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Methods of punching shear strength analysis of reinforced concrete flat plates-A comparative study

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Summary. The punching shear strength of concrete flat plates is one of the topics of intensive research in recent years by various concrete structures researchers. This paper reviews four current methods of analysing the punching shear strength at the corner-and edge-column positions of reinforced concrete flat plates. They include those recommended in the Australian Standard AS3600-1988, the American Concrete Institute ACI318-89 and the British Standard on Concrete Practices (BS8110) as well as the approach developed at the University of Wollongong, Australia. Based on half-scale model test results, a comparative study of these four analysis methods is made with regard to their limitation, accuracy and reliablility. It is found that the Wollongong approach in general gives the best performance in predicting the punching shear strength of flat plates with torsion strips and those with spandrel beams. The Australian Standard procedure performs just as satisfactorily for flat plates with torsion strips but tends to be unsafe for those with spandrel beams. Both the ACI and the British methods are applicable only to flat plates with torsion strips; they also tend to give unsafe predictions for the punching shear strength.

Key words: columns (supports); connections (mechanisms to transfer stress); failure; concrete flat plates; models; punching shear; reinforced concrete; shear strength; spandrel beams; strength analysis; torsion strips.

1. Introduction

1.1. General remarks

Concrete flat plates are slab structures without drop panels and capitals at the column and slab connections. From an aesthetic and economic point of view, the flat plate structure has an edge over other slab systems due to the significant savings in construction costs and a relatively pleasing appearance. In addition, the elimination of beams and girders reduces the centre-to-centre floor spacing of multistorey buildings thus creating more storeys per given building height. For these reasons, flat plates are widely used for multistorey structures such as office

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buildings and car-parks. To enhance the structural performance of reinforced concrete flat plates, spandrel beams are normally constructed along the free edges of the slab. A typical layout of a flat plate floor with spandrel beams is shown in Fig. 1.

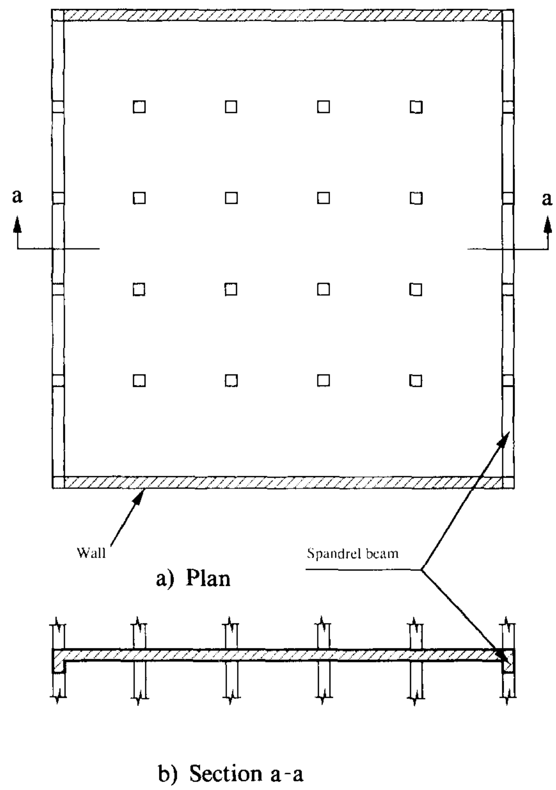


Fig. 1 Typical flat plate with spandrel beams

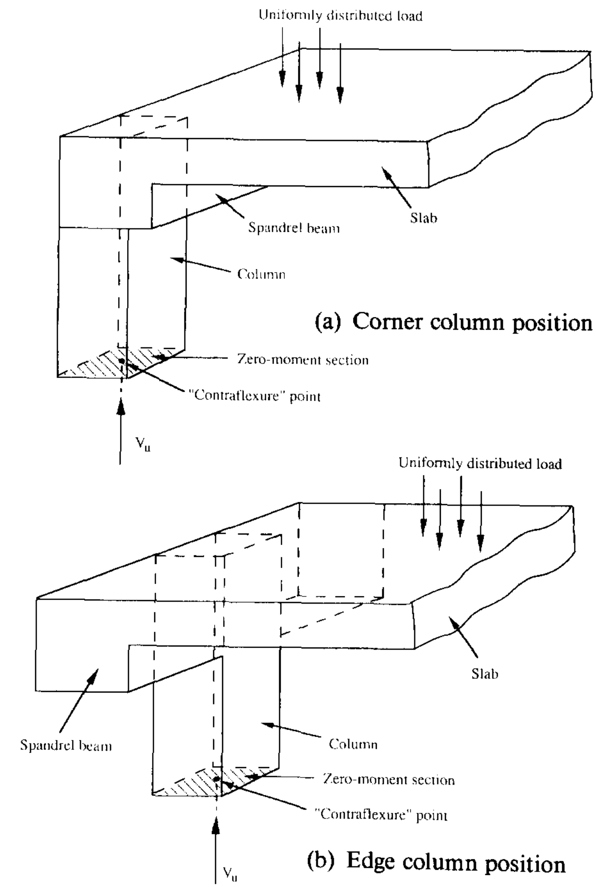


Fig. 2 Slab-column-spandrel connection

Figs. 2(a) and (b) show typical slab-column-spandrel beam connections at the corner-and edge-column positions respectively. The punching shear strength, V_u, is defined as the ultimate vertical reaction at the column contraflexure point. A spandrel beam having an overall depth equal to that of the slab is referred to as the torsion strip. The existence of the spandrel beam at the slab-column connections tends to make the already difficult analysis of V_u more complex.

1.2. Objectives and scope

Many major national codes of practice for concrete structures include recommendations for the analysis of punching shear strength of concrete flat plates. However, there appears to be no published material available on the relative accuracy and reliability of these methods of analysis. The relative merits or otherwise of the various methods are important information for the practising engineers involved in the design and construction of concrete flat plate structures in geographical regions covered by these codes.

A brief review is given herein of the analytical method recommended in

- the Australian Standard AS 3600-1988: Concrete Structures (Standard Association of Australian, SAA 1988);
- the American Concrete Institute, Building Code Requirements for Reinforced Concrete Structures ACI 318-89 and Commentary-ACI 318R-89, (ACI 1989);
- the British Standard BS 8110: Part 1, "Code of Practice for Design and Construction", (British Standards Institute, BSI 1985).

In addition, an analytical method recently developed by Loo and Falamaki (1992) at the University of Wollongong, Australia (hereinafter referred to as the Wollongong approach) is also discussed.

Based on the half-scale model test results published by Rangan and Hall (1983a), Rangan (1990b) and Falamaki and Loo (1992), the relative accuracy and reliability of all four methods of analysis are investigated. The comparison is carried out by plotting the experimental punching shear strength values against the predicted values due to each of the four methods, where applicable. Pearson correlation coefficients are also obtained to illustrate the strength of the relationship between each prediction method and the experimental values. The comparative study provides considerable insight into the relative merits or otherwise of the four methods of analysis.

2. Methods of analysis-A review

Of the four methods included in this study, the Australian Standard procedure and the Wollongong approach cover reinforced concrete flat plates with spandrel beams or torsion strips whereas the ACI and the British methods are limited to cases with torsion strips only. These methods are briefly described in the following sections.

2.1. Australian Standard procedure (SAA 1988)

The Australian Standard procedure is mainly the work of Rangan (1990a). It is based heavily on the experimental study conducted by Rangan and Hall (1983a) on half-scale models of the edge panels of flat plate floors. According to Rangan (1987), the equations developed are deemed to consider the following cases:

(a) a slab in the vicinity of an interior column, without spandrel beams and closed ties;
(b) a slab in the vicinity of an edge column, with a spandrel beam on the side provided with minimum closed ties;
(c) a heavily loaded slab in the vicinity of an edge column, with a spandrel beam as in (b) above but having more than the minimum closed ties;
(d) a slab in the vicinity of an edge column, without spandrel beam and closed ties, i.e. a free edge.

The analysis procedure for an edge-column connection is assumed valid for a corner-column connection (Rangan 1987).

When the moment to be transferred is zero, the equation for punching shear only incorporates four parameters, namely the critical perimeter, the effective depth of slab, the column dimensions and the compressive strength of concrete. The critical shear perimeter is defined as the (failure) section half the effective depth away from the faces of the column.

The punching shear strength where there is no moment transfer is given as

$$V_{uo} = udf_{cv} \tag{1}$$

where u is the critical shear perimeter, d is the effective depth of the slab and f_{cv} is the limiting concrete shear stress for which relevant formulas are given in the Australian Standard.

When there is moment transfer to the column, in addition to the abovementioned parameters, the equation also consider the total unbalanced moment, and the shear force to be transferred to the column support.

In the presence of spandrel beam and closed ties, other parameters taken into account include the dimensions and yield stength of closed ties, as well as the area and perimeter defined by the longitudinal bars at the four corners of the closed ties.

For a typical slab with moment transfer to the column, and having a spandrel beam with the minimum requirement of closed ties, the punching shear strength is given as

$$V_u = \frac{1.2\,(\frac{D_b}{D_s})\,V_{uo}}{\left[1 + \frac{M_v^* u}{2V^* ab_w}\right]} \tag{2}$$

where (D_b/D_s) is the ratio of the depths of the beam and the slab; M_v^* and V^* are respectively the design bending moment and shear force at the column; a is the segment length of the critical shear perimeter parallel to the vector direction of M_v^* and b_w is the width of the spandrel beam.

This approach has been derived based on two assumptions (Rangan 1987):

(a) the distribution of the shear stress is taken to be uniform along the critical perimeter;

(b) for both the edge-column and corner-column positions, a constant slab restraining factor of 4 is adopted for the slab-column connections with spandrel beams or torsion strips with closed ties.

Some shortcomings of these assumptions have been identified and discussed by Loo and Falamaki (1992). For further details of this method, the referred to the Australian Standard (SAA 1988).

2.2. The ACI method (ACI 1989)

The method recommended in the ACI code for punching shear design under combined shear and moment is mainly based on the work of Di Stasio and Van Buren (1960). In this method, the moment is assumed to be transferred to the column by flexure of the slab and by torsion of the slab edge strip or torsion strip (Cope and Clark 1984). Also, the moment is transferred along a critical section around the column, similar to the Australian Standard procedure, over which the punching shear failure is assumed to occur.

In the presence of an unbalanced moment, consideration is given to the effect of torsion by applying an analogous polar moment of inertia of the critical section, and a calculated fraction of the net moment to be transferred to the column by this torsion. The maximum shear stress at the critical section is expressed as

$$v^*_{max} = \frac{V^*}{A} + \frac{\gamma_v M^*_v y}{J} \tag{3}$$

where A is the area of the critical section, J is the analogous polar moment of inertia, $\gamma_v M^*_v$ is the portion of the unbalanced moment transferred to the column and y is the distance from the centroid of the critical section to the point where v^*_{max} acts.

Note that in this approach, the effects of the torsion strips with or without closed ties are taken into consideration but those of the spandrel beams cannot be included. Furthermore, according to ACI-ASCE Committee 426(1977), the predicted results are safe where the ratio of the long and short sides of a rectangular column or loaded area is higher than 2.0.

In this simplistic approach, the main parameters are the compressive strength of concrete and the dimensions of the critical perimeter. It does not take into account other factors such as the slab restraining effect and the transverse strength of the torsion strip or the spandrel beam.

2.3. The British Standard method (BSI 1985)

The general approach in determining the punching shear strength recommended in BS8110:1985 is similar to the previous British Concrete Code CP 110:1972 (BSI 1972). The values for the design nominal shear stresses in CP110 are based on the recommendations made by the Institution of Structural Engineers Shear Study Group (1969). As in the Australian Standard, it also assumes that the distribution of shear around the column is uniform, although this is not the case in general (Loo and Falamaki 1992).

According to this method, the punching shear stress is given as

$$V_c = C\sqrt[3]{\rho f_c'}\ \sqrt[4]{\frac{400}{d}} \tag{4}$$

where ρ is the percentage of tension steel of the slab in the direction perpendicular to the edge.

In contrast to the Australian and the ACI codes, BS8110 takes the shear failure perimeter to be a rectangle with its sides at 1.5 times the average effective depth from the column faces. The parameters included in calculating the punching shear are the steel ratio of slab, effective depth and cube root of the compressive strength of concrete (where the concrete strength must exceed 25 N/mm^2). Further allowance is made in the form of a reduction factor of 1.25 to be applied to the calculation in determining the punching shear strength.

As in the ACI method, the effects of spandrel beam on the punching shear strength cannot be accounted for in the BS8110 procedure. There are other limitations:

(a) compressive strength of concrete not to exceed $40N/mm^2$;
(b) effective depth of slab not to exceed $400mm$;
(c) main steel ratio of the slab not to exceed 3%.

Similar to the ACI approach, the British method is also considered simplistic in that factors such as the restraining factor of the slab and the transverse strength of the torsion strip are not taken into consideration.

2.4. The Wollongong approach (Loo and Falamaki 1992)

The Wollongong approach is the result of a comprehensive research into the punching shear strength of reinforced concrete flat plates. Applicable to flat plates with spandrel beams or torsion strips, the approach comprises explicit formulas through which the punching shear strength at the corner- and edge-column connections can be determined.

A detailed presentation of the method has been made elsewhere (Loo and Falamaki 1992) and it is not repeated herein. However, it may be noted that the Wollongong approach takes full account of the following important parameters and effects that were neglected or left out in the other three methods:

(a) The slab restraining factor is not a constant but a function of the strength of the spandrel beam.
(b) The distribution of shear force around the critical perimeter is not uniform but a function of the slab reinforcement ratio, clear span of the slab, and the size of the spandrel beam in relation to the slab.
(c) The interaction of torsion, shear and moment in the spandrel beam.
(d) Flat plates with deep spandrels have a bending mode of failure very much different from those with shallow beams and they are carefully defined.

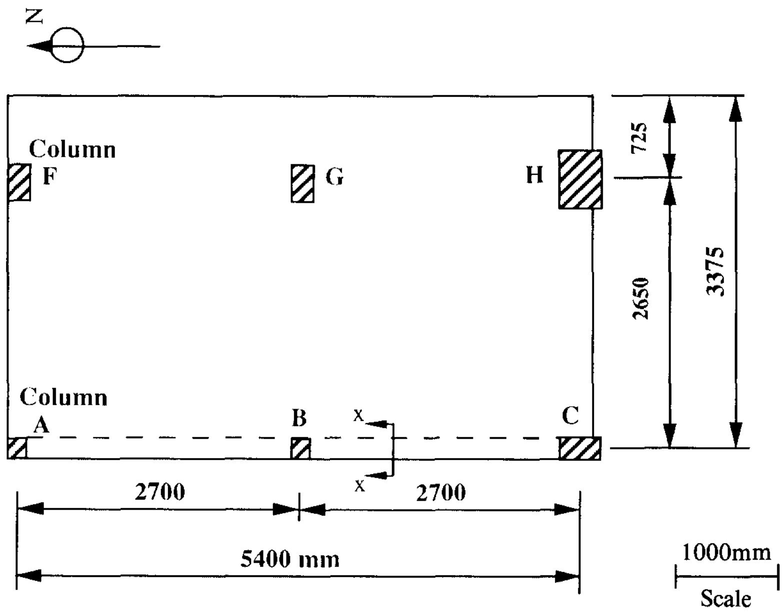
3. Presentation and comparison of results

3.1. Model tests

As part of a comprehensive study of the punching shear strength of concrete flat plates, a total of nine half-scale reinforced concrete models were constructed and tested to failure over a period of three years (from 1987 to 1990). A test program of such a scale is not believed to have been attempted elsewhere previously. Full details of this experimental work and its results have been reported by Falamaki and Loo (1992). For ease of reference, the plan dimensions of the models and a typical cross section of the slab-column-beam connection are presented in Fig. 3.

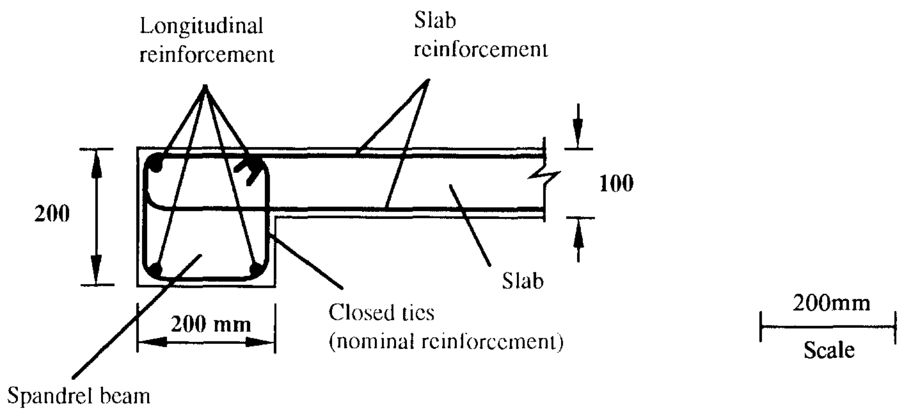
The test to failure of these nine models, each weighing about five tonnes, provides a total of 24 punching shear strength results, including 16 for corner-column and 8 for edge-column positions. Also included in the present study are the punching shear strength results from two similar models tested by Rangan and Hall (1983a) and from four single edge-column specimens tested by Rangan (1990b). These make a total of 16 results for corner-column connections and 14 for edge-column connections.

These test results are tabulated in column 3 of Tables 1 and 2. Note that the Wollongong results are those prefixed with the letter W or M and those of Rangan and associates, with an R. Note also that the four results R90-A, B, C and D are for single column specimens. Out of these 30 results, 11 are for flat plates with torsion strips while the rest are for those with spandrel beams of various sizes.



Note: Columns A, C and F are corner columns, whereas column B is an edge column.
Columns G and H are simulated as interior columns.

(a) Plan



(b) Section x-x

Fig. 3 Flat plate model W1 with a typical spandrel beam section details

Table 1 Comparison of punching shear results-models with torsion strips

Column	Type*	Measured V_u	AS 3600-1988		ACI318-89		BS 8110:1985		Wollongong	
			Predicted V_u.	Measured / Predicted	Predicted V_u.	Measured / Predicted	Predicted V_u.	Measured / Predicted	Predicted V_u.	Measured / Predicted
W5-A	C/NS	32.59	37.01	0.88	57.50	0.57	39.54	0.82	35.21	0.93
M5-A	C/NS	34.42	40.70	0.85	69.95	0.49	58.09	0.59	29.37	1.17
W5-B	E/NS	71.24	53.04	1.34	91.18	0.78	76.83	0.93	74.62	0.95
M5-B	E/NS	87.81	49.79	1.76	144.23	0.61	156.90	0.56	94.21	0.93
W5-C	C/NS	37.30	43.66	0.85	80.22	0.46	51.13	0.73	35.07	1.06
M5-C	C/NS	38.36	42.82	0.89	90.23	0.43	71.03	0.54	53.91	0.71
R3-A	E/NS	88.76	87.23	1.02	130.17	0.68	149.09	0.60	94.90	0.94
R4-A	E/NS	75.42	85.03	0.89	103.84	0.73	110.00	0.69	83.73	0.90
R90-A	E/NS	120.20	114.95	1.05	108.02	1.11	112.93	1.06	76.60	1.57
R90-B	E/NS	108.20	148.35	0.73	136.38	0.79	123.08	0.87	144.24	0.75
R90-C	E/NS	21.90	24.12	0.91	58.58	0.37	51.18	0.43	23.00	0.95
			Mean:	1.015		0.638		0.711		0.987

Note: * C-corner column ; E-edge column
NS-without spandrel beam
All measured and predicted V_u values shown are in kN.

Table 2 Comparison of punching shear results-models with spandrel beams

Column	Type*	Measured V_u	AS 3600-1988		Wollongong	
			Predicted V_u.	Measured / Predicted	Predicted V_u.	Measured / Predicted
R90-D	E/S	36.20 $\frac{1}{1}$	51.71	0.70	45.89	0.79
W1-A	C/S	50.15	119.07	0.42	55.05	0.91
W2-A	C/S	48.08	120.94	0.40	58.23	0.83
W3-A	C/S	43.38	70.42	0.62	42.91	1.01
W4-A	C/S	47.07	96.12	0.49	55.85	0.84
W1-B	E/S	117.63	146.92	0.80	148.53	0.79
W2-B	E/S	120.36	150.05	0.80	105.87	1.14
W3-B	E/S	93.57	94.99	0.98	115.76	0.81
W2-C	C/S	45.17	113.54	0.40	44.44	1.02
W3-C	C/S	44.33	73.38	0.60	38.36	1.16
W4-C	C/S	46.32	82.93	0.56	49.55	0.93
M2-A	C/S	53.90	82.90	0.65	35.36	1.52
M3-A	C/S	25.70	127.31	0.20	10.59	2.42
M4-A	C/S	58.97	114.77	0.51	46.64	1.26
M2-B	E/S	123.22	116.24	1.06	98.15	1.26
M3-B	E/S	76.50	214.32	0.36	78.07	0.98
M4-B	E/S	130.24	137.82	0.95	97.18	1.34
M3-C	C/S	24.30	131.89	0.18	32.90	0.74
M4-C	C/S	60.09	102.75	0.58	58.58	1.03
			Mean:	0.593		1.027

Note: * C-corner column ; E-edge column ; S-with spandrel beam
$\frac{1}{1}$ This specimen did not fail in punching; the reported result is the maximum value.
All measured and predicted V_u values shown are in kN.

3.2. Predicted results

The four methods of analysis reviewed herein, namely the Australian Standard procedure, the ACI method, the British Standard recommendations and the Wollongong approach, are used to compute the punching shear stength of the slab-column and slab-column-beam connections of the models. Remember that the British and the ACI methods are applicable to flat plates with torsion strips only.

Table 1 presents the predicted V_u values by all the four methods for the models with torsion strips; Table 2 contains the predicted results by the Australian Standard procedure and the Wollongong approach for the models with spandrel beams. Also included in the tables are the ratios of the measured and predicted results.

In Fig. 4, the measured V_u are plotted against the predicted values for the models with torsion strips. The corresponding correlation plots for the models with spandrel beams are given in Fig. 5. Superimposed on these figures are the 30-percentile lines. In these plots, a correlation point falling below the 45° equality line represents an overestimate or unsafe prediction. Further, a point falling below the –30% line means that the analysis method has overestimated the strength by more than 30%. The –30% line may be taken as a safety criterion to reflect the Australian Standard stipulation of a shear capacity reduction factor of 0.7.

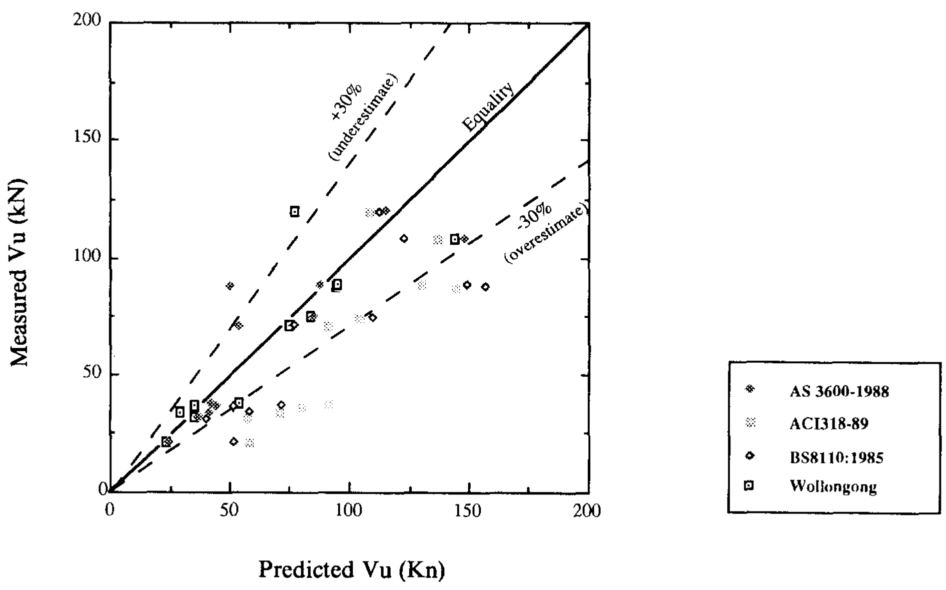


Fig. 4 Measured V_u versus predicted V_u-models with torsion strips only

The Pearson correlation coefficients (Nie, Hull and Bent 1975) for the correlation plots in Fig. 4 and 5 are summarised in Tables 3 and 4 respectively. Note that the closer the correlation coefficient is to unity the stronger is the relationship between the measured and the predicted results.

3.3. Comparison: flat plates with torsion strips

In Table 1 for models with torsion strips, it can be seen that with a mean ratio of the measured and predicted V_u very close to unity, either of the two Australian methods is superior to the ACI and British methods. The ACI method, on average, overestimates the punching shear strength by 56.7% while the British approach overestimates by 40.6%.

It is apparent in Fig. 4 that none of the predictions by the Wollongong approach or by the Australian Standard procedure falls below the –30% line and each has only one point located above the +30% line. Contrary to these, the ACI procedure has 7 out of 11 correlation points falling below the –30% line and the British method has 6 out of 11.

A survey of the Pearson correlation coefficients in Table 3 leads to the conclusion that the strength of the relationship between each of the four prediction method is almost as high as the other three. This means that even the ACI or the British Standard procedure may be safely used with an appropriate correction factor (<1) being applied to the predicted results.

Tabel 3 Pearson correlation coefficients for models with torsion strips only

Methods of Analysis	Correlation coefficient
AS 3600-1988	0.868
ACI318-89	0.849
BS 8110:1985	0.838
Wollongong	0.860

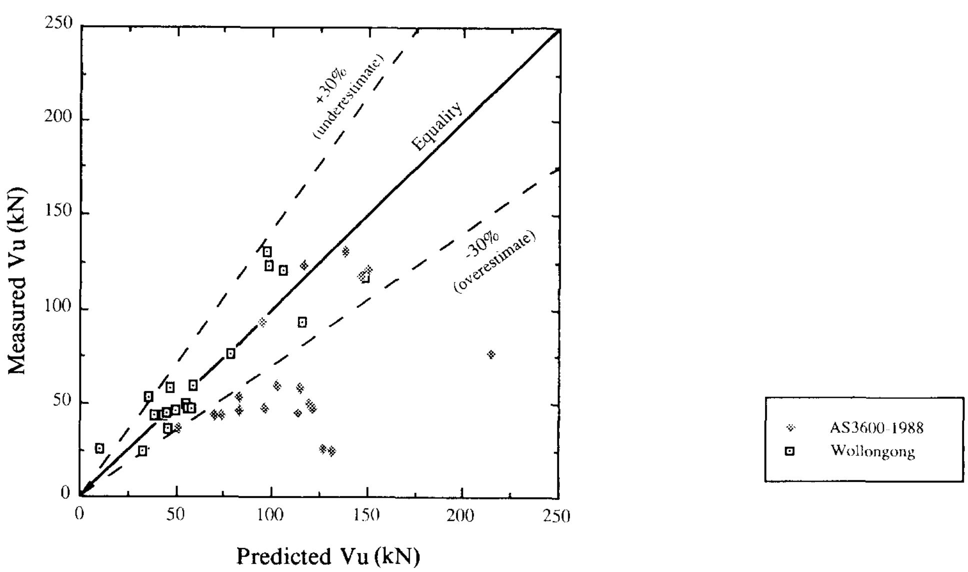


Fig. 5 Measured V_u versus predicted V_u-models with spandrel beams

3.4. Comparison: flat plates with spandrel beams

For the Wollongong approach, the mean ratio of the measured and predicted V_u given in Table 2 is 1.027. This is far more satisfactory than the Australian Standard procedure which has a mean ratio of 0.593 indicating that, on average, the procedure overestimates the punching shear strength of flat plates with spandrel beams by 68.6%. The superiority of the Wollongong approach is confirmed in Fig. 5 where all the correlation points except one, lie within the ± 30 percent lines; the exception is a conservative prediction. For the Australian Standard procedure, out of 19 correlation points, 13 are below the -30% line. This confirms the unsafe tendency of its predictions.

It is obvious in Table 4 that the Wollongong approach correlate well with the test results. The Pearson correlation coefficient of 0.891 is much higher than that for the Australian Standard procedure (0.434). Falamaki and Loo (1991) have observed that the Australian Standard procedure was developed without adequate support of model test results. In particular, none of the test results quoted by Rangan (1990a) was for models with spandrel beams. This may be the reason for the poor performance by the Australian Standard procedure.

Table 4 Pearson correlation coefficients for models with spandrel beams only

Methods of Analysis	Correlation coefficient
AS 3600-1988	0.434
Wollongong	0.891

4. Conclusions

For the punching shear strength analysis of flat plates at the corner- and edge-column positions, the Wollongong approach is found to be accurate and reliable. The performance is consistent for flat plates with spandrel beams and with torsion strips. The Australian Standard procedure on the other hand gives accurate and reliable predictions for flat plates with torsion strips but at times gravely overestimates the strength of flat plates with spandrel beams.

Both the ACI and the British Standard methods are applicable only to flat plates with torsion strips. Further, they tend to give unsafe predictions. On average, the ACI prediction is 1.57 times the measured strength and the corresponding value for the British Standard method is 1.41 times.

Acknowledgements

This study forms an integral part of a comprehensive research into the punching shear strength of concrete flat plates with spandrel beams. The authors are grateful to the Australian Research Council and the University of Wollongong for the financial support, without which the said research would not have been carried out.

Notation

A = cross sectional area of the critical shear section

a = segment length of the critical shear perimeter
b_w = width of the spandrel beam
D_b = depth of the spandrel beam
D_s = depth of the slab
d = effective depth of the slab
f_c = characteristic compressive strength of concrete at 28 days
f_{cv} = limiting concrete shear stress
J = analogous polar moment of inertia
M^*_v = design bending moment at the column
u = critical shear perimeter around the column
v^*_{max} = maximum shear stress at the critical section
V^* = design shear force at the column
V_u = punching shear strength of a slab
V_{uo} = ultimate shear strength of a slab without moment transfer
y = distance from centroid of critical section to the point of maximum shear stress
γ_V = ratio of unbalanced moment transferred to the column
ρ = tension steel ratio

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