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Nonlinear vibration of Timoshenko beam due to moving loads including the effects of weight and longitudinal inertia of beam

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Abstract. The effects of weight and axial inertia of a beam are taken into account for studying the nonlinear vibration of the Timoshenko beam due to external loads. The combination of Galerkins method and Runge-Kutta method are employed to obtain the dynamic responses of the beam. A concentrated force and a two-axle vehicle traversing on the beam are taken as two examples to investigate the response characteristics of the beam. Results show that the effect of axial inertia of the beam increases the fundamental period of the beam. Further, both the dynamic deflection and the dynamic moment of the beam obtained with including the effect of axial inertia of the beam are greater than those of the beam without including that effect of the beam.

Key words: weight; axial inertia; period; vehicle; dynamic moment.

1. Introduction

The responses of a beam due to external loads are normally obtained via the small deflection theory. Neglecting the effects of weight and longitudinal inertia of the beam, the small deflection theory implies that both the maximum deflections as well as the maximum moment of the structure induced by a moving load are greater than those induced by the load in a static situation (Wang 1997). The responses of the beam caused by a large load may, however, be too large via the small deflection theory. Employing the large deflection theory can correct these overestimated results.

The large deflection theory for beams indicates that the coupling effect between longitudinal force and transverse deflection of a beam stiffens the structure. The effects of initial imperfections (Ilanko 1990, Kim and Dickinson 1986, Plaut and Johnson 1981), large amplitudes (Mei 1973, Reddy and Singh 1981) and longitudinal extension (Bhashyam and Prathap 1980) of the beam, consequently, increase the fundamental frequency of the structure. Using the large deflection theory and neglecting the effect of weight of a beam, Hino, *et al.* (1986) have demonstrated that the fundamental frequency of the beam increases as the magnitude of the load traversing on the structure increases. Neglecting the effect of weight of beam, Xu, *et al.* (1997) have demonstrated that the effect of the friction force between the moving mass and beam is significant on the longitudinal motion of the Bernoulli-Euler beam.

Sometimes, the weight of the beam is heavier than the magnitude of the external load acting on the structure. Therefore, the effect of the weight of the beam cannot be neglected while studying the

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dynamics of the beam. The transverse deflection is normally greater than the longitudinal displacement. Moreover, the inertia of transverse motion is more dominant on the transverse vibration of beam than the longitudinal inertia is. Therefore, the effect of longitudinal inertia of beam is neglected in most case studies of nonlinear vibration of beam (Özkaya *et al.* 1997). Neglecting the longitudinal inertia of beam, Wang and Chou (1998) have put the weight effect of the beam into the study of nonlinear vibration of the Timoshenko beam due to a set of two moving forces. Their results indicated (1) a force with a large magnitude traversing on the beam at a high velocity will cause the Timoshenko beam to exhibit a small fundamental period and (2) the effects of weight on the dynamic responses of the beam is not important for a short and thick beam.

Neglecting the longitudinal inertia adds a constrained condition on the vibration of beam. Therefore, the effect of neglecting the longitudinal inertia stiffens the beam. In such a situation, the responses of the beam will, however, be underestimated. The large deflection theory and the effects of weight and axial inertia of the beam should be, therefore, taken into account simultaneously in order to obtain accurate results.

The large deflection theory will be adopted to derive the equations of motion of the Timoshenko beam caused by external loads. The static responses of the beam due to its own weight are obtained. Due to the coupling effect of longitudinal force with the transverse deflection, the equations of motion of the beam cannot be solved analytically. Therefore, a set of mode shape functions obtained from the small deflection theory for the beam is incorporated in the Galerkins method to solve the nonlinear problems. A concentrated force and a two-axle vehicle traversing on the beam are taken as two examples. The dynamic responses of a beam obtained via the large deflection theory, including the effects of weight and axial inertia of the beam, are discussed. Further, the dynamic responses of the vehicle are also investigated.

2. Governing equations of beam

An external force $F(x, t)$ acts on a simply supported Timoshenko beam is depicted in Fig. 1. Both ends of the beam are immovable. The beam is considered to be homogeneous and isotropic with Young's modulus E, Poissons ratio v, shear modulus G, mass density ρ, length L, thickness h and width b. The co-ordinate of the neutral axial of the beam is denoted as x. The static deformations of longitudinal displacement, transverse deflection and bending slope of the beam caused by its own weight are denoted as $u_o(x), w_o(x)$, and $\Psi_o(x)$, respectively. Further, the

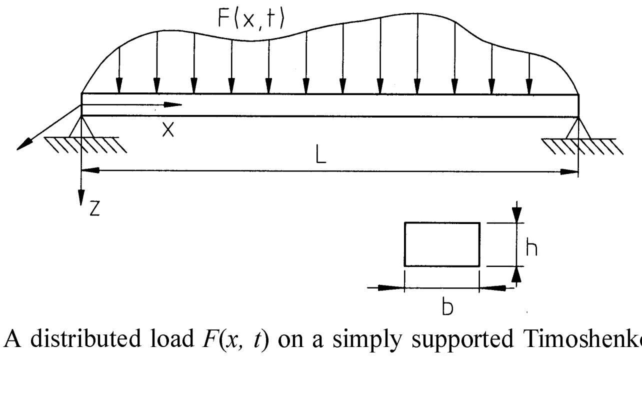

Fig. 1 A distributed load $F(x, t)$ on a simply supported Timoshenko beam

corresponding longitudinal strain, shearing strain, longitudinal force, shear force and bending moment of the beam are denoted as $\varepsilon_o(x), \gamma_o(x), n_o(x), q_o(x)$ and $m_o(x)$, respectively. The corresponding dynamic components of the beam induced by the external force are denoted as $u_*(x, t), w_*(x, t), \Psi_*(x, t)$ as $\varepsilon_*(x, t), \gamma_*(x, t), n_*(x, t), q_*(x, t)$ and $m_*(x, t)$, respectively.

The displacement fields u and w of the beam due to the combined action of its own weight and the external force are

$$u(x, z, t) = u_o(x) + u_*(x, t) - z\Psi_o(x) - z\Psi_*(x, t), \tag{1a}$$

$$w(x, z, t) = w_o(x) + w_*(x, t) \tag{1b}$$

According to the large deflection theory for beams, the strain fields of the beam are

$$\varepsilon = \varepsilon_o + \varepsilon_* - z\psi_o' - z\psi_*', \gamma = \gamma_o + \gamma_* \tag{2a, b}$$

in which

$$\varepsilon_o = u_o' + 0.5(w_o')^2, \gamma_o = w_o' - \psi_o, \tag{3a, b}$$

$$\varepsilon_* = u_*' + w_*'w_o' + 0.5(w_*')^2, \gamma_* = w_*' - \psi_o, \tag{4a, b}$$

where ( )' denotes differentiation with respect to the spatial x. The longitudinal force n, shear force q and bending moment m of the beam are

$$n = n_o + n_*, q = q_o + q_*, m = m_o + m_*, \tag{5a, b, c}$$

where,

$$(n_o, n_*) = EA(\varepsilon_o, \varepsilon_*), (q_o, q_*) = \kappa GA(\gamma_o, \gamma_*), (m_o, m_*) = -EI(\psi_o', \psi_*'), \tag{6a, b, c}$$

in which κ is the shear coefficient, $A(= bh)$ is the cross-sectional area and $I(= bh^3/12)$ is the second moment of area about the y-axis of the beam.

The strain energy V and kinetic energy KE of the beam are

$$V = \int_0^L \left(\frac{n^2}{EA} + \frac{m^2}{EI} + \frac{q^2}{\kappa GA}\right)dx/2, KE = \int_0^L \rho(A\dot{u}^2 + A\dot{w}^2 + I\dot{\Psi}^2)dx/2 \tag{7a, b}$$

in which $(\dot{\ })$ denotes differentiation with respect to time t. The work P done on the beam by the combination of its own weight and the distributed force is

$$P = \int_0^L (\rho gAw + Fw_*)dx \tag{7c}$$

Substituting V, KE and P into Hamilton's principle yields the equations of static equilibrium as

$$n_0' = 0, \quad -m_0' + q_o = 0, n_o w_o'' + q_o' + \rho gA = 0, \tag{8a, b, c}$$

and the equations of motion as

$$n_*' - \rho A\ddot{u}_* = 0, \quad m_*' - q_* + \rho I\ddot{\psi}_* = 0, \tag{9a, b}$$

$$-n_o w'' - n_*(w_o'' + w_*'') - q_*' + \rho A\ddot{w}_* = F(x, t) \tag{9c}$$

The boundary conditions at both simply supported and immovable ends of the beam are

$$w_o = w_* = 0, u_o = u_* = 0, m_o = m_* = 0 \tag{10a, b, c}$$

3. Static responses

The solution of Eq. (8a) that satisfies the boundary conditions at immovable ends is

$$n_o = EA\int_0^L w_o' \, dx/2 \tag{11}$$

Eliminating Ψ_o between Eqs. (8b) and (8c) and simplifying the result yield

$$w_o^{iv} - c^2 w_o'' = f \tag{12}$$

where,

$$\alpha = \frac{n_o}{\kappa GA}, c^2 = \frac{n_o}{EI(\alpha+1)}, f = \frac{\rho gA}{EI(\alpha+1)}$$

The solution w_o of Eq. (12) and the corresponding moment that satisfy the boundary conditions at both simply supported ends are

$$w_o = \frac{f}{c^2}\left\{a_1\left[\text{cish}(cx) + \frac{x}{L} - \frac{x\cosh(cL)}{L} - 1\right] + a_2\left[\sinh(cx) - \frac{x\sin(cL)}{L}\right] - \frac{x(x-L)}{2}\right\} \tag{13a}$$

$$m_o = -EIf\left[(a_2 + \alpha c a_1)\sinh(cx) + (a_1 + \alpha c a_2)\cosh(cx) - \frac{1}{c^2}\right] \tag{13b}$$

where,

$$a_1 = \frac{1}{1-\alpha^2 c^2}\left\{\frac{1}{c^2} - \alpha c\left[\frac{1-\cosh(cL)}{\sinh(cL)}\right]\right\}, a_2 = \frac{1}{1-\alpha^2 c^2}\left\{-\frac{\alpha}{c} + \frac{1-\cosh(cL)}{\sinh(cL)}\right\}$$

Further, the shear force obtained from Eq. (8b) is

$$q_o = -EIcf[(a_2 + \alpha c a_1)\cosh(cx) + (a_1 + \alpha c a_2) sinh(cx)] \tag{13c}$$

Substituting Eq. (13a) into Eq. (11) yields the nonlinear equation in terms of n_o as the symbolic form

$$n_o = N(n_o, \rho gA, L) \tag{14}$$

The solution n_o of Eq. (14) can be obtained by the numerical method.

4. Dynamic responses

Solutions of the set of nonlinear partial differential Eqs. (9a)-(9c) cannot be obtained exactly. Therefore, Galerkin's method is adopted here to find the approximate solutions of Eqs. (9a)-(9c). Any two distinct sets of mode shape functions of the Timoshenko beam, obtained from the small deformation theory, have shown to be orthogonal (Wang 1997), i.e.,

$$\int_0^L U_i(U_j, U_j'')dx = (0, 0), \int_0^L [\Psi_i(M_j'' - Q_j) - W_i Q_j']dx = 0,$$

$$\sum_{0}^{L} r(AW_iW_j + I\Psi_i\Psi_j)dx = 0, \quad i \neq j \tag{15a, b, c}$$

in which U_j is the jth mode shape function of the longitudinal displacement, and W_i, Ψ_i, Q_i and M_i are the ith mode shape functions of transverse deflection, bending slope, shear force and bending moment of the beam, respectively. Therefore, the dynamic responses of transverse deflection, bending slope, shear force and bending moment of the beam can be expressed in the following form for Galerkin's method

$$\{w_*, \psi_*, q_*, m_*\}(x, t) = \sum_{i=1}^{M} B_i(t)\{W_i, \Psi_i, Q_i, M_i\}(x) \tag{16a}$$

$$u_*(t) = \sum_{i=1}^{N} C_i(t)U_i(x) \tag{16b}$$

in which B_i and C_i are needed to be determined. Moreover, the dynamic longitudinal force listed in Eq. (5a) is expressed as

$$n_*(t) = EA\left[\sum_{i=1}^{N}\left(C_i(t)U_i' + \sum_{i=1}^{N} B_i(t)w_o'W_i'\right) + 0.5\sum_{i=1}^{N}\sum_{j=1}^{N} B_i(t)B_j(t)W_i'W_j'\right] \tag{16c}$$

Substituting Eqs. (16a)~(16c) into Eqs. (9a)~ (9c), respectively, yields

$$\rho A\sum_{i=1}^{N} U_i\ddot{C}_j - n_*' = 0, \quad \sum_{i=1}^{N}\{B_i(-Q_i + M_i') + \rho I\Psi_i\ddot{B}_i\} = 0 \tag{17a, b}$$

$$\sum_{i=1}^{N}\{-B_iQ_i' + \rho AW_i\ddot{B}_i\} - (n_o + n_*)w_*'' - n_*w_o'' = F(x, t) \tag{17c}$$

Multiplying Eq. (17a) by U_j and integrating the result from x=0 to x=L yield the nonlinear differentiation equation in the symbolic form

$$m_jC_j - g_j(C_j, B_i, B_l) = 0, \quad j = 1, 2, \ldots \tag{18}$$

in which

$$m_j = \int_0^L \rho AU_i^2 \mathrm{dx}, \quad g_j(C_j, B_i, B_l) = \int_0^L U_jn_*'dx \tag{19a, b}$$

Further, multiplying Eq. (17b) by Ψ_j, Eq. (17c) by W_j and integrating the summation from x=0 to x=L yield the nonlinear differentiation equation in the symbolic form

$$s_j\ddot{B}_j + q_jB_j - p_j(C_i, B_i, B_k, B_l) = F_j(t), \quad j = 1, 2, \ldots \tag{20}$$

in which

$$s_j = \int_0^L \rho(AW_j^2 + I\Psi_j^2)dx, \quad q_j = \int_0^L [\Psi_j(M_t' - Q_i) + W_i(-Q_i')]dx \tag{21a, b}$$

$$p_j(C_i, B_i, B_k, B_l) = \int_0^L W_j[(n_o + n_*)w_*'' + n_* w_o'']dx, \quad F_j(t) = \int_0^L F(x, t)W_j dx. \tag{21c, d}$$

The set of nonlinear differential Eqs. (18) and (20) can be solved by the Runge-Kutta method.

5. Moving loads

In this section, two types of moving loads are considered: concentrated load and two-axle vehicle.

5.1. Concentrated Force

A concentrated force traversing at a constant velocity v on the beam is depicted in Fig. 2. The form of the force is

$$F(x, t) = F_o\delta(x - vt), \quad 0 \le t \le T(= L/v) \tag{22}$$

where T is the duration of the force traversing on the beam and is the impulse function. The equations of motion of the beam are

1. $0 \le t \le T$

$$m_j\ddot{C}_j - g_j(C_j, B_i, B_l) = 0 \tag{23a}$$

$$s_j\ddot{B}_j + q_jB_j - p_j(C_i, B_i, B_k, B_l) = F_oW_j(vt) \tag{23b}$$

2. $T \le t$

$$m_j\ddot{C}_j - g_j(C_j, B_i, B_l) = 0$$

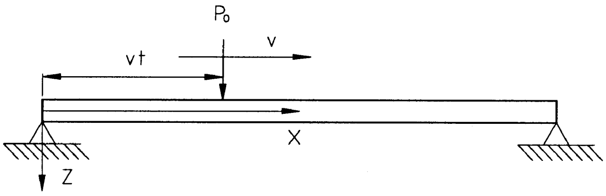

Fig. 2 A concentrated force traversing on the Timoshenko beam at a constant velocity v

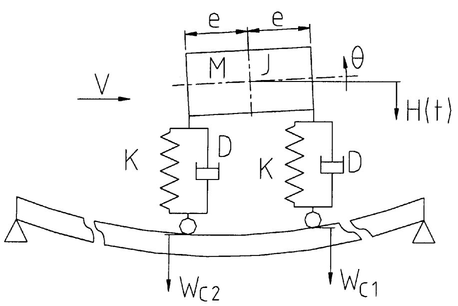

Fig. 3 A two-axle vehicle traversing on the Timoshenko beam at a constant velocity v

$$s_j\ddot{B}_j + q_jB_j - p_j(C_i, B_i, B_k, B_l) = 0 \tag{24b}$$

The initial conditions of the beam are set at zeros.

5.2. Two-axle vehicle

A two-axle vehicle travelling on the Timoshenko beam at a constant velocity v is depicted in Fig. 3. The vehicle has the mass M and the polar moment of inertia J. Each axle has a suspension with the spring constant K and the damper coefficient D. The distance from the mass center to both axles is e. The vertical displacement and the pitch angle about the mass center of the vehicle are, respectively, denoted as H and θ. The transverse displacement at the contact point between the front (or rear) wheel and the beam is denoted as w_{c1} (or w_{c2}). The frictional forces between the vehicle and the beam are neglected in the following analysis. The contact force acting on the beam due to the vehicle's movement is

1. $0 \le t \le t_1 (= 2e/v)$

$$F(x, t) = [K(H - e\theta - w_{c1}) + D(\dot{H} - e\dot{\theta} - \dot{w}_{c1})\delta(x - vt)], \tag{25}$$

where

$$w_{c1}(t) = (w_o + w_*)\big|_{x = vt}, \quad \ddot{w}_{c1}(t) = [v(w_o' + w_*') + \ddot{w}_*]_{x = vt} \tag{26a, b}$$

2. $t_1 \le t \le T$

$$F(x, t) = [K(H - e\theta - w_{c1}) + D(\dot{H} - e\dot{\theta} - \dot{w}_{c1})]\delta(x - vt) + K(H - e\theta - w_{e2})] \\ \delta[x - v(t - t_1)], \tag{27}$$

where

$$w_{c2}(t) = (w_o + w_*)\big|_{x = vt - 2e}, \quad \dot{w}_{c2}(t) = [v(w_o' + w_*') + \dot{w}_*]_{x = vt - 2e}, \tag{28a, b}$$

3. $T \le t \le t_1 + t_1$

$$F(x, t) = [K(H + e\theta - w_{c2}) + D(\dot{H} + e\dot{\theta} - \dot{w}_{c2})]\delta[x - v(t - t_1)], \tag{29}$$

4. $T + t_1 \le t$

$$F(x, t) = 0, \tag{30}$$

The equations of motion of the whole system are

1. $0 \le t \le t_1$

$$M\ddot{H} + 2(KH + D\dot{H}) - (Kw_{c1} + D\dot{w}_{c1}) = Mg \tag{31a}$$

$$J\ddot{\theta} + 2e^2(K\theta + D\dot{\theta}) - e(Kw_{c1} + D\dot{w}_{c1}) = 0 \tag{31b}$$

$$m_j\ddot{C}_j - g_j(C_j, B_i, B_l) = 0, \tag{31c}$$

$$s_j\ddot{B}_j + q_jB_j - p_j(C_i, B_i, B_k, B_l) = F_j(t), \tag{31d}$$

where

$$F_j(t) = W_j(vt)K(H - e\theta - w_{c1}) + D(\dot{H} - e\dot{\theta} - \dot{w}_{c1}) \tag{32}$$

2. $t_1 \le t \le T$

$$M\ddot{H} + 2(KH + D\dot{H}) - [K(w_{c1} + w_{c2}) + D(\dot{w}_{c1} + \dot{w}_{c2})] = Mg, \tag{33a}$$

$$J\ddot{\theta} + 2e^2(K\theta + D\dot{\theta}) + e[K(w_{c1} - w_{c2}) + D(\dot{w}_{c1} - \dot{w}_{c2})] = 0, \tag{33b}$$

$$m_j\ddot{C}_j - g_j(C_j, B_i, B_l) = 0, \tag{33c}$$

$$s_j\ddot{B}_j + q_jB_j - p_j(C_i, B_i, B_k, B_l) = F_j(t), \tag{33d}$$

where

$$F_j(t) = W_j(vt)[K(H - e\theta - w_{c1}) + D(\dot{H} - e\dot{\theta} - \dot{w}_{c1})] + W_j[v(t - t_1)][K(H + e\theta - w_{c2}) + D(\dot{H} + e\dot{\theta} - \dot{w}_{c2})] \tag{34}$$

3. $T \le t \le T + t_1$

$$M\ddot{H} + 2D\dot{H} + 2KH - D\dot{w}_{c2} - Kw_{c3} = Mg \tag{35a}$$

$$J\ddot{\theta} + 2e^2(K\theta + D\dot{\theta}) - e(Kw_{c2} + D\dot{w}_{c2}) = 0 \tag{35b}$$

$$m_j\ddot{C}_j - g_j(C_j, B_i, B_l) = 0 \tag{35c}$$

$$s_j\ddot{B}_j + q_jB_j - p_j(C_i, B_i, B_k, B_l) = F_j(t) \tag{35d}$$

where

$$F_j(t) = W_j[v(t - t_1)][K(H + e\theta - w_{c2}) + D(\dot{H} + e\dot{\theta} - \dot{w}_{c2})] \tag{36}$$

4. $T + t_1 \le t$

$$M\ddot{H} + 2D\dot{H} + 2KH = Mg, \tag{37a}$$

$$J\ddot{\theta} + 2e^2(D\dot{\theta} + K\theta) = 0, \tag{37b}$$

$$m_j\ddot{C}_j - g_j(C_j, B_i, B_l) = 0, \tag{37c}$$

$$s_j\ddot{B}_j + q_jB_j - p_j(C_i, B_i, B_k, B_l) = 0, \tag{37d}$$

The initial conditions of the beam are set at zeros. Further, the initial conditions of the vehicle are set as $H(0)=Mg/2K$, $\theta(0)=0$, $\dot{H}(0)=0$ and $\dot{\theta}(0)=0$.

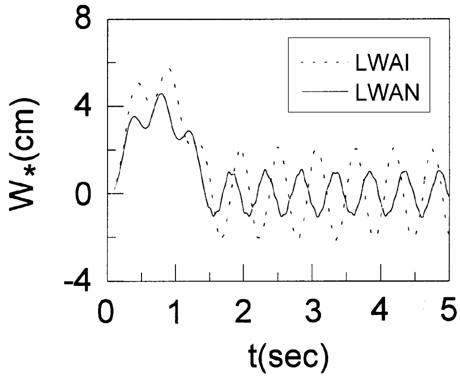

Fig. 4 Comparison of two deflection theories on the history of deflection at the mid-point of the Timoshenko beam (L=20 m, b=0.5 m, h=0.34 m) due to a moving concentrated force (P_o=3000 kg, v=50 km/hr)

6. Examples and discussion

The material properties ν=0.2, E=30 Gpa, ρ=2400 kg/m^3 and κ=0.85 (Dym and Shames 1973) of the beam are considered in the numerical computation. Based on the small deflection theory, the dynamic responses of beam obtained by the method of modal analysis converge rapidly. Therefore, it is sufficient to employ the number N=10 in the numerical computation. The velocity range considered here is from 0 to 200 km/hr. The following nomenclature and parameters are defined to illustrate the numerical results: large deflection theory excluding both effects of weight and axial inertia of the beam, LDTN (Wang and Chou 1998); large deflection theory including only the effect of weight of the beam, LWAN; large deflection theory including the effects of weight and axial inertia of the beam, LWAI; maximum dynamic deflection of the beam during motion of the force or vehicle, $W_{*\max}$; maximum dynamic moment of the beam during motion of the force or vehicle, $m_{*\max}$; vertical acceleration of the vehicle, $\ddot{H}$, maximum of the absolute value of vertical acceleration of vehicle, $|\ddot{H}|_{\max}$; angular acceleration of the vehicle, $\ddot{\theta}$; maximum of the absolute value of angular acceleration of vehicle, $|\ddot{\theta}|_{\max}$.

6.1. Concentrated force

A comparison of the results obtained by two theories of deflection for the history of w_* at the mid-span of the Timoshenko beam (L=20 m, b=0.5 m, h=0.34 m) due to a moving concentrated force (v=50 km/hr, P_o =3000 kg) is presented in Fig. 4. The coupling effect of n_o with w_* and that of w_o with n_* stiffen the beam. However, the effect of axial inertia softens the beam. Due to this reason, w_* at the mid-span of the Timoshenko beam predicated by LWAI is always greater than that by LWAN as indicated in the figure. The period of w_* of the beam during free vibration after the force has left the beam is called the fundamental period of the beam. In Table 1 it is indicated that a thick beam (L=20 m, b=0.5 m) has a small value of the fundamental period due to the moving force (v=50 km/hr, P_o=3000 kg). The effects of two different velocities of the moving force concentrated force (P_o=3000 kg) and three theories of deflection for the beam on the fundamental period of the beam (L=20 m, b=0.5 m, h=0.34 m) are listed in Table 2. A rapidly

Table 1 Both effects of the value of h and the deflection theory on the fundamental period of the Timoshenko beam (L=20 m, b=0.5 m) due to a moving concentrated force (P_o=3000 kg, v=50 km/hr)

Theories	h=0.34 m	h=0.5 m
LDTN	0.751 sec.	0.515 sec.
LWAN	0.472 sec.	0.442 sec.
LWAI	0.545 sec.	0.463 sec.

Table 2 Both effects of the velocity of the moving concentrated force (P_o=3000 kg) and the deflection theory on the fundamental period of the Timoshenko beam (L=20 m, b=0.5 m, h=0.34 m)

Theories	v=50 km/hr	v=100 km/hr
LDTN	0.751 sec.	0.721 sec.
LWAN	0.472 sec.	0.460 sec.
LWAI	0.545 sec.	0.463 sec.

Table 3 Both effects of the magnitude of the moving concentrated force (v=50 km/hr) and the deflection theory on the fundamental period of the Timoshenko beam (L=20 m, b=0.5 m, h=0.34 m)

Theories	P_o=1500 kg	P_o=3000 kg
LDTN	0.753 sec.	0.751 sec.
LWAN	0.489 sec.	0.472 sec.
LWAI	0.551 sec.	0.545 sec.

moving force excites a larger number of modes of the beam than a slowly moving force does. The coupling between high frequency modes and low frequency modes causes the magnitude of the fundamental period of the beam to be small for a rapidly moving force. A moving force with a larger magnitude will cause a strong coupling of low frequency modes with high frequency modes of the beam. Consequently, as can be seen from Table 3, the beam (L=20 m, b=0.5 m, h=0.34 m) exhibits a smaller value of the fundamental period due to a larger moving force (v=50 km/hr).

In the Tables 1~3 it is shown that LDTN leads to the largest value of the fundamental period of the beam, however, LWAN leads to the smallest value of the fundamental period of the beam. These results indicate that the effect of the weight of the beam stiffens the beam; however, the effect of the axial inertia of the beam softens the beam.

The effects of two different magnitudes of thickness and two different deflection theories (LWAI, LWAN) on $w_{*\max}-v$ and $m_{*\max}-v$ distributions of the Timoshenko beam due to a moving

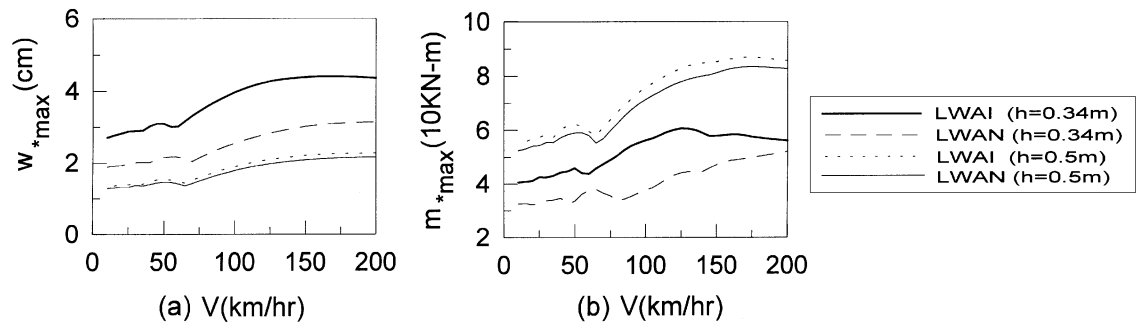

Fig. 5 Comparisons of two h values on (a) $w_{*\max}-v$ distribution and (b) $m_{*\max}-v$ distribution of the beam (L=20 m, b=0.5 m) due to a moving concentrated force (P_o=1500 kg)

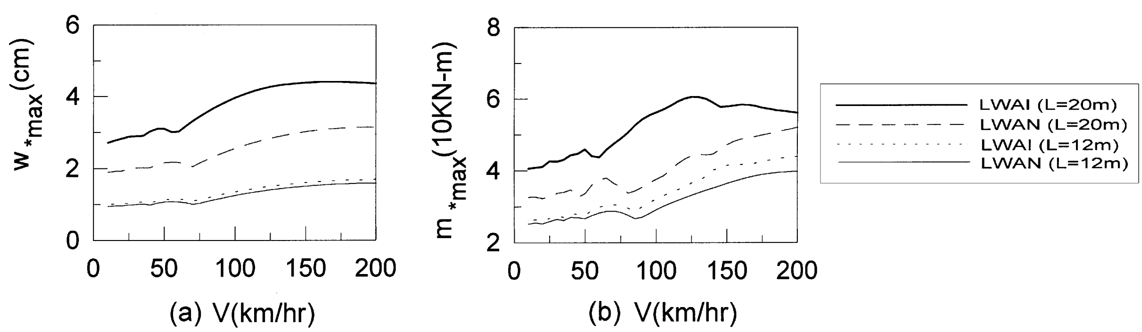

Fig. 6 Comparisons of two L values on (a) $w_{*\max}-v$ distribution and (b) $m_{*\max}-v$ distribution of the beam (b=0.5 m, h=0.34 m) due to a moving concentrated force (P_o=1500 kg)

concentrated force (P_o=1500 kg) are displayed in Figs. 5(a) and 5(b), respectively. A thicker beam exhibits a larger bending rigidity. Therefore, a thicker beam leads to a smaller $w_{*\max}$, however, a larger $m_{*\max}$. The effect of axial inertia causes the beam to have large transverse deflection and bending slope. Consequently, in Figs. 5(a) and 5(b) it is shown that both $w_{*\max}$ and $m_{*\max}$ of the beam based on LWAI are greater than those based on LWAN. In Fig. 5(a) it is also shown that the difference between the magnitude of $w_{*\max}$ as obtained by LWAI and LWAN decreases as the thickness increases. However, the difference between the magnitude of $m_{*\max}$ as obtained by LWAI and LWAN increases as the thickness decreases as displayed in Fig. 5(b). These results indicate that LWAN can be adopted to approximately obtain $w_{*\max}$ of a thick beam, however, $m_{*\max}$ of a thin beam due to a moving concentrated force.

The effects of two different magnitudes of length and two different deflection theories (LWAI, LWAN) on $w_{*\max}-v$ and $m_{*\max}-v$ distributions of the Timoshenko beam due to a moving concentrated force (P_o=1500kg) are displayed in Figs. 6(a) and 6(b), respectively. A longer beam leads to both larger $w_{*\max}$ and $m_{*\max}$. In Figs. 6(a) and 6(b) it is shown that both $w_{*\max}$ and $m_{*\max}$ of the beam based on LWAI are greater than those based on LWAN. In Fig. 6(a) it is also shown that the difference between the magnitude of $m_{*\max}$ as obtained by LWAI and LWAN decreases as the length decreases. Moreover, the difference between the magnitude of as obtained by LWAI and LWAN decreases as the length decreases as displayed in Fig. 6(b). These results indicate that LWAN can be adopted to approximately obtain both $w_{*\max}$ and $m_{*\max}$ of a short beam due to a moving concentrated force.

6.2. Two-axle vehicle

The data L=20 m, b=0.5 m and h=0.34 m, and LWAI of the beam are considered in the following discussions. The vehicle has the mass M=1500 kg and the moment of inertia J=507.8 kg-m^2.

The effects of two different velocities of the moving two-axle vehicle on the histories of vertical acceleration and angular acceleration of the vehicle (K=20 kN/m, D=800 N-s/m, e=1 m) during the motion of the vehicle are displayed in Figs. 7(a) and 7(b), respectively. Both figures show that the vehicle gets large vertical acceleration and angular acceleration while the vehicle is traversing on the beam. Moreover, the larger velocity is, the larger maximum vertical accelerations and the angular acceleration are. The angular acceleration of the vehicle changes abruptly when the front

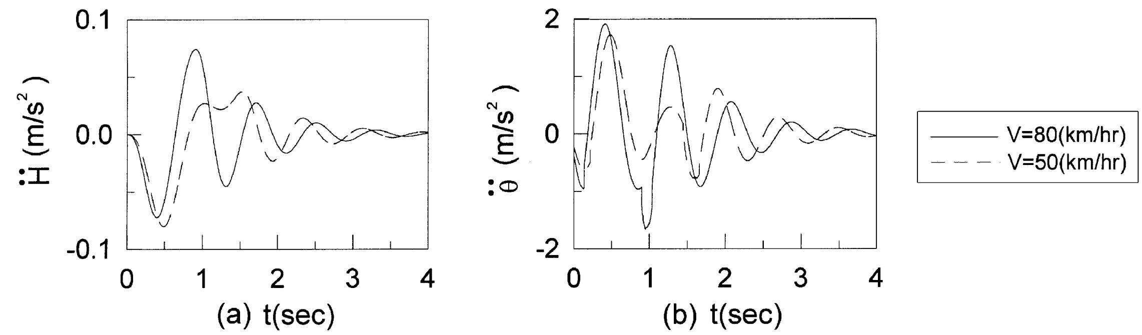

Fig. 7 Comparisons of two values on the histories of (a)$\ddot{H}$ and (b)$\ddot{\theta}$ of a two-axle vehicle (M=1500kg, J=507.8 kg-m^2, K=20 kN/m, D=800 N-s/m, e=1 m) traversing on the beam (L=20 m, b=0.5 m, h=0.34 m)

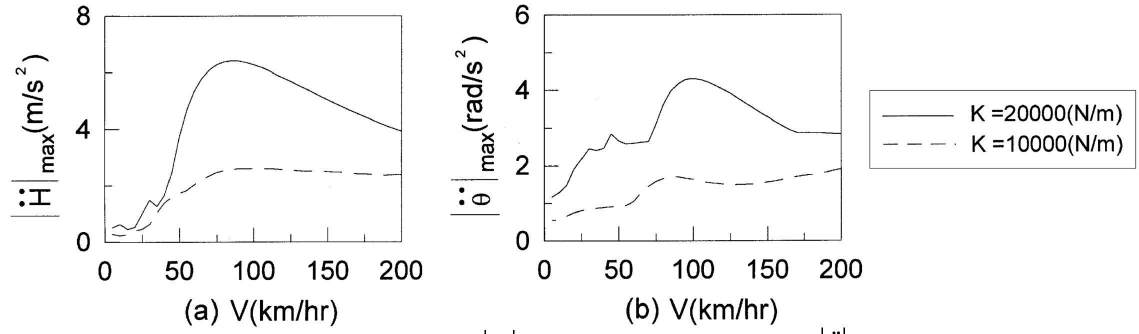

Fig. 8 Comparisons of two K values on (a) $|\ddot{H}|_{\max}-v$ distribution and (b) $|\ddot{\theta}|_{\max}-v$ distribution of a two-axle vehicle (M=1500 kg, J=507.8 kg-m^2, D=800 Ns/m, e=1 m) traversing on the beam (L=20 m, b=0.5 m , h=0.34 m)

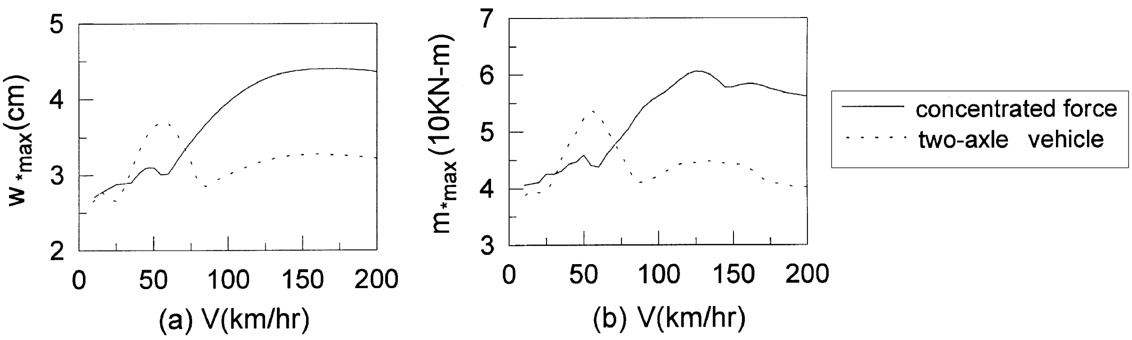

Fig. 9 Comparisons of two load types on (a) $w_{*\max}$ distribution and (b)$m_{*\max}$ distribution of the beam (L=20 m, b=0.5 m, h=0.34 m) due to a concentrated force and a two-axle vehicle (M=1500 kg, J=507.8 kg-m^2, D=800N-s/m, e=1 m)

wheel of the vehicle is leaving the beam.

The effects of two different magnitudes of spring constants of suspension (D=800 N-s/m, e=1 m) on $|\ddot{H}|_{\max}-v$ and $|\ddot{\theta}|_{\max}-v$ distributions of the vehicle are displayed in Figs. 8(a) and 8(b), respectively. A stiffer suspension leads to both larger $|\ddot{H}|_{\max}-v$ and $|\ddot{\theta}|_{\max}-v$. Two different magnitudes of critical velocities of the vehicle exist at which $|\ddot{H}|_{\max}$ and $|\ddot{\theta}|_{\max}$ become absolute maximum, respectively. Moreover, a stiffer suspension leads to more obvious critical velocities of the vehicle.

The comparisons of a concentrated moving force (P_o=1500 kg) and a moving two-axle vehicle (K=20 kN, D=800 N-s/m, e=1 m) on $w_{*\max}-v$ and $m_{*\max}-v$ distributions of the Timoshenko beam are displayed in Figs. 9(a) and 9(b), respectively. Usually, the interaction between the moving vehicle and the beam causes both $w_{*\max}-v$ and $m_{*\max}-v$ of the beam to be less than those by a moving concentrated force. However, the resonance of vehicle and beam at a specific velocity will cause $w_{*\max}$ and $m_{*\max}$ of the beam to be greater than those by a moving concentrated force. Both figures show that the system of beam and two-axle vehicle is in the state of resonance at the velocity approximates 50 km/hr.

7. Conclusions

Based on the present analysis for the nonlinear vibration of the Timoshenko beam due to moving loads, the following conclusions can be made: (1) the effect of weight stiffens the beam; (2) the effect of axial inertia softens the beam; (3) a rapidly moving force of large magnitude induces a small value of fundamental period of the beam; (4) the effect of axial inertia on the deflection is slight for a thick and short beam; (5) the effect of axial inertia on the moment cannot be neglected for a long and thin beam; (6) a critical velocity exists at which the vertical acceleration of vehicle becomes absolutely large and; (7) another critical velocity exists at which the angular acceleration of vehicle becomes absolutely large.

Acknowledgments

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