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Dynamic response of empty steel tanks with dome roof under vertical base motion

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Abstract. This paper reports results of the structural response of empty steel tanks under vertical ground motions. The tanks are modeled using a finite element discretization using shell elements, and the vertical motion is applied and analyzed using nonlinear dynamics. Several excitation frequencies are considered, with emphasis on those that may lead to resonance of the roof. The computational results illustrate that as the base motion frequency is tuned with the frequency of the first roof-mode of the tank, the system displays large-amplitude displacements. For frequencies away from such mode, small amplitude displacements are obtained. The effect of the height of the cylinder on the dynamic response of the tank to vertical ground motion has also been investigated. The vertical acceleration of the ground motion that induces significant changes in the stiffness of the tank was found to be almost constant regardless of the height of the cylinder.

Keywords: dynamics; earthquakes; harmonic loads; shell structures; steel tanks.

1. Introduction

The earthquake response of aboveground steel storage tanks in oil facilities is of vital importance not only for the oil industry but also for the population at large. Regarding the oil industry, if the tank is out of service due to an earthquake, then the owner suffers important economic losses for each day that the facility does not operate.

Most previous studies in this field consider the worst case scenario, i.e. a tank full with liquid, in which case the motion at the base is amplified by the movement of the liquid (Haroun and Housner 1981, Housner 1963, Ito *et al*. 2003, Malhotra 2000, Morita *et al*. 2003, Natsiavas and Babcock 1987, Rammerstorfer *et al*. 1990, Veletsos 1984, Veletsos and Shivakumar 1997, Veletsos and Yang 1977). However, since earthquakes occur without any warning, not all tanks may have liquid inside during an earthquake. In order to have realistic estimates of probable losses in refineries, plants, or even in geographical regions as modeled by HAZUS (2004), it is important to understand the behavior of tanks with and also without liquid.

Previous studies (Virella *et al.* 2006a, 2008) have shown that for tank-liquid systems subjected to horizontal ground motion, damage in the form of buckling is typically observed in the cylinder. Tank roofs have also experienced damage during past earthquakes and this effect has been frequently attributed to liquid sloshing. Also, the vertical component of acceleration during an earthquake may

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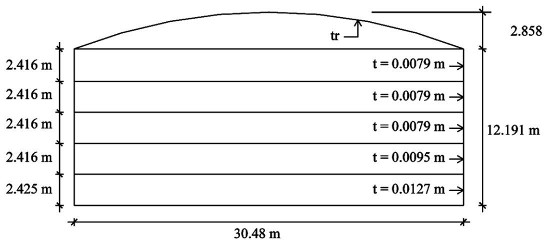
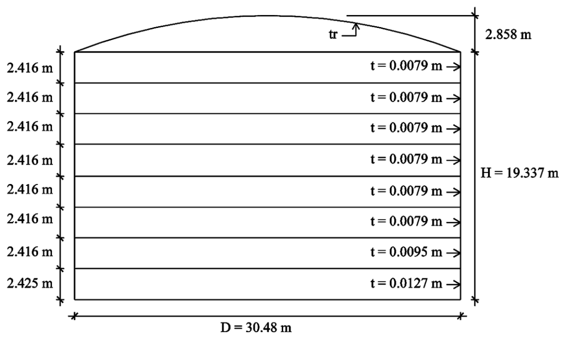
induce a significant dynamic response in the roof, even though the possibility of buckling of the roof has not been explored in detail at present. Finally, the influence of the liquid has been considered by the authors in other publications (Virella *et al.* 2005, 2006a, 2008, 2008). This paper concentrates on the behavior of empty tanks subjected to vertical acceleration in order to investigate if there are real possibilities of buckling and for what levels of excitation this would occur.

2. Methodology

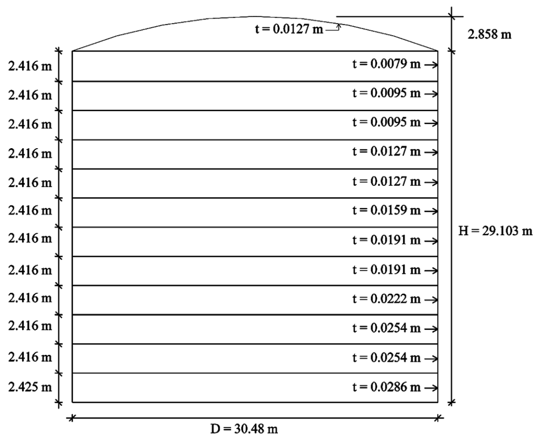
The tanks illustrated in Fig. 1, with a self-supported dome roof, height to diameter ratios of H/D = 0.40, 0.63 and 0.95, and tapered shell thicknesses, have been considered in this work. For all the models the diameter was held constant and the height was modified. These geometries are typical of tanks constructed in oil refineries and petrochemical plants (i.e. 0.24 < H/D < 1.0), and correspond to theme structures previously studied by the authors (Virella *et al*. 2003).



(a)



(b)



(c)

Fig. 1 Self-supported dome roof tanks. (a) H/D = 0.40; (b) H/D = 0.63; (c) H/D = 0.95; roof thickness, tr = 0.0127 m.

The self-weight of the tank is included in the computations by means of an initial geometrically non-linear static analysis, and in a second step, the tank is subjected to a harmonic base motion in the form of a vertical acceleration. Although an earthquake excitation does not follow a harmonic pattern, an upper bound to a failure mode could be obtained as the tank is excited in its resonant frequencies. The harmonic base acceleration $\ddot{X}g$ used in this work takes the form

$$\ddot{X}g = VPGA\sin(\omega t) \tag{1}$$

where *VPGA* is the vertical component of the peak ground acceleration, and ω is the frequency of the vertical base motion. Time history analyses with different frequencies (ω) of the input base motion have been carried out to investigate the dynamic response.

The first 30 natural frequencies of the tanks have been previously obtained by Virella *et al*. (2003), and based on these results the frequencies illustrated in Table 1 were selected to model the vertical base motion. These frequencies presented for each model, coincide with that of the first mode shape involving large displacements in the roof (designated here as roof mode).

The finite element general purpose package ABAQUS (2002) was used to carry out the computations, using quadrilateral shell elements (designated as S4R in ABAQUS) for the discretization of the cylinder and triangular shell elements (designated as S3R in ABAQUS) for the roof. The S4R is a four-node, doubly-curved shell element with reduced integration, hourglass control, and finite membrane strain formulation. The S3R element is a three-node degenerated version of the S4R, with a finite membrane strain formulation. Finite element meshes of 5,138, 6,290 and 7,826 elements for Tanks A, B and C respectively, were employed to assure convergence of the solution.

Viscous damping was introduced in the model by means of a Rayleigh mass-proportional damping. The damping coefficient was selected based on the frequency of excitation, assigning a damping ratio $\xi = 2\%$ to that frequency. The damping coefficients selected for each case are presented in Table 1. Plasticity of the shell was considered by means of an elasto-plastic (von Mises type) model, with a yield stress of 248 MPa.

The evaluation of dynamic buckling is a complex problem and does not follow the same criteria as static buckling (Godoy 2000). It is now generally accepted that dynamic buckling may be investigated by considering different levels of excitation until there is a sudden change in the response for a small change in the excitation parameter. The studies employ peak amplitudes in the transient response versus the excitation parameter. Such a criterion was originally proposed by Budiansky and Roth (1962) and has been used by a number of researchers for different transient loads. According to the Budiansky-Roth Criterion, dynamic buckling occurs whenever there is a large change in the amplitude of the displacements for a small change in the excitation amplitude such as *VPGA*.

Table 1 Parameters for the analyses in which the frequency of excitation is coincident with a natural frequency of the roof.

Tank	H/D	T [sec]	ω [rad/s]	ζ [%]	α
A	0.40	0.163	38.601	2.0	1.54
B	0.63	0.163	38.638	2.0	1.54
C	0.95	0.163	38.596	2.0	1.54

Legend:
T = period of excitation, ω = frequency of excitation,
ζ = damping ratio, α = mass proportional damping coefficient.

Sosa and Godoy (2005) computed dynamic buckling of empty tanks under wind pressures which had fluctuating pressures, but with the fluctuation oscillating from the average value of the pressure. The Budiansky-Roth criterion was used in that study and very significant jumps in the displacements were observed, so that buckling was easily identified in the tank shell. However, loading and unloading effects did not occur, as in the case of a seismic or harmonic excitation.

For problems with reverse loading cycles, such as those investigated in this paper, the dynamic response does not always take the form of a sudden jump in the peak displacements leading to a zero value of the slope (as in the cases discussed by Budiansky and Roth) and what is observed in the response is a sudden change in the stiffness of the structure. This change is reflected in a plot of peak displacement versus excitation parameter (or "pseudo-equilibrium paths", as identified by Virella *et al*. 2006a) by a jump that occurs with a non-zero slope. In those cases, a bilinear pseudo-equilibrium path is obtained, and the value of excitation at which the two lines intersect is here called a "transition load". Because of the alternating nature of the load in a case that is not governed by a snap-through behavior, the change in stiffness does not necessarily mean that the system becomes unstable following a transition load; for this reason, the term "critical load" will not be employed in the remainder of the paper.

3. Results for a frequency of excitation coincident with a natural frequency of the roof

3.1. Tank with H/D = 0.40

To evaluate the vulnerability of the tank with H/D = 0.40 under vertical ground motion, increasing amplitudes of VPGA were applied for the case in which the frequency of the base motion coincides with the first natural frequency in a roof mode (a resonance frequency) and dynamic response was evaluated for the cylinder and the roof. Large displacements were observed in different zones of the tank, and two nodes have been selected (one in the roof and one in the cylinder) to illustrate the dynamic response. The node chosen to monitor the response of the cylinder has been selected at the top of the cylinder near to the roof, as shown in Fig. 2.

The transient response for the tank with H/D = 0.40 was computed for several vertical ground motions,

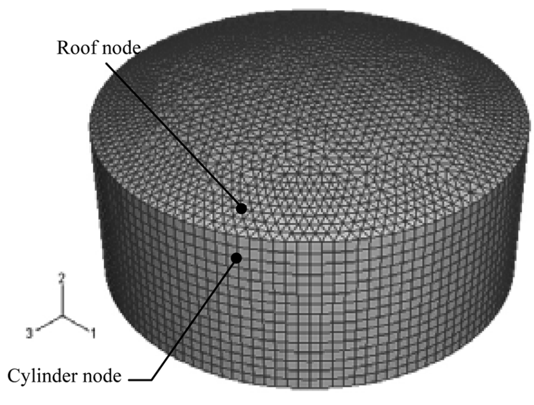


Fig. 2 Localization of nodes in the cylinder and in the roof used to investigate dynamic response.

with VPGA ranging from 0.10 g to 0.50 g, and selected results of radial displacements at the characteristic cylinder node are plotted in Fig. 3. Such displacement components occur because the motion in the roof is transmitted through the roof-cylinder connection and induces bending in the cylinder.

The displacement peaks observed in the displacement time histories (Fig. 3) were plotted in Fig. 4 for several VPGA. At small excitation levels, the VPGA versus radial displacement curve (pseudo-equilibrium

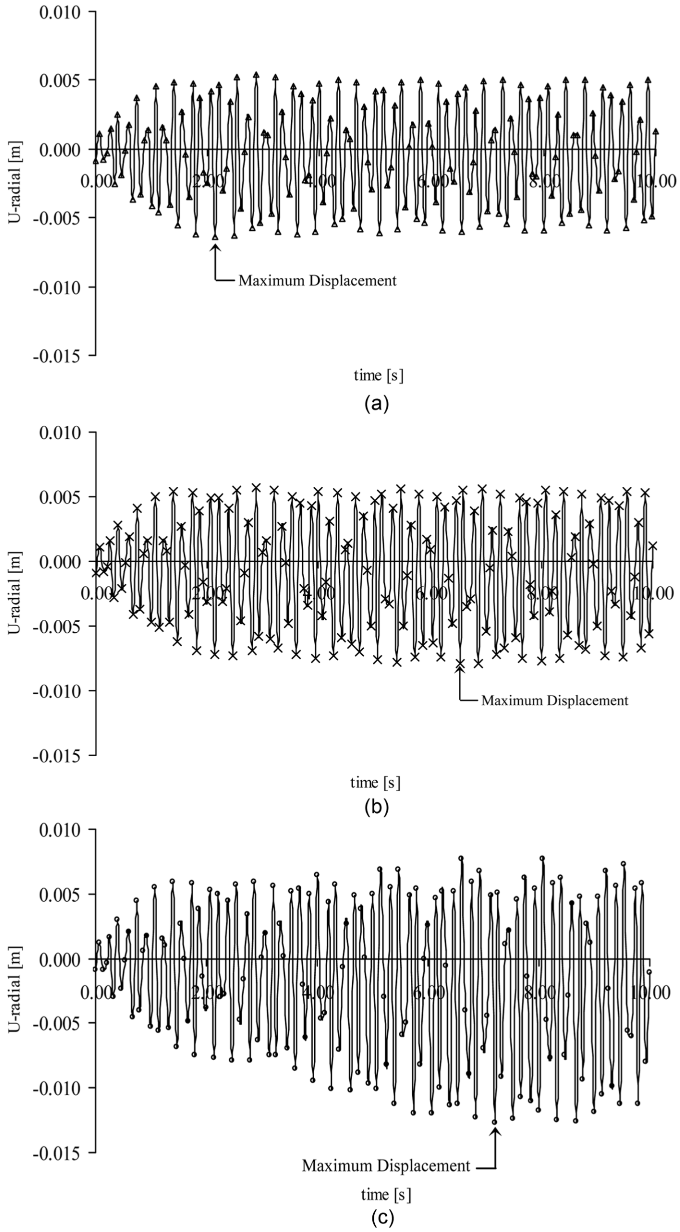


Fig. 3 Time histories for a node in the cylinder of Tank A; $\omega = 38.601$ rad/s; (a) VPGA = 0.40 g, (b) VPGA = 0.45 g, (c) VPGA = 0.50 g.

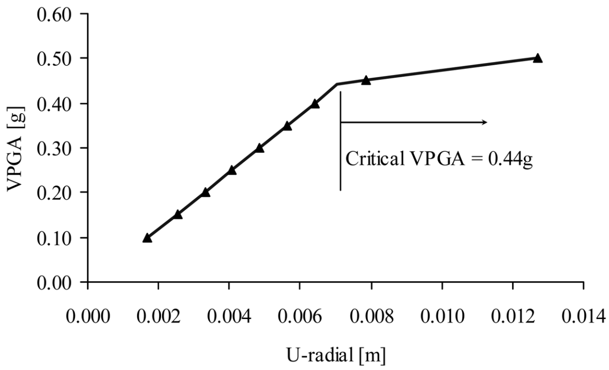


Fig. 4 VPGA versus Displacement curve for a node in the cylinder; Tank A, $\omega = 38.601$ rad/s.

path) follows an initial path, with the slope corresponding to the initial stiffness of the system. As displacements continue to grow with the load, a second path is observed, which shows a reduction in the stiffness of the system. The pseudo-equilibrium path may be represented with a bilinear approximation, and the intersection of the two lines indicates the load that establishes the transition from the initial to the reduced stiffness. This transition occurs for VPGA = 0.44 g in Fig. 4, and Fig. 5 illustrates the dynamic mode for this tank in which large deflections occur near the top of the cylindrical shell.

The dynamic response was also evaluated for the roof of the tank considering increasing values of VPGA from 0.10 g to 0.50 g. Fig. 6 presents some of the displacement time histories from which the pseudo-equilibrium path was constructed for the reference node in the roof (see Fig. 2). The pseudo-equilibrium path was performed by plotting the peaks in the displacement time histories to evaluate whether a jump in the displacement was observed for a small increase in the load. However, no displacement jump was observed in the pseudo-equilibrium path (see Fig. 7) for the characteristic node at the roof; instead a single path was obtained up to the maximum VPGA of 0.50 g considered. This means that the roof would not exhibit stiffness drops, at least for the range of the VPGA considered.

As mentioned before, the computations included both material and geometric non-linearity. However, plasticity was never reached, neither in the roof nor in the cylinder, for all the cases investigated for the tank with H/D = 0.40.

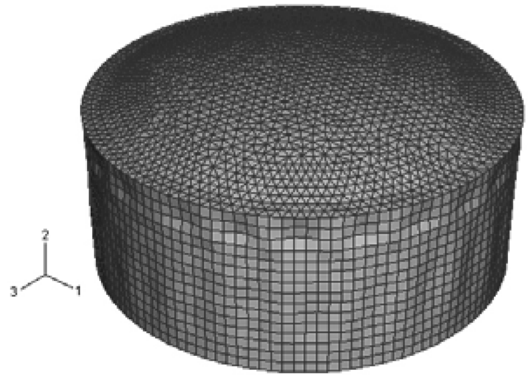


Fig. 5 Figure showing large amplitude vibration for the cylinder of Tank A.

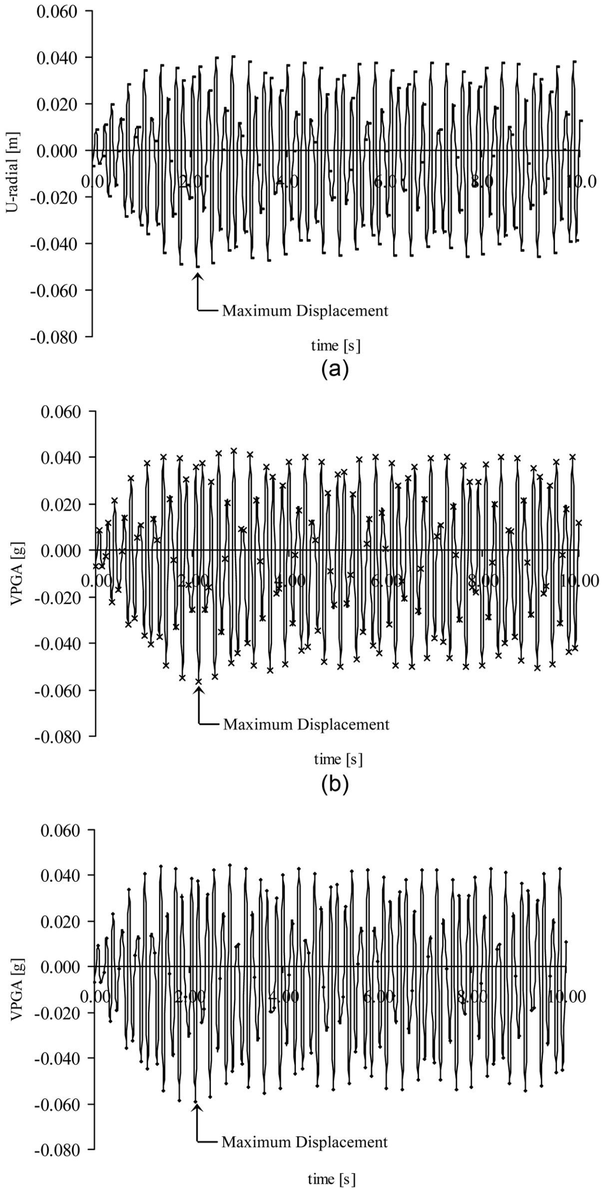


Fig. 6 Time histories for a node in the roof of Tank A; $\omega = 38.601$ rad/s; (a) VPGA = 0.40 g, (b) VPGA = 0.45 g, (c) VPGA = 0.50 g.

3.2. Tanks with H/D = 0.63 and H/D = 0.95

The same procedure described before for the tank with H/D = 0.40 was performed to assess the vulnerability to harmonic loads of the models with different H/D ratios. Figs. 8 and 9 present the pseudo-equilibrium paths for the two tanks corresponding to a reference node in the cylinder. The pseudo-

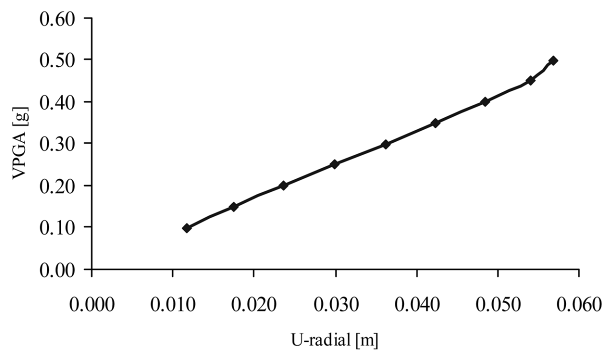


Fig. 7 VPGA versus peak displacement curve for a node in the roof; Tank A, $\omega = 38.601$ rad/s.

equilibrium path was plotted in Figs. 8 and 9 using a bilinear representation. At the intersection between both lines it was possible to identify a transition load occurring for VPGA = 0.39 g (for the tank with H/D = 0.63) and VPGA = 0.40 g (for the tank with H/D = 0.95). For a VPGA larger than the transition value, a large change in the peak displacements is observed in the pseudo-equilibrium path. For the tanks with

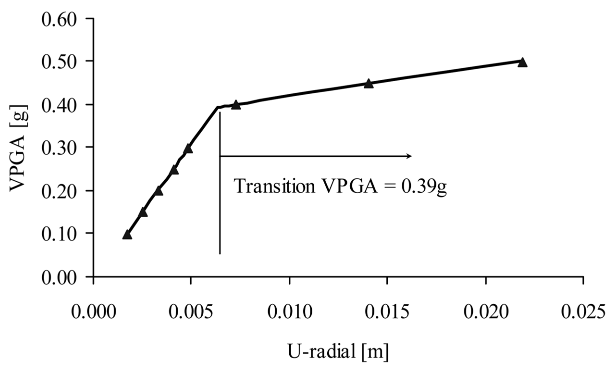


Fig. 8 VPGA vs. peak displacement curve for a node in the cylinder; Tank B, $\omega = 38.638$ rad/s.

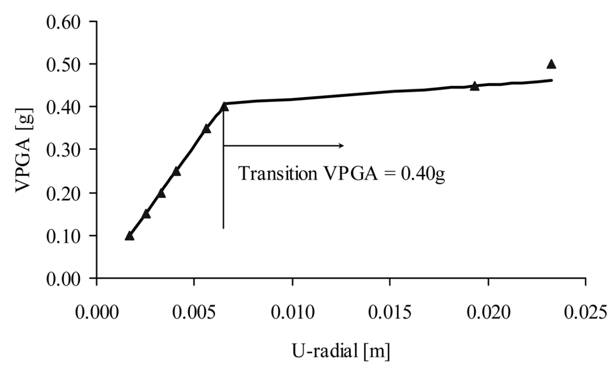


Fig. 9 VPGA vs. Displacement curve for a node in the cylinder; Tank C, $\omega = 38.596$ rad/s.

H/D = 0.63 and H/D = 0.95, large deflections were observed near the top part of the cylindrical shell.

The dynamic response was also evaluated at the roof for the tanks with H/D = 0.63 and 0.95 using the same procedure described for the tank with H/D = 0.40. The same results were obtained for these two tanks, thus having a single path up to the maximum VPGA of 0.50 g considered.

The tanks with H/D = 0.63 and H/D = 0.95 did not display plasticity at any part of the shell for all the dynamic cases considered in this section.

The results show that the VPGA required to induce significant changes in the structure is practically the same for all the tanks. Another common feature is that large deflections occurred near the top of the cylinder and not in the roof, regardless of the H/D ratio of the tank.

4. Results for a frequency of excitation not coincident with the natural frequency of the roof

The vulnerability of the tanks under vertical ground motions was previously evaluated for a resonance case, in which the frequency of the base motion coincided with the first natural frequency in a roof mode. In this section, the dynamic response of the tank with H/D = 0.40 is evaluated, considering the same harmonic base motion described in Eq. (1), but with frequencies different than the resonance one. Table 2 presents the four cases of frequencies for the harmonic base motions considered in this section.

For the base motion frequencies of 30.4 rad/s and 45.5 rad/s (Cases 2 and 3 in Table 2), the dynamic response of the cylinder has been evaluated for increasing VPGA ranging from 0.20 g to 1.0 g. No

Table 2 Parameters for the analyses in which the frequency of excitation is not coincident with a natural frequency of the roof.

Case	VPGA [g]	T [sec]	ω [rad/s]	ζ [%]	α
1	0.45	0.258	24.35	2.0	0.974
2	0.45	0.206	30.44	2.0	1.220
3	0.45	0.138	45.53	2.0	1.821
4	0.45	0.115	54.64	2.0	2.186

Legend:
VPGA = vertical peak ground acceleration, T = period of excitation, ω = frequency of excitation, ζ = damping ratio, α = mass proportional damping coeficient.

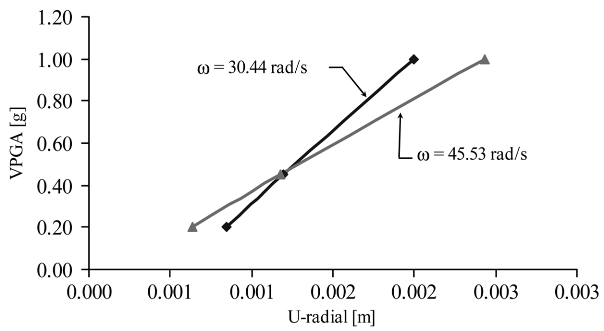


Fig. 10 VPGA versus Displacement curve for a node in the cylinder of Tank A. for case 2 (ω = 30.44 rad/s) and case 3 (ω = 45.53 rad/s).

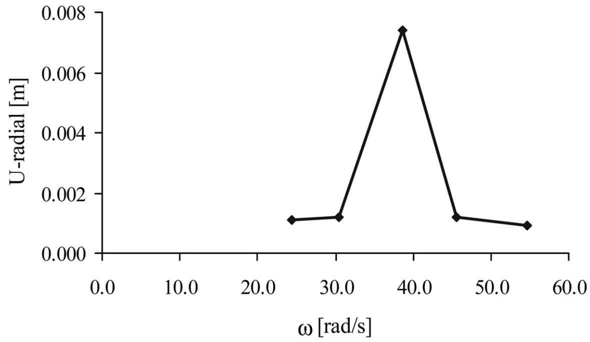


Fig. 11 Variation of maximum radial displacement U-radial in the cylinder of Tank A with base motion frequency

displacement jump or change in the stiffness is observed in the pseudo-equilibrium path of Fig. 10 for the cylinder; instead a single path was obtained up to the maximum VPGA of 1g. A similar behavior was obtained in the roof and plasticity was never reached at any part of the tank for all the VPGA considered.

The same results were obtained for the harmonic base motion frequencies of Cases 1 and 4 of Table 2, thus for neither of these non-resonance cases the tank shows large deflections. Furthermore, plasticity was not identified in the tank up to the maximum VPGA of 1g considered. A VPGA of 1g is very large and probably an unrealistic value, thus implying that a bilinear path would not occur for any of the frequencies of excitation that are not close to the first natural frequency of the roof.

Fig. 11 presents a summary of the maximum displacement response in the cylinder of the tank with H/D = 0.40, for a VPGA of 0.45 and for the different frequencies of excitation in the base. The results illustrate that, as the base motion frequency is tuned with the first roof mode natural frequency of the tank, the system enters into resonance and a maximum displacement is obtained. The results clearly show that dynamic amplification should only be expected if the tank is excited in a resonant frequency.

5. Discussion and conclusions

The behavior of empty aboveground steel storage tanks was considered in this paper under a vertical motion at the base. The specific excitation assumed was a harmonic variation in time, in which the frequency was tuned to resonant and non resonant values. Several aspects may be highlighted from this research:

1. The dynamic buckling criterion itself was extended in order to deal with the harmonic load, which is an oscillatory (cyclic) load. Notice that the Budiansky-Roth criterion was originally developed for step or impulsive loading, but a jump in the displacements is not clearly observed under cyclic load, in which the structure is loaded and unloaded in each cycle. The criterion was here extended to include a jump in the stiffness of the structure, which is similar to what would be obtained in a static bifurcation. The results show that jumps in the stiffness can be investigated in this problem employing this extended criterion.
2. For the class of tanks/excitations investigated, the frequency of excitation is close to the natural frequency of the roof. In the three cases investigated (H/D = 0.40, 0.63 and 0.95), a significant

change in the stiffness of the tank occurred only for frequencies of excitation of ω = 38.601 rad/s, 38.638 rad/s and 38.596 rad/s respectively. However, a small amplitude behavior was obtained for non resonant frequencies, such as ω = 30.44 rad/s and ω = 43.53 rad/s for the short tank with H/D = 0.40. The values of VPGA for non resonant frequencies were increased up to 1g, and neither loss of stiffness nor plasticity were reached.

3. The transition value of VPGA is almost independent of the height of the cylinder considered. For the tank with H/D = 0.40, the VPGA at the change in stiffness was found to be 0.44g. Further studies were conducted considering tanks with H/D = 0.63 and 0.95, in which the diameter of the tank was held constant and the height was modified. In both cases, transition values of VPGA = 0.39g and 0.40g were computed, i.e. a similar behavior as that obtained for the tank with H/D = 0.40.
4. The dynamic response induces large deflections only in the cylindrical shell of the tanks. Specifically, the large deflections are computed near the top part of the cylinder but not in the roof.

Finally, the results show that the tanks resisted with small amplitude linear displacement for all non-resonant frequencies considered, and it was only for resonant frequencies and VPGA > 0.39 g that large changes in the stiffness of the structure occurred. Because 0.39g is a large excitation if it is applied entirely in just one resonant mode, the practical implications of this study are that, for the range of tank geometries studied, there is a low probability that empty steel tanks will suffer large changes in the stiffness or buckle under vertical ground motions.

The study has several limitations: First, the geometries investigated do not represent all ranges of practical dimensions of tanks with a dome roof. Second, harmonic excitations (rather than seismic records) were considered. Third, only few excitation frequencies were considered, and modes other than the lowest-frequency roof mode have not been considered. Finally, the geometry of as-designed tanks was considered, so that imperfections in the geometry have not been included.

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