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Stochastic finite element analysis of structural systems with partially restrained connections subjected to seismic loads

Özlem Çavdar[1]*, Alemdar Bayraktar[2†], Ahmet Çavdar[1‡] and Murat Emre Kartal[3‡†]

[1]*Gumushane University, Department of Civil Engineering, 29000, Gumushane, Turkey*
[2]*Karadeniz Technical University, Department of Civil Engineering, 61080, Trabzon, Turkey*
[3]*Zonguldak Karaelmas University, Department of Civil Engineering, 67100, Zonguldak, Turkey*

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Abstract. The present paper investigates the stochastic seismic responses of steel structure systems with Partially Restrained (PR) connections by using Perturbation based Stochastic Finite Element (PSFEM) method. A stiffness matrix formulation of steel systems with PR connections and PSFEM and MCS formulations of structural systems are given. Based on the formulations, a computer program in FORTRAN language has been developed, and stochastic seismic analyses of steel frame and bridge systems have been performed for different types of connections. The connection parameters, material and geometrical properties are assumed to be random variables in the analyses. The Kocaeli earthquake occurred in 1999 is considered as a ground motion. The connection parameters, material and geometrical properties are considered to be random variables. The efficiency and accuracy of the proposed SFEM algorithm are validated by comparison with results of Monte Carlo simulation (MCS) method.

Keywords : perturbation based stochastic finite element method; steel structures; PR connections; stiffness matrix; Monte Carlo simulation; stochastic dynamic analysis.

1. Introduction

Traditionally, approaches to structural design assume that beam-to-column joints are either fully rigid or ideally pinned. Dynamic behavior of steel structure systems with PR connections may be significantly different from rigidly connected steel structures, particularly under strong earthquake ground excitations. Therefore, the conventional methods of analysis and design of steel structures with idealized connections (fully rigid or pinned) are inadequate, as these often cannot represent real structural behavior. There has been considerable (Kishi and Chen 1990, Ang and Morris 1984, Sekulovic, *et al*. 2002, Suarez, *et al*. 1996, Cabrero and Bayo 2005, Chen, *et al*. 1996, Liu and Xu 2005, Nader and Astaneh-Asl 1991) works on modeling the behavior of PR frames under both static and dynamic loads. However, it is essential to include uncertainties in initial connection stiffness, material and geometrical properties when investigating the seismic response of steel frames with PR connections.

Very few researchers (Gao and Haldar 1995, Sakurai, *et al*. 2001, Hadianfard and Razani 2003) studied

*Corresponding Author, Email: ozlem_cavdar@hotmail.com
†E-mail: alemdar@ktu.edu.tr
‡E-mail: ahmcavdar@hotmail.com
‡†E-mail: murat_emre_kartal@hotmail.com

stochastic static analysis of steel structures with PR connections. Gao and Haldar (1995) proposed an efficient nonlinear stochastic finite-element method for the design of frame structures, specifically applicable to steel structures. The material properties, geometry, connections parameters and external loads are considered to be random variables. Sakurai, *et al*. (2001) investigated probabilistic analysis of steel frames with type PR connections. The contribution of variability to the uncertainty in frame deformation is examined using Pertubation based Stochastic Finite Element Method (PSFEM) to compare the effect of initial connection stiffness to that of bending stiffness of the beams and columns. Hadianfard and Razani (2003) studied the effects of semi-rigid behavior of the connections in the finite element analysis and in the reliability analysis of steel frames.

This paper presents the effect of the variation in material and geometrical properties on stochastic seismic responses of structural systems modeled with PR beam-to-column connections by using PSFEM and Monte Carlo Simulation (MCS) methods. A computer program for PR connections was developed in FORTRAN language and incorporated into a general-purpose computer program (Kleiber and Hien 1992, Çavdar, *et al*. 2008) for dynamic deterministic and stochastic analysis of medium and large-scale three-dimensional frames. This program is modified for the stochastic dynamic analysis of steel structure systems including PR connections based on the PSFEM. This program is combined to MCS method. The analysis results obtained from both two methods are compared with each other.

2. Formulation

In this section, stiffness matrix formulation of structural systems with PR connections and dynamic analysis formulation of PSFEM are given.

2.1 Stiffness matrix formulation of structural systems with PR connections

Structural elements and joints are modelled considering some idealizations. The joints of idealized frame elements are assumed to be constituted by ideally rigid connections. However, another assumption is that structural members of frame systems have ideally pinned connection at joints. Actually, structural connections should be named according to their moment-rotation curves. These curves are usually derived

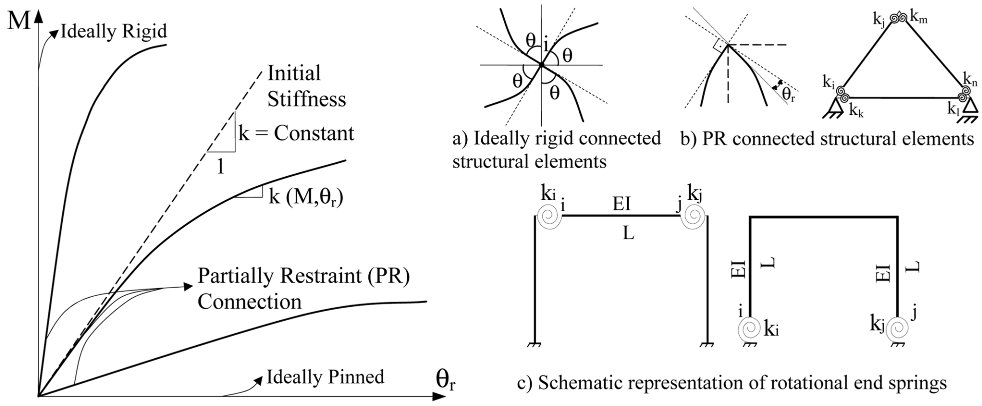


Fig. 1. Structural connections

by fitting suitable curves to the experimental data. Various types of M-θ_r models have been developed as described by Chen and Lui (1991). As seen from M-θ_r curves given in Fig. 1, the moment (M) is depended on a function of relative rotation between structural members connected to the same joint.

Connection flexibility is defined by various methods. Linear approximation and initial modulus is used for partially restraint connections in this study. To obtain an initial opinion on stiffness of rotational springs, using the modulus of elasticity (E), moment of inertia (I) and length (L) of related beam with constant cross-section is very effective and understandable approach. Stiffness matrix of a beam in local coordinates can be written using these attributes as follows (McGuire, *et al*. 1999, Çavdar 2009).

$$[K] = \begin{bmatrix} [k_{11}] & [k_{12}] \\ [k_{21}] & [k_{22}] \end{bmatrix} \tag{1}$$

$$[k_{11}] = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI_z}{L^3}\frac{U_{1z}}{D_z} & 0 & 0 & 0 & \frac{6EI_z}{L^2}\frac{U_{2z}}{D_z} \\ 0 & 0 & \frac{12EI_y}{L^3}\frac{U_{1y}}{D_y} & 0 & -\frac{6EI_y}{L^2}\frac{U_{2y}}{D_y} & 0 \\ 0 & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & 0 & -\frac{6EI_y}{L^2}\frac{U_{2y}}{D_y} & 0 & \frac{4EI_y}{L}\frac{U_{3y}}{D_y} & 0 \\ 0 & \frac{6EI_z}{L^2}\frac{U_{2z}}{D_z} & 0 & 0 & 0 & \frac{4EI_z}{L}\frac{U_{3z}}{D_z} \end{bmatrix} \tag{2}$$

$$[k_{12}] = \begin{bmatrix} -\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{12EI_z}{L^3}\frac{U_{1z}}{D_z} & 0 & 0 & 0 & \frac{6EI_z}{L^2}\frac{U_{4z}}{D_z} \\ 0 & 0 & -\frac{12EI_y}{L^3}\frac{U_{1y}}{D_y} & 0 & -\frac{6EI_y}{L^2}\frac{U_{4y}}{D_y} & 0 \\ 0 & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\ 0 & 0 & \frac{6EI_y}{L^2}\frac{U_{2y}}{D_y} & 0 & \frac{2EI_y}{L}\frac{U_{5y}}{D_y} & 0 \\ 0 & -\frac{6EI_z}{L^2}\frac{U_{2z}}{D_z} & 0 & 0 & 0 & \frac{2EI_z}{L}\frac{U_{5z}}{D_z} \end{bmatrix} \tag{3}$$

$$[k_{22}] = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI_z}{L^3}\frac{U_{1z}}{D_z} & 0 & 0 & 0 & -\frac{6EI_z}{L^2}\frac{U_{4z}}{D_z} \\ 0 & 0 & \frac{12EI_y}{L^3}\frac{U_{1y}}{D_y} & 0 & \frac{6EI_y}{L^2}\frac{U_{4y}}{D_y} & 0 \\ 0 & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & 0 & \frac{6EI_y}{L^2}\frac{U_{4y}}{D_y} & 0 & \frac{4EI_y}{L}\frac{U_{6y}}{D_y} & 0 \\ 0 & -\frac{6EI_z}{L^2}\frac{U_{4z}}{D_z} & 0 & 0 & 0 & \frac{4EI_z}{L}\frac{U_{6z}}{D_z} \end{bmatrix} \tag{4}$$

$$[k_{21}] = [k_{12}]^T \tag{5}$$

where $U_{1\text{-}6}$, D_y, and D_z are the coefficients and given as follows,

$$\begin{aligned} U_{1y} &= \frac{k_{iy}L}{4EI_y} + \frac{k_{jy}L}{4EI_y} + \frac{1}{4}\frac{k_{iy}k_{jy}L^2}{E^2I_y^2} \\ U_{1z} &= \frac{k_{iz}L}{4EI_z} + \frac{k_{jz}L}{4EI_z} + \frac{1}{4}\frac{k_{iz}k_{jz}L^2}{E^2I_z^2} \end{aligned} \tag{6}$$

$$\begin{aligned} U_{2y} &= \frac{k_{iy}L}{2EI_y}\left(1 + \frac{k_{jy}L}{2EI_y}\right) \\ U_{2z} &= \frac{k_{iz}L}{2EI_z}\left(1 + \frac{k_{jz}L}{2EI_z}\right) \end{aligned} \tag{7}$$

$$\begin{aligned} U_{3y} &= \frac{k_{iy}L}{4EI_y}\left(3 + \frac{k_{jy}L}{EI_y}\right) \\ U_{3z} &= \frac{k_{iz}L}{4EI_z}\left(3 + \frac{k_{jz}L}{EI_z}\right) \end{aligned} \tag{8}$$

$$\begin{aligned} U_{4y} &= \frac{k_{jy}L}{2EI_y}\left(1 + \frac{k_{iy}L}{2EI_y}\right) \\ U_{4z} &= \frac{k_{jz}L}{2EI_z}\left(1 + \frac{k_{iz}L}{2EI_z}\right) \end{aligned} \tag{9}$$

$$U_{5y} = \frac{1}{4}\frac{k_{iy}k_{jy}L^2}{E^2I_y^2}$$

$$U_{5z} = \frac{1}{4}\frac{k_{iz}k_{jz}L^2}{E^2I_z^2} \tag{10}$$

$$U_{6y} = \frac{k_{jy}L}{4EI_y}\left(3 + \frac{k_{iy}L}{EI_y}\right)$$

$$U_{6z} = \frac{k_{jz}L}{4EI_z}\left(3 + \frac{k_{iz}L}{EI_z}\right) \tag{11}$$

$$D_y = 3 + (k_{iy} + k_{jy})\frac{L}{EI_y} + \frac{1}{4}\frac{k_{iy}k_{jy}L^2}{E^2I_y^2}$$

$$D_z = 3 + (k_{iz} + k_{jz})\frac{L}{EI_z} + \frac{1}{4}\frac{k_{iz}k_{jz}L^2}{E^2I_y^2} \tag{12}$$

where y and z is the axes, k_{im} and k_{jm} are the rotational spring stiffness at i and j ends of the beam that varies from 0 in the case of pinned connection to ∞ for the case of fully rigid connection , respectively.

Partially restrained connection, which is considered in this study, may also be identified by connection percentage as follows (Sekulovic, *et al*. 2002, Monforton and Wu 1963).

$$k_{i,j} = \frac{3EI\,\nu_{i,j}}{(1-\nu_{i,j})L} \tag{13}$$

where $\nu_{i,j}$ is the fixity factor, which represents the connection percentage whose values are normalized form 0 to 1.

2.2 Perturbation based Stochastic Finite Element Method (PSFEM)

The perturbation method is the most widely used technique for analyzing uncertain system. The basic idea behind the perturbation method is to express the stiffness and mass matrices and the responses in terms of Taylor series expansion with respect to the parameters centered at the mean values.

A deterministic equation of motion can be written as

$$M_{\alpha\beta}\ddot{q}_\beta + C_{\alpha\beta}\dot{q}_\beta + K_{\alpha\beta}q_\beta = Q_\alpha \tag{14}$$

where $K_{\alpha\beta}$, $M_{\alpha\beta}$, $C_{\alpha\beta}$ denote the stiffness matrix, mass matrix and damping matrix, $\ddot{q}_\beta$, $\dot{q}_\beta$, q_β denote the acceleration, velocity, displacement, respectively. The stochastic perturbation based approach consists usually of the up to the second order equations obtained starting from the deterministic ones.

The perturbation stochastic finite element equations describing dynamic response of the single

random variable system for the zeroth, first and second order is given below (Kleiber and Hien 1992):
Zeroth-order equation ($\in^0$ terms, one system of N linear simultaneous ordinary differential equations for $q^0_\alpha(b^0_l;\tau)$, $\alpha = 1, 2, \ldots, N$)

$$M^0_{\alpha\beta}(b^0_l)\ddot{q}^0_\beta(b^0_l;\tau) + C^0_{\alpha\beta}(b^0_l)\dot{q}^0_\beta(b^0_l;\tau) + K^0_{\alpha\beta}(b^0_l)q^0_\beta(b^0_l;\tau) = Q^0_\alpha(b^0_l;\tau) \tag{15}$$

First-order equations, rewritten separately for all random variables of the problem ($\in^1$ terms, $\bar{N}$ systems of N linear simultaneous ordinary differential equations for $q^{,\rho}_\alpha(b^0_l;\tau)$, $\rho = 1, 2, \ldots, \bar{N}$, $\alpha = 1, 2, \ldots, N$)

$$\begin{aligned} M^0_{\alpha\beta}(b^0_l)\ddot{q}^{,\rho}_\beta(b^0_l;\tau) + C^0_{\alpha\beta}(b^0_l)\dot{q}^{,\rho}_\beta(b^0_l;\tau) + K^0_{\alpha\beta}(b^0_l)q^{,\rho}_\beta(b^0_l;\tau) &= Q^{,\rho}_\alpha(b^0_l;\tau) \\ -[M^{,\rho}_{\alpha\beta}(b^0_l)\ddot{q}^0_\beta(b^0_l;\tau) + C^{,\rho}_{\alpha\beta}(b^0_l)\dot{q}^0_\beta(b^0_l;\tau) &+ K^{,\rho}_{\alpha\beta}(b^0_l)q^0_\beta(b^0_l;\tau)] \end{aligned} \tag{16}$$

Second-order ($\in^2$ terms, one system of N linear simultaneous ordinary differential equations for $q^2_\alpha(b^0_l;\tau)$, $\alpha = 1, 2, \ldots, N$)

$$\begin{aligned} M^0_{\alpha\beta}(b^0_l)q^{,\rho}_\beta(b^0_l;\tau) + C^0_{\alpha\beta}(b^0_l)q^{,\rho}_\beta(b^0_l;\tau) + K^0_{\alpha\beta}(b^0_l)q^{,\rho}_\beta(b^0_l;\tau) &= \{Q^{,\rho\sigma}_\alpha(b^0_l;\tau) \\ -2[M^{,\rho}_{\alpha\beta}(b^0_l)q^{,\sigma}_\beta(b^0_l;\tau) + C^{,\rho}_{\alpha\beta}(b^0_l)q^{,\sigma}_\beta(b^0_l;\tau) &+ K^{,\rho}_{\alpha\beta}(b^0_l)q^{,\sigma}_\beta(b^0_l;\tau)] \\ -[M^{,\rho\sigma}_{\alpha\beta}(b^0_l)q^0_\beta(b^0_l;\tau) + C^{,\rho\sigma}_{\alpha\beta}(b^0_l)q^0_\beta(b^0_l;\tau) &+ K^{,\rho\sigma}_{\alpha\beta}(b^0_l)q^0_\beta(b^0_l;\tau)]\}Cov(b_r, b_s) \end{aligned} \tag{17}$$

where b^0_ℓ is the vector of nodal random variables, q_α is the vector of nodal displacement-type variables, τ is forward time variable, $\bar{N}$ is the number of nodal random variables. $M^0_{\alpha\beta}$, $C^0_{\alpha\beta}$ and $K^0_{\alpha\beta}$ are system mass matrix, damping matrix and system stiffness matrix, respectively. Q^0_α, q^0_β and $Cov(b_r, b_s)$ are load vector, displacement and the covariance matrix of the nodal random variable, respectively. N is the number of degrees of freedom in the system. $(.)^0$ is zeroth-order quantities, taken at means of random variables, $(.)^{,\rho}$ is first partial derivatives with respect to nodal random variables, $(.)^{,\rho\sigma}$ is second partial derivatives with respect to nodal random variables.
The first two statistical moments for the random fields $b_r(x_k)$, $r = 1,2,\ldots,R$, are defined as (Kleiber and Hien 1992)

$$E[b_r] = b^0_r = \int_{-\infty}^{+\infty} b_r p_1(b_r)db_r \tag{18}$$

$$Cov(b_r, b_s) = S^{rs}_b = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(b_r - b^0_r)(b_s - b^0_s)p_2(b_r, b_s)db_r\, db_s \tag{19}$$

$r,s = 1,2,\ldots,R$

The latter definition can be replaced by

$$S^{rs}_b = \alpha_{b_r}\alpha_{b_s}b^0_r b^0_s \mu_{b_r b_s} \tag{20}$$

with

$$\alpha_{b_r} = \left[\frac{Var(b_r)}{b_r^0}\right]^{1/2} \quad \mu_{b_r b_s} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} b_r b_s p_2(b_r, b_s) db_r db_s \tag{21}$$

where, $E[b_r]$, $Cov(b_r, b_s)$, $Var(b_r)$, $\mu_{b_r b_s}$, α_{b_r}, $p_1(b_r)$ and $p_2(b_r, b_s)$denote the expectation, covariance, variance, correlation functions, the coefficients of variation, probability density function (PDF) and the joint PDF, respectively. R is the random fields, which can represent randomness in the cross sectional area, length of truss and beam members, elastic modulus, and mass density of the material, connection stiffness, etc.

2.3 Monte Carlo Simulation (MCS) method

The Monte Carlo Simulation (MCS) method is a quite versatile mathematical tool capable of handling situations where all other methods fail to succeed; in structural dynamics, it has attracted intense attention only recently following the widespread availability of inexpensive computational systems. The MCS generates a set of random values of X according to its probability distribution function. The set can be written as $X = \{x_1, x_2, ..., x_N\}$, where N is the number of simulation. For each values of X, the stiffness and mass matrices are computed. At the end of N simulations, we have a random set of displacement and stress values, $\{\{q_\beta\}_1, \{q_\beta\}_2, \{q_\beta\}_3, ..., \{q_\beta\}_N\}$, $\{\{\sigma\}_1, \{\sigma\}_2, \{\sigma\}_3, ..., \{\sigma\}_N\}$ *for* X^i. From this finite set of solutions, the expected values of displacement and stress are computed using the following formulas (Melchers 1999):

$$\mu_{\{q\beta\}} = \frac{1}{N}\sum_{i=1}^{N} \{q_\beta\}_i \tag{22}$$

$$\mu_{\{\sigma\}} = \frac{1}{N}\sum_{i=1}^{N} \{\sigma\}_i \tag{23}$$

3. Numerical examples

Based on the above theoretical considerations, a computer program has been developed and stochastic dynamic analysis of a steel frame and steel bridge, as well as different types of connections, has been performed. In the first numerical example (steel frame), flexible connections were located at the intersection of beam and column. In the second model (steel bridge), flexible connections were located at the ends of the beam. Stochastic dynamic properties were investigated with random material and geometrical properties and initial stiffness. The probability density function of random variables is assumed as normal (or Gaussian) distribution. The Mode Superposition Method considering the Wilson-θ algorithm is used for solving the dynamic equilibrium equations. The damping ratio is chosen as 2% in all analyses. YPT330 component of Yarimca station records of 1999 Kocaeli Earthquake (Fig. 2) is utilized as ground motion (PEER 2007). This ground motion continued up 35.0 s is applied to the systems in a horizontal direction. The dynamic responses of the systems are obtained for a time interval of 0.005 s.

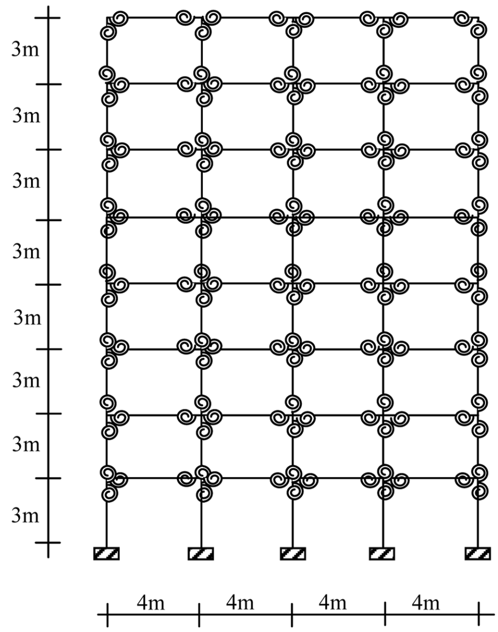


Fig. 2 The layout and dimensions of the steel frame systems

3.1 A steel frame with four stories

Eight-storey and four spans plane steel frame with 24.00 m high and 16.00 m wide subjected to earthquake ground motion has been analyzed (Fig. 2). The geometrical and material properties of this frame are given in Table 1. Two types of semi-rigid beam-to-column connections (respectively stiff and weak connections) with linear moment-rotation relations were considered. For comparison the same frame with rigid and pinned joints were also analyzed.

The steel frame is modeled by 72 stochastic finite elements. Monte Carlo Simulation (MCS) method was simulated for 10000 simulation. Mean of maximum displacements and internal forces are determined according to PSFEM and MCS methods for the steel frame system using Case A, Case B and Case C:

Case A. Elastic module from material properties is chosen as random variable for steel frame system. The other variables are considered as deterministic. This random variable is assumed to follow a normal distribution with the coefficient of variation 0.10 (Melchers 1999). The respective expectation and correlation function and coefficient of variation (Kleiber and Hien 1992) for the elastic modulus E_ρ

Table 1 Sectional properties for the frame system

Member	Elastic modulus (kN/m^2)	Mass Density (kg/m^3)	Cross-section area (m^2)	Inertia moment (m^4)	
	E	ρ	A	I_y	I_z
Column	2.1×10^8	7860	0.0703	2.264×10^{-3}	8.283×10^{-4}
Beam	2.1×10^8	7860	0.0248	1.673×10^{-3}	1.415×10^{-4}

are assumed as follows:

$$E[E_\rho] = 2.1 \times 10^8 \ \lambda = 10$$

$$\mu(E_\rho, E_\sigma) = \exp\left(-\frac{|x_\rho - x_\sigma|}{\lambda l}\right) \quad \rho, \sigma = 1, 2, \ldots, 72$$

$$\alpha = 0.10$$

where x_ρ, l and λ are ordinates of the element midpoints (n random variable, $\rho, \sigma = 1, 2, \ldots, n$), structural member length and decay factor, respectively.

Case B. Cross-sectional area from geometrical properties is chosen as random variable for the steel frame system. The respective expectation and correlation function and coefficient of variation for the cross-sectional areas (A_ρ) are assumed as follows:

$$E[A_\rho] = 0.0248 \ \lambda = 10$$

$$\mu(A_\rho, A_\sigma) = \exp\left(-\frac{|x_\rho - x_\sigma|}{\lambda}\right) \quad \rho, \sigma = 1, 2, \ldots, 72$$

$$\alpha = 0.10$$

Case C. The variation in connection rigidity, *k,* was modeled as random variables for the frame system with partially-restraint connections. The respective expectation and correlation function and coefficient of variation for the connection rigidity k_ρ are assumed as follows:

$$E[k_1] = 1.74 \times 10^4 \qquad \lambda = 10$$

$$E[k_2] = 1.16 \times 10^4 \qquad \lambda = 10$$

$$\mu(E_\rho, E_\sigma) = \exp\left(-\frac{|x_\rho - x_\sigma|}{\lambda l}\right) \qquad \rho, \sigma = 1, 2, \ldots, 72$$

$$\alpha = 0.10$$

3.1.1 Responses of steel frame with PR connections

The steel frame with PR connections is assumed to be subjected to the ground motion shown in Fig. 3. The stochastic dynamic analysis of the frame with various connection types according to random connection stiffness, material and geometrical properties has been carried out. Results of natural frequencies, horizontal displacements along story height of frame as well as bending moments and shear forces at the base of the columns for the various types of connections are presented in tables and figures.

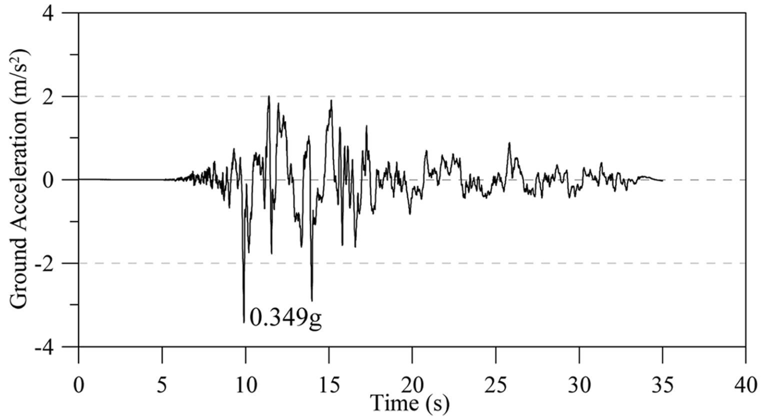


Fig. 3 Acceleration time history of Kocaeli earthquake (YPT330), 1999 (PEER 2007)

3.1.1.1 Natural frequencies

The natural frequencies for the first three modes are determined using the cases of fully rigid, pinned and linear partially restrained connections (relatively stiffness, weak connections) and shown in Table 2 for Cases A-C. As seen from Table 2, the connection flexibility has a significant influence on variation of the natural frequencies particularly on the lower frequencies. This fact can be very important for seismic analysis of steel frame structures, as the lower modes may generally have the principal influence on seismic response of buildings. Also in Table 2, the maximum natural frequency values that are obtained from the random cross section area are greater than those calculated by the random elastic module and connection rigidity.

3.1.1.2 Horizontal displacements

The mean of maximum horizontal displacements along the right border of frame system according to PSFEM and MCS methods are presented in Fig. 4 for Cases A-C. The overall horizontal displacements values obtained from PSFEM are greater than those of the MCS method for all random variables. At the

Table 2 Natural frequencies of the frame system for Cases A-C

Type of connection	Method	Natural frequencies (Hz)								
		Case A			Case B			Case C		
		First mode	Second mode	Third mode	First mode	Second mode	Third mode	First mode	Second mode	Third mode
Rigid (k_0)	PSFEM	1.863	6.075	11.536	1.865	6.086	11.636	1.863	6.075	11.536
	MCS	1.859	6.062	11.515	1.867	6.090	11.565	1.863	6.075	11.536
Relatively Stiffness(k_1)	PSFEM	1.791	5.819	10.984	1.798	5.868	11.132	1.792	5.825	10.984
	MCS	1.787	5.807	10.962	1.795	5.834	11.011	1.790	5.817	10.978
Weak connection(k_2)	PSFEM	1.668	5.392	10.081	1.698	5.481	10.432	1.668	5.395	10.091
	MCS	1.664	5.380	10.061	1.672	5.405	10.106	1.667	5.386	10.070
Pinned	PSFEM	1.040	3.294	5.954	1.047	3.334	5.984	1.040	3.297	5.958
	MCS	1.038	3.290	5.947	1.043	3.303	5.969	1.038	3.286	5.940

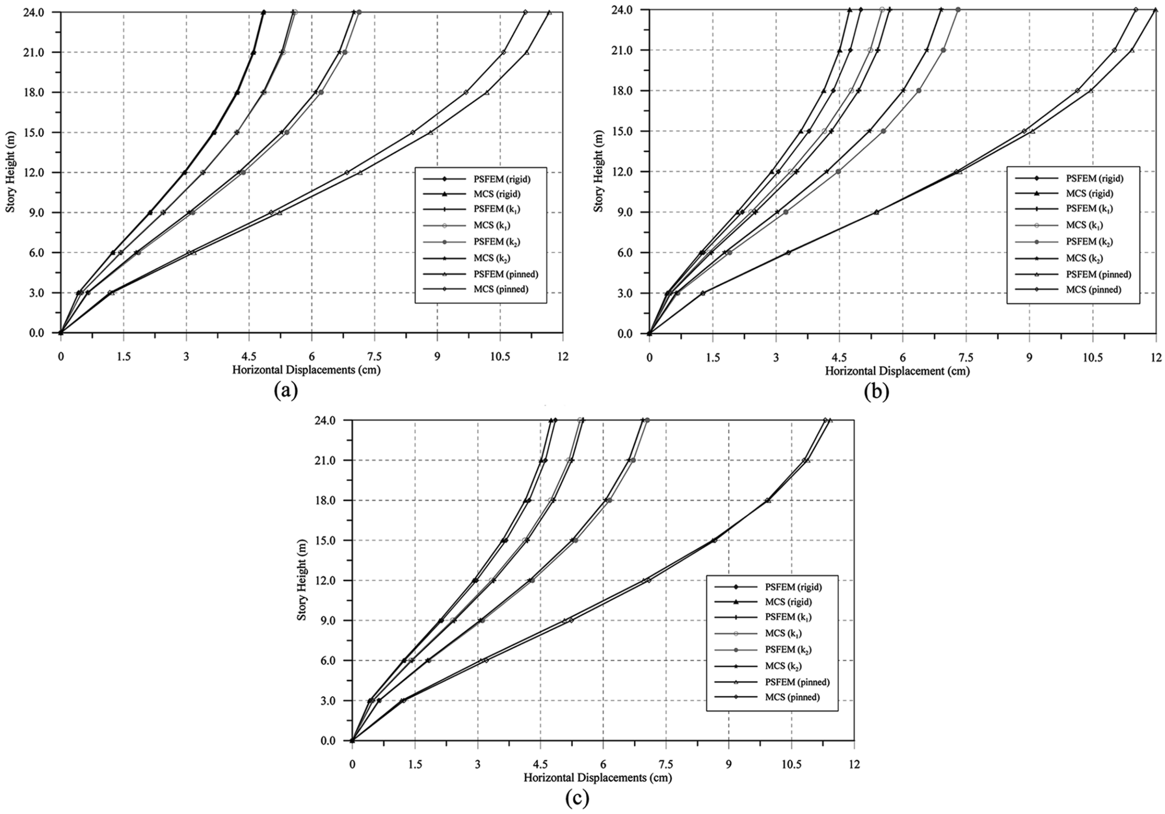


Fig. 4 Mean of maximum horizontal displacement along the story height of the steel frame system with PR connections for Case A (a), Case B (b), Case C (c)

top of frame system where maximum horizontal displacement takes place, it can be observed that the maximum differences between PSFEM and MCS method are 0.72%, 3.75% and 1.47%, respectively for Cases A-C. Also in Fig. 4, the maxima horizontal displacement values that are obtained from the random cross section area are greater than those calculated by the random elastic module and connection rigidity.

It can be seen from Fig. 4 that the frame with PR connections has a greater lateral displacement, when compared with the fully rigid connection. These differences increase with decreasing the connection stiffness. Consequently, the differences in maximum displacement at the top of the frame with rigid joints (k_o) and PR type of joints are 12.53%, 12.42% and 12.33% for relatively stiff connections (k_1) or 31.91%, 32.22% and 31.62% for weak connections (k_2) of PR connections, respectively for Cases A-C.

3.1.1.3 Internal forces

The mean of maximum shear forces at the top joint of columns in every floor for the steel frame system are plotted in Fig. 5 for Cases A-C. The maximum shear forces occur in the columns at second axes for this frame system (Fig. 2). It is seen from Fig. 5 that the steel frame with PR connections has a smaller shear forces (k_1) when compared with the fully rigid connection (k_0). These differences increase with decrease in the connection stiffness. The maximum differences between k_0 and k_2 according to k_1 connections in the shear forces at the base of the frame with rigid joints and PR type of joints are

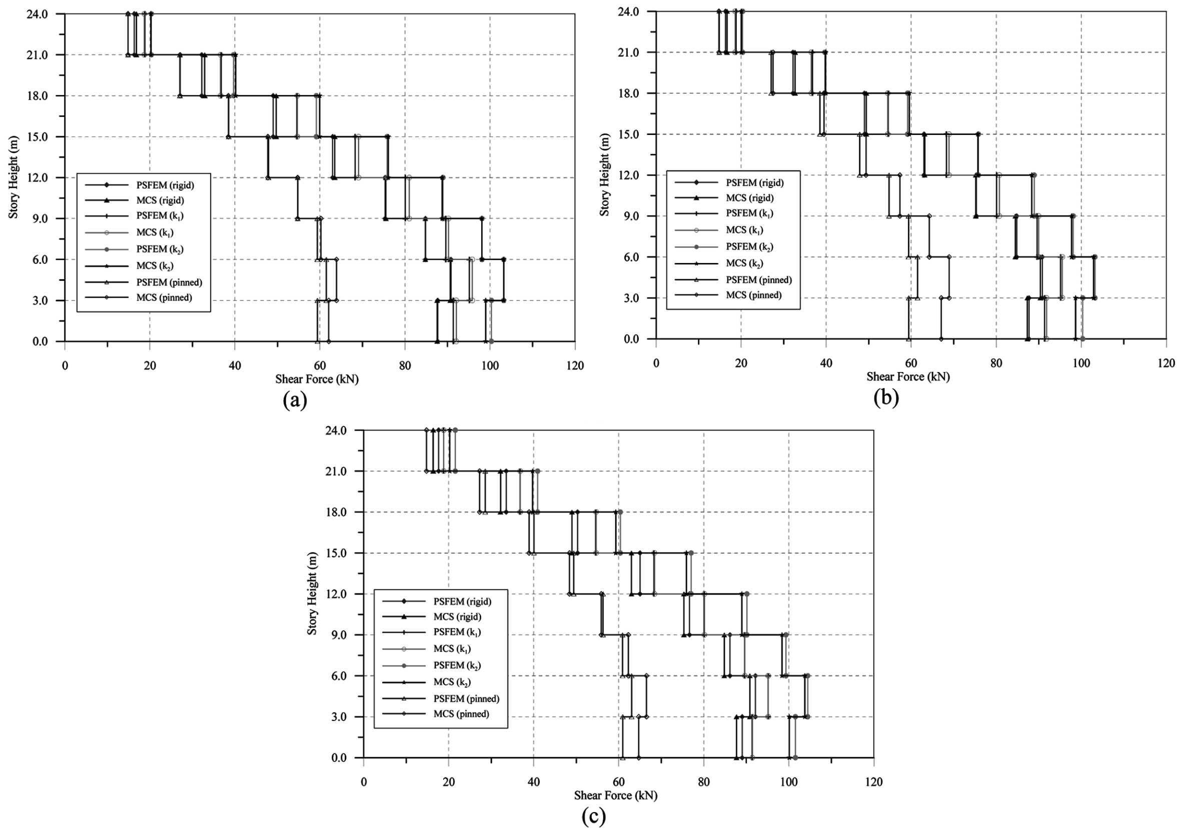


Fig. 5 Mean of maximum shear forces at the top joint of the columns along the story height of the steel frame system with PR connections for Case A (a), Case B (b), Case C (c)

19.3%, 11.7% and 8.6% for Cases A-C, respectively. Also in this figure, the maximum shear force values, which are obtained from the random cross-sectional area, are greater than those calculated by the random elastic module and connection stiffness. Changes in the bending moments are similar to shear forces for full rigidity and PR connections of Cases A-C (Fig. 6).

3.2 A steel bridge with four spans

A two-dimensional bridge that has three piers and four spans, subjected to earthquake ground motion (Fig. 3), is chosen as second application (Fig. 7). The geometrical and material properties of this bridge are given in Table 3. In this model, flexible connections were located at the ends of the beam. Two types of partially restraint beam-to-column connections (respectively stiff and weak connections) with linear moment-rotation relations were considered. The same bridge with rigid and pinned joints was also analyzed for comparison. Four different coefficients were used for representing the connection flexibility.

The steel bridge is modeled by 114 stochastic finite elements. Monte Carlo Simulation (MCS) method was simulated for 10000 simulation. Mean of maximum displacements and internal forces occurred on the steel bridge are determined for PSFEM and MCS methods considering Cases A-C:

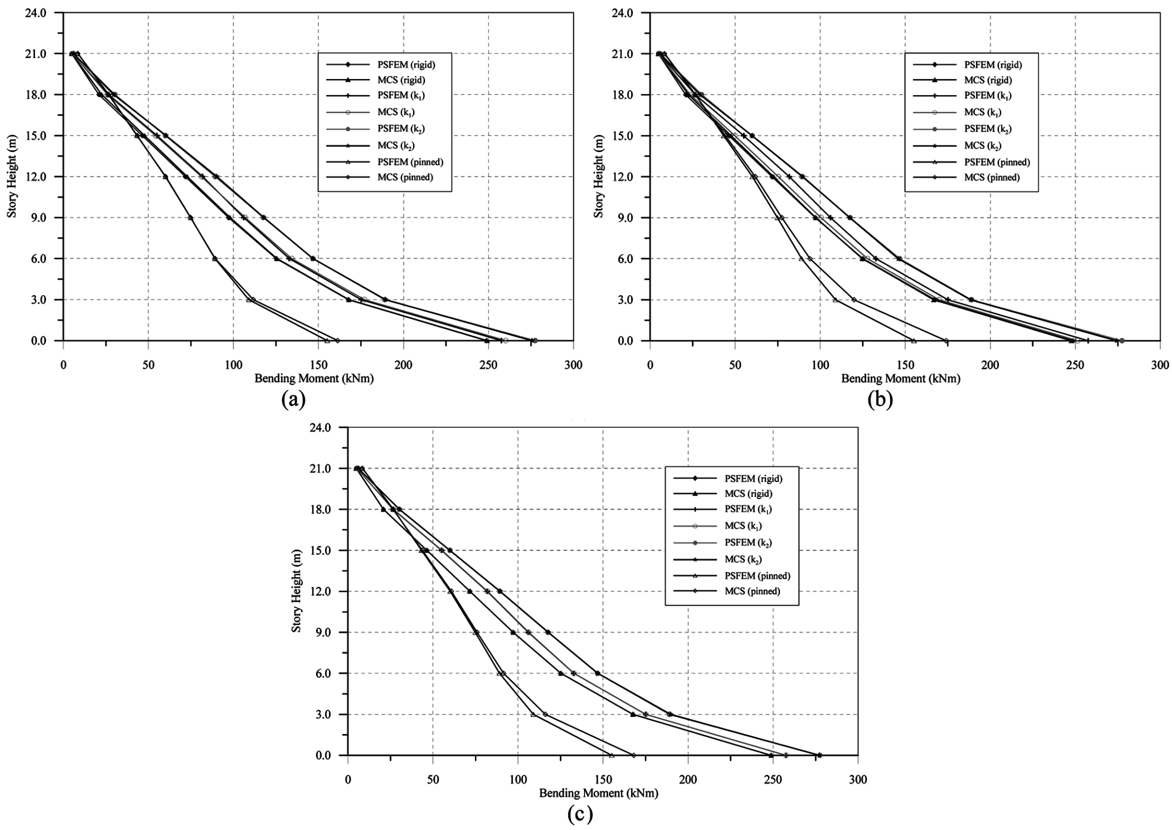


Fig. 6 Mean of maximum bending moment at the top joint of the columns along the story height of the steel frame system with PR connections for Case A (a), Case B (b), Case C (c)

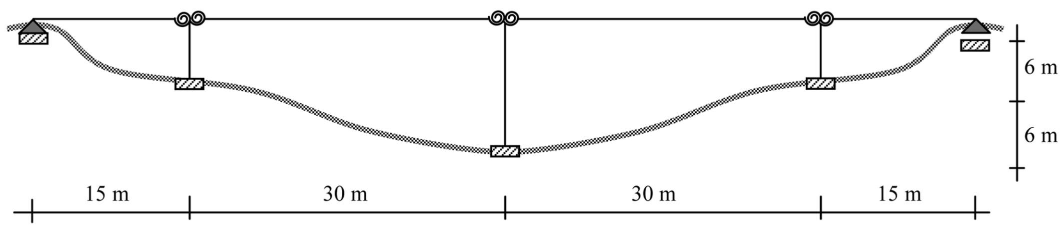


Fig. 7 The layout and dimensions of the bridge

Table 3 Sectional properties for the bridge system

Member	Elastic modulus (kN/m^2)	Mass density (kg/m^3)	Cross-section area(m^2)	Inertia moment (m^4)	
	E	ρ	A	I_y	I_z
Column	2.1×10^8	7860	0.00727	1.627×10^{-4}	1.043×10^{-5}
Beam	2.1×10^8	7860	0.0133	2.769×10^{-4}	7.436×10^{-5}

Case A. The respective expectation and correlation function and coefficient of variation (Kleiber and Hien 1992) for the elastic modulus Er are assumed as follows:

$$E[E_\rho] = 2.1 \times 10^8 \qquad \lambda = 10$$

$$\mu(E_\rho, E_\sigma) = \exp\left(-\frac{|x_\rho - x_\sigma|}{\lambda l}\right) \quad \rho, \sigma = 1, 2, \ldots, 114$$

$$\alpha = 0.10$$

Case B. Cross-sectional area from geometrical properties is chosen as random variable for steel bridge system. The respective expectation and correlation function and coefficient of variation for the cross-sectional areas (A_ρ) are assumed as follows:

$$E[A_\rho] = 0.0133 \qquad \lambda = 10$$

$$\mu(A_\rho, A_\sigma) = \exp\left(-\frac{|x_\rho - x_\sigma|}{\lambda}\right) \quad \rho, \sigma = 1, 2, \ldots, 114$$

$$\alpha = 0.10$$

Case C. The variation in connection rigidity, *k,* was modeled as random variables for bridge system's beam with semi-rigid connections. The respective expectation and correlation function and coefficient of variation for the connection rigidity k_ρ are assumed as follows:

$$E[k_1] = 2.62 \times 10^4 \qquad \lambda = 10$$

$$E[k_2] = 1.16 \times 10^4$$

$$\mu(k_\rho, k_\sigma) = \exp\left(-\frac{|x_\rho - x_\sigma|}{\lambda l}\right) \quad \rho, \sigma = 1, 2, \ldots, 114$$

$$\alpha = 0.10$$

The natural frequencies, mean of maximum displacements and internal forces are determined according to PSFEM and MCS method for the steel bridge model. The analysis results obtained from Case A, Case B and Case C are compared with each other.

3.2.1 Natural frequencies

The natural frequencies for the first three modes are determined for the cases of fully rigid, pinned and linear partially restrained connections (relatively stiff, weak connections) and shown in Table 4 for

Table 4 Natural frequencies of the steel bridge system for Cases A-C

Type of connection	Method	Natural frequencies (Hz)								
		Case A			Case B			Case C		
		First mode	Second mode	Third mode	First mode	Second mode	Third mode	First mode	Second mode	Third mode
Rigid (k_0)	PSFEM	1.177	1.447	3.315	1.185	1.457	3.338	1.177	1.447	3.315
	MCS	1.174	1.443	3.307	1.183	1.454	3.331	1.177	1.447	3.315
Relatively Stiffness(k_1)	PSFEM	0.980	1.059	3.132	0.986	1.067	3.147	0.980	1.059	3.132
	MCS	0.978	1.057	3.125	0.983	1.062	3.143	0.980	1.059	3.132
Weak connection(k_2)	PSFEM	0.866	0.891	2.996	0.872	0.898	3.010	0.866	0.891	2.996
	MCS	0.865	0.889	2.989	0.869	0.894	3.000	0.866	0.891	2.996
Pinned	PSFEM	0.730	0.732	2.545	0.735	0.737	2.558	0.730	0.732	2.545
	MCS	0.728	0.730	2.538	0.733	0.734	2.553	0.730	0.732	2.545

Cases A-C. As seen from Table 4, the connection flexibility has a significant influence on variation of the natural frequencies particularly on the higher frequencies. For each spring coefficient, natural frequencies of the fully rigid connection model are higher than natural frequencies of the PR connections model in all modes.

3.2.2 Vertical displacements

Mean of maximum vertical displacements are plotted in Fig. 8 along the decks of steel bridge system with PR connections according to MCS and PSFEM methods for Cases A-C. As shown in Fig. 8, the displacement values obtained from the PSFEM are close to those calculated using MCS method for random variables elastic module, cross-section area and connection stiffness. At the deck of bridge system where maximum vertical displacement takes place, it can be observed that the average differences between PSFEM and MCS method are 0.72%, 3.75% and 1.47%, respectively for Cases A-C. Also in Fig. 8, the maximum vertical displacement values that are obtained from the random cross section area are greater than those calculated by the random elastic module and connection rigidity.

It can be seen from Fig. 8 that the frame with PR connections has a greater vertical displacement, when compared with the fully rigid connection. These differences increase with decreasing the connection stiffness. Consequently, the differences in maximum displacement at the deck of the bridge with rigid joints (k_o) and PR type of joints are 30.42%, 31.08% and 27.88% for relatively stiff connections (k_1) or 49.24%, 49.30% and 47.21% for weak connections (k_2) of PR connections, respectively for Cases A-C. The example indicates that connection models have influences on the stochastic dynamic displacements of steel bridge.

3.2.3 Internal forces

The changing of the mean of maximum shear forces of the bridge along the decks is plotted in Fig. 9. The differences in maximum shear forces at the deck of the bridge with rigid joints (k_o) and PR type of joints are 38.76%, 56.14% and 38.55% for relatively stiff connections (k_1) or 33.89%, 55.03% and 32.59% for weak connections (k_2) of PR connections, respectively for Cases A-C. At the deck of bridge

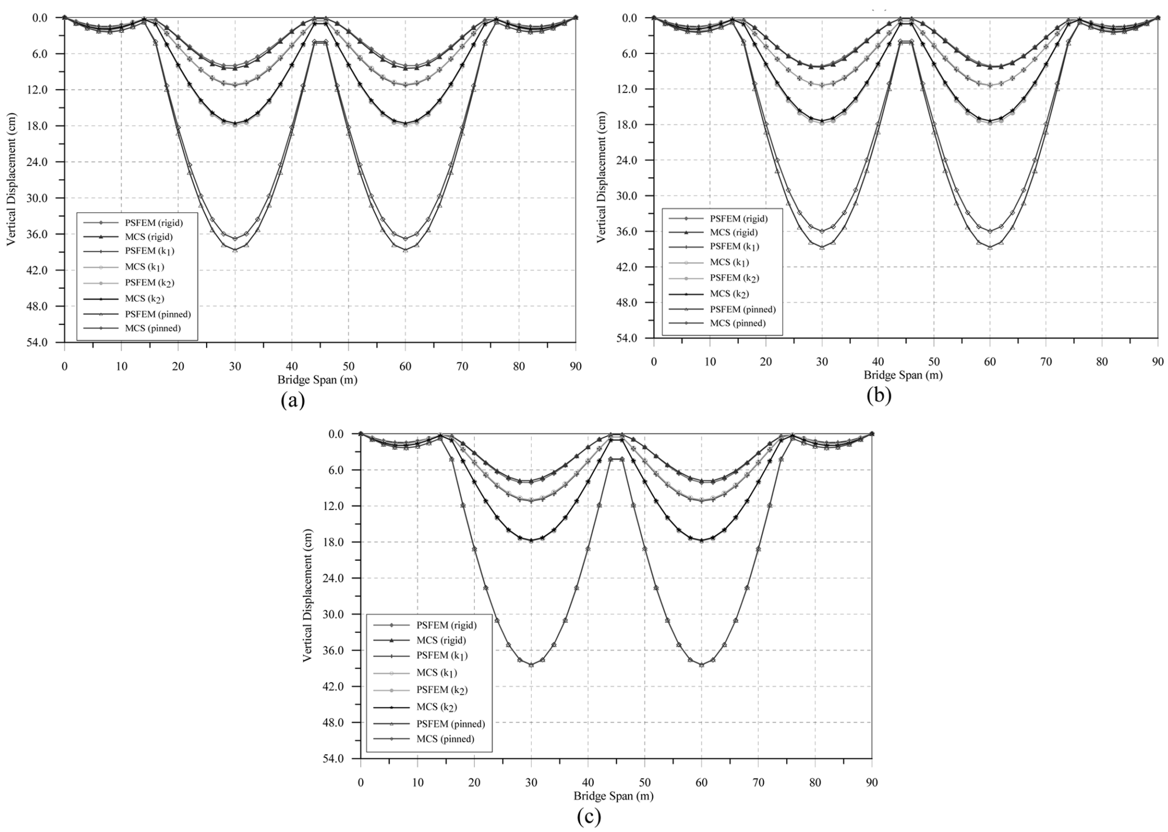
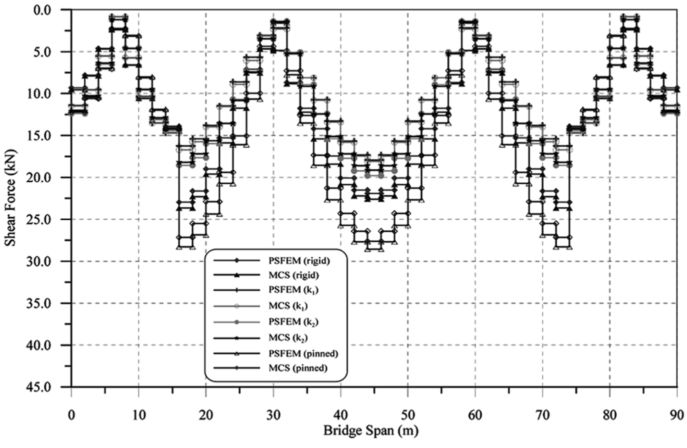


Fig. 8 Mean of maximum vertical displacements along the deck of the steel bridge system with PR connections for Case A (a), Case B (b), Case C (c)

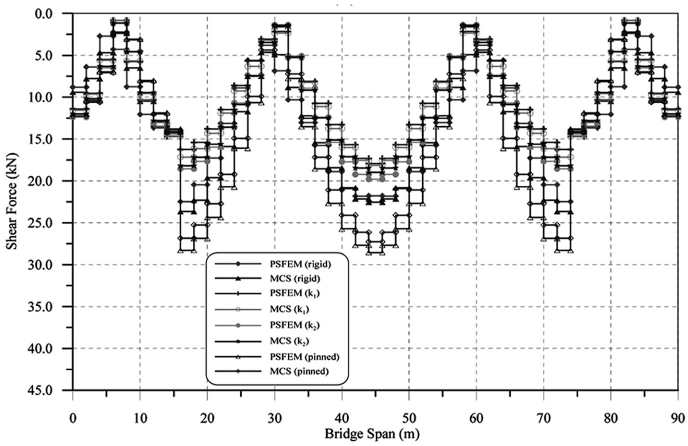
system where maximum shear forces takes place, it can be observed that the average differences between PSFEM and MCS methods are 2.88%, 3.12% and 0.53%, respectively for Cases A-C.

Maximum bending moments of deck spans along for the bridge system are plotted in Fig. 10. It is seen from Fig. 10 that values obtained from MCS and PSFEM methods are closed to each other. At the deck of bridge system where maximum bending moments takes place, it can be observed that the average differences between PSFEM and MCS method are 2.98%, 3.11% and 1.15%, respectively for Cases A-C. Consequently, the differences in maximum bending moment at the deck of the bridge with rigid joints (k_o) and PR type of joints are 39.32%, 56.27% and 38.95% for relatively stiffness connections (k_1) or 85.17%, 155.10% and 83.32% for weak connections (k_2) of PR connections, respectively for Cases A-C, respectively. Also in Figs. 9~10, the mean of maximum bending moments and shear forces that are obtained from the random cross section area are greater than those calculated by the random elastic module and connection rigidity.

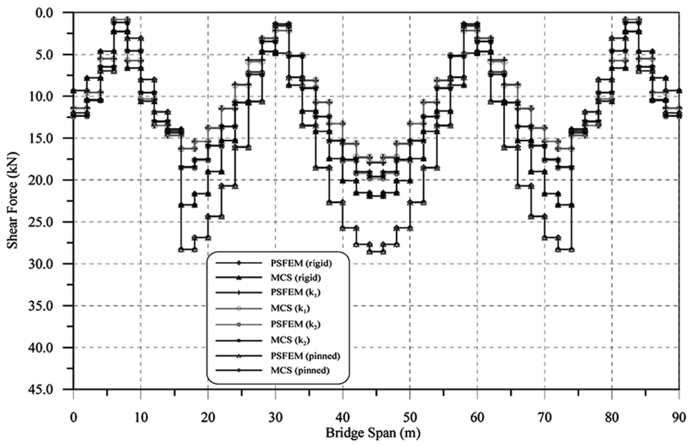
In addition to the results obtained from these two examples; for the analysis of this bridge system (Fig. 7) presented its numerical properties, it needs about 13 seconds for perturbation based stochastic analysis, however, it needs about 11 hours for MCS analysis with the PC which have Intel Pentium (R) 2.40 GHz CPU and 768 MB RAM. On the other hand, for the analysis of the frame system (Fig. 2), it needs about four seconds for perturbation based stochastic analysis, and, it needs about six hours for MCS analysis.



(a)



(b)



(c)

Fig. 9 Mean of maximum shear forces in the first joint of the each element along the bridge's deck of the steel bridge system with PR connections for Case A (a), Case B (b), Case C (c)

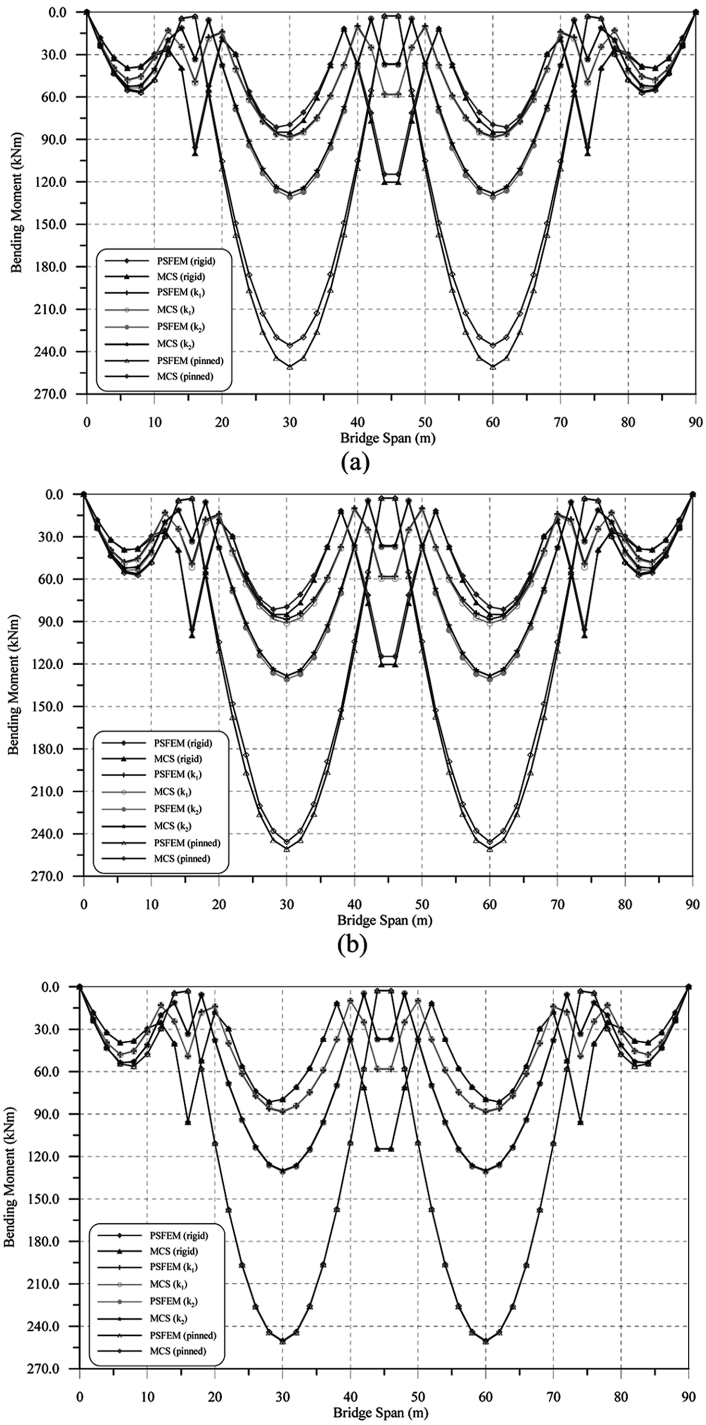


Fig. 10 Mean of maximum bending moments in the first joint of the each element along the bridge's deck of the steel bridge system for Case A (a), Case B (b), Case C (c)

4. Conclusions

In this paper, the effect of random variability in elastic module, cross-sectional area and initial connection stiffness on the stochastic seismic responses of the steel structure systems with type PR connections subjected to ground motion is investigated using PSFEM and MCS methods. A computer program developed by the authors was applied to the steel frame and bridge systems. Some conclusions drawn for the analysis of the systems are given as follows:

- It can be stated that frequencies, mean of maximum displacements and internal forces obtained from PSFEM and MCS methods are close to each other for all random variables (elastic module, cross-sectional area and connection stiffness).
- The dynamic response values obtained for the random variable cross section area are generally higher than those of the other random variables for chosen structural systems.
- It can be generally concluded that the stochastic structural responses of the steel structure systems with conventional and PR connections are considerably different. Therefore, the stochastic variation of PR connections should be considered in design of real steel structure systems.
- For the steel frame and bridge systems, MCS requires much more time effort than perturbation based stochastic finite element method.
- It can be mentioned that PSFEM proposed herein is the most efficient and economical numerical solution procedure for the stochastic seismic analysis of steel structure systems with PR connection.

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