height h_c. The overturning moment exerted by P_c acts for a significant time to uplift the tank if there is insufficient dead weight.

The dynamic lateral forces above the base are calculated as:

Inertia force on tank wall	$P_w = C_i I x (\epsilon W_w)/R_{wi}$	
Inertia force on roof	$P_r = C_i I x W_r/R_{wi}$	
Impulsive Force	$P_i = C_i I x W_i/R_{wi}$	(2)
Convective Force	$P_c = C_c I x W_c/R_{wi}$	

where I=importance factor for the tank structure defined in Table 1, W_w and W_r are the weights of the tank wall and roof respectively, W_i and W_c are the impulsive and convective components of the stored liquid, and C_i and C_c are the seismic response coefficients as defined above. The factor I provides the engineer a means to increase the safety factor for structures. The factors R_{wi} and R_{wc} reduce the elastic response spectrum to account for the structures ductility, energy-dissipating properties and redundancy. Recommended values in Table 2.

The vertical distribution, per foot of wall height, of the dynamic pressures acting on the wall are assumed as shown below (Fig. 4):

$$\begin{aligned} &P_{wy} = Ix(C_i/R_{wi})\ x\epsilon\ (\gamma_c B t_w)/12 \\ &P_{iy} = (P_i/2)[4H_L - 6h_i - (6H_L - 12h_i)\ x(y/H_L)]\ (1/H_L^2) \\ &P_{cy} = (P_c/2)[4H_L - 6h_c - (6H_L - 12h_c)\ (y/H_L)]\ (1/H_L^2) \end{aligned} \qquad (3)$$

The horizontal distribution of the hydrodynamic pressures across the tank width B is,

Table 1 Importance factor I

Tank use	Factor
Tanks containing hazardous materials	1.5
Tanks that must remain usable, with slight structural damage, for emergency purposes after an earthquake; or tanks that are part of lifeline systems	1.25
Tanks that must remain usable without significant leakage, but may suffer repairable structural damage	1.0

Table 2 Response modification factor R_w

Type of structure	R_w		R_{wc}
(a) Anchored flexible-base tanks	4.5	4.5	1.0
(b) Fixed or hinged-base tanks	2.75	4.0	.0
(c) Unanchored, contained or uncontained tanks	2.0	2.75	.0
(d) Elevated tanks	3.0	-	.0

$$P_{wy}=P_{wy}/B$$
$$P_i=P_{io}/B$$
$$P_c=P_{co}/B \qquad (4)$$

4.2. Cylindrical tanks

Fig. 5 shows a ground supported circular tank of radius R (diameter D), height H and the wall thickness t_w. The tank is filled with a liquid of density ρ up to a height H_L. A cylindrical coordinate system is used with the center of the base being the origin. The radial, circumferential and axial coordinates are denoted by r, θ, and z coordinates. The tank is subjected to ground motion $x_g(t)$ in the constant direction of θ=0. The vertical distribution, per foot of the wall height, of the dynamic pressures acting on the wall are assumed as shown below:

$$P_{wy}=P_w/2H_w$$
$$P_{iy}=P_i/2[4H_L-6h_i-(6H_L-12h_i)\ x(y/H_L)]\ (1/H_L^2)$$
$$P_{cy}=P_c/2[4H_L-6h_c-(6H_L-12h_c)\ (y/H_L)]\ (1/H_L^2) \qquad (5)$$

The horizontal distribution across the tank diameter D, of the dynamic pressures is:

$$p_{wy}=(P_{wy}/\Pi R)$$
$$p_{iy}=(2P_{iy}/\Pi R)\cos\theta$$
$$p_{cy}=(16P_{cy}/9\ \Pi R)\cos\theta \qquad (6)$$

The impulsive pressure increases from zero at the liquid surface to a maximum at the base, whereas the convective pressure is maximum at the liquid surface and decreases with depth. The total hydrodynamic wall pressure is generally obtained by taking the sum of the numerical values of the maximum impulsive pressure at the point and the maximum convective pressure. The total pressure are then calculated by adding the hydrostatic pressures to the total hydrodynamic pressures. as stated earlier, the convective pressure component is quite small when compared to the impulsive pressure and hence are neglected in the present investigation.

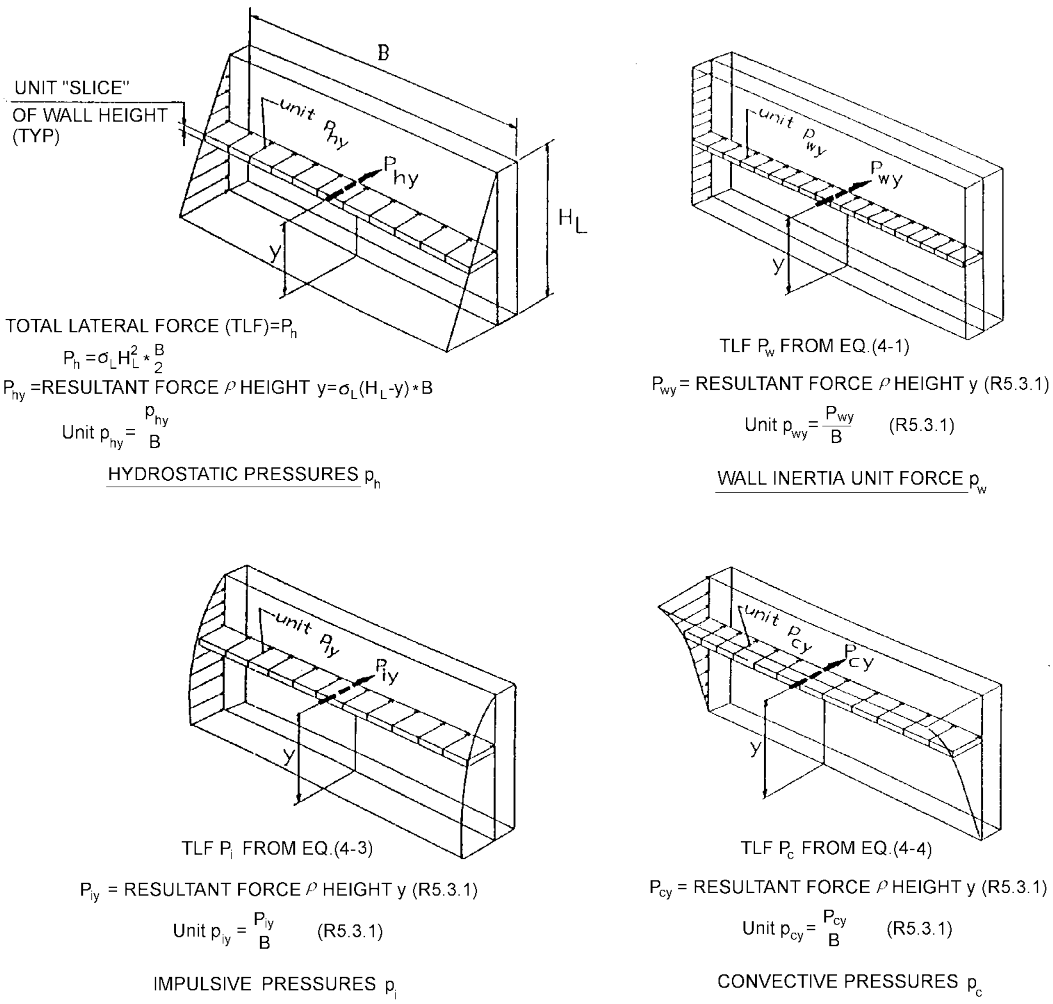

Fig. 4 Distribution of hydrostatic and hydrodynamic pressures and inertia forces

5. Tank geometry and location

5.1. Rectangular tanks

Two rectangular tanks with 0.24 million gallon (20’×40’×40’) and 0.48 million gallon (40’×80’×20’) capacities are considered. The tanks are assumed to rest on a rigid foundation. The base slabs of the tanks are assumed rigid so as to provide fixed support to the base of the walls. The tank location is assumed to be seismic map zone 4 with a seismic zone factor Z of 0.4, I=1.25, S_s=0.4 from maps, site class D, F_a=1.5, R_{wi}=2.75 and R_{wc}=1.0. The total weight of the tanks with full water content are 2048 kips and 4096 kips respectively. Both the tanks are analyzed for two conditions: i) the short wall perpendicular to the earthquake motion and ii) the long wall perpendicular to the motion.

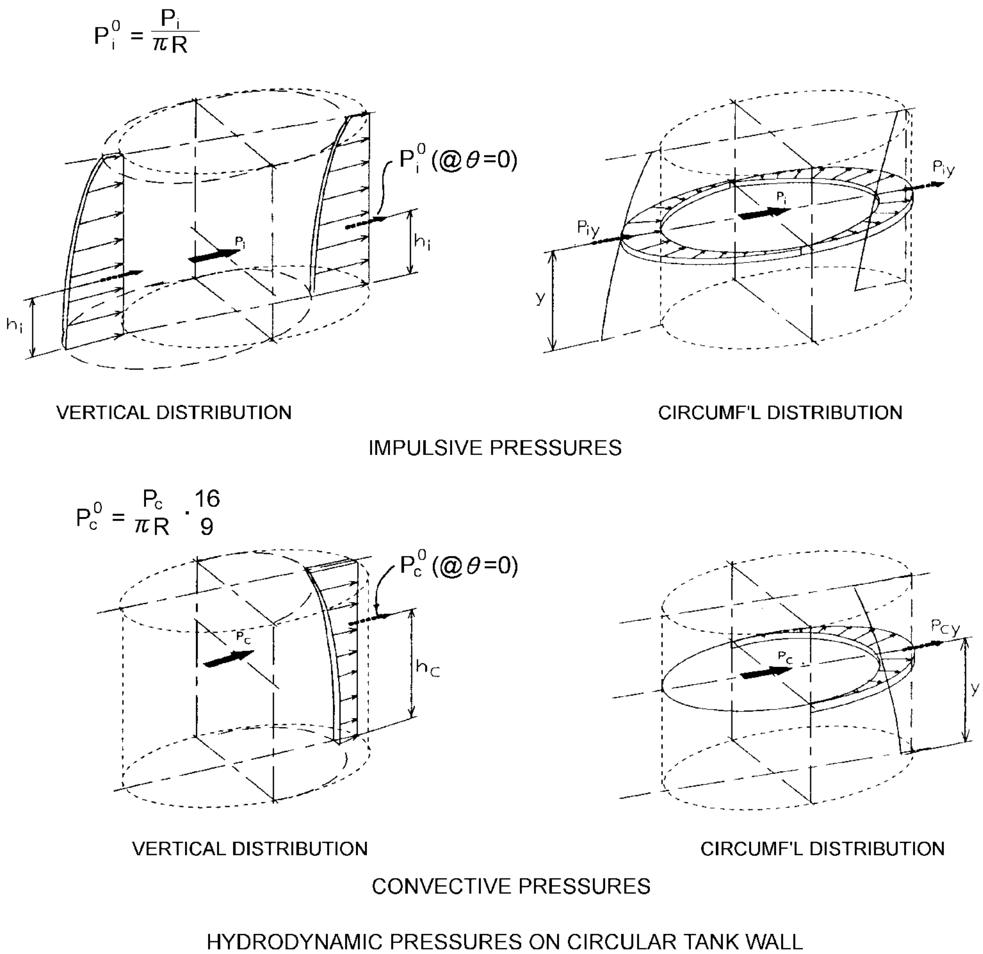

Fig. 5 Dimensions and hydrodynamic pressures on circular tank wall

5.2. Cylindrical tanks

Two cylindrical water tanks of 0.47 million gallon (40’×50’) and 1.2 million gallon capacity (100’×20’) are considered for analysis. The total weight of water when the tanks are full are 4022 kips and 10050 kips respectively.

6. Finite element analysis

The tanks are analyzed using the general purpose finite element program FINITE. For rectangular tanks, one half of the tank is analyzed using 981 nodes and 300 elements for the 0.48 MG tank and using 661 nodes and 200 elements for the 0.24 MG tank. An 8-node quadratic isoparametric flat shell element QFSHELL is used in the modelling of the structure. For circular tanks, a 16×10 mesh with 533 nodes and 160 elements is used. An isoparametric quadratic shell element QSHELL is

Table 3 Membrane contribution to base shear and overturning

Configuration	Trans. shear	Membrane shear	% base shear	N_y moment contrib	M_y contrib	% overturning
tall rect. long	117 kips	452 kips	79%	$9{,}606^{\text{ft-k}}$	$312^{\text{ft-k}}$	97%
tall rect. short	221 kips	419 kips	65%	$10{,}877^{\text{ft-k}}$	$1{,}135^{\text{ft-k}}$	90%
broad rect long	438 kips	642 kips	60%	$7{,}779^{\text{ft-k}}$	$1{,}936^{\text{ft-k}}$	80%
broad rect short	946 kips	351 kips	27%	$4{,}557^{\text{ft-k}}$	$5{,}904^{\text{ft-k}}$	44%
tall cyl	222 kips	1039 kips	82%	$25{,}666^{\text{ft-k}}$	$570^{\text{ft-k}}$	98%
broad cyl	723 kips	909 kips	56%	$10{,}180^{\text{ft-k}}$	$2{,}094^{\text{ft-k}}$	83%

used to idealize the structure. An elastic analysis is performed under a hydrodynamic pressure computed according to the proposed ACI recommendations. This load is applied on the structure as equi-static load. A reduced 2×2 integration is used for obtaining the stress resultants. Output consists of the element stresses, nodal displacements and constraint reactions.

7. Results

The membrane contributions to the earthquake resistance of the various tanks as computed from the finite element analysis are summarized in Table 3. It can be seen that membrane forces are the major resistance for most tank configurations.

7.1. Tall rectangular tank with long wall parallel to earthquake direction

Walls perpendicular to the motion respond in slab bending. Walls parallel to the motion provide support to the perpendicular walls. The N_y force in the vertical direction provides the overturning moment resistance. The inplane or membrane shear N_{xy} in the wall parallel to the wall parallel to the motion is one of the contributors resisting base shear. In this case of the long wall parallel to the motion it provides over 79% of the resistance to the base shear. The remainder is the transverse shear at the base of the walls perpendicular to the motion. M_x, and M_y are the slab bending moments. Slab bending at the base of the walls perpendicular to the motion do provide remaining resistance to overturning.

7.2. Tall rectangular tank with short wall parallel to the earthquake direction

The N_{xy} shear in the wall parallel to the motion still carries 2/3 of the base shear. The N_y contribution to overturning represents a 90% of the necessary moment. The N_y force variation looks very much like a linear stress variation at base of a cantilever beam. Walls perpendicular to the motion do provide N_y some flange contribution. The membrane forces are major contributors to the global resisting force system.

7.3. Broad rectangular tank with long wall parallel to earthquake direction

The short walls do respond in bending which does push a significant amount into reactions on the long walls. The response of the long walls is membrane and has the appearance of a deep beam. Again the major role in carrying out the resistance is that of the membrane.

7.4. Broad rectangular tank with long wall parallel to earthquake direction

When the short wall is parallel to the motion the long walls behave more like cantilevers so slab bending passes the load more in downward direction. The membrane contribution of the parallel walls is reduced but still represents about 30% of the total.

7.5. Tall cylindrica tanks

Cylindrical tanks have a much more developed membrane system of forces. Local bending is limited to that developed as a consequence of the displacement constraints at the base of the tank. Their response to earthquake forces is therefore dominated by membrane actions. The deformed configuration of the circular tanks viz., i) tall tank with D/H=0.8 (i.e., <1.33) and the broad tank with D/H=5 (i.e., >1.33) show sharp variation near the base. The membrane axial force N_y acts in the global vertical direction as overturning moment resistance. The inplane shear N_{xy}, is the major contributor to base shear resistance, and M_y the moments in the vertical strips. Although the transverse, through the wall, shear does provide a small contribution N_{xy} does constitute over 82% of base shear.

7.6. Broad cylindrical tanks

As the diameter to height ratio increases the dominance of the membrane contribution decreases. The M_y moments take on a larger share. The curvature of the shell does mean that action of vertical strips as cantilever beams does not occur.

8. Conclusions

Inplane forces are the major contributors to the earthquake resistance of tank strutures. For cylinders they provide 80-90% of the resistance until shallow or very low height/diameter ratio tanks are employed. For rectangular tanks even though the local effect to the normal pressure is slab bending, membrane forces are still important to the global resistance. For tall tanks their share is also in the 70-90% range and even for squat tanks, in the short direction, 25% of their earthquake resistance is achieved by membrane forces.

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References

American Concrete Institute (1994), "Code requirements for environmental engineering concrete structures-Chapter 21: Special provisions for seismic design", *ACI Committee 350*, January.
American Concrete Institute, Report, "Seismic design of liquid-containing concrete structures", *ACI Committee 350*, Seismic Design Subcommittee.
American Society of Civil Engineers (1981), "Guidelines for the seismic design of oil and gas pipline systems", Prepared by Committee of Gas and Liquid Fuel Lifelines of the Technical Council on Lifeline Earthquake Engineering, Section 7.
American Society of Civil Engineers (1984), "Fluid-structure interaction during seismic excitation", Report by Committee on Seismic Analysis.
Hanson, R.D. (1973), "Behavior of liquid storage tanks", Report No. EERL 80-04, Caltech.
Haroun, M.A. and Chen, W. (1989), "Seismic large amplitude liquid sloshing theory", *Seismic Engineering Structures Congress*, San Francisco, May, 418-427.
Haroun, M.A. and Housner, G.W. (April 1981), "Seismic design of liquid storage tanks", *Journal of Structural Engg., ASCE*, **107**, 191-203.
Haroun, M.A. and Housner, G.W. (1980), "A Procedure for seismic design of liquid storage tanks", Earthquake Engineering Research Lab. Report, Caltech, October.
Haroun, M.A. and Tayel, M.A. (1984), "Dynamic behavior of cylindrical liquid storage tanks under vertical earthquake excitation", *8th World Conf. Earthquake Engineering*, **7**, 421-428.
Housner, G.W. (1957), "Dynamic pressures on accelerated fluid containers", Bull. Seism. Soc., America, **47**, 15-35.
Housner, G.W. (1963), "Dynamic pressures on fluid containers", *TID Document 7024, U.S. Atomic Energy Commission*.
Jennings, P.C. (1971), "Engineering features of the San Fernando earthquake of February 9, 1971", Report EERL 71-02, California Institute of Technology.
Kelly, T.E. and Mayers, R.L. (1989), "Seismic isolation of storage tanks", *Seismic Engineering Structures Congress*, SanFrancisco, May 408-417.
Luft, R.W. (1984), "Vertical accelerations in prestressed concrete tanks", *Journal of Structural Engg. ASCE*, **110**, 706-714.
Manos, G.C. and Clough, R.W. (1985), "Tank damage during the may 1983 Coalinga earthquake", *Earthquake Engineering in Structural Dynamics*, **13**, 449-466.
Nash, W.A., Balendra, T., Shaaban, S.H. and Mouzakis, T. (1978), "Finite element analysis of seismic response of cylindrical tanks", *ASCE Convention and Exposition*, Preprint 3315, Chicago, Illinois.
Veletsos, A.S. and Tang, Y. (1986), "Dynamics of vertically excited liquid storage tanks", *Journal of Structural Engg. ASCE*, **112**, June, 1228-1246.
Veletsos, A.S. and Yang, J.Y. (1976), "Dynamics of fixed-base liquid storage tanks", *U.S.-Japan Seminar for Earthquake Engineering Research*, Tokyo, Japan, 1976.
Yang, J.Y. (1976), "Dynamic behaviour of fluid-tank systems", Ph.D., Thesis, Rice University.