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Analysis of restrained heated steel beams during cooling phase

Guo-Qiang Li[1,2*] and Shi-Xiong Guo[1]

[1]Department of Structural Engineering, Tongji University, 1239 Siping Road, Shanghai, 200092, PR China
[2]State Key Laboratory for Disaster Reduction in Civil Engineering, Tongji University, 1239 Siping Road, Shanghai 200092, PR China

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Abstract. Observations from experiments and real fire indicate that restrained steel beams have better fire-resistant capability than isolated beams. Due to the effects of restraints, a steel beam in fire condition can undergo very large deflections and the run away damage may be avoided. However disgusting damages may occur in the beam-to-column connections, which is considered to be mainly caused by the enormous axial tensile forces in steel beams resulted from temperature decreasing after fire dies out. Over the past ten years, the behaviour of restrained steel beams subjected to fire during heating has been experimentally and theoretically investigated in detail, and some simplified analytical approaches have been proposed. While the performance of restrained steel beams during cooling has not been so deeply studied. For the safety evaluation and repair of steel structures against fire, more detailed investigation on the behaviour of restrained steel beams subjected to fire during cooling is necessary. When the temperature decreases, the elastic modulus and yield strength of steel recover, and the contraction force in restrained steel beams will be produced. In this paper, an incremental method is proposed for analyzing the behaviour of restrained steel beams subjected to cooling. In each temperature decrement, the development of deformation and internal forces of a restrained beam is divided into four steps, in order to consider the effect of the recovery of the elastic modulus and strength of steel and the contraction force generated by temperature decrease in the beam respectively. At last, the proposed approach is validated by FE method.

Keywords : steel structure; restrained beam; fire resistance; cooling.

1. Introduction

Observations from experiments and real fire indicate that restrained steel beams have better fire-resistant capacity than isolated beams(Steel Construction Institute 1991, Wang 2002). Over the past ten years, the behaviour of restrained steel beams during heating has been investigated in detail through experimental and theoretical studies (Li, *et al.* 2000, Liu, *et al*. 2002, Huang and Tan 2002, Yin and Wang 2004), and some practical and simplified approaches for predicting the behaviour of restrained steel beams subjected to elevated temperature have been proposed (Yin and Wang 2005a, 2005b). On the other hand, the damages of connections of restrained beams are noticeable in fire, which is thought to be mainly caused by the enormous axial tensile forces in restrained steel beams resulted from temperature decreasing after fire extinguished. Fig. 1 illustrates the damage in beam-to-column connection found in

* Corresponding Author, Email: gqli@mail.tongji.edu.cn

Fig. 1 Damage to beam-column connection of a tower steel building in Taipei Science and Technology Park

the fire attack to a tall steel building in Taipei Science and Technology Park. While few research on the performance of restrained steel beams during cooling has been conducted, except some studies on the strain reversal of steel during cooling carried out by El-Rimawi, *et al.*(1996) and Bailey, *et al.*(1996). Considering many damages have occurred during cooling in fire-attacked steel buildings, investigation on the behaviour of restrained steel beams during cooling is necessary.

2. The behaviour of restrained steel beams during heating and cooling

The behaviour of a restrained beam in fire during heating can be divided into 4 stages according to the development of axial force in the beam, as shown in Fig. 2. The development of the deflection in accordance with each stage is shown in Fig. 3.

In stage I, the beam is in elastic state. The deflection increases slowly. Compressive axial force is induced in the beam because thermal expansion is restrained, and it increases with temperature elevation. At the end of stage I, the axial force reaches the maximum and plasticity occurs. In stage II, the compressive force in the beam begins to decrease. At the end of this stage the compressive force is reduced to zero and the deflection increases sharply. In stage III, the axial force changes into tension,

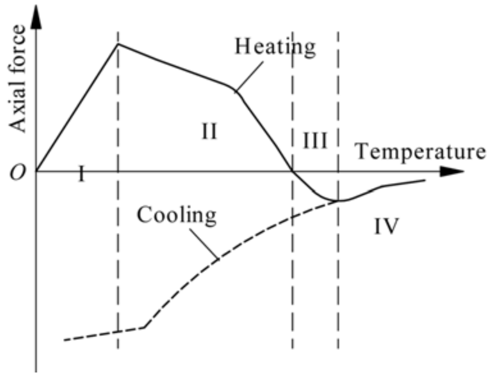

Fig. 2 Development of axial force in a restrained beam subjected to heating and cooling

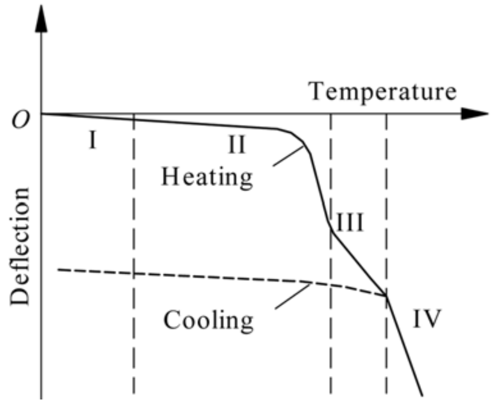

Fig. 3 Development of deflection of a restrained beam subjected to heating and cooling

and the rate of the deflection is slowed down, which indicates that catenary action take into effect. At the end of stage III the tensile axial force reaches the maximum. In stage IV, the axial tensile force begins to decrease and the rate of the deflection increases again. In stage II~IV, plastic strain is produced and accumulated in the beam with temperature increasing. When temperature begins to decrease, because the plastic strain is maintained and the beam is restrained at the ends, the contraction force in the beam will be increased, as indicted by the dashed line in Fig 2. At the same time, the recovery of the elastic modulus steel, together with the effect of increasing tensile axial force, reduce the deflection of the beam, as shown by the dashed lines in Fig. 3.

3. Equilibrium of restrained steel beam in large deflection state

Restrained steel beams in fire condition can undergo very large deflection, so the effect of geometric non-linearity, which is normally ignored when studying isolated beams, must be considered. A typical model of a restrained beam is shown in Fig. 4, in which L is the span of the beam; k_a is the stiffness of the axial restraint at the ends of the beam; k_r is the stiffness of the rotational restraint; F is the axial force in the beam; M_{end} and M_{mid} are the resistant bending moments at the ends and mid-span, respectively.

The equilibrium equation of moments in the beam, involving the effect of geometric non-linearity, can be expressed as:

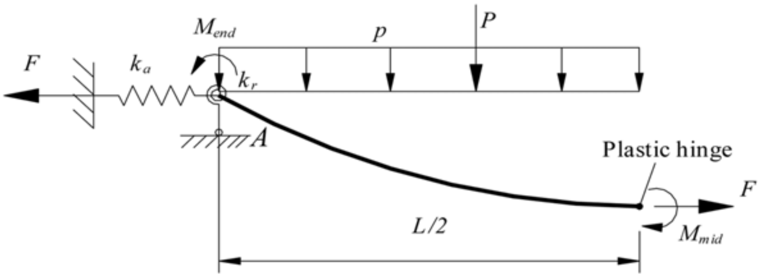

Fig. 4 The model of a restrained steel beam

$$M_{end} - M_{mid} - M_{eff} + F\delta = 0 \tag{1}$$

where δ is the vertical mid-span displacement and M_{eff} is the moment about end A induced by the applied load on the beam.

According to previous studies about restrained steel beams(Yin and Wang 2005a), the axial force in the beam can be determined by:

$$F = \left(\alpha L \Delta T - \lambda \frac{\delta^2}{L}\right) k_{e,a} \tag{2}$$

where α is the coefficient of thermal expansion of steel; ΔT is the temperature increase; λ is a factor relative to the load type and restraint condition; $k_{e,a}$ is the effective axial stiffness of the beam, which can be expressed as:

$$\frac{1}{k_{e,a}} = \frac{2}{k_a} + \frac{1}{k_{bT}} \tag{3}$$

where k_{bT} is the axial stiffness of the beam at temperature T.

4. The stiffness of restrained steel beams

In this study, the axial stiffness of the restraint to the beam, k_a, is assumed to be constant, but the axial stiffness of the beam, k_{bT}, varies widely in different stages. Since k_{bT} is one of the most important factors that affect the behaviour of restrained beams, it is studied firstly. In order to simplify the study, the temperature distribution at the beam cross section is assumed to be uniform. Three states are discussed as follows.

4.1. Elastic state with small deflection

Given the beam is elastic and its deflection is very small, the axial stiffness of the beam can be approximately determined by

$$k_{bT} = \frac{E_T A}{L} \tag{4}$$

where E_T is the elastic modulus of steel at temperature T; and A is the cross-sectional area of the beam.

4.2. Elastic state with large deflection

However, if the deflection of the beam is very large, the effect of deflection on the axial stiffness of the beam must be taken into account. Since it is very difficult to determine the accurate axial stiffness of the beam, the following approximately approach is proposed.

For a simply supported and curved beam shown in Fig. 5(a), the horizontal displacement of the end of the beam under the action of a horizontal force, P, can be determined by:

$$\Delta_P = P\int_0^l \frac{\sqrt{1+f'(x)^2}f^2(x)}{E_T I}dx + \int_0^l \frac{P}{E_T A\sqrt{1+f'(x)^2}}dx \tag{5}$$

where $f(x)$ is the profile of the beam, as proposed by Yin and Wang (Yin and Wang 2005a).

In Eq.(5), the first part is the displacement induced by the moment, and the second part is that by the axial force.

If the rotation of the beam is restrained, as shown in Fig. 5(b), the moment M_P will be induced at the ends under the axial force, P. If rotation of the ends of the beam is fully restrained, the rotation of the ends is zero. Then based on the virtual work principle, the following equation can be obtained:

$$\int_0^l \frac{M_P\overline{M}}{E_T I}ds - \int_0^l \frac{f(x)P\overline{M}}{E_T I}ds = 0 \tag{6}$$

where $\overline{M}$ is the virtual unit moment.

The moment M_P can be worked out through Eq.(6) and expressed as

$$M_P = \frac{P\int_0^l \sqrt{1+f'(x)^2}f(x)dx}{\int_0^l \sqrt{1+f'(x)^2}dx} \tag{7}$$

If the rotation of the ends of the beam is not fully restrained and the effective stiffness of the rotational restraints is $k_{e,r}$, the expression of M_p may be modified as

$$M_P = \frac{k_{e,r}}{EL/L}\frac{P\int_0^l \sqrt{1+f'(x)^2}f(x)dx}{\int_0^l \sqrt{1+f'(x)^2}dx} \tag{8}$$

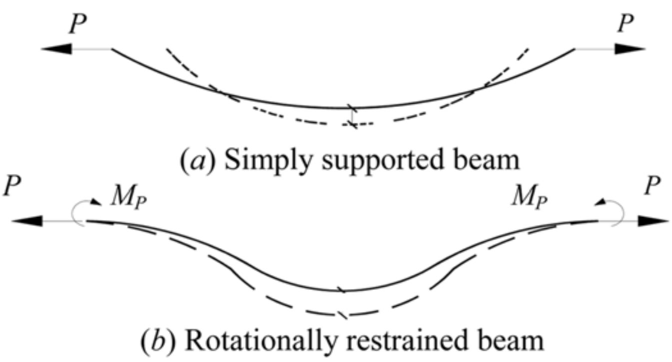

Fig. 5 The sketch of the curved beams

where $k_{e,r}$ is the effective stiffness of the restrained beam, which can be determined with

$$\frac{1}{k_{e,r}} = \frac{L}{EI} + \frac{2}{k_r} \tag{9}$$

where k_r is the rotational stiffness of the restraints to the ends of the beam.

Because M_{p} also induces horizontal displacement of the ends, the total horizontal displacement of the ends of the beam with rotational restraint can be determined by:

$$\Delta_P = P\int_0^l \frac{\sqrt{1+f'(x)^2}f^2(x)}{E_T I}dx + \int_0^l \frac{P}{E_T A\sqrt{1+y'^2}}dx$$

$$- \frac{k_{e,r}}{EL/L}\frac{P\int_0^l \sqrt{1+f'(x)^2}f(x)dx}{\int_0^l \sqrt{1+f'(x)^2}dx}\int_0^l \frac{\sqrt{1+f'(x)^2}f(x)}{E_T I}dx \tag{10}$$

Then the axial stiffness of the curved beam, considering the effect of deflection, can be determined by:

$$\frac{1}{k_{bT}} = \frac{\Delta_P}{P} = \int_0^l \frac{\sqrt{1+f'(x)^2}f^2(x)}{E_T I}dx + \int_0^l \frac{P}{E_T A\sqrt{1+f'(x)^2}}dx$$

$$- \frac{k_{E,R}}{EL/L}\frac{\int_0^l \sqrt{1+f'(x)^2}f(x)dx}{\int_0^l \sqrt{1+f'(x)^2}dx}\int_0^l \frac{\sqrt{1+f'(x)^2}f(x)}{E_T I}dx \tag{11}$$

4.3. Plastic state with large deflection

If plastic hinge occurs in the beam, the axial stiffness of the beam will be reduced sharply. According to plastic theory, the relationship between the moment, M, and axial force, F, at the plastic hinge in the beam can be expressed as:

$$\begin{cases} \dfrac{M}{M_p} + \omega\left(\dfrac{F}{F_p}\right)^2 = 1 & \text{Neutral axis in the web} \\ \zeta\dfrac{M}{M_p} + \left|\dfrac{F}{F_p}\right| = 1 & \text{Neutral axis in the flange} \end{cases} \tag{12}$$

where M_p is the plastic moment capacity of the beam; F_p is the plastic axial force capacity of the beam; ω and ζ are factors relevant to the type of the cross-section of the beam, given by(Yin and Wang 2005b)

$$\omega = \frac{(2+\mu_t)^2}{\mu_t(4+\mu_t)},\ \zeta = \frac{4+\mu_t}{2(2+\mu_t)} \tag{13}$$

where μ_t is the ratio of the cross-sectional area of the web to that of one flange of the beam.

For a beam without rotational restraints at the ends, $M_{\text{end}} = 0$, so Eq.(1) changes to:

$$F\delta = M_{mid} + M_{eff} \tag{14}$$

Work out the expression of M_{mid} from Eq.(12) and substitute it into Eq.equation 14(14), then the derivative of F with respected to δ can be obtained by:

$$\frac{dF}{d\delta} = \begin{cases} -\dfrac{F}{2\omega\dfrac{M_pF}{F_p^2} + \delta} & \text{Neutral axis in the web} \\ -\dfrac{F}{\dfrac{M_p}{\zeta F_p} + \delta} & \text{Neutral axis in the flange} \end{cases} \tag{15}$$

Given the temperature is kept in constant, the relationship of the relative displacement of the two ends of the beam, u, and the deflection at the mid span of the beam, δ, can be expressed as:

$$u = \lambda\frac{\delta^2}{L} \tag{16}$$

Then

$$\frac{du}{d\delta} = 2\lambda\frac{\delta}{L} \tag{17}$$

Therefore, according to the differential Eqs. (15) and (17), the axial stiffness of the beam with plastic hinge and large deflection can be determined by:

$$k_{bT,p} = \frac{dF}{du} = \begin{cases} -\dfrac{F}{2\omega\dfrac{M_pF}{F_p^2} + \delta}\cdot\dfrac{L}{2\lambda\delta} & \text{Neutral axis in the web} \\ -\dfrac{F}{\dfrac{M_p}{\zeta F_p} + \delta}\cdot\dfrac{L}{2\lambda\delta} & \text{Neutral axis in the flange} \end{cases} \tag{18}$$

Similarly, for a beam with end rotation fully restrained, the axial stiffness of the beam with plastic hinge and large deflection can be derived as:

$$k_{bT,p}=\frac{\mathrm{d}F}{\mathrm{d}u}=\begin{cases}-\dfrac{F}{4\omega\dfrac{M_pF}{F_p^2}+\delta}\cdot\dfrac{L}{2\lambda\delta} & \text{Neutral axis in the web}\\[2ex] -\dfrac{F}{2\dfrac{M_p}{\zeta F_p}+\delta}\cdot\dfrac{L}{2\lambda\delta} & \text{Neutral axis in the flange}\end{cases} \tag{19}$$

5. Analyses of restrained beams during cooling

In order to simplify the analysis of heated restrained steel beams during cooling, the temperature distribution over the cross-section of the beam is assumed to be uniform.

5.1. Stress–strain relationship during cooling

The stress-strain relation of steel when unloading is supposed to be linear as shown in Fig. 6. According to the proposal by El-Rimawi, *et al.*(1996), the stress-strain relation of steel during cooling can be assumed to vary as shown in Fig. 7. At temperature T_1 or T_2, the elastic modulus of steel is E_{T1} or E_{T2}, and the stress may increase up to f_{yT1} or f_{yT2}. If the strain of steel reaches ε_{T1} with the stress of f_{yT1} at T_1 and then the temperature begins to decreases from T_1 to T_2, the change of the strain and stress of steel can be divided into two steps. In the first step, unload at temperature T_1, and strain decreases to ε_y with a slop equal to E_{T1}. In the second step, decrease the temperature to T_2 and then increase the load, strain and stress increase again with a slop equal to E_{T2}. When the stress reaches $f_{y\mathrm{T1}}$, the corresponding strain is ε_{T2}, which is equal to $(\varepsilon_{T1}-f_{y1}/E_{T1}+f_{y1}/E_{T2})$.

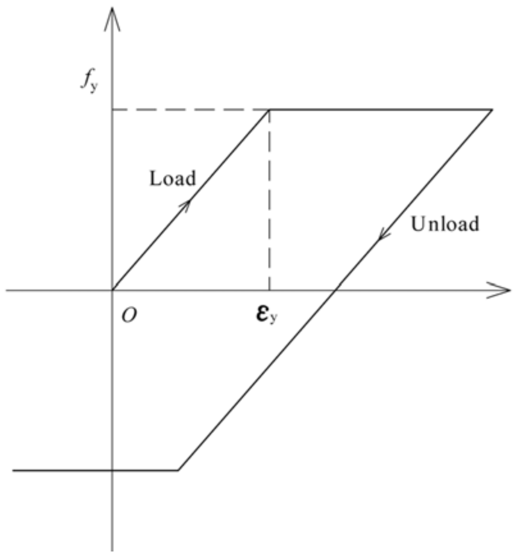

Fig. 6 Stress-strain curve of steel when unloading

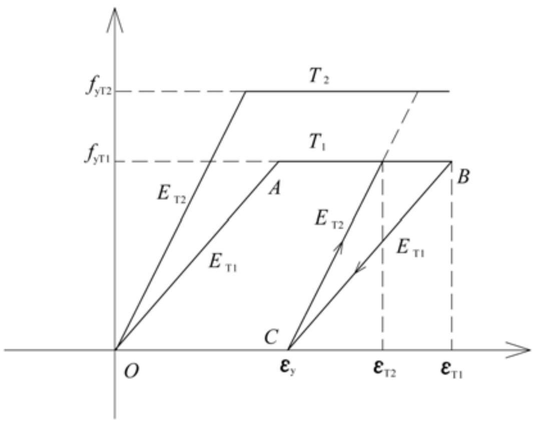

Fig. 7 Reversal of stress in steel during cooling

5.2. Development of the restrained beam during cooling

When steel temperature decreases from T_1 to T_2, steel elastic modulus and yield strength recover, which increases the stiffness and the load bearing capacity of the beam. At the same time, contraction force, ΔF, will be produced, as shown in Fig. 8(*a*).

In order to study the behaviour of restrained beams during cooling, a temperature negative incremental

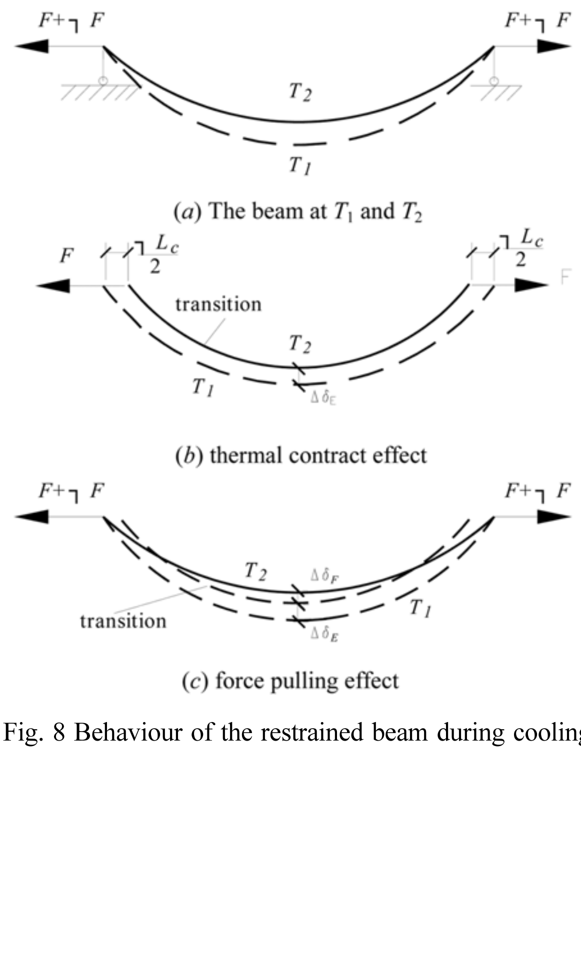

Fig. 8 Behaviour of the restrained beam during cooling

approach is employed. The completed process of cooling can be divided into many decrements. In each temperature decrement, $\Delta T = T_2 - T_1$, the change of a beam can be divided into four steps. In the first step, the applied vertical load is unloaded at temperature T_1, with the axial force in the beam and moment at the beam ends being kept constantly. In the second step, the steel temperature decreases from T_1 to T_2, the beam contracts, and the steel elastic modulus and yield strength recover, as shown in Fig. 8(*b*). In the third step, applied load on the beam is recovered to previous value. In the fourth step, the incremental tensile axial force, ΔF, resulted from contraction, is applied at the ends of the beam, as shown in Fig. 8(*c*).

5.3. Deflection induced by recovery of the elastic modulus of steel

During cooling, because the plastic strain can not be recovered, the beam keeps to be a bowed beam. The defection reversal of the restrained beam resulted from the recovery of the steel elastic modulus during step 1~3 can be determined according to the process shown in Fig. 9

For an isolated bowed beam with a profile function f(x), as shown in Fig. 10, the applied vertical load will induce an increment of deflection, $d_{p,T}$, determined by:

$$d_{p,T} = \frac{pc_L}{E_T} \tag{20}$$

where c_L is a factor dependent on the span of the beam and the type of the vertical load on the beam.

If horizontal restraints are applied on the ends of the bowed beam, deflection increment will produce axial force, F_u. The horizontal tensile force F_u will induce a decrement of deflection, $d_{F,T}$. Then the deflection increment will be reduced to d_T.

According to the law of energy conservation, $d_{F,T}$ can be determined by:

$$d_{F,T} = \frac{2F_u}{E_T I}\int_0^{\frac{L}{2}}\left(\frac{1}{2}x\right)[f(x)]\sqrt{1+(f'(x))^2}\,\mathrm{d}x$$

for simply-supported beams (21*a*)

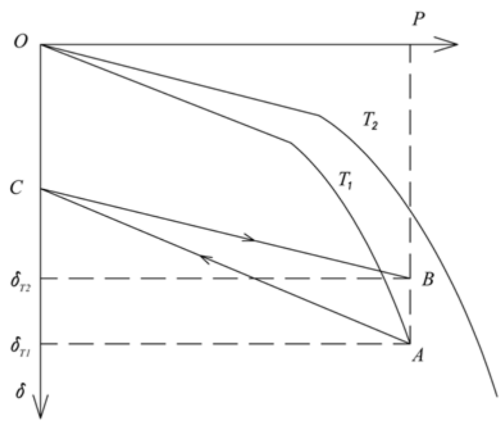

Fig. 9 Deflection reversal resulted from elastic modular recover

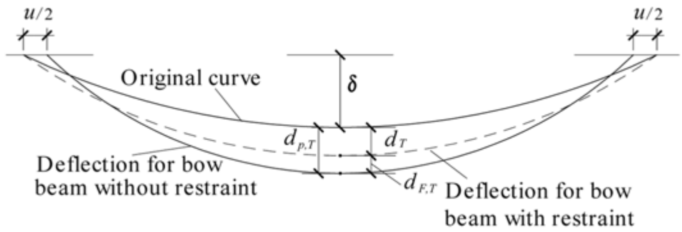

Fig. 10 Deflection for bow beam

$$d_{F,T} = \frac{2F_u}{E_T I}\int_0^{\frac{L}{2}}\left(\frac{L}{8}-\frac{1}{2}x\right)\left[\frac{\int_0^l \sqrt{1+f'(x)^2}f(x)dx}{\int_0^l \sqrt{1+f'(x)^2}dx}-f(x)\right]\sqrt{1+(f'(x))^2}dx$$

for rotationally restrained beams (22*b*)

Above equations can be simplified as:

$$d_F = \frac{2F}{E_T I}C_d \tag{22}$$

$$C_d = \int_0^{\frac{L}{2}}\left(\frac{1}{2}x\right)[f(x)]\sqrt{1+(f'(x))^2}\mathrm{d}x$$

for simply-supported beams

$$C_d = \int_0^{\frac{L}{2}}\left(\frac{L}{8}-\frac{1}{2}x\right)\left[\frac{\int_0^l \sqrt{1+f'(x)^2}f(x)dx}{\int_0^l \sqrt{1+f'(x)^2}dx}-f(x)\right]\sqrt{1+(f'(x))^2}dx$$

for rotationally restrained beams

Approximately, the horizontal displacement of beam end u can be expressed as :

$$u = \frac{2\lambda\delta_T}{L}d_T \tag{23}$$

Considering the relationship of $d_{p,T}$, d_T, $d_{F,T}$ and F_u, one has

$$d_T + d_{F,T} = d_{p,T} \tag{24}$$

$$F_u = u \cdot k_g = \frac{2\lambda\delta_T}{L}d_T \cdot k_g \tag{25}$$

Substituting Eq.(25) into (22) gives

$$d_{F,T} = \frac{4\lambda\delta}{E_T IL} d \cdot k_g C_d \tag{26}$$

Substitute Eq. (26) into (24), d_T can be worked out:

$$d_T = \frac{d_{p,T}}{1 + \frac{4\lambda\delta_T}{E_T IL} \cdot k_g C_d} \tag{27}$$

At temperature T_1, unloading will produce a deflection reduction, d_{T1}. At temperature T_2, reloading will induce a deflection increase, d_{T2}. Then the deflection reversal, resulted from recovery of elastic modulus of steel can be determined by:

$$\delta_{rev,E} = d_{T1} - d_{T2} = \frac{\frac{pc_L}{E_{T1}}}{1 + \frac{4\lambda\delta_{T1}}{E_{T1} IL} \cdot k_g C_F} - \frac{\frac{pc_L}{E_{T2}}}{1 + \frac{4\lambda\delta_{T1}}{E_{T2} IL} \cdot k_g C_F} \tag{28}$$

where δ_{T1} and δ_{T2} are the mid-span deflection of the beam at T_1 and T_2, respectively.

It should be noted that because the difference between δ_{T1} and δ_{T2} is relatively small, δ_{T2} in the second part of above equation is substituted by δ_{T1}.

Given the stiffness of axial restraint is not larger than 0.1k$_b$, $\frac{4\lambda\delta_{T1}}{E_{T1} IL} \cdot k_g C_F$ and $\frac{4\lambda\delta_{T1}}{E_{T2} IL} \cdot k_g C_F \approx 0$. Then Eq.(28) can be simplified as:

$$\delta_{rev,E} = \frac{pc_L}{E_{T1}} - \frac{pc_L}{E_{T2}} = \frac{pc_L}{E_0}\left(\frac{E_0}{E_{T1}} - \frac{E_0}{E_{T2}}\right) = d_0\left(\frac{E_0}{E_{T1}} - \frac{E_0}{E_{T2}}\right) \tag{29}$$

5.4. Contraction force generated by temperature decrease

In the second step as shown in Fig. 8(*c*), because the steel strength recovers with temperature decreasing, as shown in Fig. 11, and the strain induced by ΔF is reverse, the axial displacement change of the beam ends can be treated as elastic at the beginning. During temperature decreasing from T1 to T2, if the plastic hinge occurs, the complete relative displacement change of the ends due to ΔF can be divided into to two parts, i.e. the elastic part, Δu_e, and the plastic part, Δu_p. Accounting for the geometric compatibility, the following equation must be satisfied:

$$\Delta u_e + \Delta u_p = \alpha \Delta TL \tag{30}$$

in which $\Delta T = T_2 - T_1$, and ΔF can be determined by:

$$\Delta F = \Delta u_e k_{e,ae} + \Delta u_p k_{e,ap} \tag{31}$$

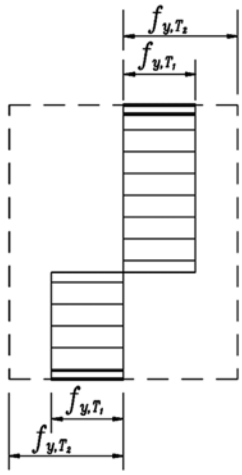

Fig. 11 Recovery of steel strength with temperature decreasing

where $k_{e,ae}$ is the effective axial restraint stiffness when the beam is in elastic state; and $k_{e,ap}$ is the effective axial restraint stiffness when the beam is in plastic state.

5.5. Moment at the beam ends during cooling

For beams with end rotation fully restrained, when temperature decreases from T_1 to T_2, incremental axial tension force will lead to the change of moment at beam ends.

Given plastic hinge does not occur, the change of the moment can be determined by Eq. (32) according to Eq.(8) as

$$\Delta M_{end} = \frac{\Delta F \int_0^l \sqrt{1+f'(x)^2} f(x)\,dx}{\int_0^l \sqrt{1+f'(x)^2}\,dx} \tag{32}$$

Then the moment at T_2 can be expressed as:

$$M_{end,T2} = M_{end,T1} - \Delta M_{end}$$

Given plastic hinge occurs, the moment at the beam ends can be worked out from Eq.(12) as

$$M_{end,T2} = \begin{cases} \left(1-\omega\left(\frac{F}{F_\rho}\right)^2\right) M_\rho & \text{Neutral axis in the web} \\ \left(1-\left|\frac{F}{F_\rho}\right|\right) \frac{M_\rho}{\zeta} & \text{Neutral axis in the flange} \end{cases} \tag{33}$$

5.6. The deflection reversal due to contraction force

When the steel temperature decreases from T_1 to T_2, and if ΔF is not so large to lead the beam into plastic, the deflection reversal of the beam induced by ΔF can be obtained with the energy conservation principle (Li and Guo 2005) by:

$$\delta_{rev,F} = \frac{2\Delta F}{E_{T_2} I} C_d \tag{34}$$

5.7. The total deflection during cooling

When temperature decreases from T_1 to T_2, given the plastic hinge does not occur, the total deflection of the beam at temperature T_2 can be determined by

$$\delta_{T_2} = \delta_{T_1} - \delta_{rev,E} - \delta_{rev,F} \tag{35}$$

If the plastic hinge occurs, the deflection of the beam has to be worked out through the equilibrium equation of the beam.

For a hinged supported beam, the deflection of the beam at temperature T_2 can be derived from Eq.(1) as:

$$\delta_{T_2} = \frac{M_{eff} + M_{mid}}{F} \tag{36}$$

where M_{mid} can be worked out from Eq. (12).

Similarly, for a beam with end rotation fully restrained, the deflection of the beam at temperature T_2 can be expressed, assuming M_{mid} is equal to M_{end}, as:

$$\delta_{T_2} = \frac{M_{eff} + 2M_{end,T_2}}{F} \tag{37}$$

The complete behaviour of the restrained beam during fire cooling can then be analyzed step by step, following the steps presented hereinabove, until the steel temperature decreases to ambient.

6. Validation

In order to validate the method proposed in this paper, the performance of a heated restrained steel beam during cooling, under different restraint conditions, is predicted by this method and compared with the results obtained by FE method. The section of the beam is H400X200X8X13, and the span is 8m. The lateral displacement of the beam is supposed to be fully restrained, so no lateral buckling will occur. The steel of the beam is Q235, of which the yield strength is 235 MPa, and the steel property at elevated temperatures is determined in accordance with the Eurocode 3(2001). The FE analysis is carried out by ANSYS 6.0 (2002), a commercial FE package. In the FE analysis, shell element is employed to simulate the flanges and web of the beam.

Four types of restraints are applied on the beam respectively. In the first and second type, the beam ends are hinged supports, and the axial stiffness of the restraints is 0.1 k_b and 0.4 k_b respectively. The uniformly distributed load on the beam is 26.25 kN/m

In the third and fourth type of restraint, the rotations of the beam ends are fully restrained, and the axial stiffness of the restraints is 0.2 k_b and 0.4 k_b respectively. The uniformly distributed load on the beam of these two types is 35.5 kN/m.

For the beam with the first and second types of restraint, the developments of deflection and axial force during cooling, predicted by the proposed method and FE method respectively, are compared in Figs.12~13.

For the beam with the third and fourth types of restraint, the developments of deflection, axial force and moment at the ends during cooling, predicted by the proposed method and FE method respectively, are compared in Figs.14~16.

The results of deflection, axial force and moment during heating predicted by the FE method are employed for the comparisons carried out for those during cooling.

It can be seen that during cooling the predictions by the method proposed in this paper and the FE method are in good accordance.

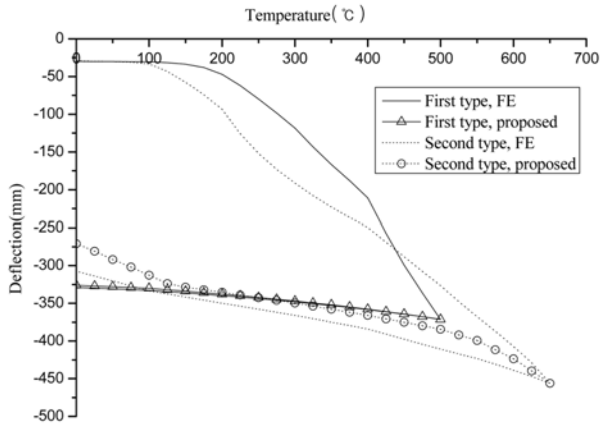

Fig. 12 Comparison of defections predicted by FE method and the proposed method (Hinged support)

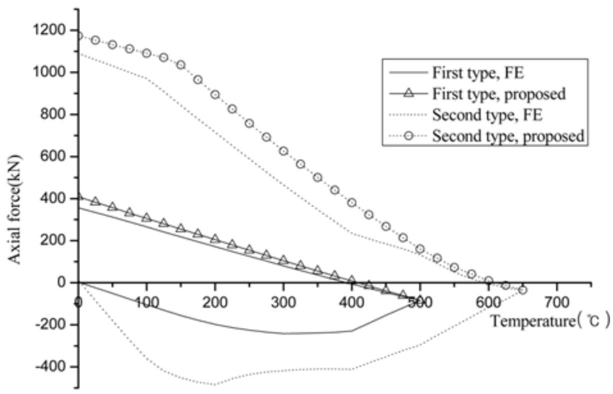

Fig. 13 Comparison of axial forces predicted by FE method and the proposed method (Hinged support)

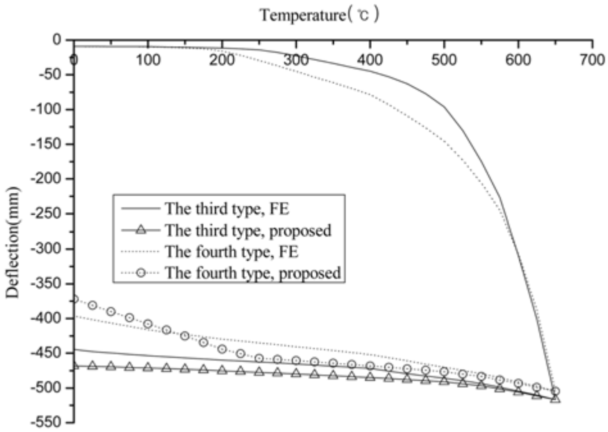

Fig. 14 Comparison of defections predicted by FE method and the proposed method (Rotation of ends fully restrained)

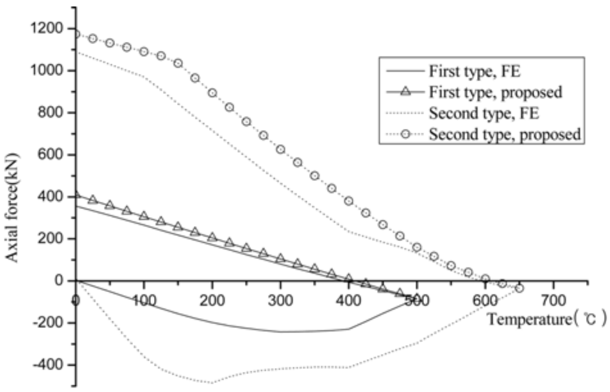

Fig. 15 Comparison of axial forces predicted by FE method and the proposed method (Rotation of ends fully restrained)

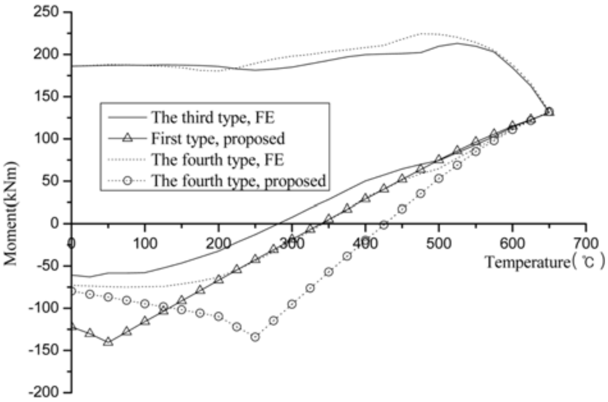

Fig. 16 Comparison of end moment predicted by FE method and the proposed method (Rotation of ends fully restrained)

Through the above analysis of the restrained steel beam during cooling, it can be found that axial tensile force increases with temperature decreasing. The larger is the stiffness of the axial restraint, the larger is the increment of the axial tensile force. However, too large axial tensile force will lead to plastic hinge in the restrained steel beam. After plastic hinge occurs, the increase rate of the axial force will decrease, and the deflection reversal will increase sharply.

In addition, if the rotation is restrained at the beam ends, the moments at the beam ends will vary reversely. Eventually, the moments at the beam can change into negative. Together with the effect of the axial tensile force, the negative moment may lead to tremendous tensile stress at the bottom flange near the ends of the restrained steel beam.

7. Conclusion

In this paper, the performance of the fire-heated restrained steel beams during cooling is investigated and a practical incremental method is proposed. By this method, the developments of deflection, axial force and moment in the restrained steel beams during cooling can be explicitly obtained. The effectiveness of the method proposed is validated by FE method.

In addition, the following points are noted through the analysis of restrained steel beams subjected to cooling after fire dies out:

(1) Very large axial tensile force will be induced in the beam and increase with temperature decreasing, but its increasing rate will be sharply reduced when plastic hinge occurs.

(2) Under the effect of the axial tensile force, the moment at the beam ends will change reversely and may turn into negative eventually. Then the bottom flange at the beam ends may experience very high tensile stress, which will threaten the safety of beam-to-column connections.

8. Acknowledgement

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